Raman transition radiation in a medium excited by an electromagnetic field

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The radiation of a uniformly moving charged particle in a medium excited by an electromagnetic field is considered with allowance for the interaction between the electromagnetic wave and the optical-phonon wave. The frequencies near which the two-wave approximation must be used to determine the radiation field are obtained. It is shown that in the vicinity of these frequencies the radiation differs greatly from Cerenkov radiation.

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The radiation of a charged particle moving uniformly in an inhomogeneous nonstationary medium has been considered in a number of papers (see, e.g., [1,2]). The medium becomes nonstationary and inhomogeneous if a polarization wave is excited in it, say by an electromagnetic wave. The ensuing transition radiation brought about by the nonlinear interaction of the electromagnetic waves in the medium was considered by Ryazanov.[3] However, as noted by Ginzburg and Tsytovich,[2] other mechanisms that cause the medium to become nonlinear are possible. One of them is the interaction of an electromagnetic wave with an optical-phonon wave. The radiation of a charge uniformly moving in an excited medium, with account taken of such an interaction, can be naturally called Raman transition radiation. If the interaction is weak this radiation introduces only small corrections in the Vavilov-Cerenkov radiation. As we shall show, however, there are frequency regions in which the Cerenkov radiation is strongly distorted.

Let the medium be a cubic (but not ionic) crystal. In analogy with the procedure used by Bloembergen and Shen,[4] we derive the wave equations from the Lagrangian of the interaction of the electromagnetic waves and the optical vibrations:

$$L_{int}=\xi_{ijk}E_iE_jQ_k,\qquad \xi_{ijk}=N\left(\frac{\partial\alpha_{ij}}{\partial Q_k}\right)_{Q=0},\tag{1}$$

where N is the number of molecules per unit volume, α_{ij} is the molecule polarizability tensor, $\mathbf{Q}=\mathrm{R}(2\rho)^{1/2}$ is the normal coordinate, R is the relative displacement of the atoms of the molecule, and ρ is the reduced density. Let the total field in the medium be

$$\mathbf{E}^{\text{tot}}(\mathbf{r},t)=\mathbf{E}_0(\mathbf{r},t)+\mathbf{E}(\mathbf{r},t),$$

where $\mathbf{E}_0$ is an external exciting field in the form $\mathbf{E}_0=\mathbf{e}E_0\cos(\mathbf{k}_0\mathbf{r}-\omega_0t)$, and $\mathbf{E}(\mathbf{r},t)$ is the field of the particle in the medium. Since $E\ll E_0$ we can, using (1), write down a system of equations for the Fourier components of the electric field $\mathbf{E}(\mathbf{k},\omega)$ and of the normal coordinate $\mathbf{Q}(\mathbf{k},\omega)$ (u is the particle velocity)

$$\left(k^2\delta_{ij}-k_ik_j-\frac{\omega^2}{c^2}\varepsilon_0\delta_{ij}\right)E_j(\mathbf{k},\omega)=\frac{ie}{2\pi^2c^2}\omega u_i\delta(\omega-\mathbf{ku})$$
$$+2\pi\omega^2c^{-2}\xi_{ijk}e_jE_0\{Q_k(\mathbf{k}+\mathbf{k}_0,\omega+\omega_0)+Q_k(\mathbf{k}-\mathbf{k}_0,\omega-\omega_0)\},\tag{2}$$

$$(\omega_v^2-\omega^2-2i\omega\Gamma(k))Q_i(\mathbf{k},\omega)$$
$$=\xi_{kji}e_jE_0\{E_k(\mathbf{k}+\mathbf{k}_0,\omega+\omega_0)+E_k(\mathbf{k}-\mathbf{k}_0,\omega-\omega_0)\}.\tag{3}$$

Equation (3) contains the phenomenological damping constant $\Gamma=\Gamma(k)$. We are interested in the optical wavelength region; consequently only long-wave phonons take part in the process, and their frequency ω_v is pratically constant and independent of k; this enables us to write down the equation for $\mathbf{Q}$ in the form (3).

