On the theory of electrosound waves in a plasma

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We study the nature of electrosound waves in a plasma with a negative permittivity in strong hf fields in which the electrons of the plasma may acquire relativistic velocities. We show that under conditions when the pressure of the hf wave is larger than the hydrodynamic pressure the electrosound wave is a compression wave, while for weak hf fields we have a rarefaction wave.

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The success attained in recent times by laser techniques enable us to obtain electromagnetic fields in which electrons may acquire relativistic velocities. The electromagnetic radiation by cosmic objects (galactic nuclei, radio-galaxies, quasars, and so on) may serve as a source for similar strong fields in cosmic space. When such strong electromagnetic radiation acts upon a plasma the electron mass becomes dependent on the amplitude of the pumping wave and this can lead to considerable changes in the vibrational properties of the plasma. The results of[1] indicate the possibility of a parametric amplification of longitudinal waves in a plasma, owing to the relativistic oscillation of the electron mass with the frequency of the external field. A peculiar feature of this parametric effect, in contrast to the well-known results of[2] (see the literature cited in[2]) is the fact that parametric resonance can occur in a purely electronic plasma when the wave motion of the ions can be neglected. The effect of the relativistic motion of the electrons on the non-linear processes in the plasma is considered in[3]. It follows from the results of these papers that the relativistic nature of the electron motion can lead to the development of shock waves.

It is well known that the propagation of high-frequency electromagnetic waves in a plasma may be accompanied by electrostrictive effects which manifest themselves in that the pressure of the hf field changes the density and thereby the permittivity of the medium. Hf waves with a frequency lower than the plasma frequency, $\omega_0 < \omega_{Le}$, can not propagate in the plasma (we have a plasma with a negative permittivity). However, if the intensity of the hf wave is sufficiently large, density modulation processes result in the region where the radiation is localized moving in the plasma in the form of electrosound solitons—see[4,5]. In those papers the hf electromagnetic wave was assumed to be weak:

$$E_m^2 < 2|\varepsilon(\omega_0)|\frac{\omega_0^2}{\omega_{Le}^2}E_c^2,$$
$$E_c^2 = \frac{4m_e^2\omega_0^2}{e^2}v_{Te}^2, \quad |\varepsilon(\omega_0)| = \frac{\omega_{Le}^2}{\omega_0^2} - 1, \tag{1}$$

where E_m is the maximum value of the hf field strength and v_{Te} is the electron thermal velocity. Inequality (1) is equivalent to the condition that the hf pressure force be small compared to the hydrodynamic pressure force. The total pressure force (hydrodynamic plus hf pressure and hence also the velocity of the wave motion of the particle is in that case always in the direction opposite to that of the wave propagation. Under condition (1) a solitary wave must always have the character of a rarefaction wave.

We study in the present paper the effect of the dependence of the electron mass on the pumping wave amplitude on the nature of the propagation of electrosound waves in a plasma with a negative dielectric permittivity. In such strong hf fields the hf pressure force is larger than the hydrodynamic force and the total pressure force changes sign. The plasma particles are accelerated in the direction of the wave propagation and a solitary wave has the character of a compression wave.

1. In this part we formulate the basic equations of the plasma hydrodynamics in strong hf fields when it is necessary to take into account the relativistic nature of the electron motion. The propagation of an electrosound wave is described by the system of the Maxwell equations and the plasma hydrodynamics equations. We shall not restrict the amplitude of the hf field and assume that the electrons in the hf field may acquire large (up to relativistic) velocities. We have

$$\frac{\partial \mathbf{p}_\alpha}{\partial t} + \mathbf{v}_\alpha\frac{\partial \mathbf{p}_\alpha}{\partial \mathbf{r}} = e_\alpha\mathbf{E} + \frac{e_\alpha}{c}[\mathbf{v}_\alpha \times \mathbf{B}] - T_\alpha\frac{\partial}{\partial \mathbf{r}}\ln n_\alpha,$$
$$\frac{\partial n_\alpha}{\partial t} + \operatorname{div} n_\alpha\mathbf{v}_\alpha = 0, \quad \alpha = (e, i),$$
$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$
$$\operatorname{rot}\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi e}{c}\{n_i\mathbf{v}_i - n_e\mathbf{v}_e\},$$
$$\mathbf{p}_e = m_e\mathbf{v}_e\{1 - v_e^2/c^2\}^{-1/2}, \quad \mathbf{p}_i = m_i\mathbf{v}_i.$$

