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Induced Compton scattering by relativistic electrons

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The behavior of an electron moving with arbitrary velocity in a given random field of intense, low-frequency radiation is considered under conditions when the dominant electron-radiation interaction mechanism is induced Compton scattering. The evolution of the electron energy spectrum is investigated in the diffusion approximation, and the equilibrium spectrum of relativistic electrons in the field of radiation with a high brightness temperature is found. The induced light-pressure force acting on a moving electron and the rate of induced heating of an electron gas in an isotropic radiation field are calculated. It is shown that, in contrast to the well-known spontaneous retarding force, the direction of the induced force depends on the radiation spectrum. Radiation spectra for which the induced force accelerates an electron in a given direction right up to ultrarelativistic energies are found.

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INTRODUCTION

The process of induced Compton scattering of electromagnetic radiation by free electrons (see the review article[1]) can play an important role in astrophysics in the interaction of high-power radio emissions of pulsars, quasars, and other objects with the surrounding tenuous plasma,[2–4] as well as under laboratory conditions in the investigation of plasma heating by radiation from lasers, masers, and superhigh-frequency devices.[5–7] It is well-known that this process[8] leads to electron heating,[5,9] the appearance of an induced light-pressure force,[10,11] a change in the radiation spectrum[12] and, in particular, to the appearance in the continuous radiation spectrum of spectrally narrow components and solitons,[13] to the divergence or convergence of the radiation beam,[14] etc.

In the present paper we consider the question of the behavior of electrons in a given radiation field, i.e., we shall be interested in: 1) the distribution of the electrons over energy in a situation in which the plasma is sufficiently rarefied and the major role in the formation of the electron distribution over energy is played by scattering processes leading to the diffusion of the electrons in momentum space; 2) the induced light pressure acting on a free electron; 3) the heating of relativistic electrons during induced scattering of light.

For all the quantities characterizing the behavior of electrons in a statistical isotropic radiation field, we have obtained exact relativistic formulas that are valid for arbitrary electron energies and arbitrary radiation spectra. The obtained formulas are fairly simple and convenient for the computation of induced effects, and allow the investigation of both nonrelativistic and ultrarelativistic asymptotic behaviors.

To the question of the induced interaction of radiation with a relativistic plasma have been devoted a large number of papers.[15–17] The dependence, obtained in these papers, of the rate of heating of monoenergetic ultrarelativistic electrons on their energy is valid only for radiation spectra of a definite form. However, it is precisely for these spectra that the pattern of plasma heating is qualitatively different from the heating of monoenergetic electrons. This is connected with the presence in momentum space, as a result of induced scattering, of electron diffusion,[10] which, in these spectra, washes out the monoenergetic distribution much more rapidly than heats it.[18] At the same time, the pattern of electron heating in a wide class of spectra differs from the results obtained by Ochelkov and Charugin,[16] Dedkov,[17] and Blandford and Scharlemann.[18] In our paper we analyze the induced effects for arbitrary radiation spectra.

In the case of a high brightness temperature of the radiation field, i.e., for $kT_b \gg mc^2$, allowance for the induced heating of the electrons and their cooling in spontaneous scattering leads to the establishment of the equilibrium[1)] distribution over energy of the relativistic electrons

$$\frac{dN_e}{d\varepsilon} \sim \varepsilon^2 \exp(-A\varepsilon^n), \qquad 4 \leq n \leq 5$$

as a function of the radiation spectrum. The mean electron energy in this distribution

$$\langle\varepsilon\rangle \approx mc^2 (kT_b/mc^2)^{1/n}.$$

Besides the systematic increase of the energy of an electron during induced light scattering, there also occurs a systematic change in the electron momentum, i.e., on the electron acts an induced pressure. In an isotropic radiation field this force may, depending on the form of the radiation spectrum, be directed either opposite the direction of the electron velocity, or along the velocity. This force is computed in the rest frame of the electron in Levich's paper,[11] and it is shown in the review article by Zel'dovich[1] that in this frame the force is always directed along the direction of the radiation flux. This result has been known for a long time for spontaneous scattering; it is also valid for induced scattering. However, the presence of induced heating in the electron rest frame leads to the result that in the laboratory system, in which the radiation is isotropic, the induced force significantly differs from the force found in Levich's paper.[11] The presence of heating leads in certain radiation spectra to a change in the direction of the induced force.

The effects which we are considering, and which arise in induced scattering of light by electrons, are classical, i.e., the Planck constant h does not enter into the final result. However, the computation of these effects in the quantum language, in which the electromagnetic radiation is regarded as a photon gas, is significantly more convenient[19] than the classical calculation.

We shall consider the electromagnetic field to be a random field and characterize it by the occupation number, $N(\nu, \mathbf{n})$, for photons of frequency ν and direction, $\mathbf{n}$, of propagation in phase space. Then the probability of scattering into this photon state from any other state is proportional to $1 + N(\nu, \mathbf{n})$. The number 1 corresponds to spontaneous scattering, while $N(\nu, \mathbf{n})$ corresponds to induced scattering. The change in the frequency of a photon upon being scattered by an electron moving with arbitrary $\mathbf{v}$ is equal to[20]

$$\frac{\nu'}{\nu} = \left(1 - \frac{\mathbf{v}}{c}\mathbf{n}\right) \Big/ \left[1 - \frac{\mathbf{v}}{c}\mathbf{n}' + \frac{h\nu}{mc^2\gamma}(1 - \mathbf{n}\mathbf{n}')\right] \tag{1}$$

where ν, ν' and $\mathbf{n}$, $\mathbf{n}'$ are the frequency and direction of propagation of the photon before and after the scattering, $\gamma = (1 - v^2/c^2)^{-1/2}$, c is the velocity of light, and m is the electron mass. When the Thomson approximation is valid,[2)] the recoil-related last term in the denominator of (1) is a small—proportional to h—correction to the Doppler effect. Thus, in low-frequency-photon scattering by electrons the frequency change is primarily connected with the Doppler effect. However, the effects of the induced scattering owe their existence to this small, recoil-related correction.

