Compton effect and pair production for relativistic particles in a plasma

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We evaluate the probabilities for the Compton effect and electron-positron pair production by two transverse quanta in an isotropic plasma. We obtain in the ultra-quantal limit an expression for the Compton energy loss of an electron in the case when the frequency of the initial quantum is close to the plasma frequency. We show that the plasma alters significantly the nature of the emission. We find the spectral power of the radiation by a system of ultra-relativistic electrons. We obtain an expression for the γ-quantum energy loss due to pair production.

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1. Recently there has been an extensive discussion of the problems of the effect of a medium on various quantum radiation processes: bremsstrahlung, the synchrotron mechanism, Compton effect, and so on. If one is dealing with high-frequency emission, any medium can be considered as a plasma, i.e., one can in practice neglect the interaction between the particles. The presence of a medium reduces not merely to effects due to the refractive index, but also leads to new emission mechanisms.[1]

The aim of the present paper is to study the Compton effect and the electron–positron pair production in the limit of ultra-relativistic particle energies, when it is well known that quantum effects are important. If the low frequency wave in the Compton effect has a frequency close to the plasma frequency, the influence of the plasma is clearly of the utmost importance in that case. We shall show below that in the ultra-quantal limit the energy losses of a particle in this case turn out to be proportional to its energy ε (in contrast to the dependence $\propto \ln\varepsilon$ for the normal Compton effect). In that sense the effect discovered below indicates a new qualitative change in the nature of the radiation, where the plasma significantly amplifies the emission processes in contrast to the well known inhibitive effects due to the influence of the refractive index. The influence of the medium on pair production turns out to be important only at very high densities $\hbar\omega_{pe}\sim 2mc^2$ (ω_{pe} is the plasma frequency).

In a medium there are possible not only spontaneous processes, but also induced processes which are non-linear processes with the same probability. It is therefore convenient to compute the probabilities for the induced processes and to obtain simultaneously the non-linear equations. We shall therefore first construct the non-linear theory. The effects of interest to us can be considered once we have worked out the theory of non-linear interactions in a relativistic plasma in the quantum limit. We therefore precede the consideration of the effect of the plasma on the Compton effect with a general theoretical discussion of non-linear processes in a quantal relativistic plasma. It enables us to study not only the processes studied in this paper, but also a broad class of processes for which the medium plays an essential role. These include transition radiation,[2] which was studied earlier in the classical limit, plasma bremsstrahlung,[3] which also has been studied only in the classical limit, and so on.

2. Non-linear interactions of waves and particles in a plasma are completely described by the probabilities of the corresponding processes. To obtain those it is necessary to find the non-linear currents of the plasma. We shall start from the kinetic equation for the density matrix $\rho_{\alpha\beta}(\mathbf{r}',\mathbf{r},t)$ in the coordinate representation[1)][4]

$$i\partial\rho_{\alpha\beta}(\mathbf{r}',\mathbf{r},t)/\partial t=H_{\beta\gamma}\rho_{\alpha\gamma}(\mathbf{r}',\mathbf{r},t)-H_{\gamma\alpha}^{*\prime}\rho_{\gamma\beta}(\mathbf{r}',\mathbf{r},t), \tag{1}$$

$H=\boldsymbol{\alpha}(\mathbf{p}-e\mathbf{A})+\beta m+e\varphi$ is the Hamiltonian operator of a relativistic electron in an external electromagnetic field acting on the unprimed variables; H' is the same operator acting on the primed variables, and the asterisk indicates the complex conjugate.

Aiming at a quantum-mechanical description which is closest to the classical one, we follow[4] and introduce the density matrix $f_{\alpha\beta}(\mathbf{p},\mathbf{r},t)$ in the Wigner representation which plays the role of a quantal distribution function

$$\begin{gathered}f_{\alpha\beta}(\mathbf{p},\mathbf{r},t)=\frac{1}{(2\pi)^3}\int d\boldsymbol{\tau}\,e^{-i\boldsymbol{\tau}\mathbf{p}}\rho_{\alpha\beta}\left(\mathbf{r}-\frac{\boldsymbol{\tau}}{2},\ \mathbf{r}+\frac{\boldsymbol{\tau}}{2},\ t\right),\\ \rho_{\alpha\beta}(\mathbf{r}',\mathbf{r},t)=\int d\mathbf{p}\,e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')}f_{\alpha\beta}\left(\mathbf{p},\ \frac{\mathbf{r}+\mathbf{r}'}{2},\ t\right).\end{gathered} \tag{2}$$

