

# Mechanism of superconductivity stimulated by microwave radiation

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The mechanism of superconductivity stimulation in film bridges by microwave radiation is investigated experimentally. An increase in the critical current is observed in both short and long bridges. It is shown that the effect is observable only in a definite frequency range with an upper and a lower limit. The magnitude of the critical current in the presence of microwaves exceeds the theoretical value of the pair-breaking current in the absence of radiation. As the microwave power is increased, the stimulation of superconductivity begins to saturate as a result of the heating of the film by phonons emitted during the energy relaxation of the quasiparticles. Agreement is obtained between experiment and theory taking this heating into account. The obtained data indicate that the most probable mechanism responsible for the effect of stimulated superconductivity is the mechanism proposed by Éliashberh<sup>5</sup>.

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## INTRODUCTION

As is well known from experiments, under certain conditions the action of microwave radiation on a superconducting film bridge can lead to increases in its critical temperature and critical current, i. e., to the stimulation of the superconductivity.<sup>[1-4]</sup> So far, however, there have not been purposeful investigations pertaining to the elucidation of the actual mechanism underlying this phenomenon. In our opinion, the most convincing theory that could explain these effects is the microscopic theory developed by Éliashberg and co-workers.<sup>[5,6]</sup> The basic physical idea of this theory is that under the action of shf radiation the "center of gravity" of the quasi-particle distribution function shifts upward with respect to energy, which leads to an increase in the energy gap in the superconductivity. The present paper is devoted to the experimental investigation of stimulated superconductivity, with the object of elucidating the mechanism underlying this phenomenon. We have studied the dependence of the magnitude of this effect on temperature and the frequency and intensity of the incident radiation in both short (of length  $\sim 1 \mu$ ) and long (of length  $\geq 100 \mu$ ), thin (of width  $\sim 1 \mu$ ) film bridges. We have demonstrated that stimulated superconductivity is observable in both short and long bridges and that the critical current of a bridge under the influence of radiation can exceed the Ginzburg-Landau pair-breaking critical current. We have also established that the heat evolved in the film under the action of the radiation (as a result of the "heating up" of the quasi-particles) can exert considerable influence on the effect of stimulated superconductivity. The obtained experimental data are, on the whole, in good agreement with calculations, based on the Éliashberg theory, in which the heat generated in the film under the influence of microwave radiation is taken into account.

## EXPERIMENTAL PROCEDURE

We have measured the dependence of the critical current  $I_c$  and the shape of the current-voltage characteristic of bridges on the temperature of the helium bath

and on the intensity and frequency of the incident radiation. The measurements were performed on thin-film tin bridges on crystalline quartz or sapphire substrates. All the observed effects, including the rise in the critical current, did not depend on the choice of the substrate (crystalline quartz or sapphire). The narrow and short bridges with width and length less than a micron were prepared by a previously described method.<sup>[4]</sup> The narrow and long bridges (of width  $\sim 1 \mu$  and length  $\geq 100 \mu$ ) were prepared by deposition through a mask, which was a strip of colloidal film on the substrate with a transverse cut made with a razor blade. After the deposition of the bridge, the mask was removed with a solvent. The parameters of the investigated bridges are given in Table I. The width,  $w$ , and length,  $L$ , of the bridges were measured with an MBI-11 optical microscope to within  $0.3 \mu$  and, in some cases, with a JX-5A electron microanalyzer to within  $0.2 \mu$ . The film thickness  $d$  was determined from measurements of the film resistance for a square of the surface at room temperature,  $R_{\square 300}$ , and the residual resistance at 4.2 K,  $R_{\square 4.2}$ .<sup>[7]</sup> The resistance of a film was measured when the current through the bridge was equal to  $10 \mu\text{A}$ . The critical temperature  $T_c$  was determined by extrapolating to zero the temperature dependence of the critical current  $I_c$ . The mean free path,  $l$ , of the electrons in a film was determined from the formula  $l = l_{300}(R_{300}/R_{4.2} - 1)$ ,<sup>[7]</sup> where  $l_{300}$  is the mean free path in the bulk pure tin at

TABLE I. Bridge parameters.