When there is a hf field present the required quantities contain not only a slow time dependence but also a fast dependence with a characteristic time $\tau \sim 1/\omega_0$. We can therefore look for each of the quantities $A \equiv (\mathbf{E}, \mathbf{B}, n_\alpha, \mathbf{p}_\alpha)$ in the form

$$A = \langle A\rangle + \tilde{A}, \tag{3}$$

where the angle brackets indicate averaging over a time interval $\sim\tau$:

$$\langle A\rangle = \frac{1}{2\tau}\int_{t-\tau}^{t+\tau} A(t')\,dt'. \tag{4}$$

The time interval τ is assumed to be much shorter than the characteristic period of the slow change t_{sl}. We shall not take into account in what follows processes which are connected with the heating of a plasma in a hf field. This assumption is valid only under conditions when the collision frequency is small compared to t_{sl}^{-1}. We confine ourselves to the case where the fast changing motion of the ions can be neglected and also when there is no magnetic field $\langle \mathbf{B} \rangle = 0$. If we assume the spatial dependence to be sufficiently smooth, $L \gg \tau |\tilde{v}_e|$ (L is the characteristic distance of the change of the slow or fast changing quantities), we can use the procedure of calculations described in[5]. Substituting Eq. (3) into the set of Eqs. (2), we can use the averaging (4) to separate the fast changing motion from the slow one. Further assuming that $\langle \mathbf{v}_e \rangle \ll c$ and $L \gg \tau v_{Te}$ we get for the fast changing quantities a set of equations

$$\frac{\partial \tilde{\mathbf{p}}_e}{\partial t} = -e\tilde{\mathbf{E}}, \quad m_e\tilde{\mathbf{v}}_e = \tilde{\mathbf{p}}_e\left(1+\frac{\tilde{\mathbf{p}}_e^2}{m_e^2c^2}\right)^{-1/2}, \tag{5a}$$

$$\mathbf{B} = \frac{c}{e}\,\mathrm{rot}\,\tilde{\mathbf{p}}_e, \tag{5b}$$

$$\frac{\partial n_e}{\partial t} + \mathrm{div}\,\langle n_e\rangle \tilde{\mathbf{v}}_e = 0, \tag{5c}$$

$$\mathrm{rot\,rot}\,\tilde{\mathbf{p}}_e + \frac{1}{c^2}\frac{\partial^2 \tilde{\mathbf{p}}_e}{\partial t^2} + \frac{4\pi e^2}{c^2}\langle n_e\rangle \tilde{\mathbf{v}}_e = 0. \tag{6}$$

Knowing the quantity $\langle n_e \rangle$ we can find $\tilde{\mathbf{p}}_e$ from Eq. (6) and then from Eqs. (5) all rapidly changing quantities.

The set of equations describing the slow motion of the plasma has the form

$$\begin{gathered}
\frac{\partial \langle \mathbf{p}_e\rangle}{\partial t} + \langle \mathbf{v}_e\rangle \frac{\partial \langle \mathbf{p}_e\rangle}{\partial \mathbf{r}} \\
= -e\langle \mathbf{E}\rangle - m_ec^2\frac{\partial}{\partial \mathbf{r}}\left\langle\left(1+\frac{\tilde{\mathbf{p}}_e^2}{m_e^2c^2}\right)^{1/2}\right\rangle - T_e\frac{\partial}{\partial \mathbf{r}}\ln\langle n_e\rangle, \\
\frac{\partial \langle n_e\rangle}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\{\langle n_e\rangle\langle \mathbf{v}_e\rangle + \langle \tilde{n}_e\tilde{\mathbf{v}}_e\rangle\} = 0, \\
\frac{\partial \mathbf{p}_i}{\partial t} + \mathbf{v}_i\frac{\partial \mathbf{p}_i}{\partial r} = e\langle \mathbf{E}\rangle - T_i\frac{\partial}{\partial \mathbf{r}}\ln n_i, \\
\frac{\partial n_i}{\partial t} + \mathrm{div}\, n_i\mathbf{v}_i = 0, \\
\mathrm{div}\,\langle \mathbf{E}\rangle = 4\pi e\{n_i - \langle n_e\rangle\}.
\end{gathered} \tag{7}$$

