Effect of impurities on electron pairing in one-dimensional metals

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The effect of electron scattering by impurities on the main logarithmic diagrams responsible for pairing phenomena in one-dimensional metals is considered. It is shown that a sufficiently high impurity concentration $\ln(2D/\omega_0)$ is replaced by $\ln(2D\tau)$ in the diagram corresponding to exciton pairing (D is the upper integration limit ($\sim\epsilon_F$ or ω_D) and τ is the collision time). Diagrams corresponding to Cooper pairing, however, are not affected by scattering by impurities.

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1. INTRODUCTION

It is known[1] that in a one-dimensional metal the formation of Cooper pairs is complicated by the process of dielectric electron–hole pairing. This leads to the need of summing diagrams of the "parquet" type instead of the "ladder" diagrams used in the three-dimensional case. In the case of the "parquet" situation, the calculation with better than logarithmic accuracy entails considerable difficulties, and even though in this approximation pairing is obtained, it is not clear the extent to which this result is reliable. It should be noted here that in the logarithmic approximation both types of pairing occur simultaneously, but in actuality this may not be the case, and moreover, the dielectric pairing may hinder the formation of Cooper pairs. This is precisely the situation with the experimental studies of quasi-one-dimensional compounds based on TCNQ. However, among compounds of this type there are also some in which, in view of the several equivalent positions for definite complexes, there is an "innate" disorder, and it is known that in these compounds the transition into a dielectric is either not observed at all or occurs at a very low temperature (see[2]).

The purpose of the present paper is to demonstrate how scattering of electrons by a random potential (in the form of impurities or incomplete order of the host substance) leads to elimination of the dielectric pairing without essential changes in the formation of Cooper pairs. We use a purely one-dimensional model. We note that the same question was investigated by Zavadovskiĭ[3] under the assumption of a slow quasi-classical random potential (in our case it corresponds to the field η, see below). It turned out that such a potential does not influence the pairing in any of the channels. The difference between our results and that of[3] is the consequence of the allowance for the "backward" scattering ($p_0 \to -p_0$).

2. COOPER PAIRING

The main element that determines the Cooper pairing is the sum of diagrams of the loop type, shown in Fig. 1. In the absence of electron scattering by impurities, there is only the diagram 1a, and it is proportional to $\ln(D/\omega_0)$, where D is the upper limit of the logarithmic integrals (on the order of the Fermi energy or the Debye frequency). This leads in final analysis to the appearance of an imaginary pole with respect to the variable ω_0 at the upper vertex and to the corresponding sign of the bare interaction, and in the latter situation this is evidence of instability of the Fermi spectrum and the need for restructuring the state (the appearance of superconductivity).

In the one-dimensional case, as already noted, the use of parquet diagrams complicates the situation. In any case, however, it can be stated that the absence of $\ln(D/\omega_0)$ means also the absence of Cooper pairing. If we assume electron scattering by impurities, then at $\omega_0 \gg \tau^{-1}$, where τ is the collision time, the scattering is obviously insignificant and the logarithm is present. At $\omega_0 \ll \tau^{-1}$, however, it is perfectly possible to have $\ln(D\tau)$ instead of $\ln(D/\omega_0)$. The pole in the total vertex with respect to the variable ω_0 then vanishes. If this is precisely the situation, this means that at a suitable impurity concentration (or at a suitable degree of disorder), the Cooper pairing is eliminated. On the other hand, if the loop yields $\ln(D/\omega_0)$ even at $\omega_0 \ll \tau^{-1}$, this means that the impurities do not influence the Cooper pairing.

We use the method of[4]. It follows from it that it is necessary to calculate the integral

$$\Pi_1=2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}dz_1[G_{11}(\omega_0-\omega,z,z_1)G_{22}(\omega,z,z_1)+G_{12}(\omega_0-\omega,z,z_1)G_{21}(\omega,z,z_1)]. \tag{1}$$

(We recall that the subscript 1 denotes the vicinity of p_0 and the subscript 2 denotes the vicinity of $-p_0$.) The integral (1) with respect to frequency can be broken up into integrals containing only retarded and advanced functions, namely

$$\int_{-\infty}^{\infty}G(\omega_0-\omega)G(\omega)d\omega=\int_{\omega_0}^{\infty}G_A(\omega_0-\omega)G_R(\omega)d\omega+\int_{0}^{\omega_0}G_R(\omega_0-\omega)G_R(\omega)d\omega+\int_{-\infty}^{0}G_R(\omega_0-\omega)G_A(\omega)d\omega. \tag{2}$$

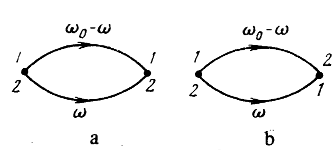

FIG. 1.

