Critical behavior of the Hall coefficient near the percolation threshold

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The dependence of the Hall coefficient on the composition of a system of two components with strongly differing electrical conductivities (a "dielectric" and a "metal") is considered. In the case of equal mobilities of the current carriers in the two components the behavior near the percolation threshold is described on the basis of the scaling hypothesis. Critical indices are introduced. Relations between the indices, and their numerical values for two and three dimensions, are found. In the two-dimensional case, by means of a method of Dykhne, an exact relation, valid for an arbitrary ratio of the mobilities of the components and for arbitrary composition of the two-component system, is found between the effective Hall constant and the effective electrical conductivity. In the three-dimensional case, a model theory of the Hall effect is constructed for an arbitrary ratio of the mobilities. Unlike the effective-medium method, the theory predicts a sharp maximum of the Hall coefficient near the percolation threshold if the mobility in the dielectric is much smaller than that in the metal.

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1. INTRODUCTION

Much attention has been attracted recently by materials which, to a good approximation, can be regarded as two-component systems consisting of macroscopic regions with large and small electrical conductivities. [1–3] For brevity, we shall call the former regions metallic, and the latter dielectric. Granular metals or cermets[4] can serve as an example of such systems. In this example the volume fraction x of metal is fixed by the technology of the preparation of the cermet. As another example, we can mention substances that undergo a first-order metal-insulator phase transition (e.g., transition–metal oxides) with change of temperature. Because of inhomogeneity of the material the transition can be smeared out. It occurs at lower temperatures in certain regions, and metallic nuclei form in a background of dielectric. The volume fraction of the metallic phase in this case is determined by the temperature and with increase of temperature varies from zero to unity. [5] A third example could be the appearance of electrical conduction in a system of metallic exciton droplets in semiconductors. Here the quantity x is determined by the light-pumping intensity.

An increase, for whatever reason, of the volume fraction x of metal leads to a sharp increase of the effective electrical conductivity $\sigma(x)$ near a certain critical value $x=x_c$, at which an infinite cluster of metallic regions first forms. This value is called the percolation threshold. [1] The behavior of the electrical conductivity near the percolation threshold resembles the behavior of the order parameter of a second-order phase transition. If the electrical conductivity σ_d of the dielectric regions is equal to zero, then for $x<x_c$ we have $\sigma(x)=0$ and for $x \geqslant x_c$

$$\sigma(x)=\sigma_m(x-x_c)^t. \tag{1}$$

where σ_m is the conductivity of the metallic regions and the index t for two and three dimensions is equal to[1,2]

$$t_2=1.3,\ t_3=1.6. \tag{2}$$

The decrease of $\sigma(x)$ by the law (1) as $x \to x_c$ is connected with the gradual cutting-up of the infinite metallic cluster. [6,7] If the parameter $h=\sigma_d/\sigma_m$ is very small, but finite, then, for sufficiently small $x-x_c$, the dielectric begins to shunt the infinite metallic cluster. In this case the singularity of $\sigma(x)$ at $x=x_c$ is removed and $\sigma(x)$ becomes a smooth function, increasing monotonically from σ_d to σ_m. Thus, the parameter h plays a role analogous to that of the magnetic field in the theory of ferromagnetic transitions. Starting from this analogy, Straley[8] and Éfros and the author[9] have constructed a theory of the critical behavior of $\sigma(x)$, based on a scaling hypothesis. They introduced the power laws

$$\sigma(x_c)=\sigma_m h^s \tag{3}$$

$$\sigma(x)=\sigma_d(x_c-x)^{-q}=\sigma_m h(x_c-x)^{-q} \quad \text{for } x<x_c. \tag{4}$$

The growth of σ with increase of x by the law (4) is related to the gradual increase of the sizes of the metallic clusters and of the area of the thin dielectric layers between neighboring clusters (Fig. 1). In the immediate vicinity of x_c the power laws (1) and (4) are violated and there is a crossover from one to the other. In the papers indicated it was assumed that in the crossover region the behavior of $\sigma(x)$ is determined by the single parameter $\Delta=h^m$, which may be called the width of the transition (Fig. 2). In other words, it was assumed

FIG. 1. Metallic clusters (shaded) for $x<x_c$ near the percolation threshold. The direction of the current is indicated by the arrow. The "active" dielectric regions that determine the electrical conductivity are indicated by the letter A.

