Possibility of observing nonstationary quantum-mechanical effects by means of ultracold neutrons

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(Submitted May 21, 1976)
Zh. Eksp. Teor. Fiz. 71, 1700–1711 (November 1976)

Nonstationary phenomena arising from the interaction between ultracold neutrons (UCN) and a time-dependent potential are considered. One possibility of observing these phenomena is to record UCN emerging from the region between two plane-parallel time-dependent potential barriers. The variation of the UCN momentum distribution due to interaction with a weakly oscillating potential is also discussed.

PACS numbers: 03.65.Nk, 28.20.Cz

1. INTRODUCTION

Ultracold neutrons (UCN), owing to their extremely low energy E ($\lesssim 10^{-7}$ eV), serve as a unique object for experimental investigations of nonstationary quantum-mechanical effects. In this connection, particular interest attaches to the case of the interaction of UCN with a nonstationary force field that varies significantly within a time

$$\tau \ll \hbar/E \sim 10^{-8} \text{ sec}. \tag{1}$$

It is precisely in this case that one observes most pronounced manifestations of the specific properties of the solution of the nonstationary Schrödinger equation, properties connected with the energy-time uncertainty relations. We are dealing here with a large group of phenomena that have not been investigated at all, and it appears that an experimental observation of these phenomena is presently possible only with the aid of UCN.

Of definite interest is also the interaction of UCN with an oscillating potential. Owing to the possibility of spatially separating the fluxes of neutrons with different momenta, experimental observation of this phenomenon with UCN is even simpler than with charged particles (say, electrons). A study of the interaction of UCN with an oscillating potential is also important from the point of view of applications, since the walls of the traps used for UCN, especially magnetic traps, practically always oscillate weakly and this leads to "heating" and additional losses of the UCN.

The present paper is devoted to an examination of quantum-mechanical effects arising when UCN interact with a nonstationary field.

2. PASSAGE OF UCN THROUGH A TIME-DEPENDENT RECTANGULAR POTENTIAL BARRIER

One possible nonstationary action on UCN is to pass them through a time-dependent rectangular potential barrier.[1)] Such a barrier acts only on the particle-momentum component normal to its surface, and to describe the particle motion it suffices to consider the one-dimensional Schrödinger equation, which in this case takes the form

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + U(x)f(t)\psi, \tag{2}$$

$$U(x) = U_0\theta(\varepsilon - |x|) \tag{2'}$$

(we are using units with particle mass $m = \hbar = 1$; $\theta(x) = 0$ at $x<0$ and $\theta(x) = 1$ at $x<0$).

We consider first the ideal case of instantaneous vanishing of an infinitely high barrier

$$U_0 \to \infty, \quad \varepsilon \to 0 \quad (\text{but } U_0\varepsilon^2 \to \infty), \quad f(t) = \theta(-t). \tag{3}$$

Assume that a plane wave with momentum p_0 is incident on the barrier. Then, apart from a normalization constant, we have

$$\begin{aligned} \psi &= \psi_1(x,t) \equiv (e^{ip_0x} - e^{-ip_0x}) \exp\left(-i\frac{p_0^2 t}{2}\right)\theta(-x) \quad \text{at} \quad t<0, \\ \psi &= \psi_2(x,t) \equiv (2\pi)^{-1/2}\int_{-\infty}^{\infty} dp\, A(p) \exp\left(ipx - i\frac{p^2t}{2}\right) \quad \text{at} \quad t>0, \end{aligned} \tag{4}$$

and from the condition $\psi_1(x, 0) = \Psi_2(x, 0)$ it follows that

$$A(p) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} dx\, e^{-ipx}\psi_1(x,0)$$

$$= \left(\frac{\pi}{2}\right)^{1/2}[\delta(p-p_0) - \delta(p+p_0)] - \frac{1}{(2\pi)^{1/2}i}\mathrm{P}\left(\frac{1}{p-p_0} - \frac{1}{p+p_0}\right), \tag{5}$$

where the symbol P means that the integral is taken in the sense of the principal value. Hence, taking into account the following chain of equations

$$\frac{1}{2\pi i}\mathrm{P}\int_{-\infty}^{\infty}\frac{dp}{p-p_0}\exp\left(ipx - i\frac{p^2t}{2}\right)$$

$$= \frac{1}{\pi}\exp\left(ip_0x - i\frac{p_0^2t}{2}\right)\int_0^{\infty}\frac{dp'}{p'}\exp\left(-i\frac{p'^2t}{2}\right)\sin[p'(x-p_0t)]$$