Expressing $\mathbf{Q}(\mathbf{k},\omega)$ in terms of $\mathbf{E}(\mathbf{k},\omega)$ with the aid of (3) and substituting the result in (2), we obtain an equation for $\mathbf{E}$:

$$\left(k^2\delta_{ij}-k_ik_j-\frac{\omega^2}{c^2}\varepsilon_0(\omega)\delta_{ij}\right)E_j(\mathbf{k},\omega)=\frac{ie}{2\pi^2c^2}\omega u_i\delta(\omega-\mathbf{ku})$$
$$+2\pi\frac{\omega^2}{c^2}E_0{}^2\xi_{ijk}e_j\xi_{lmk}e_m$$
$$\times\left\{\frac{E_l(\mathbf{k}+2\mathbf{k}_0,\omega+2\omega_0)+E_l(\mathbf{k},\omega)}{\omega_v^2-(\omega+\omega_0)^2-2i(\omega+\omega_0)\Gamma}+\frac{E_l(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)+E_l(\mathbf{k},\omega)}{\omega_v^2-(\omega-\omega_0)^2-2i(\omega-\omega_0)\Gamma}\right\}.\tag{4}$$

At small ξ the system (4) can be solved by successive approximations. We shall not write out here the results of using this method, since they are analogous to those presented by Ryazanov,[3] namely: radiation with absorption or emission of two quanta of the wave is possible and (as is immediately clear) the intensity of this radiation is smaller by a factor $(\xi^2\mathbf{E}_0^2/[\omega_v^2-(\omega\pm\omega_0)^2])^2$ than the intensity of the Cerenkov radiation. We note that the damping constant $\Gamma(k)$ turns out to be as a rule a large quantity, so that as $\omega_v^2-(\omega\pm\omega_0)^2\to 0$ the intensity of the radiation with absorption and emission of wave quanta increases, but not so much as to exceed the intensity of the Cerenkov radiation.

3. We consider now for $\mathbf{E}(\mathbf{k}-2\mathbf{k}_0,\ \omega-2\omega_0)$, an equation analogous to (4) ($\mathbf{E}(\mathbf{k}-2\mathbf{k}_0,\ \omega-2\omega_0)$ was chosen for the sake of argument; the entire reasoning that follows can be applied also to the quantity $\mathbf{E}(\mathbf{k}+2\mathbf{k}_0,\ \omega+2\omega_0)$):

$$\left(k'^2\delta_{ij}-k_i'k_j'-\frac{\omega'^2}{c^2}\varepsilon_0(\omega')\delta_{ij}\right)E_j(\mathbf{k}',\omega')$$
$$=\frac{ie}{2\pi^2c^2}\omega'u_i\delta(\omega'-\mathbf{k}'\mathbf{u})+2\pi\frac{\omega'^2}{c^2}E_0{}^2\xi_{ijk}e_j\xi_{lmk}e_m$$
$$\times\left\{\frac{E_l(\mathbf{k},\omega)+E_l(\mathbf{k}',\omega')}{\omega_v^2-(\omega'+\omega_0)^2-2i(\omega'+\omega_0)\Gamma}+\frac{E_l(\mathbf{k}',\omega')+E_l(\mathbf{k}'-2\mathbf{k}_0,\omega'-2\omega_0)}{\omega_v^2-(\omega'-\omega_0)^2-2i(\omega'-\omega_0)\Gamma}\right\}\tag{5}$$
$$\mathbf{k}'=\mathbf{k}-2\mathbf{k}_0,\qquad \omega'=\omega-2\omega_0.$$