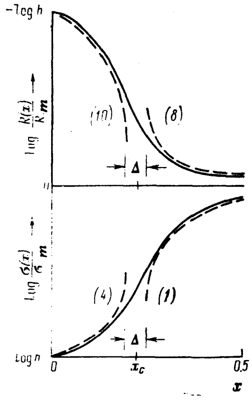

FIG. 2. Schematic graphs of the functions $\sigma(x)$ and $R(x)$ for the case $\mu_d=\mu_m$ and $h=\sigma_d/\sigma_m\ll 1$ (solid lines). Each dashed line corresponds to the formula whose label is indicated alongside. The three-dimensional case is plotted, and $g\neq 0$.

that the function $\sigma(\tau, h)$ satisfied the scaling hypothesis

$$\sigma(\tau, h)=\sigma_m h^s\varphi(\tau/h^m), \tag{5}$$

where $\tau=x-x_c$, and $\varphi(z)=1$ for $z=0$ and is a power function of z for $z\to\pm\infty$. A comparison of (5) with the laws (1), (3) and (4) led to relations between the indices:

$$m=s/t, \quad q=t/s-t. \tag{6}$$

Thus, all the indices of $\sigma(x)$ are expressed in terms of the indices t and s. The latter, according to[9], is equal to

$$s_2=0.5, \quad s_3\approx 0.62. \tag{7}$$

The relations (6) agree with numerical calculations for three-dimensional lattices[3] and are well fulfilled for the Bethe lattice.[8]

In an analogous way we can introduce critical indices in the problem of the effective Hall constant $R(x)$ of a two-component medium and relate them to each other. Section 2 is devoted to this question.

2. CRITICAL INDICES OF $R(x)$ IN THE CASE OF EQUAL MOBILITIES OF THE METAL AND DIELECTRIC

Let μ_d, μ_m and R_d, R_m be the mobilities and Hall constants, respectively, in the dielectric and metallic regions. We shall consider first the simple case when $\mu_d=\mu_m\equiv\mu$ and, consequently, the ratio $R_m/R_d=\sigma_d/\sigma_m=h\ll 1$ is the only parameter describing the decrease of $R(x)$ with increase of x. In this case the following picture of the behavior of $R(x)$ seems natural. In the critical region to the "right" of the threshold ($\Delta\ll x-x_c\ll 1$), corresponding to the power law (1) for $\sigma(x)$ there is a power law for $R(x)$:

$$R(x)=R_m(x-x_c)^{-g}=R_d h(x-x_c)^{-g} \quad (\Delta\ll x-x_c\ll 1). \tag{8}$$

This law is connected with the gradual cutting-up of the metallic infinite cluster as x_c is approached, and the dielectric plays no role in its origin. It was derived in[6] on the basis of a network model of the infinite cluster and in[7] on the basis of ideas of geometric scaling in percolation theory. It was found that

$$g_2=0, \quad g_3=\nu_3, \tag{9}$$

where $\nu_3\approx 0.9$ is the correlation-length index of percolation theory.[2] In the region of x to the "left" of the threshold ($1\gg x_c-x\gg\Delta$), corresponding to the power law (4) for $\sigma(x)$ there is a power law for $R(x)$:

$$R(x)=R_d(x_c-x)^f \quad (1\gg x_c-x\gg\Delta). \tag{10}$$

In this region of values of x, in view of the large value of R_d, the Hall effect is determined by the dielectric. The decrease of $R(x)$ is connected with the fact that as the threshold is approached the current is concentrated in an ever smaller part of the volume of the dielectric (Fig. 1). The fraction of current flowing outside the active dielectric layers decreases continuously.

