Statistical characteristics of the heating of a transparent medium with random absorbing inclusions by laser radiation

M. I. Tribel'skiĭ

Research Institute for Organic Semifabricates and Dyes
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A theoretical investigation is reported of the statistical characteristic of the random temperature field produced in a transparent medium containing transparent microscopic inclusions and acted upon by laser radiation. It is shown that the probability density describing such a field can be reduced to a certain universal function, which has been calculated numerically in some actual cases. It is indicated that the form of the size distribution function of the inclusions can be reconstructed from the experimentally measured deviation of the most probable temperature from its mean value.

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The question of the probability characteristics of the laser heating of transparent media that contain random absorbing inclusions (inhomogeneities) of size R that is distributed in some manner has already been investigated before.[1,2] The statistical characteristics of the optical breakdown initiated by such inhomogeneities have also been investigated.[3] In all the cited studies we considered the case of sufficiently long laser pulses, such that the temperature fields produced by the individual inclusions overlapped strongly. It was shown that different statistical characteristics have a number of universal properties that are determined only by the asymptotic behavior of the probability density that describes the size distribution $f(R)$ of the inclusions in the region of large values of R. It was indicated, in particular, that the probability distribution for the random temperature field produced in the medium is generally speaking not Gaussian, and certain features of such a non-Gaussian distribution were studied.

It is of interest to investigate this distribution in detail. This is the problem solved in the present paper. The formulation of the problem, the notation, and the conditions for the applicability of the results are the same as in the preceding paper.[1] For the sake of brevity, they are omitted here.

We introduce the dimensionless variable

$$\xi=(T-\overline{T})/(nVF)^{\nu},\quad \nu=\begin{cases} l/m; & {}^1/_2\leq l/m<1, \\ {}^1/_2; & l/m<{}^1/_2. \end{cases} \qquad (1)$$

Here $\overline{T}$ is the mean value of the temperature[1)]; nV is the mean number of inhomogeneities in the total volume V of the sample; F is a certain function constructed of the Green's functions of the heat-conduction problem; the parameters l and m characterize respectively the power-law growth of the cross section for the absorption of the radiation by the inhomogeneity and the decrease of the size distribution function of the inhomogeneities in the region of large values of R ($R\gg\overline{R}$).

It follows then from the earlier results[1] that the normalized probability density, which describes the distribution of the random quantity ξ, is equal to

$$P_\nu(\xi)=\frac{1}{\pi}\int_0^\infty \exp[-\omega^{1/\nu}\cos\pi p]\cos[\omega\xi+\omega^{1/\nu}\sin\pi p]\,d\omega,$$
$$p=1-1/2\nu, \qquad (2)$$

i.e., it is a universal function that is completely determined by the value of a single parameter ν. The integral in the right-hand side of (2) was calculated numerically for different values of ν. The calculation results are shown in Fig. 1 At $\nu=1/2$ the distribution $P_\nu(\xi)$ becomes Gaussian. It is seen from the figure that this transition takes place continuously, despite the fact that $P_\nu(\xi)$ at arbitrary $\nu>1/2$ has a power-law asymptotic form as $\xi\to\infty$ and not an exponential one as in the Gaussian case.[1]

It follows from the definition (see (1)) that $\overline{\xi}=0$. The reason why the modal (most probable) value ξ_m of ξ does not agree with $\overline{\xi}$ at $\nu>1/2$ is that in this case a small number of inhomogeneities, the size of which greatly exceeds $\overline{R}$, introduces an anomalously large contribution in the formation of the temperature field.

Reconverting from $P_\nu(\xi)$ to $P_{\mathbf{x},t}(T)$, the probability density that the temperature at the point $\mathbf{x}$ is equal to T at the instant of time t, we conclude that $(nVF)^\nu$ plays at all values of ν the same role as the square root of the variance in the Gaussian case,[2)] i.e., it is the characteristic "dimension," on the temperature scale, over which $P_{\mathbf{x},t}(T)$ varies.

At short times, where $(\chi t)^{1/2}$ (χ is the thermal diffu-

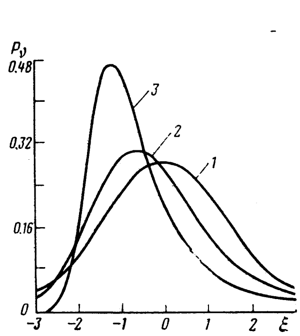

FIG. 1. Probability density that describes the random temperature field at different values of the parameter ν: 1) $\nu=\frac{1}{2}$; 2) $\nu=\frac{3}{5}$; 3) $\nu=\frac{3}{4}$.

sivity of the medium) is small in comparison with the focal radius, and the Green's function in the heat-conduction problem "does not feel" the influence of the boundary conditions, we have apart from a numerical factor

$$(nVF)^{\nu}/\overline{T} \sim [n(\chi t)^{3/2}]^{\nu-1}. \tag{3}$$

We see therefore that the probability density $P_{\mathbf{x},t}(T)$ is more "smeared out" the larger ν, i.e., the slower the decrease of the "tail" of the size distribution function of the inhomogeneities. This statement remains valid for arbitrary times of action of the laser pulse on the sample, even though relation (3) may not be satisfied in this case.

Concluding our analysis, we state without proof that the expression for the correlation radius of the investigated temperature field (r_c) contains neither small nor large parameters of the problem. In particular, in the case of short times, accurate to a numerical factor of the order of unity, we have $r_c \approx (\chi t)^{1/2}$, i.e., it increases with increasing time of action of the laser radiation.

We note in conclusion that since ν is an unambiguous function of $\Delta\xi \equiv \overline{\xi} - \xi_m$, it follows that to determine the actual form of the size distribution of the inhomogeneities there is no need to investigate experimentally the detailed form of $P_\nu(\xi)$, and it suffices to measure the quantity $\Delta\xi$. This seems to me very important, since direct methods of experimentally investigating $f(R)$ meet with considerable difficulties, in view of the small characteristic dimensions of the inhomogeneities.[4,5]

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1) The superior bar denotes throughout the mean value of the corresponding quantity.

2) At $\nu > 1/2$ the distribution $P_{\mathbf{x},t}(T)$ has no variance, since $\overline{T^2}$ does not exist in this case (the corresponding integral diverges).[1]

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