Since Π_1 is a diverging expression, and our formulas for the G functions are valid only for the vicinity of the Fermi limit, we add and subtract the corresponding expression for the free electrons. If the integration is performed in correct order, we should write in this case:

$$\Pi_{10}=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-D}^{D}\frac{dp_z}{2\pi}[\omega_0-\omega-\xi+i\delta\operatorname{sign}\xi]^{-1}[\omega-\xi+i\delta\operatorname{sign}\xi]^{-1}$$
$$=\frac{i}{\pi v}\ln\left(\frac{2D}{-i\omega_0}\right). \tag{3}$$

We now subtract an expression of the type (1), but for the free electrons. We introduce for the time being an upper cutoff limit with respect to the frequencies D_1. Using the representation of the integral in the form (2) and the expressions for the free Green's functions

$$\begin{aligned}G_{R11}(zz'\omega)&=-(i/v)\theta(z-z')\exp[i\omega(z-z')/v],\\ G_{R22}&=-(i/v)\theta(z'-z)\exp[-i\omega(z-z')/v],\\ G_{A11}&=(i/v)\theta(z'-z)\exp[i\omega(z-z')/v],\\ G_{A22}&=(i/v)\theta(z-z')\exp[-i\omega(z-z')/v],\end{aligned} \tag{4}$$

we obtain

$$\Pi_{10}'=\frac{i}{\pi v}\ln\frac{2D_1}{\omega_0}.$$

Thus, we must calculate expression (1), limiting the integration with respect to ω to the interval $(-D_1, D_1)$, and adding to it

$$\Pi_{10}-\Pi_{10}'=\frac{i}{\pi v}\ln\left(\frac{D}{-iD_1}\right). \tag{5}$$

Changing over to expression (1), we note first that the field η corresponding to forward scattering of the electrons exerts no influence on this expression. Indeed, if we change over to the interaction representation with respect to η (see[3]), then factors of the type

$$\exp\left(\pm 2i\int\eta(z_1)\frac{dz_1}{v}\right)$$

cancel out from the operators ζ and ζ^* upon averaging, and the external factors of the G functions, of the type

$$\exp\left(i\int\eta(z_1)dz_1\frac{\sigma_3}{v}\right)$$

cancel out in expression (1). We can therefore perform the calculations by putting $\eta=0$.

The expressions for the Green's functions G_R and G_A were obtained in[4]. Substituting them, we obtain

$$\Pi_1=\frac{1}{\pi v^2}\left\{\int_{\omega_0}^{D_1}d\omega\left[\int_{-\infty}^{z}dz_1\,S_{12}^{(1)}(\infty z)S_{21}(\infty z)[S_{11}^{(1)}(z_1-\infty)S_{22}(z_1-\infty)\right.\right.$$
$$+S_{21}^{(1)}(z_1-\infty)S_{12}(z_1-\infty)]+\int_z^{\infty}dz_1S_{11}^{(1)}(z-\infty)S_{22}(z-\infty)[S_{11}^{(1)}(\infty z_1)$$
$$\left.\times S_{22}(\infty z_1)+S_{12}^{(1)}(\infty z_1)S_{21}(\infty z_1)]\right][S_{11}^{(1)}(\infty-\infty)S_{22}(\infty-\infty)]^{-1}$$
$$+\int_0^{\omega_0}d\omega\left[\int_{-\infty}^{z}dz_1\,S_{22}^{(1)}(\infty z)S_{21}(\infty z)[S_{22}^{(1)}(z_1-\infty)S_{12}(z_1-\infty)\right.$$
$$+S_{12}^{(1)}(z_1-\infty)S_{22}(z_1-\infty)]+\int_z^{\infty}dz_1S_{12}^{(1)}(z-\infty)S_{22}(z-\infty)[S_{21}^{(1)}(\infty z_1)$$
$$\left.S_{22}(\infty z_1)+S_{22}^{(1)}(\infty z_1)S_{21}(\infty z_1)]\right][S_{22}^{(1)}(\infty-\infty)S_{22}(\infty-\infty)]^{-1}$$
$$+\int_{-D_1}^{0}d\omega\left[\int_{-\infty}^{z}dz_1\,S_{22}^{(1)}(\infty z)S_{11}(\infty z)[S_{22}^{(1)}(z_1-\infty)S_{11}(z_1-\infty)\right.$$
$$+S_{12}^{(1)}(z_1-\infty)S_{21}(z_1-\infty)]+\int_z^{\infty}dz_1\,S_{12}^{(1)}(z-\infty)S_{21}(z-\infty)[S_{21}^{(1)}(\infty z_1)S_{12}(\infty z_1)$$
$$\left.\left.+S_{22}^{(1)}(\infty z_1)S_{11}(\infty z_1)]\right][S_{22}^{(1)}(\infty-\infty)S_{11}(\infty-\infty)]^{-1}\right\}, \tag{6}$$