Sample No.	$w, \mu$	$L, \mu$	$d, \text{Å}$	$T_c, \text{K}$	$R_{300}/R_{4.2}$	$R_{\square 4.2}, \Omega$	$l, \text{Å}$	$\xi_0/l$
1E	1.2	1	1000	3.83	—	—	—	—
2G	2.5	1.5	1000	3.82	—	—	—	—
5B	2	0.8	1000	3.83	—	—	—	—
4-2	1.6	470	500	3.77	13.9	0.18	1200	1.9
6-1*	2.1	720	290	3.93	7.0	0.67	550	4.2
8-1	1.5	240	710	3.78	12.5	0.14	1100	2.1
9-1	1.5	1420	980	3.78	14.1	0.09	1200	1.9
10-1	2.4	1220	900	3.79	15.6	0.09	1300	1.8
10-2	0.9	1360	900	3.79	15.6	0.09	1300	1.8

\*The bridge 6-1 was prepared by deposition onto a substrate cooled down to the temperature of liquid nitrogen.

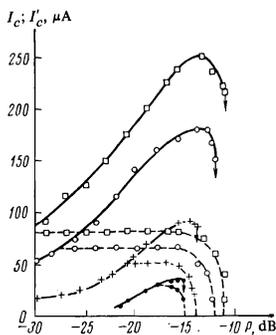


FIG. 1. Dependence of the critical currents  $I_c$  and  $I'_c$  of the bridge 1E on the power of 23.1 GHz incident radiation: ● 3.833; + 3.817; ○ 3.801; □ 3.784 K.

300 K, equal in our case to  $90 \text{ \AA}$  (the calculations were performed with the data of<sup>[8]</sup>).

The substrates with the bridges were placed across a 3-cm waveguide, so that the electric field was parallel to the transport current in the sample. The stabilization of the temperature of the helium bath was accomplished with the aid of a manostat and the temperature measurement was performed with a germanium thermometer. The temperature was measured and maintained to within 0.002 K. The critical current  $I_c$  was measured oscillographically, using the current-voltage characteristic (CVC) of the bridge. The rise in  $I_c$  was measured under the influence of radiation of frequencies from 8 to 78 GHz. Quantitative comparisons of the experimental data with the calculation were carried out mainly for a frequency  $\sim 10$  GHz, for which the incident-radiation power was known exactly. To determine the magnitude of the shf currents induced in a bridge, we measured with the aid of a measuring line the impedance of the bridge at the same frequency and its temperature dependence.

## EXPERIMENTAL RESULTS

In Fig. 1 we show the dependence of the critical current,  $I_c$ , of a bridge (continuous curves) on the power,  $P$ , of incident radiation of frequency 23.1 GHz for different temperatures. Let us note the main distinctive features of these curves.

1) The  $I_c(P)$  plots for different temperatures are similar to each other:  $I_c$  increases with increasing radiation power, attains a maximum, and, upon a further increase of the power by an amount  $\sim 0.1$  dB, the current drops abruptly to zero.

2) In the absence of radiation the critical current is equal to zero at temperatures slightly exceeding  $T_c$  ( $T - T_c \leq 0.01$  K; the curve for  $T = 3.833$  K). Under the influence of radiation, however, a critical current (the vertical section of the CVC for  $V = 0$ ) appears even at these temperatures, i. e., stimulation of the superconducting state by radiation is observed.

3) At temperatures below some value  $T_h$  (in our case  $T_c - T_h \sim 50$  mK) hysteresis appears in the CVC of the bridge, i. e., after the transition of a bridge into the normal state at some value of the critical current  $I_c$ , the inverse transition into the superconducting state is realized at a critical-current value  $I'_c$  less than  $I_c$ .<sup>[9,10]</sup> The

quantity  $I'_c$  (the dashed curves) always only decreases with increasing radiation power. It can also be seen from Fig. 1 that, under the influence of radiation, hysteresis also appears at temperatures above  $T_h$  (the curves corresponding to  $T = 3.833$ , 3.817, and 3.801 K), the hysteresis arising at lower radiation-power values as the temperature is lowered.