Let us go over to the electron rest frame and show that, without allowance for recoil during induced scattering, there is no exchange of energy or momentum between the electron and the radiation. This can easily be seen from the following example. Let us place an electron in crossed radiation beams. Induced scattering of photons from the first beam is possible only in the direction of the second beam, and the number of photons from the first beam scattered into the second in unit time is proportional to N_1N_2. Photons from the second beam are scattered at the same rate (proportional to N_1N_2) into the first, the frequency remaining unchanged in the scattering when the recoil is neglected. Thus, the radiation field does not change in induced scattering, and the electron gains neither momentum nor energy, i.e., both induced heating and induced light pressure are absent. In the inverse transition to the laboratory frame (in which the electron moves with velocity $\mathbf{v}$) the rate of induced heating and the induced force are found according to the Lorentz transformation $Q = W + \mathbf{L}\cdot\mathbf{v}$, $\mathbf{f} = \mathbf{L} + \mathbf{v}W/c^2$, where Q and $\mathbf{f}$ (W and $\mathbf{L}$) are the rates of change of energy and momentum in the laboratory system (in the rest frame). It is clear that, if the heating and the force are equal to zero in the rest frame, then they will be equal to zero in any other inertial coordinate system. Thus, the pure Doppler effect does not contribute to induced electron heating, and only allowance for the recoil effect leads to the indicated effects.

1. ELECTRON DIFFUSION IN MOMENTUM SPACE

Let us consider a system of electrons located in a given field of radiation with a high brightness temperature, when the effects of induced scattering are important. An electron, in scattering light, changes its momentum both systematically and randomly. If, as a result of each scattering event, the momentum of the electron changes by a relatively small amount, then we can use for the determination of the electron distribution function in momentum space, φ, the Fokker–Planck equation, according to which

$$\frac{\partial\varphi}{\partial t} + v_i\frac{\partial\varphi}{\partial x_i} = \frac{\partial}{\partial p_i}\left[\frac{\partial}{\partial p_k}D_{ik}\varphi - f_i\varphi\right], \tag{2}$$

where f_i, the mean (induced and spontaneous) force exerted on the electron by the radiation field, and D_{ik}, the electron-diffusion tensor defined in momentum space for this radiation field, are determined by the formulas

$$\begin{pmatrix} f_i \\ D_{ik} \end{pmatrix} = \int \begin{pmatrix} \Delta p_i \\ {}^1/_2\Delta p_i \Delta p_k \end{pmatrix} N(\nu, \mathbf{n}) \times [1 + N(\nu', \mathbf{n}')]\, c\sigma \frac{2\nu^2}{c^3} d\nu\, d\mathbf{n}'\, d\mathbf{n}, \tag{3}$$

where

$$\Delta p_i = hc^{-1}(\nu n_i - \nu' n_i') \tag{4}$$

is the momentum transfer to the electron as a result of

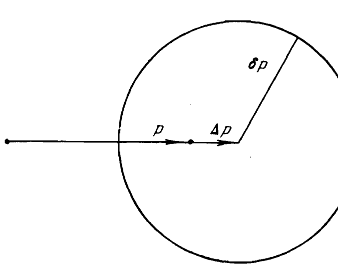

FIG. 1. The systematic, $\Delta\mathbf{p}=\mathbf{f}_{ind}\Delta t$, and random, $\delta p=\overline{[(\Delta p)]}^{1/2}=(6D\,\Delta t)^{1/2}$, changes, connected respectively with the induced force and diffusion, in the initial momentum p of a group of electrons over a short interval of time Δt. The induced force leads to a relatively small value of Δp as compared to the rondom value δp. The figure depicts the situation in the case of an accelerating induced force and nonrelativistic electrons. In this case Δp is proportional to, while δp does not depend on, p.

one scattering act, while the Thomson scattering cross section

$$\sigma=\left(\frac{e^2}{mc^2}\right)^2\frac{1-\beta\mathbf{n}}{2\gamma^2(1-\beta\mathbf{n}')^2}\times\left[1+\left(1-\frac{1-\mathbf{n}\mathbf{n}'}{\gamma^2(1-\beta\mathbf{n})(1-\beta\mathbf{n}')}\right)^2\right]. \tag{5}$$

In the formula (3) the number 1 in the square brackets corresponds to spontaneous scattering, while $N(\nu',\mathbf{n}')$ corresponds to induced scattering. The quantity $\Delta p_i\Delta p_k \sim h^2$; therefore, spontaneous diffusion is a quantum quantity, while induced diffusion is classical, [10] which can be seen if the coefficient of diffusion is written in terms of the classical quantity—the spectral intensity F (in place of N):

$$F(\nu,\mathbf{n})=\frac{2h\nu^3N(\nu,\mathbf{n})}{c^2},\quad [F]=\mathrm{erg/cm^2\cdot sec\cdot sr\cdot Hz}. \tag{6}$$

In such notation the constant h vanishes from the expression for the induced D_{ik}.

The light pressure—both spontaneous and induced—acting on the electron is a classical quantity. The computation of f_{ind} with the aid of the formula (3) should be carried out with allowance for the first correction in $h\nu/mc^2$ in the expression for the frequency ν' in Δp_i and $N(\nu',\mathbf{n}')$. The Klein–Nishina–Tamm scattering cross section[20] has a first correction in $h\nu/mc^2$ to the cross section (5). However, this correction does not contribute to the induced processes, since it does not destroy reversibility and enters in like manner into the direct and inverse scattering processes. In computing the diffusion coefficient with the aid of the formula (3), it is, in general, not necessary to take the quantum corrections into account, assuming $\nu'=\nu(1-\boldsymbol{\beta}\cdot\mathbf{n})/(1-\boldsymbol{\beta}\cdot\mathbf{n}')$. Taking these remarks into account, and using the formulas (4) and (5), we can show by direct differentiation of the formula (3) that the induced force $f_{i\,ind}=\partial D_{ik}/\partial p_k$ and that Eq. (2) coincides with a well-known equation in quasi-linear plasma theory. [21] In the case of homogeneous electron and photon distributions the equation has the form

$$\frac{\partial\varphi}{\partial t}=\frac{\partial}{\partial p_i}\left[D_{ik}\frac{\partial\varphi}{\partial p_k}-f_{i\,sp}\varphi\right]. \tag{7}$$

In the case of an isotropic radiation field the coefficient D_{ik} can be represented in the form

$$D_{ik}=D_l(p)\frac{p_ip_k}{p^2}+D_t(p)\left(\delta_{ik}-\frac{p_ip_k}{p^2}\right), \tag{8}$$

where D_l and D_t are respectively the longitudinal and transverse diffusion coefficients.