where $S^{(1)}$ denotes $S_{\omega_0-\omega}$, and S stands for S_ω.

We shall show that the term with the integral

$$\int_0^{\omega_0}d\omega$$

is equal to zero. This follows from the fact that it is impossible to cancel out the extreme left ζ^* in the second integral in the interval (∞z) at $z > z_1$ or in the interval (∞z_1). The first term in (6) depends only on the frequency $\Omega=2\omega-\omega_0$, and the limits of this variable are ω_0 and $2D_1$. Similarly, the third term of (6) depends likewise only on Ω, and in this case Ω changes from $-2D_1$ to $-\omega_0$. The interchange $\zeta\rightleftarrows\zeta^*$ in this term leads, as is well known,[4] to the substitutions $1\rightleftarrows 2$ and $\Omega\to-\Omega$. The limits of the integral with respect to the new frequency are now ω_0 and $2D_1$. Thus, the remaining expression can be written in the form

$$\Pi_1=\frac{1}{2\pi v^2}\int_{\omega_0}^{2D_1}d\Omega\left\{\int_{-\infty}^{z}[S_{22\Omega}(\infty z)S_{11}(\infty z)+S_{21\Omega}(\infty z)S_{12}(\infty z)]\right.$$
$$\times[S_{22\Omega}(z_1-\infty)S_{11}(z_1-\infty)+S_{12\Omega}(z_1-\infty)S_{21}(z_1-\infty)]$$
$$+\int_z^{\infty}dz_1[S_{22\Omega}(z-\infty)S_{11}(z-\infty)+S_{12\Omega}(z-\infty)S_{21}(z-\infty)]$$
$$\left.\times[S_{22\Omega}(\infty z_1)S_{11}(\infty z_1)+S_{21\Omega}(\infty z_1)S_{12}(\infty z_1)]\right\}[S_{22\Omega}(\infty-\infty)S_{11}(\infty-\infty)]^{-1}. \tag{7}$$

After the averaging, the expressions under the integral sign will obviously depend on $z-z_1$. If we introduce the variable $z-z_1$ in the first integral and z_1-z in the second integral, then obviously the two terms will coincide. Thus, it suffices to calculate one of these terms and double the results. Just as in[4], we expand in the last factor of (7) $S_{22}(\infty-\infty)$ in terms of

$$S_{21}(\infty z)S_{12}(z-\infty)/[S_{22}(\infty z)S_{22}(z-\infty)]$$

and proceed analogously with $S_{11}(\infty-\infty)$. We then obtain from the condition of the cancellation of ζ and ζ^*:

$$\Pi_1=\frac{1}{\pi v^2}\int_{\omega_0}^{2D_1}d\Omega\int_{-\infty}^{z}dz_1\sum_n[B_n(\infty z)+B_{n+1}(\infty z)]P_n(zz_1-\infty); \tag{8}$$

where

$$B_n(z_1z_2)=\left[\frac{S_{21\Omega}(z_1z_2)S_{12}(z_1z_2)}{S_{22\Omega}(z_1z_2)S_{11}(z_1z_2)}\right]^n,$$
$$P_n(z_1z_2z_3)=\frac{[S_{12\Omega}(z_1z_3)S_{21}(z_1z_3)]^n}{[S_{22\Omega}(z_1z_3)S_{11}(z_1z_3)]^{n+1}}[S_{22\Omega}(z_2z_3)S_{11}(z_2z_3)+S_{12\Omega}(z_2z_3)S_{21}(z_2z_3)]. \tag{9}$$