In the critical region $|x-x_c|\lesssim\Delta$ the dielectric and metal produce comparable contributions to the conductivity and Hall effect. In this case a gradual crossover from the law (10) to the law (8) occurs (Fig. 2). We can introduce a further critical index by writing $R(x_c)$ in the form

$$R(x_c)=R_d h^k. \tag{11}$$

The pattern described for the behavior of $R(x)$ contains essentially two assumptions:

1. The smearing-out of the transition for $R(x)$ is determined by the same interval $\Delta=h^{s/t}$ as for $\sigma(x)$.

2. For all $|\tau|\ll 1$ to the right and left of the region of smearing, $R(x)$ is a power function of τ.

In other words, we take for $R(x)$ the scaling hypothesis

$$R(x)=R_d h^k\psi(\tau/h^{s/t}), \tag{12}$$

where $\psi(z)=1$ for $z=0$ and $\psi(z)$ is a power function of z for $z\to\pm\infty$. It follows from a comparison of (12) with (8) and (10) that for $z\gg 1$ we have $\psi\approx z^{-g}$ and for $z\ll -1$, $\psi\approx(-z)^f$. Then, in order that the dependence on the parameter h as comprised in (12) coincide with the dependences (8) and (10), the following relations between the indices should be fulfilled:

$$k=1-sg/t, \quad f=t/s-g. \tag{13}$$

Using the values of t, s and g from (2), (7) and (9), we obtain the remaining critical indices of the Hall constant:

$$k_2=1, \quad k_3=0.65, \tag{14}$$

$$f_2=2t_2\approx 2.6, \quad f_3\approx 1.7. \tag{15}$$

Knowing the behavior of $\sigma(x)$ and $R(x)$, it is not difficult to find all the critical indices of the effective mobility $\mu(x)=R(x)\,\sigma(x)$. In particular, by writing $\mu(x_c)$ in the form $\mu(x_c)=\mu h^l$, we obtain $l=s+k-1$, i.e.,

$$l_2=0.5, \quad l_3\approx 0.37. \tag{16}$$

It is interesting to compare the values of the indices in (9), (14) and (16) with the predictions of the effective-medium theory. According to[12], this theory gives

$$g_2=g_3=0,\ f_2=f_3=2,\ k_2=k_3=1,\ l_2=l_3=0.5. \tag{17}$$

Thus, except for g_2, k_2, and l_2, the effective-medium theory predicts critical-index values that differ from ours.

3. EXACT RESULTS FOR TWO-DIMENSIONAL SYSTEMS

For two-dimensional systems with $\mu_d=\mu_m\equiv\mu$ there exists an exact relation between $R(x)$ and $\sigma(x)$, which is essentially contained in the work of Dykhne[10]:

$$R(x)=\frac{\mu}{\sigma_d+\sigma_m}\left[1+\frac{\sigma_d\sigma_m}{\sigma^2(x)}\right]. \tag{18}$$

It can be obtained in the limit of weak magnetic fields from the formula (D.20) (this is the way in which we shall refer to formulas in Dykhne's article[10]). According to formulas (3) and (7), in the two-dimensional case $\sigma^2(x_c)\approx\sigma_m\sigma_d$ (for a symmetric spatial distribution of the components the exact equality $\sigma^2(x_c)=\sigma_m\sigma_d$ holds[11]). It can be seen from (18) that for $x-x_c\ll-\Delta$, when $\sigma(x)\ll\sigma(x_c)$, we have $R(x)=R_d\sigma_d^2/\sigma^2(x)$. According to (4) this means that $f_2=2t_2$. For $x-x_c\gg\Delta$, on the other hand, $\sigma(x)\gg\sigma(x_c)$ and $R=R_m$, i.e., $g_2=0$. For $x=x_c$ we have $R(x_c)\approx R_m$, so that $k=1$. Thus, in the two-dimensional case the values of the critical indices in (9), (14) and (15) agree with the exact relation (18).