$$= \frac{1}{\pi}\exp\left(ip_0x - \frac{ip_0^2t}{2}\right)\int_0^{x-p_0t}dy\int_0^{\infty}dp'\exp\left(-i\frac{p'^2t}{2}\right)\cos p'y$$

$$= \frac{1}{2}\exp\left(ip_0x - i\frac{p_0^2t}{2}\right)F\left(\frac{x-p_0t}{(2t)^{1/2}}\right), \tag{6}$$

$$F(z) = \left(\frac{2}{\pi}\right)^{1/2}(1-i)\int_0^z e^{iy^2}dy, \tag{7}$$

we obtain

$$\psi_2 = \frac{1}{2}\exp\left(-i\frac{p_0^2t}{2}\right)\{[1-F(z_-)]e^{ip_0x} - [1-F(z_+)]e^{-ip_0x}\}, \tag{8}$$

$$z_\pm = \frac{1}{\sqrt{2t}}(x \pm p_0t). \tag{9}$$

An analysis of this expression can be easily carried out by taking into account the relations

$$F(-z)=-F(z),\quad F(z)\underset{z\to 0}{\longrightarrow}\left(\frac{2}{\pi}\right)^{1/2}(1-i)z,$$
$$F(z)\underset{z\to\infty}{\longrightarrow}1+\frac{1+i}{(2\pi)^{1/2}z}e^{iz^2}. \tag{10}$$

Particular interest attaches to the behavior of ψ_2 in the region $p_0t\,|x|\lesssim 1$, or in the customary units $(v_0=p_0/m)$

$$v_0t/|x|\leqslant 1, \tag{11}$$

in which ψ_2 is essentially nonstationary. The condition $v_0t<x$ determines the classically unattainable region. If $p_0^2t\gg 1$ (meaning in ordinary units $t\gg\hbar/E$), then in the classically unattainable region $|\psi_2|^2$ is close to zero. On the other hand, if

$$p_0^2t\ll 1, \tag{12}$$

i.e., quantum-mechanical effects should be significant, then in the region (11) we have

$$\psi_2=\frac{1}{2}\exp\left(-i\frac{p_0^2t}{2}\right)\left\{\left[1-(1-i)\frac{x-p_0t}{(\pi t)^{1/2}}\right]e^{ip_0x}-\left[1-(1-i)\frac{x+p_0t}{(\pi t)^{1/2}}\right]e^{-ip_0x}\right\}. \tag{13}$$

It is easily seen therefore that the better the condition (12) is satisfied, the wider the relative range of times in which $|\psi_2|^2$ differs from zero in the classically unattainable region.

The results yield also a criterion for the applicability of the model of "instantaneous vanishing of an infinitely high barrier" considered by us. Namely, beside the obvious conditions $U_0\varepsilon^2\gg 1$, and $\varepsilon p_0\ll 1$, the barrier vanishing time τ_0 should satisfy the requirement $\tau_0\ll p_0^{-2}$, which in the customary units agrees with (1).

We consider now the case of the vanishing of two parallel barriers separated by a distance $2a$, i.e., the case when $U(x)$ takes in place of (2′) the form

$$U(x)=U_0\{\theta(\varepsilon-|x-a|)+\theta(\varepsilon-|x+a|)\} \tag{14}$$

(and the conditions (3) are satisfied). Accordingly, the wave function at $t<0$ is chosen to be

TABLE I. Values of $|\psi(x,t)|^2$ as functions of t (in units of $(x-a)/v$) at $a=5\times10^{-4}$ cm and at different $x-a$ and velocities $v=p_0/m$.

$\frac{tv}{x-a}$	$x-a=2\cdot10^{-4}$ cm, $v=1\ \frac{m}{sec}$	$v=3\ \frac{m}{sec}$	$v=5\ \frac{m}{sec}$	$\frac{tv}{x-a}$	$x-a=5\cdot10^{-4}$ cm, $v=1\ \frac{m}{sec}$	$v=3\ \frac{m}{sec}$	$v=5\ \frac{m}{sec}$
0.50	2.1	0.7	0.4	0.70	4.8	1.3	1.1
0.60	4,8	1.7	1.1	0.80	12.6	3.3	3.1
0.70	10,7	4.2	2.6	0.85	20.8	5.7	6.0
0.80	23.8	11.2	7.4	0.90	35,7	10.7	13.7
0.90	51.2	34.2	26.4	0.95	61.8	21.9	37.2
1.00	106.1	111.4	116.5	1.00	106.4	49.4	116.2
1,50	721.2	563,8	475.4	1.20	561.9	687.0	434.9
2.00	568,2	476.7	536.0	1.40	486.0	560.4	528.9
2.50	549.0	487.8	530.5	1.60	609.1	565.0	478.4
3,00	562.0	455,6	485.1	1.80	522,7	528.2	494.1
3.50	581.0	529,7	526.6	2.00	489,7	467.6	499.9
4.00	549.1	469.6	506.6	2.20	396,6	511,5	524.9
4.50	550.9	481.4	536.4	2.40	401.3	552.5	475.8
5.00	511,2	495.6	522.8	2.60	607.4	529.3	444.9
5,50	473.6	708.7	551.2	2,80	435.8	431.4	645.5
6.00	179.3	138,5	148.5	3.00	164.1	502.1	135,9
6.50	66.4	28.3	10.8	3.20	58.2	87.7	14.7
7.00	30.2	8.8	3.4	3.40	19.6	23.4	5,6
7.50	11,0	3.1	1,1	3.60	6,3	6,0	2.2