To find the radiation field (the field at large distances from the charge) we use the formula

$$\int d^3p \frac{e^{i\mathbf{pR}}f(\mathbf{p})}{p^2-p_0^2-i0} = 2\pi^2 \frac{e^{ip_0R}}{R} f\left(p_0 \frac{\mathbf{R}}{R}\right), \tag{6}$$

which is valid when $p_0R \gg 1$. Since ξ is small, it is clear that the poles of $\mathbf{E}(\mathbf{k}, \omega)$ lie in the vicinity of $k^2 = \omega^2\varepsilon_0(\omega)/c^2$, while the poles of $\mathbf{E}(\mathbf{k}', \omega')$ lie in the vicinity of $k'^2 = \omega'^2\varepsilon_0(\omega')/c^2$. Assume now that the condition

$$k^2 - \frac{\omega^2}{c^2}\varepsilon_0(\omega) \approx k'^2 - \frac{\omega'^2}{c^2}\varepsilon_0(\omega') \tag{7}$$

is satisfied. Then $\mathbf{E}(\mathbf{k}, \omega)$ and $\mathbf{E}(\mathbf{k}', \omega')$ are simultaneously large in the vicinity of the pole. On the other hand, since (7) cannot be simultaneously satisfied with

$$k^2-\omega^2\varepsilon_0(\omega)/c^2 \approx (\mathbf{k}+2\mathbf{k}_0)^2-(\omega+2\omega_0)^2\varepsilon_0(\omega+2\omega_0)/c^2,$$

it follows that $\mathbf{E}(\mathbf{k}+2\mathbf{k}_0, \omega+2\omega_0)$ is small and we can retain in (4) and (5) only the terms containing $\mathbf{E}(\mathbf{k}, \omega)$ and $\mathbf{E}(\mathbf{k}', \omega')$, i.e., we can replace (4) and (5) by a system of equations for two coupled waves.

Let $\mathbf{k}_0 \parallel \mathbf{u}$. Then in the presence of δ functions in the right-hand sides of (4) and (5) the condition (7) determines two frequencies ω_1 and ω_2, in the vicinities of which we can use the system of equations for two coupled waves:

$$4k_0^2-4\frac{\omega_1}{u}k_0+\frac{\omega_1^2}{c^2}\varepsilon_0(\omega_1)-\frac{(\omega_1-2\omega_0)^2}{c^2}\varepsilon_0(\omega_1-2\omega_0)=0, \tag{8a}$$

$$4k_0^2+4\frac{(\omega_2-2\omega_0)}{u}k_0-\frac{\omega_2^2}{c^2}\varepsilon_0(\omega_2)+\frac{(\omega_2-2\omega_0)^2}{c^2}\varepsilon_0(\omega_2-2\omega_0)=0. \tag{8b}$$

The frequencies ω_1 and ω_2 are not independent. Invoking the frequencies ω_3 and ω_4, which arise when

$$k^2-\frac{\omega^2}{c^2}\varepsilon_0(\omega) \approx (\mathbf{k}+2\mathbf{k}_0)^2-\frac{(\omega+2\omega_0)^2}{c^2}\varepsilon_0(\omega+2\omega_0),$$

$$4k_0^2+4\frac{\omega_3}{u}k_0+\frac{\omega_3^2}{c^2}\varepsilon_0(\omega_3)-\frac{(\omega_3+2\omega_0)^2}{c^2}\varepsilon_0(\omega_3+2\omega_0)=0, \tag{9a}$$

$$4k_0^2-4\frac{(\omega_4+2\omega_0)}{u}k_0-\frac{\omega_4^2}{c^2}\varepsilon_0(\omega_4)+\frac{(\omega_4+2\omega_0)^2}{c^2}\varepsilon_0(\omega_4+2\omega_0)=0, \tag{9b}$$

and considering Eqs. (8a), (8b), (9a), and (9b) simultaneously, we readily obtain $\omega_2 = -(\omega_1 - 2\omega_0)$, $\omega_3 = -\omega_1$, and $\omega_4 = \omega_1 - 2\omega_0$.