It turns out that, in the general case $\mu_m\neq\mu_d$, in two-dimensional systems there exists an exact relation between $R(x)$ and $\sigma(x)$:

$$R(x)=\frac{\mu_d-\mu_m}{\sigma_d-\sigma_m}+\frac{\sigma_d\mu_m-\sigma_m\mu_d}{\sigma_d^2-\sigma_m^2}\left[1+\frac{\sigma_d\sigma_m}{\sigma^2(x)}\right]. \tag{19}$$

To obtain (19) one must change the derivation of formula (D.20) slightly. The derivation of (D.20) was based on a transformation of the local currents and fields that does not change the local values of the conductivity but reverses the sign of the Hall parameter $\beta=\mu H/c$ (here H is the magnetic field and c is the velocity of light). In going over from the case $\mu_d=\mu_m\equiv\mu$ to the case $\mu_d\neq\mu_m$ we must generalize this transformation in such a way that both local values β_m and β_d of the Hall parameter reverse their signs without changing their absolute values. In weak fields (β_m, $\beta_d\ll1$), the transformation sought is effected by means of the following values of the coefficients in (D.14):

$$a=c=1,\quad b=\frac{2(\sigma_d\beta_m-\sigma_m\beta_d)\sigma_m\sigma_d}{\sigma_d^2-\sigma_m^2},$$
$$d=\frac{2(\beta_m-\beta_d)}{\sigma_d-\sigma_m}-\frac{2(\sigma_d\beta_m-\sigma_m\beta_d)}{\sigma_d^2-\sigma_m^2} \tag{20}$$

One can convince oneself of this by substituting (20) into the expressions (D.15) for β'' and σ'' (it is necessary, however, to take into account that there is a misprint in the numerator of β'' in[10]: the term $cd\sigma^2$ was omitted). The subsequent chain of arguments repeats exactly the arguments of Dykhne and, in weak magnetic fields, leads to the relation (19).

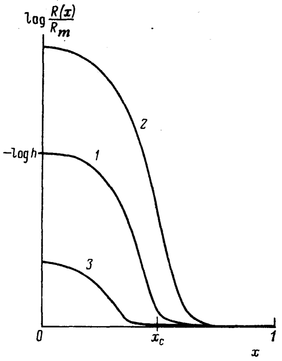

FIG. 3. Schematic graphs of the function $R(x)$ for the two-dimensional case for fixed values of the quantity R_m and the parameter $h=\sigma_d/\sigma_m\ll1$ and different values of the ratio μ_d/μ_m: 1) $\mu_d=\mu_m$, 2) $\mu_d\gg\mu_m$, 3) $\mu_d\ll\mu_m$, $R_d\gg R_m$.

We turn to an investigation of the relation (19). We shall consider, e.g., the situation when

$$\sigma_d\ll\sigma_m\quad R_m\sigma_m^2/\sigma_d^2\gg R_d\gg R_m$$

Then from (19) we obtain

$$R(x)=R_m\left[1+\frac{\mu_d}{\mu_m}\frac{\sigma_d\sigma_m}{\sigma^2(x)}\right]=R_m+R_d\frac{\sigma_d^2}{\sigma^2(x)}. \tag{21}$$

This formula can be interpreted on the basis of the so-called two-band model, i.e., of a model in which there are two parallel conduction mechanisms 1 and 2. In this case,

$$R=(R_1\sigma_1^2+R_2\sigma_2^2)/(\sigma_1+\sigma_2)^2. \tag{22}$$

We shall visualize our two-component system as a homogeneous dielectric ($\sigma_1=\sigma_d$, $R_1=R_d$) and a conducting network, connected in parallel. By the conducting network we mean the connected region delineated by the current lines; this, for $x>x_c$, coincides with the infinite metallic cluster, and, for $x<x_c$, consists of large metallic clusters linked by dielectric layers. The conductivity of this network is $\sigma(x)$, and the Hall constant for $x>x_c$ is, according to[6,7], equal to R_m. If we neglect the Hall current that arises in the thin dielectric layers, then, for $x<x_c$, to calculate the Hall constant of the network we can repeat exactly the arguments given in[6,7]. We then obtain that, in the two-dimensional case, the Hall constant of the conducting network is R_m in the whole range of values of x. Substituting $R_2=R_m$, $\sigma_2=\sigma(x)$, $\sigma_1=\sigma_d$ and $R_1=R_d$ into formula (22) and neglecting σ_d in comparison with $\sigma(x)$, we obtain (21).