Using Eqs. (2) we get from Eq. (1) a kinetic equation for $f_{\alpha\beta}(\mathbf{p},\mathbf{r},t)$, which is the analog of the collisionless Boltzmann equation in the classical theory:

$$\begin{aligned}\frac{\partial f_{\alpha\beta}(\mathbf{p},\mathbf{r},t)}{\partial t}&=\frac{i}{(2\pi)^6}\int d\boldsymbol{\tau}\,d\mathbf{p}'\,d\boldsymbol{\eta}\,d\mathbf{r}'\exp\{i[\boldsymbol{\tau}(\mathbf{p}'-\mathbf{p})+\boldsymbol{\eta}(\mathbf{r}'-\mathbf{r})]\}\\ &\times\{[(\boldsymbol{\alpha},(\mathbf{p}'+\boldsymbol{\eta}/2)-e\mathbf{A}(\mathbf{r}'-\boldsymbol{\tau}/2))+e\varphi(\mathbf{r}'-\boldsymbol{\tau}/2)+\beta m]_{\beta\gamma}f_{\alpha\gamma}(\mathbf{p}',\mathbf{r}',t)\\ &-[(\boldsymbol{\alpha},-(\mathbf{p}'-\boldsymbol{\eta}/2)+e\mathbf{A}(\mathbf{r}'+\boldsymbol{\tau}/2))-e\varphi(\mathbf{r}'+\boldsymbol{\tau}/2)-\beta m)_{\gamma\alpha}f_{\gamma\beta}(\mathbf{p}',\mathbf{r}',t)\}.\end{aligned} \tag{3}$$

In[4] this equation was used to study the static magnetic susceptibility of a relativistic electron gas. We shall assume the electromagnetic field to be strong (classical). In that case Eq. (3) together with the Maxwell equations forms a set of equations with a self-consistent field for a quantal relativistic plasma. Changing in (3) to the Fourier components $f_{\alpha\beta}(\mathbf{p}, k)$, $\mathbf{A}(k)$, and $\varphi(k)$ we get

$$-i\omega f_{\alpha\beta}(\mathbf{p},k)+\{i\beta[i\boldsymbol{\gamma}(\mathbf{p}+\mathbf{k}/2)+m]\}_{\alpha\gamma}f_{\gamma\beta}(\mathbf{p},k)$$
$$-f_{\alpha\gamma}(\mathbf{p},k)\{[-i\boldsymbol{\gamma}(\mathbf{p}-\mathbf{k}/2)+m]i\beta\}_{\gamma\beta}$$
$$=e\int dk_1\,dk_2\delta(k-k_1-k_2)\left[f_{\alpha\gamma}\left(\mathbf{p}+\frac{\mathbf{k}_2}{2},k_1\right)(\beta\hat{A}_{k_2})_{\gamma\beta}\right.$$
$$\left.-(\beta\hat{A}_{k_2})_{\alpha\gamma}f_{\gamma\beta}\left(\mathbf{p}-\frac{\mathbf{k}_2}{2},k_1\right)\right], \tag{4}$$
$$k=\{\mathbf{k}, i\omega\},\ dk=d\mathbf{k}d\omega,\ \delta(k)=\delta(\mathbf{k})\delta(\omega),\quad \hat{A}=\gamma_\alpha A_\alpha,$$

where $A_\alpha=\{\mathbf{A}, i\varphi\}$ and $\gamma_\alpha(\boldsymbol{\gamma}, \beta)$ are the Dirac matrices. Further we use the expansion[5]

$$f_{\alpha\beta}(\mathbf{p},\mathbf{k})=\sum_{\mu\mu',\Lambda\Lambda'}u^{\mu\Lambda}_{\alpha,\mathbf{p}+\mathbf{k}/2}\overset{*}{u}{}^{\mu'\Lambda'}_{\beta,\mathbf{p}-\mathbf{k}/2}f^{\mu\mu'\Lambda\Lambda'}(\mathbf{p},k), \tag{5}$$

where the $u^{\mu\Lambda}_{\alpha,\mathbf{p}}$ are four linearly independent bispinors which describe states with well-defined values of the spin component along the direction of motion ($\mu=\pm\frac{1}{2}$) and a well-defined energy law ($\Lambda=\pm1$). Substituting the expansion (5) into (4) and using the well known relations[5]

$$(i\boldsymbol{\gamma}\mathbf{p}+m)u_{\mathbf{p}}^{\mu\Lambda}=\beta\Lambda\varepsilon_{\mathbf{p}}u_{\mathbf{p}}^{\mu\Lambda},\qquad u_{\mathbf{p}}^{*\mu\Lambda}(-i\boldsymbol{\gamma}\mathbf{p}+m)=\beta\Lambda\varepsilon_{\mathbf{p}}u_{\mathbf{p}}^{*\mu\Lambda},$$
$$\sum_\mu u^{\mu,1}_{\alpha,\mathbf{p}}u^{*\mu,1}_{\beta,\mathbf{p}}=(\Lambda_{\mathbf{p}}^+\beta)_{\alpha\beta},\qquad \sum_\mu u^{\mu,-1}_{\alpha,\mathbf{p}}u^{*\mu,-1}_{\beta,\mathbf{p}}=-(\Lambda_{\mathbf{p}}^-\beta)_{\alpha\beta}, \tag{6}$$