The investigations of the hysteresis in the CVC of superconducting film bridges show that its appearance is due to the processes of Joule-heat generation in the bridges.<sup>[9,10]</sup> The action of shf radiation leads to two main effects: to an increase in the critical current and, consequently, to increased Joule heating of the bridge for  $I > I_c$  (the bridge resistance virtually does not depend on  $T$  in a narrow temperature range) and to additional heating by the microwave field of the bridge after it had gone over into the normal state (on the resistive part of the CVC). Both these factors lead to a situation in which the hysteresis appears under the influence of radiation at higher temperatures and correspondingly lower critical-current values than in the absence of radiation. Such a conclusion is confirmed also by a direct calculation, performed in analogy to<sup>[9]</sup> but with allowance for the heating of the bridge by the shf radiation, of the temperature  $T_h$  at which the hysteresis appears. As a result of such a computation, we obtained for a short bridge with a linear dependence of  $I_c$  on temperature ( $I_c^0 = \alpha(T_c - T)$ ) the expression

$$\Delta T_h = T_c - T_h \approx \frac{1}{\alpha(1 + \gamma P)} \left( \frac{2FA\theta}{R_0} - kP \right)^{1/2}. \quad (1)$$

In deriving this formula we used the fact that, as shown by experiment for low powers, the critical current, under the influence of radiation, grows linearly with power:  $I_c^P = I_c^0(1 + \gamma P) = \alpha(1 + \gamma P)(T_c - T)$ . The remaining symbols have the following meanings:  $\theta$  is the width of the superconducting transition in a direct current,  $A$  is the area of the bridge neck,  $R_0$  is the bridge resistance,  $F$  is the coefficient of heat transfer, and  $k$  is the circuit factor, taking into account the fraction of power absorbed in the bridge. As can be seen from the formula (1), the hysteresis temperature  $T_h$  rises with increasing power  $P$ .

The absence of increase in  $I'_c$  and its decrease with increasing  $P$  are also well understood, since the quantity  $I'_c$  is determined by the heating of the bridge in the normal state, decreasing as the intensity of heating by the microwave field increases. Notice, incidentally, that a sharp decrease in  $I_c$  occurs at the same power value at which  $I'_c$  becomes equal to zero. At somewhat lower powers the CVC of the bridge is X-shaped.

The rise in the critical current  $I_c$  was also measured by us at different frequencies  $f$  in the range 8–78 GHz in the temperature range  $T_c - T \leq 0.8$  K. In Fig. 2 we show the dependence of the ratio of the quantity  $I_c(P)$  under the influence of radiation to  $I_c(0)$  without the influence of radiation on the power of 53.5-GHz radiation at different temperatures. At this frequency, in contrast to the case shown in Fig. 1 (frequency 23.1 GHz), there is no rise in  $I_c$  near  $T_c$ ; the rise appears only at some

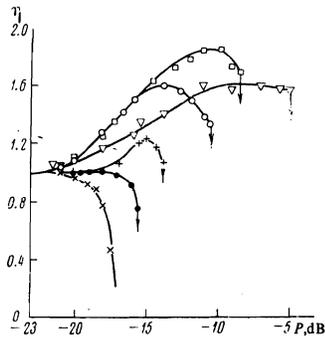


FIG. 2. Dependence of the normalized critical current  $\eta = I_c(P)/I_c(0)$  of the bridge 1E on the power of 53.5 GHz incident radiation:  $\times$ ) 3.816;  $\bullet$ ) 3.804;  $+$ ) 3.793;  $\square$ ) 3.767;  $\nabla$ ) 3.650 K.

temperature such that  $T_c - T \gtrsim 30$  mK, and does not disappear right down to the lowest temperatures at which we made measurements. It follows from this that there exists at the 53.5-GHz radiation frequency a limiting temperature,  $T_c - T \sim 30$  mK, characterized by the fact that a rise in  $I_c$  occurs only at lower temperatures. In the low frequency case, i. e., for  $f \lesssim 30$  GHz, the effect, lowered (for  $f = 23.1$  GHz this occurred at  $T_c - T < 0.8$  K). The measurements carried out in the 8–78 GHz range showed that each frequency has its own limiting points for the onset and disappearance of the growth effect.<sup>[4]</sup> Plotting all these limiting points for the onset and disappearance of the growth of  $I_c$  with  $T_c - T$  as the abscissa and  $f$  as the ordinate (Fig. 3), we obtain two curves that delineate the region of existence of the effect in bridges. The growth of  $I_c$  is observed in the band between these curves, and is not observed outside this band.