It can be seen from (21) that for $\mu_d/\mu_m\ll1$, i.e., $R_d\ll R_m\sigma_m/\sigma_d$, owing to the comparatively small value of R_d the dielectric ceases to play a role in the creation of the Hall emf when $x-x_c$ is still $\ll-\Delta$, when the conductivity is essentially determined by the dielectric layers. For $\mu_d/\mu_m\gg1$, on the other hand, the value of R_d is so large that even after the conductivity begins to be determined by the infinite metallic cluster the dielectric continues to play a determining role in the Hall effect (Fig. 3).

For $\mu_d=\mu_m$ the exact solution (18) has justified the assumptions 1 and 2. The assumption 1 is also justified for $\mu_d\neq\mu_m$, as can be seen from (19). The entire critical behavior of $R(x)$ is contained in $\sigma(x)$. However, as we should expect, assumption 2 is no longer justified. Owing to the presence of the parameter μ_d/μ_m, the distance from the threshold over which the change in the power laws occurs turns out to differ from Δ.

4. THREE-DIMENSIONAL SYSTEMS WITH $\mu_d \neq \mu_m$

In the three-dimensional case exact relations analogous to (19) are absent. For $\mu_d \neq \mu_m$ it is not possible to apply the theory of critical indices developed in Sec. 2. In fact, in this case there are no reasons for assumption 2 to be fulfilled. Using the example of a two-dimensional system it can be seen clearly how the presence of the parameter μ_d/μ_m leads to a futher change in the power laws outside the region $|\tau| \leq \Delta$ of smearing of the transition. At the same time we are sure that assumption 1 remains valid. This certainty is based, in particular, on a consideration of a system in which $\sigma_d = \sigma_m \equiv \sigma$ and $R_d \gg R_m$. In a weak magnetic field, using perturbation theory in the nondiagonal components of the conductivity tensor $(\sigma_{xy})_{d,m} = R_{d,m} H\sigma^2$, it is easy to show that the exact relation

$$R(x) = xR_m + (1-x)R_d \tag{23}$$

holds, irrespective of the number of space dimensions. For a two-dimensional system the relation (23) can also be obtained from (19). For this we must assume that $\delta = (\sigma_m - \sigma_d)/(\sigma_d + \sigma_m) \ll 1$ and make use of the fact that, in first order in δ,

$$\sigma(x) = x\sigma_m + (1-x)\sigma_d = \sigma_d + x\delta(\sigma_d + \sigma_m). \tag{24}$$

Substituting (24) into (19) and expanding in δ, we obtain (23).

From (23) it is particularly clear that sharp drops in the mobilities or Hall coefficients from one medium to the other do not in themselves produce any critical behavior near x_c. The critical behavior of $R(x)$ arises only when $\sigma_d \ll \sigma_m$ and is entirely determined by the critical behavior of $\sigma(x)$. Therefore, it is natural to assume that for $|\tau| \ll 1$ and $h \ll 1$ the quantity $R(x)$ is determined by formula (12), although, as already stated, $\psi(z)$, generally speaking, is no longer a power function in both the regions $z \gg 1$ and $z \gg -1$. The ratio μ_d/μ_m, which determines the point at which the power laws change, can appear in the function $\psi(z)$ as a parameter. There are not enough of these general properties of the function $R(x)$ to determine the function. Therefore, to determine $R(x)$ we shall have to use the "two-band" model, used successfully above to interpret the formula (19).