we get the equation

$$f_{\alpha\beta}(\mathbf{p},k)=ie\int dk_1\,dk_2\delta(k-k_1-k_2)\left\{\frac{B^{++}_{\alpha\beta}}{\omega-\varepsilon_{\mathbf{p}+\mathbf{k}/2}+\varepsilon_{\mathbf{p}-\mathbf{k}/2}}+\frac{B^{--}_{\alpha\beta}}{\omega+\varepsilon_{\mathbf{p}+\mathbf{k}/2}-\varepsilon_{\mathbf{p}-\mathbf{k}/2}}\right.$$
$$\left.-\frac{B^{+-}_{\alpha\beta}}{\omega-\varepsilon_{\mathbf{p}+\mathbf{k}/2}-\varepsilon_{\mathbf{p}-\mathbf{k}/2}}-\frac{B^{-+}_{\alpha\beta}}{\omega+\varepsilon_{\mathbf{p}+\mathbf{k}/2}+\varepsilon_{\mathbf{p}-\mathbf{k}/2}}\right\}, \tag{7}$$

where

$$B^{\pm\pm}_{\alpha\beta}=[\Lambda^{\mp}_{\mathbf{p}+\mathbf{k}/2}\beta f(\mathbf{p}+\mathbf{k}_2/2,k_1)\beta\hat{A}_{k_2}\Lambda^{\pm}_{\mathbf{p}-\mathbf{k}/2}\beta-\Lambda^{\pm}_{\mathbf{p}+\mathbf{k}/2}\hat{A}_{k_2}f(\mathbf{p}-\mathbf{k}_2/2,k_1)\Lambda^{\pm}_{\mathbf{p}-\mathbf{k}/2}\beta]_{\alpha\beta},$$
$$\varepsilon_{\mathbf{p}}=+(\mathbf{p}^2+m^2)^{1/2},\qquad \Lambda_{\mathbf{p}}^{\pm}=(m-i\hat{p}^{\pm})/2\varepsilon_{\mathbf{p}},\qquad \hat{p}^{\pm}=\gamma_\alpha p_\alpha^{\pm},\qquad p_\alpha^{\pm}=\{\mathbf{p},\pm i\varepsilon_{\mathbf{p}}\}.$$

Equation (7) will play a fundamental role in the study of non-linear effects in a quantal plasma.

The equation for the Fourier components $j_{k,i}$ of the plasma current has the form[2)]

$$j_{k,i}=\sum_s e_s\int \mathrm{Sp}\{\alpha_i f_s(\mathbf{p},k)\}\,d\mathbf{p}. \tag{8}$$

The summation is here over the kinds of particles and e_s is the charge of particles of kind s. We shall find the quantal distribution functions $f_s(\mathbf{p}, k)$ from Eq. (7). To do this we write $f_s(\mathbf{p}, k)$ as a power series in the Fourier component E_k of the electrical field:

$$f_s(\mathbf{p},k)=\sum_{n=0}^{\infty}f_s^{(n)}(\mathbf{p},k),\qquad f_s^{(n)}(\mathbf{p},k)\sim E_k^n. \tag{9}$$

The first term $f_s^{(0)}(\mathbf{p}, k)$ is independent of the electromagnetic field and thus corresponds to the unperturbed equilibrium state of the plasma. We shall assume that $f_s^{(0)}(\mathbf{p}, k)$ is given. Substituting the expansion (9) into Eq. (7) and equating terms with the same powers of E_k we get a set of coupled equations for the non-equilibrium parts $f_s^{(n)}(\mathbf{p}, k)$ which are perturbed by the electromagnetic field:

$$f_s^{(n)}(\mathbf{p},k)=J[f_s^{(n-1)}(\mathbf{p}+\mathbf{k}_2/2,k_1),\ f_s^{(n-1)}(\mathbf{p}-\mathbf{k}_2/2,k_1)],\qquad n=1,2,\ldots, \tag{10}$$

$J[f(\mathbf{p}+\frac{1}{2}\mathbf{k}_2, k_1), f(\mathbf{p}-\frac{1}{2}\mathbf{k}_2, k_1)]$ is a symbolical way of writing down the right-hand side of Eq. (7). The first equation of this chain corresponds to the linear theory of the oscillations of a relativistic plasma, and the following ones are connected with non-linear effects. Using (9) we can rewrite Eq. (8) in the form

$$j_{k,i}=\sum_{n=1}^{\infty}j_{k,i}^{(n)},\qquad j_{k,i}^{(n)}=\sum_s e_s\int \mathrm{Sp}\{\alpha_i f_s^{(n)}(\mathbf{p},k)\}\,d\mathbf{p}. \tag{11}$$