In this case Eq. (7) can be written as follows:

$$\frac{\partial\varphi}{\partial t}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_l\frac{\partial\varphi}{\partial p}-f_{sp}\varphi\right]+\frac{D_t}{p^2}\Delta_{\varphi,\theta}\varphi, \tag{9}$$

where $\Delta_{\varphi,\theta}$ is the angular part of the Laplacian in spherical coordinates. Let us multiply both sides of Eq. (7) by $\varepsilon=\gamma mc^2$ and integrate it over phase space. After setting $\varphi=\delta(q-p)/4\pi p^2$, we obtain the rate of heating of monoenergetic electrons. Calculating the rate for one electron, we have

$$\frac{d\varepsilon}{dt}=Q+\mathbf{f}_{sp}\mathbf{v}=\frac{1}{p^2}\frac{d}{dp}(D_lp^2v)+\mathbf{f}_{sp}\mathbf{v}, \tag{10}$$

and, similarly, after multiplying by p and setting in (7) $\varphi=\delta(\mathbf{q}-\mathbf{p})$, we obtain an expression for the light pressure acting on a moving electron:

$$\mathbf{f}=\mathbf{f}_{sp}+\mathbf{f}_{ind}=\mathbf{f}_{sp}+\frac{\mathbf{p}}{p}\left(D_l'+\frac{2}{p}(D_l-D_t)\right). \tag{11}$$

The well-known expression for the spontaneous force[22] can be obtained from the formula (3):

$$\mathbf{f}_{sp}=-\frac{4}{3}\sigma_T\frac{\mathbf{v}}{c}\mathscr{E}_r\gamma^2,$$

where $\sigma_T=(8\pi/3)(e^2/mc^2)^2$ is the Thomson cross section and $\mathscr{E}_r=4\pi c^{-1}\int F(\nu)d\nu$ is the radiation-energy density.

2. THE INDUCED FORCE ACTING ON AN ELECTRON IN A RADIATION FIELD

For simplicity of computation, we shall find the expression for the induced force acting on an electron moving with velocity $\mathbf{v}=\boldsymbol{\beta}c$ in a given radiation field in terms of quantities written in the electron rest frame. In accordance with the Lorentz transformation,

$$\mathbf{f}_{ind}=\int\left(\Delta\mathbf{p}+\frac{\mathbf{v}}{c^2}\Delta\varepsilon\right)N(\nu,\mathbf{n})N(\nu',\mathbf{n}')c\sigma\frac{2\nu^2}{c^3}d\nu\,d\mathbf{n}'\,d\mathbf{n}, \tag{12}$$

where $\Delta\mathbf{p}=hc^{-1}(\nu\mathbf{n}-\nu'\mathbf{n}')$ and $\Delta\varepsilon=h(\nu-\nu')$ are respectively the electron momentum and energy changes that occur in the rest frame during one scattering event, $\sigma=\frac{1}{2}r_e^2(1+\cos^2\alpha)$ is the Thomson scattering cross section, $r_e=e^2/mc^2$ is the electron radius, $\cos\alpha=\mathbf{n}\cdot\mathbf{n}'$, α is the scattering angle, and $\nu'=\nu-(h\nu^2/mc^2)(1-\cos\alpha)$ is the frequency of a photon with $h\nu\ll mc^2$ after scattering by an electron initially at rest.

Expanding $N(\nu',\mathbf{n}')$ in the formula (12) in powers of $(\nu'-\nu)/\nu$, and retaining the clearly nonvanishing (after integration over the angles), lowest-order (second-order in h) terms, we obtain an expression for the induced force

$$f_{ind}=\frac{r_e^2h^2}{mc^5}\int(1-\cos\alpha)(1+\cos^2\alpha)N(\nu,\mathbf{n})$$
$$\times\left[N(\nu,\mathbf{n}')(\mathbf{n}+\beta)+(\mathbf{n}'-\mathbf{n})\nu\frac{dN(\nu,n')}{d\nu}\right]\nu^4\,d\nu\,d\mathbf{n}'\,d\mathbf{n}. \quad (13)$$

Let an electron move in an isotropic radiation field with a given spectrum $N(\nu)$. Since the occupation number N is an invariant quantity, the radiation spectrum in the electron rest frame (already naturally anisotropic) has the form

$$N(\nu,\ \mathbf{n})=N[\nu\gamma(1+\beta\cos\theta)]. \quad (14)$$

Here and below $\beta\cos\theta=\mathbf{v}\cdot\mathbf{n}/c$, and the plus sign in the brackets corresponds to a situation in which $\mathbf{n}$, the direction of the photon wave vector, differs by an angle π from the direction from which we receive the photon.

Let us find the induced force first in the nonrelativistic limit. The expression (14) in the first approximation in $\beta\ll 1$ has the form

$$N(\nu,\ \mathbf{n})=N(\nu)+\beta\mathbf{n}\nu(dN(\nu)/d\nu).$$

Substituting this formula into (13), and evaluating the integrals with allowance for the expression for the scattering angle $\cos\alpha=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\varphi-\varphi')$, we obtain, writing the answer in terms of the spectral intensity (6),

$$\mathbf{f}_{ind}=-\frac{2\pi}{15}\frac{\sigma_T}{mc}\frac{\mathbf{v}}{c}\left[14\int\left(\frac{dF}{d\nu}\right)^2d\nu-11\int\frac{F^2}{\nu^2}d\nu\right]. \quad (15)$$

Here and everywhere below, in computing the effects connected with induced scattering, we assume that $F(\nu)$ for $\nu\to 0$ falls off more rapidly than $\nu^{1/2}$.

In Zel'dovich's review paper[1] it is pointed out that the force in the electron rest frame is always directed in the direction opposite to that of the velocity. In contrast, the force (15), i.e., the force computed in the laboratory system, may, depending on the radiation spectrum, be directed either opposite or along the velocity of the electron. For example, for spectra of the form $F\sim\nu^\alpha e^{-\nu a}$ the force accelerates the electron when $\frac{1}{2}<\alpha<\frac{11}{7}$ and retards it when $\alpha>\frac{11}{7}$. The critical value of the exponent α depends on the law according to which the radiation spectrum falls off at high frequencies.