The existence of a lower temperature limit follows from the theoretical papers,<sup>[5–6]</sup> and the experimental paper<sup>[2]</sup> contains some data that point to that. Using them, we obtained some points in Fig. 3 that are in good agreement with our experimental data. The exact computation for the  $\Delta/h\omega > 1$  case leads to the following relation for the limiting frequency  $\omega_c$ <sup>[5]</sup>:

$$\omega_c^2 \ln \frac{8\Delta}{\omega_c} = 2\pi\tau_0^{-1}\Delta, \quad (2)$$

where  $\tau_0$  is the excitation relaxation time.

The obtained, according to (2), temperature dependence of  $\omega_c$  is depicted by the dashed curve in Fig. 3 for the more precise value  $\tau_0 = 1.4 \times 10^{-10}$  sec (see below). The agreement between the experimental data and the theoretical curve is sufficiently better. The magnitude of the excitation relaxation time,  $\tau_0 = 1.4 \times 10^{-10}$ , thus determined by us is, apparently, a "mean" quantity encompassing both the processes of quasi-particle relaxation in terms of energy and the processes of quasi-particle recombination into pairs. The value obtained by us is in good agreement with the value of  $\tau_0$  determined from other measurements.<sup>[8]</sup>

We have also observed experimentally the existence of an upper frequency limit for the effect of stimulated

superconductivity (Fig. 3, curve b). The absence of this effect at frequencies higher than 70 GHz is connected, apparently, with the fact that at these frequencies  $\hbar\omega$  becomes of the order of  $kT$ . As the estimates made by us on the basis of the theoretical paper<sup>[6]</sup> showed, the effect of stimulated superconductivity should vanish when  $\hbar\omega \approx kT$ . At lower frequencies the upper limit is located slightly above the energy-gap-versus-temperature curve (the dash-dot curve in Fig. 3). The parallel shapes of these curves suggest that the disappearance of the growth in this frequency region is due to the process, occurring when  $\hbar\omega \gtrsim 2\Delta$ , of quasi-particle number growth as a result of the breaking up of the pairs. One of the consequences of the increase in the number of quasiparticles is an increase in the absorption of the radiation, leading to the intensification of the thermal heating of the film and, in the final analysis, to a decrease in the energy gap. Below we shall consider in detail the influence of the thermal effects on the processes of superconductivity stimulation.

All the above-described results were obtained by us in narrow and short ( $\sim 1 \mu$ ) bridges, for which the existence of the effect of superconductivity stimulation has been related in a number of papers precisely with their small length.<sup>[3,11]</sup> In this connection, it was of interest to thoroughly investigate the influence of microwave radiation on the critical current  $I_c$  of narrow and long (above  $100 \mu$ ) bridges.

The question of the uniformity of long bridges is very important. The uniformity in width of the bridges was monitored by us with the aid of a JX-5A electron micro-analyzer to within  $0.2 \mu$ . The bridges on which we observed nonuniformities exceeding this maximum attainable—by us—resolution were rejected. Furthermore, as will be shown below, for the long, narrow bridges with sharp edges investigated by us, the experimental dependence of the critical current on temperature agrees (within the limits of experimental error) well with the theoretical dependence. This fact also attests the uniformity of these bridges. However, in order to completely clarify this question, it is apparently still necessary to perform additional experiments. In Fig. 4 we show the experimental dependences of the magnitude of the critical-current density  $j_c$  on the power of 9.4-