First we shall calculate the Hall constant $R_2(x)$ of the conducting network. According to[6,7], the Hall constant of a network is proportional to its "period" (the average distance between nodes). For $x - x_c \gg \Delta$ the "period" of the conducting network coincides with the percolation-theory correlation length and grows like $(x - x_c)^{-\nu_3}$ as the transition is approached. Correspondingly, $R_2(x)$ grows by the law (8) with $g_3 = \nu_3$. In the region $|x - x_c| \lesssim \Delta$ the period of the conducting network does not change and, consequently, $R_2(x)$ is constant. In the region $1 \gg x_c - x \gg \Delta$ to the left of the threshold the period of the network is equal to the characteristic size of the metallic cluster, i.e., it again coincides with the correlation length. With decrease of x it falls off like $(x_c - x)^{-\nu_3}$, and, therefore,

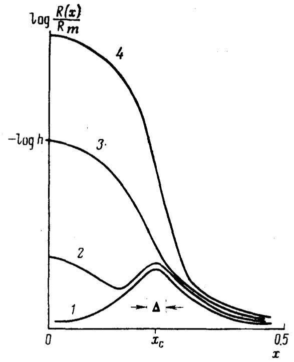

FIG. 4. The same as Fig. 3, for the three-dimensional case: 1) $\mu_d = h\mu_m$ $(R_d = R_m)$, 2) $\mu_m h \ll \mu_d \ll \mu_m$, 3) $\mu_d = \mu_m$, 4) $\mu_d > \mu_m$.

$$R_2(x) = R_m(x_c - x)^{-g_3}. \tag{25}$$

In the whole range $|x - x_c| \ll 1$ the Hall constant of the conducting network can be written by means of the interpolation formula

$$R_2(x) = R_m[(x - x_c)^2 + \Delta^2]^{-g_3/2}. \tag{26}$$

Substituting $R_1 = R_d$, $\sigma_1 = \sigma_d$, $\sigma_2 = \sigma(x)$ and $R_2(x)$ from formula (26) into (22), we obtain

$$R(x) = R_d \frac{\sigma_d^2}{\sigma^2(x)} + R_m[(x - x_c)^2 + \Delta^2]^{-g_3/2}. \tag{27}$$

Before analyzing (27) for arbitrary values of μ_d/μ_m we shall examine how well the "two-band" model describes the case $\mu_d = \mu_m \equiv \mu$. It is not difficult to see that, in order that (27) reproduce the results of Sec. 2, the equality

$$f_3 = 2q_3. \tag{28}$$

should be fulfilled. This same equality is necessary in order that, for any ratio μ_d/μ_m, the function $R(x)$ (27) satisfy the general conditions formulated above, i.e., have the form (12) with μ_d/μ_m as the parameter. According to formula (15) and[3], $f_3 \approx 1.7$ and $q_3 \approx 1$. The uncertainty in the numerical values of the indices f_3 and q_3 make the equality (28) perfectly possible. If this equality is fulfilled, we obtain a consistent picture of the dependence $R(x)$ for $\sigma_d \ll \sigma_m$.

We shall start the investigation of (27) from the case $R_m = R_d \equiv R$. In this case the contribution of the dielectric is negligibly small and we obtain a curve with a sharp maximum near x_c (curve 1 of Fig. 4). We shall increase R_d gradually, leaving R_m, σ_d and σ_m constant. Then in the interval

$$R_m\sigma_m/\sigma_d \geq R_d \geq R_m \quad (1 \geq \mu_d/\mu_m \geq \sigma_d/\sigma_m)$$

a gradual increase occurs in the role of the dielectric. This leads to smoothing of the minimum (curve 2) and to change-over to a monotonic curve. When $R_d = R_m\sigma_m/\sigma_d$ $(\mu_d = \mu_m)$ we arrive at the result of Sec. 2 (curve 3). In this case the dielectric determines the entire curve on the left of the transition and the conducting network determines that on the right. For $R_d > R_m\sigma_m/\sigma_d$ the dielectric is found to have a determining role in a certain region to the right of the transition as well (curve 4).

It is interesting to note that for $\mu_d < \mu_m$ our results differ not only quantitatively but also qualitatively from the predictions of effective-medium theory.[12] This theory leads to a monotonic decrease of $R(x)$ with increase of x. For example, in the case $R_m = R_d \equiv R$ the effective-medium theory gives $R(x) = R = \text{const}$. But in reality $R(x)$ should have a sharp maximum.