As an example, let us compare the magnitudes of the spontaneous and induced forces acting on a nonrelativistic electron moving in an isotropic quasi-Planckian,

$$N(\nu)=AN_0(\nu)=A[\exp(h\nu/kT)-1]^{-1},$$

radiation field; the induced force in this spectrum is a retarding force; the introduction of the factor $A\gg 1$ allows us to describe the situation with a high brightness temperature $T_b=h\nu N(\nu)/k=AT$ for the radiation in the low-frequency, $h\nu\ll kT$, spectral region where $N\gg 1$. For the ratio of the forces we obtain

$$\frac{f_{ind}}{f_{sp}}\approx 0.03\frac{kT_b}{mc^2}.$$

Usually, the induced force constitutes a small correction to the spontaneous force (on the surface of the sun its portion is equal to only 2×10^{-7}). However, in an astrophysical situation, near, for example, pulsars, the radiation is essentially non-Planckian, and the brightness temperature of the radio-frequency radiation is enormous: $T_b\approx 10^{30}$ K $\approx 10^{20}(mc^2/k)$. Under these conditions the induced force plays a major role in the interaction of the radiation with the surrounding plasma. It must also be taken into account in the interaction of high-power radiation beams with tenuous laboratory plasmas.

For the case of relativistic electrons moving in an isotropic radiation field, the induced force is determined by the expression (13) with allowance for the formula (14). The result of the exact evaluation of the multiple integrals entering into the expression (13) for the force in the case of arbitrary radiation spectra is given in the Appendix. As in the nonrelativistic case, the induced force may, depending on the form of the radiation spectrum, retard or accelerate the electron, the sign of the effect being dependent only on the behavior of the spectrum at low frequencies. Thus, for the spectra

$$F(\nu)\sim\nu^\alpha \quad \text{as} \quad \nu\to 0 \quad (16)$$

asymptotically in the case when $\gamma\gg 1$, the induced force retards the electron when $\alpha>1$. It has the form

$$\mathbf{f}_{ind}=-\frac{12\pi\sigma_T h^2}{\gamma^3 mc^5}\beta\int_0^1 dy\,\Phi(y)y^2\left(1-\frac{y}{3}\right) \quad (17)$$

and behaves like γ^{-3}.[18] Here

$$\Phi(y)=\int_0^\infty N(\nu)N(\nu y)\nu^4\,d\nu \quad (18)$$

is a universal function determining the dependence of the induced effects on the radiation spectrum.

In the case of the spectrum (16) with $\frac{1}{2}<\alpha<1$, the force accelerates the electron, its asymptotic form coincides with the asymptotic form of the rate of heating (see Sec. 3) in such spectra, and is equal to

$$\mathbf{f}_{ind}=\beta Q/c\sim\gamma^{-1-2\alpha}.$$

3. INDUCED HEATING OF ELECTRONS IN AN ISOTROPIC RADIATION FIELD

An electron located in a radiation field changes its energy both as a result of induced, and on account of spontaneous, scattering. Since the heating of electrons as a result of spontaneous scattering is a quantum effect, proportional to h, we shall not consider it. In the case of a nonrelativistic electron and an isotropic radiation field from A. S. Kompaneets' equation[8] follows[5,9] the induced-heating rate:

$$Q=\frac{8\pi\sigma_T h^2}{mc^4}\int N^2\nu^4\,d\nu=\frac{2\pi\sigma_T}{m}\int\frac{F^2}{\nu^2}d\nu. \quad (19)$$

Owing to spontaneous heating, an electron can gain energy of the order of the mean photon energy in the radiation. Radiation with a high brightness temperature, 3)

T_b, at low frequencies is capable of doing more: It heats up the electrons to an energy much higher than the mean photon energy, owing to induced scattering. Thus, radiation with a Rayleigh–Jeans spectrum $F=2\nu^2kT_b/c^2$ for $0<\nu<\nu_0$ and $F=0$ for $\nu>\nu_0$ ($h\nu_0 \ll kT_b$) is capable of heating an electron up to the energy $\langle\varepsilon\rangle = 3kT_{eq}/2 = 3kT_b/8$ (in the nonrelativistic treatment, see (24)). In the case when $kT_b \gg mc^2$, which is not rare under astrophysical conditions, the electrons are capable of being heated to relativistic energies. In the present section we shall find expressions determining the heating of a relativistic electron.

The expression for the induced heating of an electron moving with velocity $\mathbf{v}=\beta c$ is found in a manner similar to the way the induced force was found and has, in the same notation, the form

$$Q=\int (\Delta\varepsilon+\mathbf{v}\Delta\mathbf{p})N(\nu,\mathbf{n})N(\nu',\mathbf{n}')c\sigma\frac{2\nu^2}{c^3}d\nu\, d\mathbf{n}'\, d\mathbf{n}. \tag{20}$$

In the nonrelativistic case, for which $\beta \ll 1$, this formula gives the result (19). After the integration (see the Appendix), the formula (20) leads to the expression

$$Q=\frac{12\pi\sigma_T h^2}{mc^4}\int\limits_0^\beta d\beta'\Phi(y')G_Q(\beta,\beta'), \tag{21}$$

$$G_Q(\beta,\beta')=\frac{\beta'^2}{\gamma^5\beta^8(1+\beta')^5}\Big[(30-24\beta^2+2\beta^4)\ln\frac{y'}{y}$$
$$+28\beta^3-60\beta+5(3-\beta^2)^2\beta'+(5-\beta^2)(3+\beta'^2)\beta'(\gamma'/\gamma)^2],$$
$$y'=\frac{1-\beta'}{1+\beta'},\quad y=\frac{1-\beta}{1+\beta},$$

and the function Φ is determined by the formula (18). In the limit when β, $\beta' \ll 1$ the function

$$G_Q(\beta,\beta')=4\beta'^3[\beta^4-2\beta^2\beta'^2+2\beta'^4]\beta^{-8}. \tag{21'}$$

The behavior of Q as a function of the quantity $\gamma(\gamma\gg 1)$ is determined by the form of $\Phi(y')$ for small y', i.e., by the radiation spectrum at low frequencies. In the case of the spectrum (16) for $\alpha>2$, the quantity $Q\sim \ln\gamma/\gamma^5$. In the case when $\frac{1}{2}<\alpha<2$, however, we have $Q\sim 1/\gamma^{1+2\alpha}$. For $\alpha=2$ the quantity $Q\sim \ln^2\gamma/\gamma^5$.