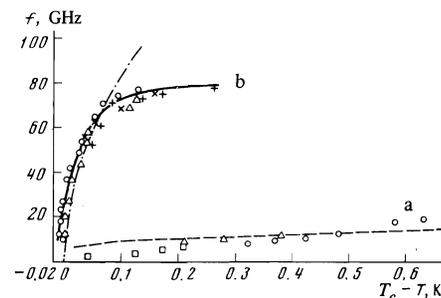


FIG. 3. The frequency limits of the existence domain for the effect of critical-current growth. The points on the curves correspond to the following bridges:  $\circ$ ) 1E,  $+$ ) 5B,  $\Delta$ ) 4-2,  $\times$ ) 8-1,  $\square$ ) according to the data of<sup>[2]</sup>.

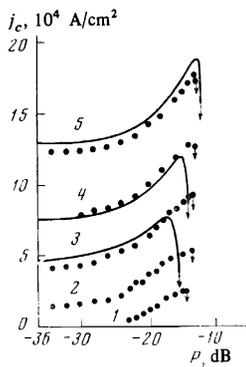


FIG. 4. Dependence of the critical current density  $j_c$  of the 10-1 sample on the power of 9.4 GHz incident radiation: 1) 3.796; 2) 3.778; 3) 3.759; 4) 3.747; 5) 3.728 K.

GHz incident radiation for the 10-1 bridge for different temperatures. It can quite well be seen that the critical-current growth is observable in a bridge of length 1.22 mm, these curves being qualitatively similar to the analogous curves for a short bridge (Fig. 1). Also observable in a long bridge are critical current induction at  $T > T_c$  (Fig. 4, curve 1), the growth of  $I_c$  at  $T < T_c$ , and an abrupt transition into the normal state ( $I_c = 0$ ) at some value of the incident-radiation power. Similar results have been obtained with other long bridges. In<sup>[12]</sup> there are also some indications of the existence in long bridges of the  $I_c$ -growth phenomenon. We also investigated the frequency dependence of the  $I_c$  growth in long bridges and obtained for the effect the limiting existence points shown in Fig. 3. It can be seen that these points lie quite close to the curves corresponding to the lower and upper frequency limits for the growth. It follows from these data that there exists in long bridges an effect of superconductivity stimulation by microwave radiation that is similar to the same effect in short bridges.

If the effects observed by us are indeed the effects of superconductivity stimulation, and not simply the suppression of some fluctuation processes,<sup>[3,11]</sup> then the magnitude of the critical current of a bridge under the influence of microwave radiation should not just exceed the experimental value of  $I_c$  in the absence of radiation, but should also exceed the theoretical value of the maximum Ginzburg-Landau pair-breaking critical current  $I_c^{GL}$ . For the purpose of verifying this supposition, we measured experimentally the temperature dependences of the  $I_c$  of a long bridge in the absence, and under the influence, of radiation and compared these dependences with the theoretical pair-breaking current versus temperature curve (Fig. 5). The computation of the critical current  $I_c^{GL}$  in the case of its uniform distribution over the bridge cross section was carried out on the basis of well-known formulas with allowance for the dependence of the penetration depth  $\lambda(T)$  and coherence length  $\xi(T)$  on the electron mean free path, i.e., on the degree of purity of the superconducting film. This dependence for the case of nonmagnetic impurities is determined by the Gor'kov function  $\chi = \chi(\xi_0/l)$ , where  $l$  is the mean free path.<sup>[13]</sup> On the basis of this we obtained

$$I_c^{GL} = \frac{c\Phi_0}{12\sqrt{3}\pi^2} \frac{wd}{\lambda_{\text{pure}}^2 \xi_{\text{pure}}} \chi^{1/2}, \quad (3)$$

where  $\Phi_0$  is the flux quantum,  $w$  is the film width,  $d$  is the film thickness,  $\lambda_{\text{pure}} = \lambda_L(0)/\sqrt{2(1 - T/T_c)^{1/2}}$ ,  $\xi_{\text{pure}} = 0.74 \xi_0(1 - T/T_c)^{-1/2}$  for  $\text{Sn } \lambda_L(0) = 355 \text{ \AA}$ , and  $\xi_0 = 2300 \text{ \AA}$ . The value of  $\chi$  was taken from<sup>[13]</sup>.