We restrict ourselves to finding the first three terms of the series (11) in the case of an isotropic electron plasma (the positrons can be taken into account similarly). Assuming that the unperturbed equilibrium state of the plasma is stationary and uniform we get, using (6), from Eq. (5)

$$f^{(0)}(\mathbf{p},k)={}^1\!/_2\Lambda_{\mathbf{p}}^+\beta\delta(k)f^{(0)}(\varepsilon_{\mathbf{p}}), \tag{12}$$

where $f^{(0)}(\varepsilon_{\mathbf{p}})$ is the usual electron distribution function, while the factor $\frac{1}{2}$ is introduced for convenience of normalization.

We find first the current $j^{(1)}_{k,i}$. Using the δ-function to integrate over dk_1dk_2 in the first of the set of Eqs. (10) and using the matrix relations

$$\Lambda_{\mathbf{p}}^+\beta\Lambda_{\mathbf{p}}^+=\Lambda_{\mathbf{p}}^+,\ \Lambda_{\mathbf{p}}^-\beta\Lambda_{\mathbf{p}}^+=0,\ \Lambda_{\mathbf{p}}^+\beta\Lambda_{\mathbf{p}}^-=0,$$
$$\Lambda_{\mathbf{p}}^-\beta\Lambda_{\mathbf{p}}^-=-\Lambda_{\mathbf{p}}^-, \tag{13}$$

we get for the matrix $f^{(1)}(\mathbf{p}, k)$ an equation that describes the linear properties of a relativistic plasma,

$$f^{(1)}(\mathbf{p},k)=ief^{(0)}(\varepsilon_{\mathbf{p}+\mathbf{k}/2})\Lambda^+_{\mathbf{p}+\mathbf{k}/2}\hat{A}_k\left[\frac{\Lambda^+_{\mathbf{p}-\mathbf{k}/2}}{\omega-\varepsilon_{\mathbf{p}+\mathbf{k}/2}+\varepsilon_{\mathbf{p}-\mathbf{k}/2}}-\frac{\Lambda^-_{\mathbf{p}-\mathbf{k}/2}}{\omega-\varepsilon_{\mathbf{p}+\mathbf{k}/2}-\varepsilon_{\mathbf{p}-\mathbf{k}/2}}\right]\beta$$
$$+ief^{(0)}(\varepsilon_{\mathbf{p}-\mathbf{k}/2})\left[\frac{\Lambda^-_{\mathbf{p}+\mathbf{k}/2}}{\omega+\varepsilon_{\mathbf{p}+\mathbf{k}/2}+\varepsilon_{\mathbf{p}-\mathbf{k}/2}}-\frac{\Lambda^+_{\mathbf{p}+\mathbf{k}/2}}{\omega-\varepsilon_{\mathbf{p}+\mathbf{k}/2}+\varepsilon_{\mathbf{p}-\mathbf{k}/2}}\right]\hat{A}_k\Lambda^+_{\mathbf{p}-\mathbf{k}/2}\beta. \tag{14}$$

The linear collective oscillations of a relativistic quantal plasma were studied in detail in[6]. A number of effects in such a plasma were studied in[7]. Substituting (14) into the first term of the series (11) and evaluating the trace of the Dirac matrices by the standard rules[5] we get the current $j^{(1)}_{k,i}$. As expected, the expression obtained is the same as the one obtained by one of us[6] using the Green functions method, and

$$f^{(0)}(\varepsilon_{\mathbf{p}})=2n_{\mathbf{p}}/(2\pi)^3, \tag{15}$$

where $n_{\mathbf{p}}$ is the average number of electrons with energy $\varepsilon_{\mathbf{p}}$. Moreover, using the set (10) and the series (11) and transformations similar to the above-mentioned ones we get the non-linear currents $j^{(2)}_{k,i}$ and $j^{(3)}_{k,i}$:

$$j^{(2)}_{k,i}=\int S_{ijl}(k,k_1,k_2)E_{k_1j}E_{k_2l}\,d\lambda^{(2)},\qquad d\lambda^{(2)}=dk_1\,dk_2\delta(k-k_1-k_2),$$
$$j^{(3)}_{k,i}=\int \Sigma_{ijlm}(k,k_1,k_2,k_3)E_{k_1j}E_{k_2l}E_{k_3m}\,d\lambda^{(3)},\qquad d\lambda^{(3)}=dk_1\,dk_2\,dk_3\delta(k-k_1-k_2-k_3). \tag{16}$$

The expressions for the quantities S_{ijl} and Σ_{ijlm} in (16) are very cumbersome and we are not able to give them here.