We confine ourselves to plasma motion such that the condition of quasi-neutrality is satisfied and that there is no slow current:

$$\begin{gathered}
\langle n_e\rangle = n_i, \\
\langle n_e\rangle\langle \mathbf{v}_e\rangle + \langle \tilde{n}_e\tilde{\mathbf{v}}_e\rangle = n_i\mathbf{v}_i.
\end{gathered} \tag{8}$$

The set of Eqs. (7) can then be reduced to two equations: to the equation of motion and the equation of continuity of the ion component of the plasma. These two equations together with Eq. (6) form the basic set of equations describing the behavior of a plasma in the field of a hf wave:

$$\partial n/\partial t + \mathrm{div}\, n\mathbf{V} = 0, \tag{9}$$

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\frac{\partial \mathbf{V}}{\partial \mathbf{r}} = -\frac{m_e}{m_i}c^2\frac{\partial}{\partial \mathbf{r}}\left\langle\left(1+\frac{p^2}{m_e^2c^2}\right)^{1/2}\right\rangle - v_s^2\frac{\partial}{\partial \mathbf{r}}\ln n, \tag{10}$$

$$\mathrm{rot\,rot}\,\mathbf{p} + \frac{1}{c^2}\frac{\partial^2 \mathbf{p}}{\partial t^2} + \frac{\omega_{Le}^2}{c^2}\frac{n}{n_0}\mathbf{p}\left\{1+\frac{p^2}{m_e^2c^2}\right\}^{-1/2} = 0; \tag{11}$$

Here $v_s^2 = T_e/m_i$ ($T_e \gg T_i$) and for the sake of simplicity we have introduced the notation: $n = n_i$, $\mathbf{V} = \mathbf{v}_i$, $\mathbf{p} = \tilde{\mathbf{p}}_e$. As one should expect, the pressure of the hf field is given by the average kinetic energy of the fast changing electron motion. In the non-relativistic limit, $p^2/m_e^2c^2 \ll 1$, Eqs. (10) and (11) are the same as the equations given in[4,5]. In the case of small density perturbations $n = n_0 + \delta n$, $\delta n \ll n_0$ (we restrict ourselves to this case in what follows) the change in the density δn is proportional to p^2 and in the non-relativistic limit the last term in Eq. (11), which takes into account the modulation of the plasma density is proportional to p^3. Taking the relativistic correction to the electron mass into account leads already in first approximation to the appearance in Eq. (11) of yet another term proportional to p^3 which under well defined conditions may change the structure of a non-linear wave in the plasma.

2. We study the solution of the set of Eqs. (9) to (11) in the case of stationary waves propagating in the plasma with a constant velocity. We restrict our consideration to a one-dimensional problem when all slowly varying quantities we are looking for depend on the argument $\xi = x - ut$:

$$n(x, t) = n(\xi), \quad V(x, t) = V(\xi). \tag{12}$$

When the hf electromagnetic field is so strong that the electrons in the plasma acquire in it relativistic velocities, the hf wave can, according to the results of[6], be purely transverse only when it is circularly polarized. We therefore look for the momentum of the rapidly changing motion in the form

$$\begin{gathered}
p_y(x, t) = p_0(\xi)\sin(\omega_0 t - \varphi(\xi)), \\
p_z(x, t) = p_0(\xi)\cos(\omega_0 t - \varphi(\xi)).
\end{gathered} \tag{13}$$

In the case of circular polarization of the wave we can omit in Eq. (10) the brackets indicating averaging over the fast changing motion as $p^2 = p_0^2(\xi)$ contains only the slow time dependence. Substituting (12) and (13) into Eqs. (9) to (11) we get

$$n(V-u) = C_1, \tag{14}$$

$$\frac{1}{2}(V-u)^2 + \frac{m_e}{m_i}c^2\left(1+\frac{p_0^2}{m_e^2c^2}\right)^{1/2} + v_s^2\ln n = C_2, \tag{15}$$

$$\frac{d\varphi}{d\xi} = \frac{C_3 + u\omega_0 p_0^2}{p_0^2(c^2-u^2)}, \tag{16}$$

$$\frac{d^2p_0}{d\xi^2} + \frac{\omega_0^2c^2}{(c^2-u^2)^2}p_0 - \frac{C_3^2}{(c^2-u^2)^2}\frac{1}{p_0^3} - \frac{\omega_{Le}^2}{c^2-u^2}\frac{n}{n_0}p_0\left(1+\frac{p_0^2}{m_e^2c^2}\right)^{-1/2} = 0, \tag{17}$$

where C_1, C_2, and C_3 are integration constants.