In a cubic crystal we have $\xi_{xyz} = \xi_{xzy} = \xi_{yzx} = \xi$, $\xi_{ijk} = \xi_{jik}$, and the remaining components of ξ are equal to zero. Let $\mathbf{e} \parallel OX$. We write down for this case the system of equations for $\mathbf{E}(\mathbf{k}, \omega)$ and $\mathbf{E}(\mathbf{k}', \omega')$ in the vicinity of ω_1:

$$\left(k^2\delta_{ij}-k_ik_j-\frac{\omega^2}{c^2}\varepsilon(\omega)\delta_{ij}-\frac{\omega^2}{c^2}\varkappa(\omega)e_ie_j\right)E_j(\mathbf{k},\omega)$$
$$=\frac{ie}{2\pi^2c^2}\omega u_i\delta(\omega-\mathbf{ku})-\frac{\omega^2}{c^2}QE_i(\mathbf{k}',\omega')+\frac{\omega^2}{c^2}Qe_ie_jE_j(\mathbf{k}',\omega'), \tag{10}$$
$$\left(k'^2\delta_{ij}-k_i'k_j'-\frac{\omega'^2}{c^2}\varepsilon(\omega')\delta_{ij}-\frac{\omega'^2}{c^2}\varkappa(\omega')e_ie_j\right)E_j(\mathbf{k}',\omega')$$
$$=-\frac{\omega'^2}{c^2}QE_i(\mathbf{k},\omega)+\frac{\omega'^2}{c^2}Qe_ie_jE_j(\mathbf{k},\omega),$$
$$\varkappa(\omega)=2\pi E_0^2\xi^2[(\omega_v^2-(\omega+\omega_0)^2-2i(\omega+\omega_0)\Gamma)^{-1}$$
$$+(\omega_v^2-(\omega-\omega_0)^2-2i(\omega-\omega_0)\Gamma)^{-1}],$$
$$\varepsilon(\omega)=\varepsilon_0(\omega)-\varkappa(\omega),$$
$$Q=2\pi E_0^2\xi^2(\omega_v^2-(\omega-\omega_0)^2-2i(\omega-\omega_0)\Gamma)^{-1}.$$

After solving the system (10), we can obtain with the aid of (6) the radiation field, and hence the radiation intensity. The exciting field introduces anisotropy into the medium. Since $\mathbf{k}_0 \parallel \mathbf{u}$ and $\mathbf{e} \perp \mathbf{k}_0$, it follows that $\mathbf{u} \perp \mathbf{e}$, and in this case both ordinary and extraordinary waves are emitted. The corresponding expressions for the radiation intensity in the vicinity of ω_1 take the form

$$dI^i(\omega)=dI_0^i(\omega)(K^i)^{-2}\{\delta(\cos\theta-\cos\theta_1^i)[{}^1/_2(-p_1^i+p_2^i+a)+K^i]^2$$
$$+\delta(\cos\theta-\cos\theta_2^i)[{}^1/_2(-p_1^i+p_2^i+a)-K^i]^2\}. \tag{11}$$

The superscript i stands here for o or e, corresponding to the ordinary and extraordinary waves.

The quantities $dI_0^o(\omega)$ and $dI_0^e(\omega)$ far from the threshold of the ordinary Cerenkov radiation take the form

$$dI_0^o(\omega)=\frac{e^2\omega u}{2\pi c^2}d\omega\, d\Omega_n\frac{(u^2\varepsilon(\omega)/c^2-1)\sin^2\varphi}{u^2c^{-2}\varepsilon(\omega)\sin^2\varphi+\cos^2\varphi},$$
$$dI_0^e(\omega)=\frac{e^2\omega u}{2\pi c^2}d\omega\, d\Omega_n\frac{\cos^2\varphi(1-c^2/u^2\varepsilon(\omega))}{u^2c^{-2}\varepsilon(\omega)\sin^2\varphi+\cos^2\varphi},$$

where φ is the angle between OX and the projection of n on the plane perpendicular to $\mathbf{u}$. The angles $\theta_{1,2}^i$ determines the emission directions (θ is the angle between $\mathbf{n}$ and $\mathbf{u}$):

$$\theta_{1,2}^i=\frac{\omega}{u}\left(\frac{\omega^2}{c^2}\varepsilon_0(\omega)-\frac{p_1^i+p_2^i+a}{2}\pm K^i\right)^{-1/2}. \tag{12}$$