In electrical measurements the film thickness is usually determined by the following expression<sup>[7]</sup>:  $d = L\rho_{300}/w(R_{300} - R_{4.2})$ , where  $\rho_{300}$  is the resistivity of bulk tin samples at 300 K,  $R_{300}$  and  $R_{4.2}$  are the film resistances at 300 and 4.2 K respectively. Substituting this expression into (3), we obtain a relation into which the film width does not enter:

$$I_c^{GL} = \frac{c\Phi_0}{12\sqrt{3}\pi^2} \frac{\rho_{300}}{R_{300} - R_{4.2}} \frac{L\chi^{1/2}}{\lambda_{\text{pure}}^2 \xi_{\text{pure}}}. \quad (4)$$

The  $T$  dependence of  $I_c^{GL}$  computed from this formula is represented by the solid line in Fig. 5. The thin dashed lines on both sides of it indicate the error made in the computation of  $I_c^{GL}$  as a result of the errors made in the measurement of all the quantities entering into (4). The solid circles in Fig. 5 represent the experimentally obtained  $I_c$  values with indications of the errors made in their measurement.

It can be seen from the figure that for our case of a long, narrow bridge, the current in which can, to a good degree of accuracy, be assumed to be uniformly distributed at  $T \sim T_c$ , the experimental values of the critical current  $I_c$  in the absence of radiation agree well with the theoretical dependence of the unpairing current  $I_c^{GL}$  (similar results were obtained in<sup>[14]</sup>). On the other hand, as can be seen from Fig. 5, the magnitude of the critical current under the influence of radiation appreciably exceeds (by a factor of two at  $T_c - T \sim 0.03 \text{ K}$ ) the thermodynamic-equilibrium values of the Ginzburg-Landau pair-breaking critical current. At certain frequencies the excess is observable up to temperatures significantly below  $T_c$  ( $T_c - T \sim 1 \text{ K}$ ), where the influence of the fluctuations on the quantity  $I_c$  is negligible.

Not all the experimental facts observed by us can be explained in the framework of the theory developed by Éliashberg and his co-workers.<sup>[5,6]</sup> Thus, the theory predicted the possibility of the critical temperature of a film increasing by a factor of up to two. In the experiment, we obtained an increase in  $T_c$  of  $\sim 0.02 \text{ K}$ . Ac-

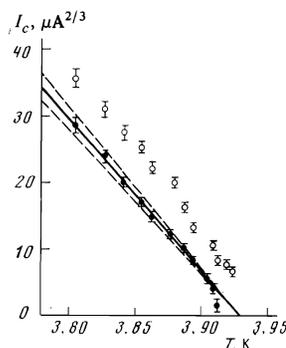


FIG. 5. Temperature dependence of the critical current  $I_c^{2/3}$  of the 6-1 sample. (●) in the absence of the influence of radiation, (○) under the influence of radiation of frequency 27 GHz. The continuous line is a theoretical line. The dashed lines indicate the computational errors in the critical-current values.

cording to the theory, the rise in the critical current should be much greater and should persist to higher powers of the incident microwaves. In experiment, as can be seen from Figs. 1 and 2, we observe saturation of the  $I_c$  growth, with a subsequent sharp decrease in  $I_c$  and a transition of the bridge into the normal state. As has already been noted, this sharp decrease in  $I_c$  occurs just at the moment when the "second" critical current  $I'_c$  (because of the hysteresis) becomes equal to zero. The Joule-heat evolution, which is responsible for the appearance of the hysteresis, occurs when  $V \neq 0$ .