4. COEFFICIENTS OF ELECTRON DIFFUSION IN MOMENTUM SPACE. THE EQUILIBRIUM DISTRIBUTION

In an isotropic radiation field the momentum distribution function of the electrons satisfies Eq. (7), where $D_l=D_{ik}p_ip_k/p^2$, while $D_t=(D_{ii}-D_l)/2$. The quantity D_l is computed from the general formulas (3) with allowance for (4) (see the Appendix), and is equal to (see also[15])

$$D_l=\frac{12\pi\sigma_T h^2}{c^4}\int\limits_0^\beta \Phi(y')G_{D_l}(\beta,\beta')d\beta',$$

$$G_{D_l}(\beta,\beta') \tag{22}$$
$$=\frac{2\beta'^2}{\gamma^4\beta^8(1+\beta')^5}\left[(3-\beta^2)\ln\frac{y}{y'}+2\beta-2\beta'\left(\frac{\gamma'}{\gamma}\right)^2+2(\gamma^2+\gamma^{-2})(\beta-\beta')\right].$$

In the limit when β, $\beta' \ll 1$, in accordance with Vinogradov and Pustovalov's result,[6]

$$C_{D_l}(\beta,\beta')=\frac{4}{15}\beta'^2[11\beta^5-15\beta'\beta^4+10\beta'^3\beta^2-6\beta'^5]\beta^{-8}. \tag{22'}$$

To find the form of D_t, let us use the relation, which follows from Eqs. (10) and (11), between D_l and D_t:

$$Q-\mathbf{f}_{ind}\mathbf{v}\equiv W\gamma^{-2}=(2D_t\gamma+D_l\gamma^{-1})/m\gamma^2, \tag{23}$$

where W has the meaning of the rate of accumulation of energy by an electron in its rest frame (see the Appendix).

In the nonrelativistic limit $D_l=D_t=mQ/3$. In the relativistic case, in which $\gamma\gg 1$, the behavior of the quantity D_l depends (as in the case of Q and $\mathbf{f}_{ind}$) on the behavior of the spectrum at low frequencies. In the case of a spectrum of the form (16), $D_l\sim\gamma^{-2}$ for $\alpha>1$ and $D_l\sim\gamma^{-2\alpha}$ for $\frac{1}{2}<\alpha<1$. As is easy to show, asymptotically, the quantity W is proportional to γ^{-1}, and the formula for W/γ^2c coincides with the formula, (17), for the force and is valid for any radiation spectra. It follows from the formula (23) and the asymptotic forms of D_l and W that, irrespective of the radiation spectrum, $D_t=W/2\gamma\sim\gamma^{-2}$.

Let us find the equilibrium momentum distribution function of the electrons. It follows from Eq. (9) that

$$\varphi_{eq}(p)\sim\exp\Big(-\int\limits_0^p (f_{sp}/D_l)\,dp'\Big).$$

In the nonrelativistic case the problem of electron distribution over energy during the Compton interaction of the electrons with radiation with a broad spectrum has been solved by Zel'dovich and Levich.[9] The steady-state function turns out to be Maxwellian with the temperature

$$T_{eq}=\frac{h}{4k}\frac{\int N^2\nu^4\,d\nu}{\int N\nu^3\,d\nu}. \tag{24}$$

Let us consider the case of relativistic electrons, which corresponds to the condition $kT_{eq}>mc^2$. Depending on the form of the radiation spectrum (16) at low frequencies, the equilibrium distribution function of the electrons has the form

$$\varphi_{eq}\sim\exp(-ap^n), \tag{25}$$

where $n=5$ for $\alpha>1$ and $n=3+2\alpha$ for $\frac{1}{2}<\alpha<1$, while the parameter $a\approx(kT_{eq}/mc^2)^{-1}(mc)^{-n}$. In this case the mean energy $\langle\varepsilon\rangle=c\langle p\rangle\approx ca^{-1/n}$, while φ_{eq}, given by (25), corresponds to a plane electron spectrum truncated on the high-energy side by the spontaneous braking force.

5. EVOLUTION OF THE ELECTRON SPECTRUM DURING THE ACTION OF INDUCED SCATTERING AND THE HEATING OF AN ELECTRON GAS

The analytic solution of Eq. (9) in the isotropic case with the diffusion coefficient D_l, which depends in a complicated manner on momentum (see (22)), is not possible. However, the asymptotic solution of the diffusion equation for large $p\gg mc$, where $D_l(p)$ is a known power function of momentum, to wit, $D_l=D_0p^{-k}$, is easy

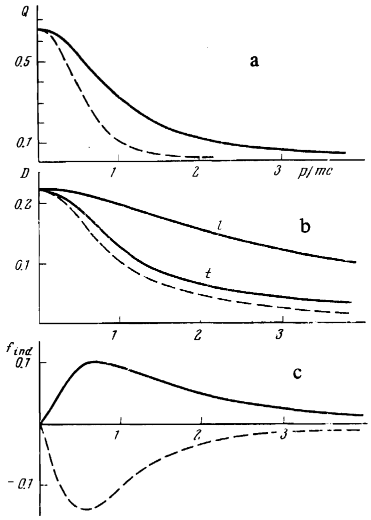

FIG. 2. The dependences on the electron momentum of: a) the electron-heating rate Q: b) the diffusion coefficients D_l and D_t: c) the induced force f_{ind} for radiation spectra of the form $F \sim \nu^{\alpha} e^{-\nu\alpha}$. The broken curve corresponds to a Wien spectrum with $\alpha = 3$, while the continuous curve corresponds to a spectrum with $\alpha = \frac{3}{4}$. In the $\alpha = 3$ case the quantity D_l virtually coincides with D_t. In the case c) positive f_{ind} corresponds to an accelerating force. The quantities are given in units of

$$Q_0 = \frac{12\pi\sigma_T h^2}{mc^4}\int N^2\nu^4\,d\nu. \quad D_0 = Q_0 m, \quad f_0 = Q_0/c$$

to find. The normalized—to unity—function

$$\varphi(p,t) = \frac{\varkappa}{4\pi\Gamma(3/\varkappa)\,(\varkappa^2 D_0 t)^{3/\varkappa}}\exp\left\{-\frac{p^{\varkappa}}{\varkappa^2 D_0 t}\right\}, \tag{26}$$

where $\varkappa = k+2$. This solution is valid in the momentum range where we can neglect the spontaneous braking force, i.e., for $p \ll \langle p\rangle$, and at times $t \ll \langle p\rangle^{\varkappa}/D_0$ when the equilibrium distribution (25) has not yet been reached.