As $z_1\to z_2$ we have $P_n\to B_n'(z_2z_3)+B_{n+1}'(z_2z_3)$. The functions B_n and B_n' coincide with those introduced in[4] and, as shown there, they are equal to each other after averaging.

To calculate (8) we shall use a somewhat different method than in our preceding paper.[4] We note first that at $\Omega \gg \tau_2^{-1}$ the impurity scattering has no significance, i.e., in this region we obtain with logarithmic accuracy

$$\Pi_1^{(1)}=\frac{i}{\pi v}\ln(2D_1\tau_2). \tag{10}$$

We must ascertain whether the part of the integral from ω_0 to $\sim\tau_2^{-1}$ yields at $\omega_0\tau_2 \ll 1$ an analogous expression with $\ln(1/\omega_0\tau_2)$. Since the logarithmic integral is accumulated over the region $\omega_0 \ll \Omega \ll \tau_2^{-1}$, it suffices to calculate expression (8) at $\Omega\tau_2 \ll 1$. However (see[4]), in this case the large values $n \gg 1$ play the essential role in the sum over n. We can therefore replace the sum by an integral and, confining ourselves to the first term, put $B_{n+1}\approx B_n$.

The equation for B_n takes the form (see[3])

$$\frac{\partial B_n}{\partial t}=n^2(B_{n-1}+B_{n+1}-2B_n)+i\beta nB_n, \tag{11}$$

where $t=z/(v\tau_2)$ and $\beta=2\Omega\tau_2$. Just as in[4], we must find a solution that does not depend on t. Changing over for large n from the discrete variable n to the continuous variable, we obtain

$$n^2\frac{d^2B_n}{dn^2}+i\beta nB_n=0. \tag{12}$$

A solution of this equation, satisfying the boundary condition (see (9)) and not increasing as $n\to\infty$, is

$$B_n=uK_1(u), \tag{13}$$

where $u=2(-i\beta n)^{1/2}$. The function P_n satisfies the same equation as C_n in[4], and differs only in the boundary condition:

$$\partial P_n/\partial t=n^2P_{n-1}+(n+1)^2P_{n+1}-[2n(n+1)+1]P_n+i\beta(n+{}^1/_2)P_n. \tag{14}$$

Taking the Laplace transform:

$$P_n(t)=\int_0^\infty P_{ns}e^{-st}\,ds$$

and changing over to the continuous variable n, we obtain

$$n^2\frac{d^2P_{ns}}{dn^2}+2n\frac{dP_{ns}}{dn}+(i\beta n+s)P_{ns}=0. \tag{15}$$

This equation has a solution that does not increase as $n\to\infty$, in the form

$$P_\lambda(u)=u^{-1}K_{2i\lambda}(u); \tag{16}$$

where

$$s=\lambda^2+{}^1/_4,\qquad u=2(-i\beta n)^{1/2}. \tag{17}$$

The functions $P_\lambda(u)$ have the orthogonality property on the segment $u=0-\infty$:

$$\int_0^\infty P_\lambda(u)P_{\lambda'}(u)u\,du=\frac{\pi^2\delta(\lambda-\lambda')}{8\lambda\,\mathrm{sh}\,2\pi\lambda}. \tag{18}$$

The boundary condition for $P_n(t)$ at $t=0$ is $2B_n$ (13).