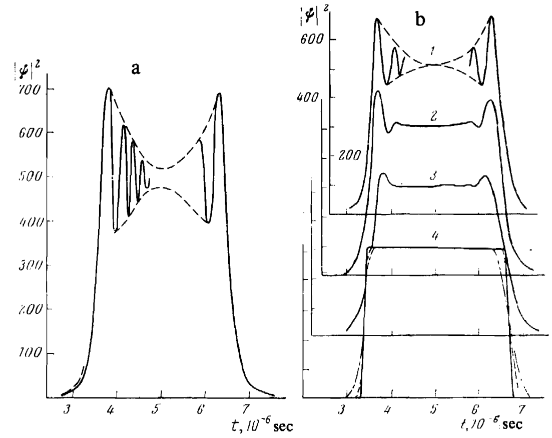

FIG. 1. Dependence of the square of the modulus of the wave function of UCN on the time t after the vanishing of the barriers at $x=3a=1.5\cdot10^{-3}$ cm, $\bar{v}=3$ m/sec. a) Calculation by formulas (17) (solid curve), and (19) (dashed), b) result of averaging curve a over the velocities and the thickness of the detector (the latter is 3×10^{-5} cm): 1) $\Delta v/\bar{v}=0.01$; 2) $\Delta v/\bar{v}=0.03$; 3) $\Delta v/\bar{v}=0.06$; 4) result of a similar averaging under the assumption of classical motion of the UCN (solid curve $\Delta v/\bar{v}=0.01$, dashed $\Delta v/\bar{v}=0.03$, dash-dot $\Delta v/\bar{v}=0.06$).

$$\psi=\psi_1(x,t)=\frac{a^{-1/2}}{2}\left[e^{ip_0(x-a)}-e^{-ip_0(x-a)}\right]\exp\left(-i\frac{p_0^2t}{2}\right)\theta(a-|x|), \tag{15}$$

with $p_0a=\pi n_0$, where n_0 is an integer. In this case we have in place of (5) and (8)

$$A(p)=(2\pi a)^{-1/2}e^{i\pi n_0}\left\{\frac{\sin(p-p_0)a}{p-p_0}-\frac{\sin(p+p_0)a}{p+p_0}\right\}, \tag{16}$$

$$\psi_2(x,t)=\frac{1}{4\sqrt{a}}\exp\left(-i\frac{p_0^2t}{2}\right)\{[F(z_{+-})-F(z_{--})]e^{ip_0(x-a)}$$
$$-[F(z_{++})-F(z_{-+})]e^{-ip_0(x-a)}\}, \tag{17}$$

$$z_{+\pm}=(2t)^{-1/2}(x+a\pm p_0t),\quad z_{-\pm}=(2t)^{-1/2}(x-a\pm p_0t). \tag{18}$$

At $a\gg\sqrt{2t}$, this expression coincides with ψ_2 for one barrier (with x replaced by $x-a$ at $x>0$ and x replaced by $x+a$ at $x<0$). In particular, at $z_{--}\gg 1$ we have

$$\psi_2=\frac{1+i}{2(\pi a)^{1/2}}\frac{p_0t^{1/2}}{(x-a)^2-p_0^2t^2}\exp\left[i\frac{(x-a)^2}{2t}\right]. \tag{19}$$

In the general case the analysis of (17) is similar to that described above.

Table I lists the values of $|\psi_2(x,t)|^2$ as functions of t for different values of x, p_0, and a. Fig. 1a shows a typical plot of this function, and also a plot of $|\psi_2(x,t)|^2$, calculated from the approximate formula (19). It is quite easy to solve the problem of the vanishing of the barrier or of two parallel barriers under certain other boundary conditions, for example in the case when the wave function of the neutron must vanish at certain points after the vanishing of the barrier. We shall not, however, describe the corresponding formulas since they do not lead to physically new results.