3. We consider the Compton scattering and the electron-positron pair production by two transverse quanta in an isotropic plasma. It is well known[5] that the matrix elements for these processes can be obtained from one another. We turn first of all to the pair production process. Following[1] we obtain the non-linear equation describing this process:

$$\frac{\partial N_{\mathbf{k}}}{\partial t}=N_{\mathbf{k}}\int v_{\mathbf{k},\mathbf{k}_1}N_{\mathbf{k}_1}\,d\mathbf{k}_1,$$

$$v_{\mathbf{k},\mathbf{k}_1}=\frac{4}{\pi\omega}\operatorname{Re}\{\Sigma_{k,k,k_1,-k_1}+\Sigma_{k,k_1,k,-k_1}\}\frac{\omega^2}{\partial(\omega^2\varepsilon_t)/\partial\omega}\,\frac{\omega_1^2}{\partial(\omega_1^2\varepsilon_t)/\partial\omega_1}\bigg|_{\omega=\omega(\mathbf{k}),\ \omega_1=\omega_1(\mathbf{k}_1)}$$

$$\Sigma_{k,k_1,k_2,k_3}=\Sigma_{ijlm}(k,k_1,k_2,k_3)\,e^{*}_{i,k}e_{j,k_1}e_{l,k_2}e_{m,k_3}. \qquad (17)$$

Here $N_{\mathbf{k}}$ and $N_{\mathbf{k}_1}$ are the numbers of transverse quanta per unit wavevector space volume, $\mathbf{e}_k$ the polarization vectors, and ε^t the transverse permittivity of the plasma. As compared to[1] we have neglected in the quantity $v_{\mathbf{k},\mathbf{k}_1}$ the contribution from the current S which, as is shown by estimates, is possible under the conditions when the electron plasma frequency ω_{pe} is appreciably less than $|\mathbf{k}|$ or $|\mathbf{k}_1|$. To fix the ideas we shall further assume that the condition $\omega_{pe}\ll|\mathbf{k}_1|$ is satisfied.

We turn to finding the probability of the process considered. To do this we get from obvious balance considerations the appropriate kinetic equation. In the case considered of an electron plasma it has the form

$$\frac{\partial N_{\mathbf{k}}}{\partial t}=-N_{\mathbf{k}}\int w(\mathbf{k},\mathbf{k}_1,\mathbf{p}_1)(1-n_{\mathbf{p}_1})N_{\mathbf{k}_1}\frac{d\mathbf{k}_1\,d\mathbf{p}_1}{(2\pi)^6}, \qquad (18)$$

where $w(\mathbf{k},\ \mathbf{k}_1,\ \mathbf{p}_1)$ is the probability for the creation of a pair $\mathbf{p}_{el}=\mathbf{p}_1$ and $\mathbf{p}_{pos}=-\mathbf{p}_2=\mathbf{k}+\mathbf{k}_1-\mathbf{p}_1$ by quanta with momenta $\mathbf{k}$ and $\mathbf{k}_1$, respectively, while the 1 inside the round brackets takes into account the contribution from the electron–positron vacuum. Moreover, using according to (17) only the imaginary parts in the denominators:

$$\operatorname{Im}(\omega+\omega_1-\varepsilon_{\mathbf{p}_1}-\varepsilon_{\mathbf{p}_1-\mathbf{k}-\mathbf{k}_1})^{-1}=-\pi\delta(\omega+\omega_1-\varepsilon_{\mathbf{p}_1}-\varepsilon_{\mathbf{p}_1-\mathbf{k}-\mathbf{k}_1}), \qquad (19)$$

using the relations

$$e^{*}_{i,k}e_{j,k}={}^1/_2(\delta_{ij}-k_ik_j/\mathbf{k}^2),\qquad e^{*}_{l,k_1}e_{m,k_1}={}^1/_2(\delta_{lm}-k_{1l}k_{1m}/\mathbf{k}_1^2) \qquad (20)$$

to averaging over the polarizations of the quanta and evaluating the contraction of Σ_{ijlm} with the tensors (20), we find the quantity $v_{\mathbf{k},\mathbf{k}_1}$. After that we determine the probability $w(\mathbf{k},\mathbf{k}_1,\mathbf{p}_1)$ by comparing (17) and (18). Performing the calculations indicated we get

$$w(\mathbf{k},\mathbf{k}_1,\mathbf{p}_1)=\frac{4e^4}{m^2}\frac{\Phi(k,k_1,p_1)\,\delta(\omega+\omega_1-\varepsilon_{\mathbf{p}_1}-\varepsilon_{\mathbf{p}_2})(2\pi)^3}{\partial(\omega^2\varepsilon^t)/\partial\omega\ \partial(\omega_1^2\varepsilon^t)/\partial\omega_1}\bigg|_{\omega=\omega(\mathbf{k}),\ \omega_1=\omega_1(\mathbf{k}_1)}, \qquad (21)$$