We consider the solution of the set of Eqs. (14) to (17) which has the form of a solitary wave. We assume that at infinity the hf electromagnetic field, the plasma density perturbation, and the velocity vanish ($V \to 0$, $p_0 \to 0$, $n \to n_0$ as $\xi \to \pm\infty$) and that $d\varphi/d\xi$ is constant. From these conditions we easily determine the integration constants:

$$\begin{gathered}
C_1 = -un_0, \quad C_3 = 0, \\
C_2 = \frac{u^2}{2} + \frac{m_e}{m_i}c^2 + v_s^2\ln n_0.
\end{gathered}$$

In the case of small density perturbations, $\delta n/n_0 = (n-n_0)/n_0 \ll 1$, we get from Eqs. (14) and (15)

$$\frac{\delta n}{n_0}=\frac{m_e}{m_i}\frac{c^2}{u^2-v_s^2}\left\{\left(1+\frac{p_0^2}{m_e^2c^2}\right)^{1/2}-1\right\}. \tag{18}$$

Substituting this last relation into (17) we find for the first integral of Eq. (17) the following expression:

$$\left(\frac{dp_0}{dz}\right)^2-\left\{|\varepsilon(\omega_0)|-\frac{u^2}{c^2-u^2}\right\}p_0^2+\frac{\omega_{Le}^2}{\omega_0^2}\left\{1+\frac{m_e}{m_i}\frac{c^2}{v_s^2-u^2}\right\}m_e^2c^2K_0^2=0, \tag{19}$$

where we have introduced the notation

$$z=\frac{\omega_0}{(c^2-u^2)^{1/2}}\xi,\quad K_0=\left(1+\frac{p_0^2}{m_e^2c^2}\right)^{1/2}-1.$$

We took the integration constant, in accordance with the conditions chosen at infinity ($dp_0/d\xi \to 0$ as $\xi \to \pm\infty$), to be equal to zero. It is not possible to find the analytical solution of Eq. (19) in the general case, but we can trace the nature of the solution by using a formal mechanical analogue. We take the momentum p_0 in Eq. (19) as the coordinate and the dimensionless coordinate z as the time. We can then identify Eq. (19) with the energy conservation law for a particle moving with zero total energy in a potential field

$$U(p_0)=-\left\{|\varepsilon(\omega_0)|-\frac{u^2}{c^2-u^2}\right\}p_0^2+\frac{\omega_{Le}^2}{\omega_0^2}\left\{1-\frac{m_e}{m_i}\frac{c^2}{u^2-v_s^2}\right\}m_e^2c^2K_0^2. \tag{20}$$

A closed motion in the field (20) is possible only provided

$$|\varepsilon(\omega_0)|>u^2/(c^2-u^2), \tag{21}$$

$$1-\frac{m_e}{m_i}\frac{c^2}{u^2-v_s^2}>0. \tag{22}$$

The maximum value of the "coordinate" $p_0=p_m\neq 0$ is determined by the condition

$$U(p_m)=0. \tag{23}$$

The potential (20) has a maximum, equal to zero, at the point $p_0=0$ and under the conditions (21), (22) a negative minimum value at the point

$$p_0=\frac{m_ec}{\sqrt{2}}\left\{\left(1+\frac{p_m^2}{m_e^2c^2}\right)^{1/2}-1+\frac{p_m^2}{2m_e^2c^2}\right\}^{1/2}.$$

The amplitude dependence of the propagation speed of the wave can be found from Eq. (23):

$$\frac{u^2}{c^2}=\frac{1}{2}\left\{-S\pm\left(S^2+2\left[\frac{\omega_0^2}{\omega_{Le}^2}\frac{v_s^2}{c^2}+\frac{m_e}{m_i}\right]K_m-4\frac{\omega_0^2}{\omega_{Le}^2}\frac{v_s^2}{c^2}|\varepsilon(\omega_0)|\right)^{1/2}\right\}, \tag{24}$$

$$S=\frac{1}{2}\left(\frac{\omega_0^2}{\omega_{Le}^2}+\frac{m_e}{m_i}\right)K_m-\frac{\omega_0^2}{\omega_{Le}^2}|\varepsilon(\omega_0)|-\frac{v_s^2}{c^2}, \tag{25}$$

where $K_m=(1+p_m^2/m_e^2c^2)^{1/2}-1$. If

$$\frac{m_e}{m_i}K_m>2\frac{\omega_0^2}{\omega_{Le}^2}\frac{v_s^2}{c^2}|\varepsilon(\omega_0)| \tag{26}$$