The above-enumerated differences between theory and experiment are connected with the measurements of the critical current of a bridge in the superconducting state, i. e., when the direct-current voltage across it is equal to zero. However, there occurs in this case absorption of the microwaves by the quasiparticles, i. e., dissipation of energy, which will eventually proceed from the quasiparticle "gas" into the thermostat. We imagine the mechanism underlying this process as follows. The absorption of the microwave radiation by the quasiparticles leads to a change in the distribution of the quasiparticles over energy, to their unusual "warmup." Let us recall that the explanation of the effect of stimulated superconductivity is based precisely on this fact.<sup>[5,6]</sup> The "heated" quasiparticles give up their excess energy to the phonons, which leads, under certain conditions, to the heating of the lattice of the film, i. e., to a rise in the temperature of the film relative to that of the substrate.<sup>1)</sup> A quantitative analysis of these processes was carried out on the basis of the simple heat equation for a long bridge of length  $L$  and width  $w$ . Under our conditions—an Sn bridge on a crystalline-quartz substrate at  $T = 3-4$  K—the main channel for the outflow of heat from the bridge is the flow into the substrate (the coefficient of heat transfer of the Sn film-quartz interface is equal to  $F = 1.5$  W/cm<sup>2</sup>-deg and that of the film-liquid helium interface is not more than 0.1 W/cm<sup>2</sup>-deg<sup>[15]</sup>). Since the current is uniformly distributed over the bridge cross section and the conditions of generation and outflow of heat do not vary along the bridge length (with the exception of small regions of the order of the quasiparticle diffusion length  $l_d \ll L$  near the "shores" of the bridge), the temperature of the bridge is the same along its entire length. Under these assumptions, the bridge temperature  $T$  is, according to the heat equation, equal to

$$T = T_0 + \frac{I_\omega^2 R_\omega(T)}{2FwL}, \quad (5)$$

where  $T_0$  is the temperature of the substrate,  $I_\omega$  is the induced microwave current in the film,  $R_\omega(T) = \text{Re} Z(T)$  is the real part of the bridge impedance. Thus, as can be seen from (5), for a finite value of the coefficient of heat transfer from the bridge into the thermostat the bridge temperature will increase with increasing power of the microwave radiation. The total bridge impedance and its temperature dependence in the region 3.9–3.5 K were experimentally measured by us with the aid of a standard measuring line at a frequency of 9.4 GHz. The

results of these measurements are in good agreement with the data of<sup>[16]</sup>. Knowing the bridge impedance and the power,  $P$ , delivered by the generator, we could determine the transfer coefficient from the power, as well as the current  $I_\omega$  induced in the bridge.

In order that the above-presented experimental data on the  $I_c$  growth could be compared with the theoretical data, we expressed with the aid of simple transformations the potential  $A_0$  in Eq. (3) of<sup>[5]</sup> in terms of the direct-current density  $j$  and the potential  $A_\omega$  in terms of the microwave current density  $j_\omega$ . Then, in dimensionless variables, Eq. (3) of<sup>[5]</sup> for the case when  $\Delta/\hbar\omega > 1$  becomes transformed into the form

$$y = \left(\frac{3\pi}{2}\right)^{1/2} x^2 \left[ 1 - t - \frac{7\zeta(3)}{8\pi^2} x^2 - \frac{\pi}{12} \frac{y_\omega^2}{\Omega^2 |\sigma(t)|^2} \times \left( 1 - \frac{\tau}{2\pi} \frac{\Omega^2}{x^2} \ln \frac{8x}{\Omega} \right) \right]^{1/2}, \quad (6)$$

where the dimensionless direct-current density  $y = j/j_0$ ,  $j_0 = (\sigma_n^2 T_c^3 / l v e^2 \hbar)^{1/2}$ ,  $\sigma_n$  is the electrical conductivity at  $T \gg T_c$ ,  $v$  is the Fermi velocity,  $l$  is the electron mean free path, the Boltzmann constant  $k = 1$ ,  $y_\omega = j_\omega/j_0$ ,  $x = \Delta/T_c$ ,  $\Omega = \hbar\omega/T_c$ ,  $\tau = \tau_0/\hbar T_c$ , and  $\sigma(t)$  is normalized to  $\sigma_n$  and was determined from the measured—by us—quantity  $Z(t)$ . For our real case, in which  $\xi_0 \geq l$  (see the table), we introduced the appropriate correction into the normalization of  $y$  in Eq. (6), which has been written for the case when  $\xi_0 \gg l$  with allowance for the Gor'kov function, as was done in the formula (3). The critical current is found from the condition  $\partial y / \partial x = 0$ , while the film temperature  $t = T/T_c$  is determined by Eq. (5).