The orthogonality relation (18) makes it possible to write down the solution in the form[1)]

$$P_n(t)=16\pi^{-2}\int_0^\infty d\lambda\lambda\,\mathrm{sh}\,2\pi\lambda\exp[-({}^1/_4+\lambda^2)t]u^{-1}K_{2i\lambda}(u)\int_0^\infty u_1K_{2i\lambda}(u_1)K_1(u_1)du_1. \tag{19}$$

Substituting this expression in (8), we obtain

$$\Pi_1^{(2)}=\frac{8i}{\pi^3v}\int_{\omega_0}^{2D_1}\frac{d\Omega}{\Omega}\int_0^\infty d\lambda\int_0^\infty dt\lambda\,\mathrm{sh}\,2\pi\lambda\exp[-({}^1/_4+\lambda^2)t]\left[\int_0^\infty u\,duK_{2i\lambda}(u)K_1(u)\right]^2. \tag{20}$$

The integral with respect to u is equal to

$$\int_0^\infty u\,duK_{2i\lambda}(u)K_1(u)=\frac{\pi^2}{2}\left(\frac{1}{4}+\lambda^2\right)\mathrm{ch}^{-2}\,\pi\lambda.$$

Substituting this expression in (20), we obtain with logarithmic accuracy

$$\Pi_1^{(2)}=\frac{i}{\pi v}\ln\left(\frac{1}{\omega_0\tau_2}\right). \tag{21}$$

Combining expression (21) with $\Pi_1^{(1)}$ (10) and adding (5) we obtain[2)]

$$\Pi_1=\frac{i}{\pi v}\ln\left(\frac{2D}{-i\omega_0}\right). \tag{22}$$

It is seen therefore that even in the case when $1/\tau_2 \gg \omega_0$, the logarithm $\ln(2D/\omega_0)$ is retained, from which it follows that scattering by impurities does not prevent pairing of the Cooper type.

3. EXCITON PAIRING

The principal element of exciton pairing is the loop shown in Fig. 2. This loop corresponds to the integral

$$\Pi_2=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}dz_1G_{11}(\omega+\omega_0,z,z_1)G_{22}(\omega,z_1,z). \tag{23}$$

Again we add and subtract the corresponding expression for the free electrons. We then obtain

$$\Pi_{20}-\Pi_{20}'=-\frac{i}{2\pi v}\ln\left(\frac{D}{-iD_1}\right), \tag{24}$$

where we have formally introduced a finite interval for the frequency $(-D_1-D_1)$.

We break up the integral (23) into three parts

$$\Pi_2=\int_{-D_1}^{-\omega_0}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}dz_1G_{11A}^{(1)}(zz_1)G_{22A}(z_1z)+\int_{-\omega_0}^{0}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}dz_1G_{11R}^{(1)}(zz_1)G_{22A}(z_1z)$$
$$+\int_0^{D_1}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}dz_1G_{11R}^{(1)}(zz_1)G_{22R}(z_1z), \tag{25}$$

where $G^{(1)}=G_{\omega+\omega_0}$ and $G=G_\omega$. In this case the field η

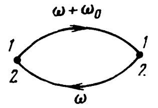

FIG. 2.

introduces into Π_2 phase factors of the type

$$\exp\left[-2i\int_{z_1}^{z}\eta(z_2)\frac{dz_2}{v}\right],$$

which do not cancel out. It must be borne in mind, however, that the element calculated by us is part of a parquet diagram and is joined by other Green's functions with their own phase multipliers. In view of the momentum conservation, the number of ends of type 1 and 2, which enter and leave a given point, is always such that the factors with η cancel out. Thus, as before, we can put $\eta=0$.