(which in the non-relativistic case corresponds to the case which is the opposite of (1)) we must take from the two solutions (24) only the solution with the positive sign in front of the radical as $u^2/c^2>0$. One verifies easily that this solution is always larger than the ion-sound velocity $u^2>v_s^2$, and, hence, that we have according to (18) a compression wave, $\delta n>0$. Inequality (21) can then be assumed to be satisfied, if

$$|\varepsilon(\omega_0)|>(\omega_{Le}/\omega_0)^4 m_e/m_i.$$

If the hf field is sufficiently weak so that the inequality

$$\frac{m_e}{m_i}K_m<2\frac{\omega_0^2}{\omega_{Le}^2}\frac{v_s^2}{c^2}|\varepsilon(\omega_0)| \tag{27}$$

(the case considered in[4]) is satisfied, we must choose from the two solutions (24) the one with the negative sign in front of the radical. Indeed, it follows from the inequality (27) that

$$|\varepsilon(\omega_0)|\gg{}^1/_2(\omega_{Le}^2/\omega_0^2)K_m\gg{}^1/_2K_m.$$

Expanding the square root in (24) in a power series in small quantities and retaining the first terms in the expansion we check that condition (21) can with a large margin be satisfied by the solution with the negative sign in front of the radical. As was shown in the first section, the presence of the term $\frac{1}{2}(\omega_0^2/\omega_{Le}^2)K_m$ in Eq. (25) is caused by taking into account the relativistic correction to the electron mass. This term does not occur in[4,5], where relativistic effects were completely neglected, and the quantity S, given by Eq. (25), is negative, $S<0$. The requirement of bounded solutions at infinity, i.e., Eq. (21), can thus under the conditions of[4,5] be satisfied only with the negative sign in front of the radical. But for that solution $u^2<v_s^2$ and a solitary wave has always the character of a rarefaction wave.

3. We consider the results obtained above in the weakly relativistic limit when the analytical solution of the problem can be carried out until the end.

The solution of Eq. (19) with the conditions (21), (22) has the form

$$v_0=\frac{p_0}{m_e}=v_m\,\mathrm{ch}^{-1}\left\{\frac{\omega_0}{c}\left[|\varepsilon(\omega_0)|-\frac{u^2}{c^2}\right]^{1/2}\xi\right\}, \tag{28}$$

where v_m is the maximum value of the electron velocity in the hf field. Expression (28) is the same as the result of[7]. If we assume that

$$\frac{m_e}{m_i}\frac{v_m^2}{c^2}\ll|\alpha|^2,$$

$$\alpha=\frac{1}{4}\frac{\omega_0^2}{\omega_{Le}^2}\left(\frac{v_m^2}{c^2}-4|\varepsilon(\omega_0)|\right)\ll 1,$$

then Eq. (24), which determines the dependence of the phase velocity on the amplitude of the fast changing velocity v_m can be simplified:

$$u^2=v_s^2+\frac{1}{4}\frac{m_e}{m_i}\frac{v_m^2}{\alpha}. \tag{29}$$

This relation is valid in two cases.

1) In the case when the hf field is sufficiently strong, so that

$$\frac{m_e}{m_i}v_m^2 \gg 4\frac{\omega_0^2}{\omega_{Le}^2}v_s^2|\varepsilon(\omega_0)|, \tag{30}$$

and $\alpha > 0$.

2) In the case when the hydrodynamic pressure force is larger than the hf pressure force. One can write that condition, according to (27), in the form

$$\frac{m_e}{m_i}v_m^2 < 4\frac{\omega_0^2}{\omega_{Le}^2}v_s^2|\varepsilon(\omega_0)|. \tag{31}$$

Then $v_m^2/c^2 \ll 4|\varepsilon(\omega_0)|$ and $\alpha \approx -(\omega_0^2/\omega_{Le}^2)|\varepsilon(\omega_0)|$.

We note that in the second case we chose from the two solutions of (24) the solution corresponding to the one considered in[4,5].