$$\Phi(k,k_1,p_1)=\frac{m^2}{\varepsilon_{\mathbf{p}_1}\varepsilon_{\mathbf{p}_2}}\frac{1}{\varkappa_1^2\varkappa_2^2}\left[\Phi_0+\frac{k^2}{\mathbf{k}^2}\Phi_1+\frac{k_1^2}{\mathbf{k}_1^2}\Phi_2+\frac{k^2k_1^2}{\mathbf{k}^2\mathbf{k}_1^2}\Phi_{1.2}\right]. \qquad (22)$$

$$\Phi_0=-4(\varkappa_1+\varkappa_2)^2+4\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)+\frac{2(k^2+k_1^2)}{m^2}[\varkappa_1^2+\varkappa_2^2-\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)]$$
$$-\frac{k^2k_1^2}{m^4}(\varkappa_1^2+\varkappa_2^2)+2\varkappa_1\varkappa_2\frac{(k^2+k_1^2)^2}{m^4}+\varkappa_1\varkappa_2(\varkappa_1^2+\varkappa_2^2),$$

$$\Phi_1=-\frac{\varkappa_1\varkappa_2}{2}(\varkappa_1^2+\varkappa_2^2)-\frac{k^2}{m^2}(\varkappa_1+\varkappa_2)^2+\frac{k^2}{m^2}\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)-\varkappa_1\varkappa_2\frac{k^2(k^2+k_1^2)}{m^4}$$
$$+\frac{k^2k_1^2}{2m^4}(\varkappa_1^2+\varkappa_2^2)-\frac{4\varepsilon_{\mathbf{p}_1}\varepsilon_{\mathbf{p}_2}}{m^2}\left[(\varkappa_1+\varkappa_2)^2\left(1-\frac{k_1^2}{2m^2}\right)+\varkappa_1\varkappa_2\frac{k^2}{m^2}\right]$$
$$+\frac{4\varepsilon_{\mathbf{p}_2}\omega_1}{m^2}\left[\varkappa_2(\varkappa_1+\varkappa_2)-\frac{k_1^2}{2m^2}\varkappa_2^2+\frac{k^2}{2m^2}\varkappa_1\varkappa_2\right]-\frac{4\varepsilon_{\mathbf{p}_1}\omega_1}{m^2}\left[-\varkappa_1(\varkappa_1+\varkappa_2)\right.$$
$$\left.+\frac{k_1^2}{2m^2}\varkappa_1^2-\frac{k^2}{2m^2}\varkappa_1\varkappa_2\right]+\frac{2\varepsilon_{\mathbf{p}_1}\omega}{m^2}\left(\varkappa_1\varkappa_2^2-\frac{k_1^2}{m^2}\varkappa_1\varkappa_2\right)$$
$$-\frac{2\varepsilon_{\mathbf{p}_2}\omega}{m^2}\left(-\varkappa_1^2\varkappa_2+\frac{k_1^2}{m^2}\varkappa_1\varkappa_2\right)-4\varkappa_1\varkappa_2\frac{\omega_1^2}{m^2}.$$

The quantity Φ_2 is obtained from Φ_1 by the replacements $k\rightleftarrows k_1$ and $\varepsilon_{\mathbf{p}_1}\rightleftarrows\varepsilon_{\mathbf{p}_2}$ in all quantities bar $\varkappa_1$ and $\varkappa_2$. Moreover,

$$2\Phi_{1.2}=\frac{\varkappa_1\varkappa_2}{2}(\varkappa_1^2+\varkappa_2^2)-\varkappa_1^2\varkappa_2^2-\frac{k^2k_1^2}{2m^4}(\varkappa_1^2+\varkappa_2^2-2\varkappa_1\varkappa_2)$$
$$+\frac{2\varepsilon_{\mathbf{p}_1}\varepsilon_{\mathbf{p}_2}}{m^2}\left[4\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)-\frac{k^2+k_1^2}{m^2}(\varkappa_1+\varkappa_2)^2\right]+\frac{2\varepsilon_{\mathbf{p}_2}\omega_1}{m^2}$$
$$\times\left[-2\varkappa_1\varkappa_2^2+\frac{k_1^2}{m^2}\varkappa_2^2+\frac{k^2}{m^2}\varkappa_1\varkappa_2\right]-\frac{2\varepsilon_{\mathbf{p}_1}\omega}{m^2}\left[2\varkappa_1\varkappa_2^2-\frac{k^2}{m^2}\varkappa_2^2-\frac{k_1^2}{m^2}\varkappa_1\varkappa_2\right]$$
$$+\frac{2\varepsilon_{\mathbf{p}_2}\omega}{m^2}\left[-2\varkappa_1^2\varkappa_2+\frac{k^2}{m^2}\varkappa_1^2+\frac{k_1^2}{m^2}\varkappa_1\varkappa_2\right]$$
$$+\frac{2\varepsilon_{\mathbf{p}_1}\omega_1}{m^2}\left[-2\varkappa_2\varkappa_1^2+\frac{k_1^2}{m^2}\varkappa_1^2+\frac{k^2}{m^2}\varkappa_1\varkappa_2\right]+\frac{8\varepsilon_{\mathbf{p}_1}\varepsilon_{\mathbf{p}_2}}{m^4}[(\omega-\varepsilon_{\mathbf{p}_2})\varkappa_2+(\omega_1-\varepsilon_{\mathbf{p}_2})\varkappa_1]^2.$$