Substituting in (25) the expressions for G_R and G_A, we get

$$\begin{aligned}\Pi_2=-\frac{1}{2\pi v^2}\Bigg\{&\int_{-D_1}^{-\omega_0}d\omega\Bigg[\int_{-\infty}^{z}dz_1S_{12}^{(1)}(\infty z)S_{12}(\infty z)S_{21}^{(1)}(z_1-\infty)S_{21}(z_1-\infty)\\&+\int_{z}^{\infty}dz_1S_{11}^{(1)}(\infty z_1)S_{11}(\infty z_1)S_{11}^{(1)}(z-\infty)S_{11}(z-\infty)\Bigg][S_{11}^{(1)}(\infty-\infty)S_{11}(\infty-\infty)]^{-1}\\&+\int_{-\omega_0}^{0}d\omega\Bigg[\int_{-\infty}^{z}dz_1S_{22}^{(1)}(\infty z)S_{12}(\infty z)S_{22}^{(1)}(z_1-\infty)S_{21}(z_1-\infty)\\&+\int_{z}^{\infty}dz_1S_{21}^{(1)}(\infty z_1)S_{11}(\infty z_1)S_{12}^{(1)}(z-\infty)S_{11}(z-\infty)\Bigg][S_{22}^{(1)}(\infty-\infty)S_{11}(\infty-\infty)]^{-1}\\&+\int_{0}^{D_1}d\omega\Bigg[\int_{-\infty}^{z}dz_1S_{22}^{(1)}(\infty z)S_{22}(\infty z)S_{22}^{(1)}(z_1-\infty)S_{22}(z_1-\infty)\\&+\int_{z}^{\infty}dz_1S_{21}^{(1)}(\infty z_1)S_{21}(\infty z_1)S_{12}^{(1)}(z-\infty)S_{12}(z-\infty)\Bigg][S_{22}^{(1)}(\infty-\infty)S_{22}(\infty-\infty)]^{-1}.\end{aligned}\qquad(26)$$

Although in this case the integral

$$\int_{-\omega_0}^{0}d\omega$$

is not equal to zero, it is not logarithmic. It can be shown that this integral yields only a real constant under the logarithm sigh. As to the first and third integrals, it can easily be seen that in these integrals it is necessary to take into consideration from each of the S only the factor

$$\prod_i(1+|\zeta_i|^2/2)$$

(see[3]), for if we take one of the terms of the type $\zeta_i^*\zeta_l\zeta_m^*\ldots$, $i\neq l\neq m\ldots$, then in the first integral the extreme left ζ will have nothing to be paired with (and the same holds true for ζ^* in the third integral). As a result we obtain

$$\Pi_2=-\frac{1}{2\pi v^2}\left\{\int_{-D_1}^{-\omega_0}d\omega\int_{z}^{\infty}dz_1\exp\left[\frac{i(2\omega+\omega_0)(z-z_1)}{v}-\frac{z_1-z}{\tau_2 v}\right]\right.$$
$$\left.+\int_{0}^{D_1}d\omega\int_{-\infty}^{z}dz_1\exp\left[\frac{i(2\omega+\omega_0)(z-z_1)}{v}-\frac{z-z_1}{\tau_2 v}\right]\right\}=-\frac{i}{2\pi v}\ln\frac{2D_1}{\omega_0+i\tau_2^{-1}}\qquad(27)$$

Combining (27) with (24), we get

$$\Pi_2=-\frac{i}{2\pi v}\ln\frac{2D}{\tau_2^{-1}-i\omega_0}.\qquad(28)$$

It is seen from this answer than at a sufficient impurity concentration, when $\tau_2^{-1}\gg\omega_0$, we have $\Pi_2\propto\ln 2D\tau_2$. In this case we are left only with $\ln(2D/\omega_0)$ from the Cooper loop, i.e., the parquet is replaced by a ladder. It is seen therefore that in either case the impurities enhance the tendency to superconductivity (at the appropriate sign of the interaction).

We note in conclusion that the present result was obtained under the assumption that the averaging over the impurities can be carried out in each loop independently. This is correct for the three-dimensional case, but not so obvious for the one-dimensional case, so that an additional analysis is necessary. We note furthermore that substantial differences can occur in real quasi-one-dimensional systems in which the electron transitions from filament to filament have a finite probability. For example, as shown by Larkin and Mel'nikov, [5] even the quasi-classical field η suppresses in this case pairing in both channels. Nonetheless, the purely one-dimensional effects considered in the present paper is apparently the principal one, i.e., if the forward scattering described by the field η suppresses to some degree both channels, then the backward scattering, not taken into account by Larkin and Mel'nikov, [5] definitely acts more strongly in the dielectric channel, and consequently the introduction of impurities can still contribute to superconductivity.

[1)]The function B_n (13) depends on the same variable u. Since the integral in (18) is taken along the real u axis, we shall consider for the time being the imaginary frequency $\beta=i\beta'$, and then make in the final result an analytic continuation to real frequencies.

[2)]One may question the validity of writing $-i$ in the logarithm if we confine ourselves to logarithmic accuracy. This validity follows from the fact that Eqs. (11) and (14) must yield real solutions if $i\beta$ is real.

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