Using (18), (28), and (29) we get for the change in the density the expression

$$\frac{\delta n}{n_0} = 2\alpha\,\mathrm{ch}^{-2}\left\{\frac{\omega_0}{c}\left[|\varepsilon(\omega_0)| - \frac{u^2}{c^2}\right]^{1/2}\xi\right\}. \tag{32}$$

The way the density depends on the coordinate ξ under the conditions (30) and (31) is thus the same, but the maximum deviation of the density from its equilibrium value has under those conditions opposite signs. Under condition (30) we have a compression wave.

We substitute Eq. (32) into the equation of motion (10) for the ions. Neglecting small terms we get

$$\frac{\partial V}{\partial t} = -\frac{1}{2}\left\{\frac{m_e}{m_i}v_m^2 - 4\frac{\omega_0^2}{\omega_{Le}^2}v_s^2|\varepsilon(\omega_0)|\right\}\frac{\partial}{\partial x}\frac{v_0^2}{v_m^2}.$$

The right-hand side of this equation determines the total pressure force acting upon the ion component of the plasma. The hf pressure and hydrodynamic pressure forces are in opposite directions. If the hf pressure force is larger than the hydrodynamic pressure force,

$$\frac{m_e}{m_i}v_m^2 > 4\frac{\omega_0^2}{\omega_{Le}^2}v_s^2|\varepsilon(\omega_0)|,$$

the total pressure (the hydrodynamic plus the hf pressure) force is directed outwards from the region where the hf field is localized. In the region of leading edge (along the direction of motion of the solitary wave) the particles in the plasma acquire a velocity directed in the direction of the wave propagation—there is a "sweeping up" of the plasma by the hf field pressure. As the phase velocity of the solitary wave is larger than the particle velocity, $u \gg V$, the wave overtakes the particles. In the region of the trailing edge of the solitary wave the particles are braked and behind the wave we have the unperturbed plasma ($V \to 0$, $\delta n \to 0$, as $\xi \to -\infty$). The effect of the sweeping up of the plasma by the hf pressure thus leads to an enhancement of the plasma particle density in the perturbed region of space.

If the hf pressure force is less than the hydrodynamic pressure force (the case considered in[4,5]) the total pressure force is directed into the region where the hf field is localized. In the region of the leading edge of the solitary wave there is an intake of plasma particles, and in the region of the trailing edge of the solitary wave the particles are braked. The direction of motion of the particles is the opposite of that of the wave propagation, and we have everywhere a rarefaction wave.

[1] N. L. Tsintsadze, Zh. Eksp. Teor. Fiz. **59**, 1251 (1970) [Sov. Phys. JETP **32**, 684 (1971)]; D. D. Tskhakaja and M. Ya. El-Ashri, Fifth European Conf. Controlled Fusion and Plasma Phys., Grenoble, Vol. 1, Commisariat à l'énergie atomique, 1972, p. 135; A. Bourdier, G. Di Bona, N. Fortin, and C. Masselet, Phys. Rev. **A13**, 887 (1976).

[2] V. P. Silin, Parametricheskoe vozdeĭstvie izlucheniya bol'shoĭ moshchnosti na plazmy (Parametric action of high-power radiation on a plasma), Nauka, 1973.

[3] J. F. Drake, Y. C. Lee, K. Nishikawa, and N. L. Tsintsadze, Phys. Rev. Lett. **36**, 196 (1976); J. F. Drake, Y. C. Lee, and N. L. Tsintsadze, Phys. Rev. Lett. **36**, 31 (1976).

[4] V. Ts. Gurevich and V. I. Karpman, Zh. Eksp. Teor. Fiz. **56**, 1952 (1969) [Sov. Phys. JETP **29**, 1048 (1969)]; V. I. Karpman, Nelineĭnye volny v dispergiruyushchikh Sredakh (Non-linear waves in dispersive media) Nauka, 1973 [Translation published by Pergamon, Oxford, 1975].

[5] L. N. Gorbunov, Usp. Fiz. Nauk **109**, 631 (1973) [Sov. Phys. Usp. **16**, 217 (1973)].

[6] A. I. Akhiezer and R. V. Polovin, Zh. Eksp. Teor. Fiz. **30**, 915 (1956) [Sov. Phys. JETP **3**, 696 (1956)].

[7] N. L. Tsintsadze and D. D. Tskhakaja, Seventh European Conf. Controlled Fusion and Plasma Phys., Lausanne, Vol. 1, Centre Rech. Phys. Plasmas, 1975, p. 151.

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