The quantities p_1^i, p_2^i, and K^i are functions of the field E_0:

$$p_1^o=\frac{\omega^2}{c^2}\varkappa(\omega),\quad p_2^o=\frac{\omega'^2}{c^2}\varkappa(\omega'),$$
$$K^o=\left[\frac{(p_1^o-p_2^o-a)^2}{4}+\frac{\omega^2\omega'^2Q^2}{c^4}\right]^{1/2},$$
$$p_1^e=-\frac{\varkappa(\omega)}{\varepsilon(\omega)}\left(\frac{\omega^2}{c^2}\varepsilon(\omega)-\frac{\omega^2}{u^2}\right)\cos^2\varphi,$$
$$p_2^e=-\frac{\varkappa(\omega')}{\varepsilon(\omega')}\left(\frac{\omega^2}{c^2}\varepsilon(\omega)-\frac{\omega^2}{u^2}\right)\cos^2\varphi,$$
$$K^e=\left[\frac{(p_1^e-p_2^e-a)^2}{4}+\frac{p_1^ep_2^eQ^2}{\varkappa(\omega)\varkappa(\omega')}\right]^{1/2}$$

The quantity a characterizes the deviation of the frequency ω from ω_1:

$$a=4k_0^2-4\frac{\omega}{u}k_0+\frac{\omega^2}{c^2}\varepsilon_0(\omega)-\frac{\omega'^2}{c^2}\varepsilon_0(\omega').$$

In the vicinity of the frequency $\omega_1 - 2\omega_0$, the following expressions hold for the radiation intensity:

$$dI^o(\omega')=dI_0^o(\omega)\omega\omega'^3Q^2(K^o)^{-2}$$
$$\times\{\delta(\cos\theta-\cos\bar\theta_1^o)+\delta(\cos\theta-\cos\bar\theta_2^o)\},$$
$$dI^e(\omega')=dI_0^e(\omega)\frac{\omega'Q^2\cos^4\varphi}{\omega\varepsilon(\omega)\varepsilon(\omega')}\left(\frac{\omega^2}{c^2}\varepsilon(\omega)-\frac{\omega^2}{u^2}\right)^2(K^e)^{-2} \tag{13}$$
$$\times\{\delta(\cos\theta-\cos\bar\theta_1^e)+\delta(\cos\theta-\cos\bar\theta_2^e)\},$$

where

$$\cos\bar\theta_{1,2}^i=\frac{\omega'+2\omega_0-2k_0u}{u}\left(\frac{\omega'^2}{c^2}\varepsilon_0(\omega')-\frac{p_1^i+p_2^i-a}{2}\pm K^i\right)^{-1/2}. \tag{14}$$

In relations (11)–(14) we have

$$\omega_v^2-(\omega\pm\omega_0)^2 \gg (\omega\pm\omega_0)\Gamma; \tag{15}$$

therefore the quantities Q and $\varkappa$ in (11)–(14) are assumed to be real. If the condition (15) is not satisfied, then the radiation field is strongly damped.