On the basis of the obtained solution we can find the rate of heating of an electron gas during induced radiation scattering. Calculating the rate for one electron, we have

$$\frac{d\varepsilon}{dt} = \int Q(p)\,\varphi(p,t)\,4\pi p^2\,dp. \tag{27}$$

Substituting $Q(p)$ from the asymptotic form of (21), we obtain that, depending on the radiation spectrum at low frequencies, (16),

$$\frac{d\varepsilon}{dt} \sim t^{(3-n)/4}$$

where $n = 0$ for $\alpha > 1$ and $n = (1-\alpha)/(1+\alpha)$ for $\frac{1}{2} < \alpha < 1$. To this corresponds the dependence

$$\frac{d\varepsilon}{dt} \sim \bar{\varepsilon}^{-h}, \tag{28}$$

where $k = 3$ for $\alpha > 1$ and $k = 1 + 2\alpha$ for $\frac{1}{2} < \alpha < 1$, of the rate of heating of relativistic electrons with a spectrum of the form (26) on the mean electron energy $\bar{\varepsilon}$.

In the case of the equilibrium electron distribution (25), the induced heating of the electrons is balanced by the spontaneous cooling. However, the cooling of the electron gas is determined by the high-energy end of the spectrum of the electrons, while the heating (for $\alpha > 1$) is determined by the presence of semirelativistic particles with $p \approx mc$, since it is just momentum values $p \approx mc$ that are important for the integral (27). In the case when $\frac{1}{2} < \alpha < 1$ the heating is determined by the energetic electrons, which leads to the same dependence of the heating rate on the mean electron energy in the distribution (26) as for monoenergetic electrons.

Besides the heating of the electron gas, let us consider the problem of the evolution in time of the mean electron momentum under the action of induced scattering. Interest to such a problem is due to the fact that the induced light pressure can, in principle, accelerate an electron right up to ultrarelativistic energies. It follows from Eq. (9) that the induced processes alone (without allowance for the spontaneous retarding force) cannot by themselves lead to an equilibrium distribution of the electrons over energy. The influence, however, of the spontaneous force is equivalent to the presence of an effective "reflecting" wall at energies of the order of $\langle\varepsilon\rangle$. Since the equilibrium distribution is a symmetric function of the momentum, the final distribution of the electrons will, irrespective of the initial conditions and the direction of the induced force, possess zero momentum. However, the direction of the induced force essentially influences the pattern of evolution of the total momentum of the distribution. Thus, in the case of a radiation spectrum leading to an accelerating—in the relativistic limit—induced force ($\frac{1}{2} < \alpha < 1$), the inequality $D_l \gg D_t$ is valid when $\gamma \gg 1$, and Eq. (9) may (as long as we can neglect the spontaneous braking force) be written in the form

$$\frac{\partial\varphi(\mathbf{p})}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}p^2 D_l\frac{\partial\varphi(\mathbf{p})}{\partial p}, \qquad D_l \sim \gamma^{-2\alpha}.$$

Such a form of the equation corresponds to strong diffusion in the direction of $\mathbf{p}$, and, in accordance with the formula (28), to the growth, according to the law

$$\frac{d\bar{p}}{dt} \sim \bar{p}^{-(1+2\alpha)},$$

of the total momentum of the electrons right up to $p \approx \langle p\rangle$, since in the case of the radiation spectrum (16) the relation

$$f_{\text{ind}} \approx \frac{Q}{c} \approx \frac{1}{p^2}\frac{d}{dp}(D_l p^2).$$

is valid for $\frac{1}{2} < \alpha < 1$.

6. SPECTRALLY NARROW RADIATION LINES

The formulas given above for the quantities describing the induced effects (the rate of electron heating, the coefficients of diffusion, etc.) allow us to compute quite simply the quantities for any specific spectrum of isotropic random radiation. In particular, this computation is easy to carry out for the case, which has characteristic features, of spectrally narrow radiation consisting of one or several lines. Such a situation is realized in cosmic masers and can be realized in a laser experiment. The case of one narrow line has been considered by Vinogradov and Pustovalov.[6] Their principal results[6] (diffusion coefficients and electron heating) follow from the formulas (21) and (22) given above upon the substitution in them of the nonrelativistic expressions for the functions $G(\beta, \beta')$ (see (21′) and (22′)). In the case of a single narrow line ($\delta\nu \ll \nu$), the overlap function $\Phi((1-\beta')/(1+\beta'))$ (see the formula (18)) decreases rapidly when $\beta' > \delta\nu/\nu$. For example, in the case of a line with the Gaussian profile $N \sim \exp[-(\nu-\nu_0)^2/2(\delta\nu)^2]$ with a dispersion $(\delta\nu)^2 \ll \nu_0^2$, the function $\Phi \sim \exp[-\nu_0^2\beta'^2/(\delta\nu)^2]$; therefore, the quantities D_l and Q decrease rapidly with increasing electron velocity. Thus, for a Gaussian line, the values of the quantities $Q = \pi^{1/2}F_0^2\sigma_T/\nu_0^2 m\,\delta\nu$ and $D_l = mQ/3$, which are large when $\beta \ll \delta\nu/\nu$, decrease when $\beta > \delta\nu/\nu$ with increasing β like

$$Q = \frac{3\pi^{1/2} F_0^2\sigma_T}{\nu_0^3 m}\left(\frac{\delta\nu}{\nu_0}\right)^3\frac{1}{\beta^4}, \quad D_l = \frac{11\pi}{10}\frac{F_0^2\sigma_T}{\nu_0^3}\left(\frac{\delta\nu}{\nu_0}\right)^2\frac{1}{\beta^3}, \tag{29}$$

where $F_0 = \int F\,d\nu = 2hc^{-2}\nu_0^3\int N\,d\nu$ is the total power radiated in the line.