The system (5), (6) was solved with an electronic computer.<sup>2)</sup> As a result, we obtained the dependence of the critical-current density of a long bridge on the incident-radiation power at different temperatures. In Fig. 4 we show these dependences (continuous curves) for the 10–1 bridge for three temperatures of the helium bath. The only adjustable parameter used in the computation was the time,  $\tau_0$ , of relaxation of the excitations in terms of energy. The curves shown in Fig. 4 are for  $\tau_0 = 1.4 \times 10^{-10}$  sec, which differs little from the value  $2 \times 10^{-10}$  sec indicated in our previous paper<sup>[4]</sup>, and does not lead to an appreciable shift of the computed curve of the lower frequency limit (Fig. 3). As can be seen from Fig. 4, the computation, based on the Éliashberg equation and supplemented by the heat-balance equation, of the rise in  $I_c$ , gives new effects—the saturation of the  $I_c$  growth and the sharp decrease at some power, which agree on the whole with experiment. Thus, the energy evolved during the energy relaxation of the heated quasiparticle gas gives rise, under normal conditions (without any special measures taken to improve the heat transfer), to the heating of the film, and may be the natural limitation on the growth of the magnitude of the gap, the critical temperatures and current of the bridge.

## CONCLUSION

The experimental and theoretical data obtained by us show that, from our point of view, the most probable

mechanism responsible for the effect of superconductivity stimulation by microwave radiation is the mechanism proposed by Éliashberg.<sup>[5,6]</sup> Let us briefly enumerate the main results supporting such a conclusion. 1) The existence of the effect of critical-current growth not only in short, but also in long, bridges. 2) The existence of a lower frequency limit consistent with the Éliashberg theory. 3) The existence of the growth effect in a temperature region fairly far below  $T_c$ , where the influence of the fluctuations is insignificant. 4) The fact that the theoretical value of the Ginzburg–Landau pair-breaking critical current is exceeded by the values of the critical current under the influence of radiation. 5) The agreement between the experimental power dependences of  $I_c$  and the theoretical dependences obtained from the Éliashberg equation with allowance for the thermal effects.

The Éliashberg mechanism is also attested by the discovery of the growth of  $I_c$  in bridges and point contacts under the influence of phonons, and not photons.<sup>[17]</sup> In fact, the stimulation of superconductivity can occur as a result of the heating of the quasiparticles relative to the state of thermodynamic equilibrium either by microwave radiation, or by phonons. The factor limiting this effect may be the heating of the film lattice by the phonons emitted during the energy relaxation.

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<sup>1)</sup>As can be shown on the basis of<sup>[6]</sup>, the mean excess energy given away by the quasiparticles during relaxation is of the order  $\Delta$ . For  $T_c - T \approx 0.1$  K, i. e., in the temperature region where we compared experiment with theory,  $\Delta \sim kT$ , and the processes of relaxation of the excess energy are close to

being equilibrium processes. In this case we can use the heat-transfer coefficients measured under equilibrium conditions.<sup>[15]</sup>

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## Van der Waals forces in liquid crystals with a large dielectric anisotropy

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Methods are proposed for the calculation of Van der Waals forces in liquid crystals whose dielectric anisotropy is not small. An expansion in terms of the deviation of the director from the equilibrium orientation or of its derivative is employed. Examples are considered. It is shown that the Van der Waals forces can induce instability of the plane disclination. Biaxial effects, which are appreciable near the point of a phase transition to an isotropic phase, are taken into account in the intensity of the light scattering.

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1. In recent years, the general theory of Van der Waals forces in condensed systems<sup>[1]</sup> has been applied to various problems of the physics of liquid crystals.<sup>[2–4]</sup> This interest is due mainly to the fact that the magnitude

of the short-range forces, which assure the stability of the liquid crystal, is not large. Therefore, the long-range forces, which are usually not very significant, can lead to appreciable effects. In the work of Dzya-