It is seen from (11) and (12) that if $a \lesssim \omega^2 Q$ and $a < \omega'^2 Q$, the radiation intensity at a frequency close to ω_1 differs strongly from the Cerenkov frequency. Furthermore, according to (13) and (14), the intensity of radiation with absorption of the wave quanta is comparable in this case with the intensity of the Cerenkov radiation. Thus, the magnitude of the field E_0 influences the width $\Delta\omega$ of the frequency region in which the raciation behaves in the manner described above. Using the definition of a, we readily obtain:

$$\frac{\Delta\omega}{\omega} \sim Q \sim \frac{\xi^2 E_0^2}{|\omega_v^2-(\omega-\omega_0)^2|}.$$

4. We indicate in conclusion conditions that favor the observation of this effect. The quantity

$$\xi^2/[\omega_v^2-(\omega-\omega_0)^2-2i(\omega-\omega_0)\Gamma]$$

is the nonlinear Raman susceptibility (see[5]). As shown by Bloembergen,[5] this susceptibility in the nonresonant region of frequencies is about one-tenth the susceptibility of third order due to pure electronic transitions in atoms. On the other hand, recognizing that usually $\omega_v/\Gamma \sim 10^3$, we can, retaining the condition (15), use the resonant character of the dependence of the Raman nonlinear susceptibility on the frequency. With the aid of the experimental data of[5] we can estimate the value of the Raman susceptibility in the region $\omega_v\Gamma \ll \omega_v^2 - (\omega - \omega_0)^2 \ll \omega_v^2$ in the following manner:

$$\xi^2/[\omega_v^2-(\omega-\omega_0)^2] \sim 10^{-12}\ [\mathrm{erg}^{-1}\ \mathrm{cm}^3],$$

which is much higher than the nonlinear third-order susceptibility ($\sim 10^{-13}\,\mathrm{erg}^{-1}\,\mathrm{cm}^3$).

The radiation considered above should arise in the vicinity of the frequency ω_1 (relation (8a)). It follows therefore that to observe this radiation it is necessary to satisfy simultaneously the two conditions:

$$\omega_v\Gamma \ll \omega_v^2-(\omega_1-\omega_0)^2 \ll \omega_v^2,$$

$$4k_0^2-4\frac{\omega_1}{u}k_0+\frac{\omega_1^2}{c^2}\varepsilon_0(\omega_1)-\frac{(\omega_1-2\omega_0)^2}{c^2}\varepsilon_0(\omega_1-2\omega_0)=0.$$

Eliminating from these expressions the frequency ω_1, we obtain the condition imposed on the frequency of the exciting field ω_0:

$$4k_0^2-4\frac{\omega_0+\omega_v}{u}k_0+\frac{(\omega_0+\omega_v)^2}{c^2}\varepsilon_0(\omega_0+\omega_v)-\frac{(\omega_v-\omega_0)^2}{c^2}\varepsilon_0(-\omega_0+\omega_v)\approx 0.$$

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Effect of a strong magnetic field on a laser plasma produced from a solid target in a gaseous atmosphere

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The effect of a magnetic field on the dynamics of the development of a laser plasma produced from a target at different pressures of the ambient gas has been investigated with the aid of a streak camera. The velocity of propagation of a laser-supported detonation wave in a magnetic field of intensity $H = 170$ kOe turned out to be higher than in the $H = 0$ case. The presence of a magnetic field leads to a more efficient generation of x rays by the laser plasma from the focal region. It has been found that a radially-confined, long-lived, hot plasma is formed in a magnetic field along the optical axis. The possibility in principle of separating with the aid of a strong magnetic field plasma formations with different parameters is demonstrated.

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Laser plasmas formed in strong magnetic fields have been investigated in a large number of papers (see, for example,[1–9]). What was mainly studied in these papers was, however, either the influence of a magnetic field on the production threshold and the dynamics of a laser spark, or the effect on the flare arising during the focusing of laser radiation onto a target surface. It is of interest to investigate the influence of a strong magnetic field on the laser plasma formed in the atmosphere of the gas surrounding the target. The performance of such an experiment can give additional information about the physical processes accompanying the dispersion of the plasma flare into the ambient gas.

A thorough investigation of the heating and dispersion of a plasma under the action of a high-power (10^{12} W/