An interesting situation arises in the presence of several narrow lines in the radiation spectrum. Let us consider the case of two lines separated in frequency from each other by $\Delta\nu \gg \delta\nu_1$, $\delta\nu_2$ under the assumption that $\Delta\nu \ll \nu_1 \approx \nu_2$, when the nonrelativistic case is the interesting case.

For small $\beta < \Delta\nu/2\nu_1$, the quantities describing the induced effects from both lines add up arithmetically. However, for $\beta \geq \Delta\nu/(\nu_1+\nu_2) = \Delta\nu/2\nu_c$ induced transitions of photons from one line to another become possible during scattering. The function Φ has, for $\beta' = \Delta\nu/2\nu_c$, a sharp (with dispersion $[(\delta\nu_1)^2 + (\delta\nu_2)^2]/4\nu_c^2 \ll 1$) peak. This leads to a considerable enhancement, when $\beta > \Delta\nu/2\nu_c$, of the induced effects:

$$\begin{aligned} Q &= \frac{3\pi}{2}\frac{F_1F_2}{\nu_c^3}\frac{\sigma_T}{m}G_Q\left(\beta, \frac{\Delta\nu}{2\nu_c}\right) \approx \frac{3\pi}{4}\frac{F_1F_2}{\nu_c^3}\frac{\sigma_T}{m}\left(\frac{\Delta\nu}{\nu_c}\right)^3\frac{1}{\beta^4}, \\ D_l &= \frac{3\pi}{2}\frac{F_1F_2}{\nu_c^3}\sigma_T G_{D_l}\left(\beta, \frac{\Delta\nu}{2\nu_c}\right) \approx \frac{11\pi}{10}\frac{F_1F_2}{\nu_c^3}\sigma_T\left(\frac{\Delta\nu}{\nu_c}\right)^2\frac{1}{\beta^3}, \end{aligned} \tag{30}$$

where F_1 and F_2 are the total powers radiated in the first and second lines and $\nu_c = (\nu_1+\nu_2)/2$. It can be seen from this that for $\beta > \Delta\nu/2\nu_c$ the difference, $\Delta\nu$, between the frequencies of the lines (and not $\delta\nu_1$ or $\delta\nu_2$) plays the role of the effective width of the radiation spectrum, so that, for example, the rate of heating of Maxwellian electrons with temperature $kT_e \gtrsim mc^2(\Delta\nu/\nu_c)^2$ significantly exceeds the rate of heating of a colder plasma with $kT_e < mc^2(\Delta\nu/\nu_c)^2$.

In conclusion, we express our gratitude to Ya. B. Zel'dovich and R. A. Syunyaev for interest in the work and for valuable comments.

APPENDIX

As an example, let us carry out the computation of the quantity W for an electron moving in an isotropic radiation field with velocity $\mathbf{v} = \beta c$. The quantity W, which has the meaning of the rate of accumulation of energy by the electron in its rest frame, is determined by the formula (23), and is equal to

$$W = \int \Delta\varepsilon N(\nu, \mathbf{n})N(\nu, \mathbf{n}')c\sigma\frac{2\nu^2}{c^3}d\nu\,d\mathbf{n}\,d\mathbf{n}'.$$

Setting $\Delta\varepsilon = h(\nu - \nu')$, and performing the integration over the azimuthal angles φ and φ', we obtain

$$\begin{gathered} W = \frac{3\pi\sigma_T h^2}{4mc^4}\int P_W(\mu, \mu')N(\nu\gamma(1+\beta\mu))N(\nu\gamma(1+\beta\mu'))\nu^4\,d\nu\,d\mu\,d\mu', \\ \mu = \cos\theta, \quad \mu' = \cos\theta', \\ P_W(\mu, \mu') = 3 - \mu^2 - \mu'^2 - 5\mu\mu' + 3\mu^2\mu'^2 + 3\mu\mu'^3 + 3\mu^3\mu' - 5\mu^3\mu'^3. \end{gathered} \tag{A.1}$$

Let us give a scheme for the computation of the angular integrals in (A.1). Let us represent (A.1) in the form

$$\begin{gathered} W = \sum_{ik} a_{ik}\int I_iI_k\nu^4\,d\nu, \\ I_k = (\nu\gamma\beta)^{-k-1}\int_{\nu\gamma(1-\beta)}^{\nu\gamma(1+\beta)} N(x)x^k\,dx, \quad 0 \leqslant i, k \leqslant 4, \end{gathered} \tag{A.2}$$

where the coefficients a_{ik} are functions of the parameter β. Differentiating I_k, we obtain

$$\nu\frac{dI_k}{d\nu} = -(k+1)I_k + \left(\frac{1+\beta}{\beta}\right)^{k+1}N(\nu\gamma(1+\beta)) - \left(\frac{1-\beta}{\beta}\right)^{k+1}N(\nu\gamma(1-\beta)). \tag{A.3}$$

Integrating the expression

$$\int I_k\frac{dI_i}{d\nu}\nu^5\,d\nu$$

by parts and using (A.3) for $\nu dI_k/d\nu$ and for $\nu dI_i/d\nu$, we obtain the formula

$$(k+i-3)\int \nu^4 I_iI_k d\nu = \left(\frac{1+\beta}{\beta}\right)^{i+1}\int I_kN(\nu\gamma(1+\beta))\nu^4\,d\nu - \left(\frac{1-\beta}{\beta}\right)^{i+1} \times \int I_kN(\nu\gamma(1-\beta))\nu^4\,d\nu + \text{the expression obtained by interchanging the indices } i \text{ and } k. \tag{A.4}$$