We assumed above that the equality (28) is fulfilled. If this is not so, the two-band model is not entirely adequate for $x < x_c$. This may be connected with the neglect of the Hall current generated in the active dielectric layers of the conducting network. However, we are confident that, even in this case, the formula (27) gives a qualitatively correct description of the behavior of $R(x)$.

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[1]S. Kirkpatrick, Rev. Mod. Phys. 45, 574 (1973).
[2]B. I. Shklovskiĭ and A. L. Éfros, Usp. Fiz. Nauk 117, 401 (1975) [Sov. Phys. Usp. 18, 845].
[3]I. Webman, J. Jortner, and M. H. Cohen, Phys. Rev. B11, 2885 (1975); B13, 713 (1976).
[4]B. Abeles, Ping Sheng, M. D. Coutts, and Y. Arie, Adv. Phys. 24, 407 (1975).
[5]Yu. M. Gerbshteĭn, T. V. Smirnova, E. I. Terukhov, and F. A. Chudnovskiĭ, Fiz. Tverd. Tela (Leningrad) 18, 503 (1976) [Sov. Phys. Solid State 18, 290 (1976)].
[6]A. S. Skal and B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn, 8, 1586 (1974) [Sov. Phys. Semicond. 8, 1029 (1975)].
[7]M. E. Levinshteĭn, M. S. Shur, and A. L. Éfros, Zh. Eksp. Teor. Fiz. 69, 2203 (1975) [Sov. Phys. JETP 42, 1120].
[8]J. P. Straley, J. Phys. C9, 783 (1976).
[9]A. L. Éfros and B. I. Shklovskiĭ, Phys. Stat. Sol. (b) 76, 475 (1976).
[10]A. M. Dykhne, Zh. Eksp. Teor. Fiz. 59, 641 (1970) [Sov. Phys. JETP 32, 348 (1971)].
[11]A. M. Dykhne, Zh. Eksp. Teor. Fiz. 59, 110 (1970) [Sov. Phys. JETP 32, 63 (1971)].
[12]M. H. Cohen and J. Jortner, Phys. Rev. Lett. 30, 699 (1973).

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Anomalous penetration of an electromagnetic field into a metal with diffuse reflection of electrons by the specimen boundary

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A theory is constructed for anomalous penetration (AP) of an electromagnetic wave into a metal placed in a magnetic field parallel to its surface. The reflection of electrons from the metal-vacuum interface is assumed to be diffuse. AP of the field occurs along a chain of electron trajectories. It is shown that under anomalous-skin-effect conditions in the radiofrequency range, the field distribution contains four spikes, at distances from the boundary of one, two, three, and four cyclotron diameters. The first three spikes have a distinct spatial structure, whereas the fourth exists against the background of a smooth quasiharmonic distribution. At distances exceeding the region of existence of the last spike, the field has a quasiharmonic character.

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1. INTRODUCTION

The effect of anomalous penetration (AP) of an electromagnetic wave into a metal along a chain of electron trajectories, in a magnetic field **H** parallel to the surface of the specimen, is well known in the physics of metals (see Fig. 1). It has been observed experimentally by Gantmakher[1] and investigated theoretically in an article by one of the authors.[2] A large number of papers have now been devoted to this phenomenon (see the review[3] and also the article[4]). There is at present extensive experimental material on the observation of AP of the trajectory type in many metals. Nevertheless there has so far been lacking a systematic theory of the trajectorial transfer of an electromagnetic wave with allowance for the interaction of the electrons with the specimen surface. The reason lies in the mathematical difficulties that arise when one takes account of this interaction and that lead to a complex character of the field distribution in the metal. In order to circumvent these difficulties, qualitative considerations have been introduced. It has been supposed that a good approximation is the distribution of the electric field **E** in an infinite specimen with a current sheet, simulating the skin layer δ (see, for example, [3]). In other words, it has been assumed that the principal role in AP is played by electrons that do not collide with the metal–vacuum interface, and for which it is possible to use the results that are valid in an infinite specimen. Thus for a wave polarized perpendicular to **H**, the spatial distri-