In the quantities Φ_0, Φ_1, Φ_2, $\Phi_{1,2}$ we have written $k^2=\mathbf{k}^2-\omega^2$, $k_1^2=\mathbf{k}_1^2-\omega_1^2$, while the invariants $\varkappa_1$ and $\varkappa_2$ have the form

$$\varkappa_1=\frac{2\varepsilon_{\mathbf{p}_1}}{m^2}\left[(\omega_1-\mathbf{k}_1\mathbf{V}_1)+\frac{k_1^2}{2\varepsilon_{\mathbf{p}_1}}\right],\qquad \varkappa_2=\frac{2\varepsilon_{\mathbf{p}_1}}{m^2}\left[(\omega-\mathbf{k}\mathbf{V}_1)+\frac{k^2}{2\varepsilon_{\mathbf{p}_1}}\right], \qquad (23)$$

where $\mathbf{V}_1=\mathbf{p}_1/\varepsilon_{\mathbf{p}_1}$ is the velocity of the electron. The quantities $\varkappa_1$ and $\varkappa_2$ can also be expressed in terms of the energy and the velocity of the positron. In the limit $k^2=k_1^2=0$ we get from (21) the well known expression for the pair creation by two photons.[5]

Using a similar approach we can obtain an expression for the probability for Compton scattering. However, we can obtain it more simply, obtaining at once the matrix element for that for pair creation putting in the latter $-p_2=p_1'$, $p_1=p_1'+k_1-k_2$, $k=k_1$, $k_1=k_2$ and changing the sign of the quantity Φ into the opposite one.

4. We consider the Compton scattering of transverse quanta of frequency $\omega_1\approx\omega_{pe}\ll m$ ($V_{ph\,1}=\omega_1/|\mathbf{k}_1|\gg 1$) by ultra-relativistic electrons ($\varepsilon_{\mathbf{p}_1}\gg m$) which may be the source of energetic x-ray- and γ quanta. In this process quantum effects begin to play a part when the energy of the secondary quantum becomes of the order of the initial electron energy $\omega\sim\varepsilon_{\mathbf{p}_1}$ ($\varepsilon_{\mathbf{p}_1}=\varepsilon$). Multiplying the probability for the Compton effect by the energy ω of the secondary quantum and the number of quanta $N_{\mathbf{k}_1}$ and averaging over the collision angle between $\mathbf{k}_1$ and $\mathbf{p}_1$ we get the spectral distribution of the emission:

$$P_\omega=\frac{\omega^3}{8\pi^2c^3\omega_{pc}}\bar{w}W^p, \qquad (24)$$

$$\bar{w}=\frac{2(2\pi)^3e^4c^4}{\omega_{pe}\omega^3\varepsilon}\left[4q^2\sigma(q-1)+2\frac{(\omega^2+q^2\sigma^2\varepsilon^2)}{\omega\varepsilon}\right]$$

which is the probability for the process averaged over angles,

$$\sigma=\frac{2\varepsilon\omega_1}{m^2},\quad q=\frac{\omega}{\sigma(\varepsilon-\omega)}=\frac{1}{(1+\sigma)\,\omega_{max}/\omega-\sigma}.$$

Here $\omega_{max}=\varepsilon(1+1/\sigma)^{-1}$ is the maximum emitted frequency, and W^p is the energy density of quanta with $|\mathbf{k}_1| \ll \omega_{pe}$. In the non-quantal region ($\sigma \ll 1$) Eq. (24) corresponds to a well known result.[1] By integrating we find the total emission by an electron in the ultra-quantal ($\sigma \gg 1$) case[3)]:

$$P=\int_0^{\omega_{max}} P_\omega\, d\omega=4\pi\left(\frac{e^2}{mc^2}\right)^2\frac{\varepsilon}{\hbar\omega_{pe}}cW^p. \tag{25}$$