Making a change of variables, we obtain

$$\int I_kN(\nu\gamma(1\pm\beta))\nu^4\,d\nu = \frac{(1\pm\beta)^{k-4}}{\gamma^5\beta^{k+1}}\int_{y_0}^{1}\Phi(y)y^{3/2\pm(k-3/2)}\,dy, \tag{A.5}$$

where $\Phi(y)$ is the overlap function introduced by the formula (18), and $y_0 = (1-\beta)/(1+\beta)$. For $i+k \neq 3$, the formula (A.4) can be written in the form

$$\int I_iI_k\nu^4\,d\nu = \frac{(1+\beta)^{i+k-3}}{(i+k-3)\gamma^5\beta^{i+k+2}}\int_{y_0}^{1}\Phi(y)(y^k+y^i)\left[1 - \left(\frac{y_0}{y}\right)^{i+k-3}\right]dy. \tag{A.6}$$

For $i+k = 3$ this formula, after the passage to the limit $i+k \to 3$, has the form

$$\int I_iI_k\nu^4\,d\nu = \frac{1}{\gamma^5\beta^5}\int_{y_0}^{1}\Phi(y)(y^k+y^i)\ln\frac{y}{y_0}\,dy. \tag{A.7}$$

And, finally, substituting the obtained formulas (A.5)–(A.7) into (A.2), we obtain after setting $y' = (1-\beta')/(1+\beta')$ and $y = y_0 = (1-\beta)/(1+\beta)$ the expression

$$W=\frac{12\pi\sigma_T h^2}{mc^4}\int_0^\beta \Phi(y')G_W(\beta,\beta')d\beta', \tag{A.8}$$

$$G_W(\beta,\beta')=\frac{1}{\gamma\beta^8(1+\beta')^5}\left\{[25-9\beta^2+3\beta'^2(\beta^2-5)]\gamma^{-4}\left[\ln\frac{y'}{y}+2(\beta'-\beta)\right]\right.$$
$$\left.+2\beta^3(5-8\beta^2+\beta^4)(\beta'^2-\beta^2)+\frac{2}{3}[(25-9\beta^2)\gamma^{-4}+2\beta^4(3\beta^2-1)](\beta'^3-\beta^3)\right\}.$$

In the nonrelativistic limit, when $\beta,\beta'\ll 1$,

$$G_W(\beta,\beta')=\left[\frac{92}{105}\beta^7-\frac{4}{3}\beta'^3\beta^4+\frac{8}{5}\beta'^5\beta^2-\frac{8}{7}\beta'^7\right]\beta^{-8}. \tag{A.9}$$

The ultrarelativistic asymptotic form yields

$$G_W(1,\beta')=\frac{4(1-\beta')^2(1+2\beta')}{3\gamma(1+\beta')^5}=\frac{y'^2}{\gamma}\left(1-\frac{y'}{3}\right)\left|\frac{dy'}{d\beta'}\right|. \tag{A.10}$$

Let us compute the quantity $D_l=D_{ik}p_ip_k/p^2$, where D_{ik} is determined by the formula (3), by going (as in the calculation of $f_{\rm ind}$ and Q) over into the electron rest frame and replacing Δp_i by the right-hand side of (4). After the integration over φ and φ', we obtain

$$D_l=\frac{3\pi\sigma_T h^2}{8c^4}\gamma\int P_{D_l}(\mu,\mu')N(\nu\gamma(1+\beta\mu))N(\nu\gamma(1+\beta\mu'))\nu^4d\nu\,d\mu\,d\mu',$$
$$P_{D_l}(\mu,\mu')=(\mu-\mu')^2(3-\mu^2-\mu'^2+3\mu^2\mu'^2).$$

Evaluating this integral according to the scheme given above, we obtain the expression (22).

The induced heating is found from the formula (see (10))

$$Q=\frac{1}{p^2}\frac{d}{dp}(D_lp^2v).$$

The result is represented by the formula (21).

Similarly, we find on the basis of the formula (23) the quantity

$$D_t=\frac{12\pi\sigma_T h^2}{c^4}\int_0^\beta \Phi(y')G_{D_t}(\beta,\beta')d\beta', \tag{A.11}$$
$$G_{D_t}={}^1/_2(G_W\gamma^{-1}-G_{D_l}\gamma^{-2}).$$

For $\beta,\beta'\ll 1$

$$G_{D_t}(\beta,\beta')=\left[\frac{46}{105}\beta^7-\frac{22}{15}\beta'^2\beta^5+\frac{4}{3}\beta'^3\beta^4-\frac{8}{15}\beta'^5\beta^2+\frac{8}{35}\beta'^7\right]\beta^{-8}. \tag{A.12}$$

In the case when $\gamma\gg 1$

$$G_{D_t}(1,\beta')=\frac{1}{2\gamma}G_W(1,\beta'). \tag{A.13}$$

Let us find the induced force acting on an electron moving in an isotropic radiation field from the relation (see (23))

$$\mathbf{f}_{ind}=\frac{\mathbf{v}}{v^2}\left(Q-\frac{W}{\gamma^2}\right).$$

From the formulas (21) and (A.8) we obtain

$$\mathbf{f}_{ind}=\frac{12\pi\sigma_T h^2}{c^5}\frac{\boldsymbol{\beta}}{\beta}\int_0^\beta \Phi(y')G_f(\beta,\beta')d\beta', \tag{A.14}$$
$$G_f=(G_Q-G_W\gamma^{-2})\beta^{-1}.$$

For $\beta,\beta'\ll 1$ the expansion of G_f has an unwieldy form. In the case of a broad radiation spectrum this expansion leads to the formula (15). For the case of spectrally narrow radiation ($\delta\nu/\nu\ll 1$; see Sec. 6), for $\beta>\delta\nu/\nu$, we have the first term of the expansion $G=-\frac{92}{105}\beta^{-2}$. In the ultrarelativistic asymptotic form, $G_f=-G_W\gamma^{-2}$ for spectra that decrease to zero more rapidly than the first power of the frequency, and $G_f=G_Q$ for a slower decrease of the spectral intensity.

[1)]Here and below we imply equilibrium with respect to the processes of radiation scattering, and not total thermodynamic equilibrium.

[2)]Notice that the Thomson approximation is applicable in induced and spontaneous scattering under different conditions. In the case of spontaneous scattering this condition has the form $h\nu\ll mc^2/\gamma$. Induced scattering of light by a relativistic electron occurs primarily at small angles $1-\mathbf{nn}'\sim\gamma^{-2}$. The Thomson approximation is in this case valid when $h\nu\ll mc^2\gamma$.

[3)]The consideration of induced scattering is equivalent to a transition to high brightness temperatures.

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