The energy losses of a particle are in this case proportional to its energy ε and not to $\ln\varepsilon$, as is the case for the normal Compton effect occurring with the participation of photons. This difference is connected with the difference in the nature of the dispersion of the quanta $\omega_1 \approx \omega_{pe} \gg |\mathbf{k}_1|$ and of the photons. It is interesting to note that in this case the plasma appreciably amplifies the emission in contrast to the known inhibitive effects due to the influence of the refractive index.[8] The effect obtained can have important applications both for cosmic and for laboratory plasmas. An elucidation of its role for the Compton scattering by the relict radiation and also for processes occurring with the participation of quanta from laser radiation in a dense plasma is of interest. We note also that in the ultra-quantal case $P \propto \varepsilon \ln\varepsilon$ when electrons, scattered by longitudinal quanta with $|\mathbf{k}_1| \gg \omega_{pe}$, radiate.[9]

We compare the mechanism considered with other emission mechanisms in the conditions when (15) is applicable. Comparing (25) with the bremsstrahlung P_{br} by an electron[5]:

$$P_{br}=4\left(\frac{e^2}{mc^2}\right)^2\alpha Z^2 n_i c\varepsilon \ln\frac{2\varepsilon}{mc^2},$$

we find the condition for the dominance of the Compton process in the form

$$W^p \gg \pi^{-1}\alpha Z^2 n_i \hbar\omega_{pe}, \tag{26}$$

Z is the nuclear charge, n_i the concentration of nuclei, and α the fine-structure constant.

It is also of interest to compare (25) with the synchrotron losses for $\xi_0 \gg 1$ [10]:

$$P_c=2^{2/3}\Gamma(2/3)P^{(0)}_{syn}\xi_0^{-4/3}/9,$$
$$P_c^{(0)}=\frac{2}{3}\left(\frac{e^2}{mc^2}\right)^2 c\mathrm{H}^2\left(\frac{\varepsilon}{mc^2}\right)^2,\quad \xi_0=\frac{3}{2}\frac{|\mathbf{H}|}{H_0}\frac{\varepsilon}{mc^2}, \tag{27}$$

$H_0=mc^3/e\hbar$ is the Schwinger magnetic field. One should draw attention to the fact that in contrast to the classical case the Compton and synchrotron losses are nonequivalent ($P \propto \varepsilon$, $P_{syn} \propto \varepsilon^{2/3}$). Comparing (25) and (27) we find

$$P/P_{syn} \approx 9\pi W^p (mc^2)^2/\mathrm{H}^2\varepsilon\hbar\omega_{pe}\xi_0^{-4/3}. \tag{28}$$

We find further the spectral power of the radiation Q_ω of a system of ultra-relativistic electrons:

$$Q_\omega=\frac{\omega^3}{8\pi^2c^3\omega_{pe}}W^p\int_{\varepsilon'}^{\infty}\overline{w}f(\varepsilon)\,d\varepsilon. \tag{29}$$

Using here the decreasing part of the distribution function $f(\varepsilon)$ [11]:

$$f(\varepsilon)=\gamma(\gamma^2-1)n_*\varepsilon_*^{\gamma-1}\varepsilon^2/2(\varepsilon_*+\varepsilon)^{\gamma+2}, \tag{30}$$

we get in the case $\sigma \gg 1$ ($\varepsilon'=\hbar\omega+\frac{1}{2}(mc^2)^2/\hbar\omega_1$)

$$Q_\omega=2\pi\gamma(\gamma^2-1)\left(\frac{e^2}{mc^2}\right)^2\left(\frac{\varepsilon_*}{\hbar\omega}\right)^{\gamma-1}\frac{cn_*}{\omega_{pe}}W^p. \tag{31}$$

In the ultra-quantal case the spectral index of the radiation $\nu=\gamma-1$ (in contrast to the classical one, where $\nu=\frac{1}{2}(\gamma-1)$) in view of the fact that the maximum frequency of the radiation ω_{max} is proportional to the first power of the electron energy.

In conclusion we dwell briefly upon the process of the creation of an electron–positron pair by a transverse quantum $\omega_1 \approx \omega_{pe}$ and a hard γ quantum ω as a possible mechanism for the energy loss of γ quanta. It may be of interest in a number of astrophysical applications.[12] Using the pair production probability and a method similar to the one used for considering the Compton effect we get an expression for the energy loss of a γ quantum:

$$P_\gamma=8\pi\left(\frac{e^2}{mc^2}\right)^2 cW^p. \tag{32}$$

This formula is obtained under the conditions $\sigma'=(2\omega_1/\omega)(\varepsilon^2/(mc^2)^2) \gg 1$.

1) Everywhere in what follows $\hbar=c=1$; $\alpha, \beta, \gamma, \delta=1, 2, 3, 4$; $i, j, l, m=1, 2, 3$.

2) We shall drop the spinor indexes in what follows.

3) We shall use in what follows the normal system of units.

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