We turn now to the case of "instantaneous vanishing of an infinitely high barrier," i.e., we seek a solution of Eq. (1) at $U_0\to\infty$, $\varepsilon\to 0$, $U_0\varepsilon^2\to\infty$, $f(t)=\theta(t)$ with the initial condition

$$\psi(x,0)=\psi_1'(x,0),\quad \psi_1'(x,t)=\exp(ip_0x-ip_0^2t/2).$$

We note first that the antisymmetrical part of the wave function is not perturbed at all by the appearance of such a barrier, i.e., at all values of t we have

$$\psi_a(x,t)=\frac{1}{2}[\psi(x,t)-\psi(-x,t)]=\frac{1}{2}\exp\left(-i\frac{p_0^2t}{2}\right)[e^{ip_0x}-e^{-ip_0x}]. \qquad (20)$$

Next, recognizing that at $t>0$ the symmetrical part of the wave function should vanish at the boundary of the barrier (at $x=0$), we can seek in the form

$$\psi_s(x,t)=\frac{1}{2}[\psi(x,t)+\psi(-x,t)]=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty B(p)\sin(p|x|)\exp\left(-i\frac{p^2t}{2}\right)dp, \qquad (21)$$

$$B(p)=\sqrt{\frac{2}{\pi}}\int_0^\infty dx\sin px\cos p_0x=\frac{1}{\sqrt{2\pi}}\mathrm{P}\left(\frac{1}{p-p_0}+\frac{1}{p+p_0}\right). \qquad (22)$$

Accordingly

$$\psi_s(x,t)=\frac{1}{\pi}\mathrm{P}\int_0^\infty\sin(p|x|)\exp\left(-i\frac{p^2t}{2}\right)\left(\frac{1}{p-p_0}+\frac{1}{p+p_0}\right)dp$$
$$=\frac{1}{2\pi i}\mathrm{P}\int_{-\infty}^\infty\frac{dp}{p-p_0}\exp\left(-i\frac{p^2t}{2}\right)(e^{ip|x|}-e^{-ip|x|})$$
$$=\frac{1}{2}\exp\left(-i\frac{p_0^2t}{2}\right)\left[e^{ip_0|x|}F\left(\frac{|x|-p_0t}{(2t)^{1/2}}\right)+e^{-ip_0|x|}F\left(\frac{|x|+p_0t}{(2t)^{1/2}}\right)\right]. \qquad (23)$$

And finally

$$\psi(x,t)=\frac{1}{2}\exp\left(-i\frac{p_0^2t}{2}\right)\left\{e^{ip_0x}-e^{-ip_0x}+F\left(\frac{|x|-p_0t}{(2t)^{1/2}}\right)e^{ip_0|x|}+F\left(\frac{|x|+p_0t}{(2t)^{1/2}}\right)e^{-ip_0|x|}\right\}. \qquad (24)$$

In this expression, the term proportional to e^{-ip_0x} at $x>0$ and the term proportional to e^{ip_0x} at $x<0$ are the same as in (8), while the term proportional to $\exp(ip_0x)$ at $x>0$ and the term proportional to e^{-ip_0x} at $x<0$ coincide with the same term in (8) if we replace x in the latter by $-x$. We shall not present the results of the detailed analysis of expression (24). We note only the following relation, which we shall need later on: at $|x|\ll\sqrt{2t}$ we have an essentially nonstationary wave function

$$\psi(x,t)=i\exp\left(-i\frac{p_0^2t}{2}\right)\left[\sin p_0x-\sin p_0|x|F\left(p_0\left(\frac{t}{2}\right)^{1/2}\right)\right]+\frac{(1-i)|x|}{(\pi t)^{1/2}}\cos(p_0x). \qquad (25)$$

In particular, at $p_0\ll(2/t)^{1/2}$

$$\psi(x,t)=ip_0x+(1-i)|x|(\pi t)^{-1/2}, \qquad (26)$$

and at $p_0\gg(2/t)^{1/2}$

$$\psi(x,t)=i\exp(-ip_0^2t/2)\{x-|x|\}p_0+p_0|x|O(1/p_0^2t). \qquad (27)$$

Plots of $|\psi(x,t)|^2$ against t at certain values of x and $v_0=p_0/m=1$ m/sec, calculated with the aid of the exact formula (24) and the approximate formula (25), are shown in Fig. 2.

The conditions for the applicability of expression (4) are obviously the same as in the case of fast vanishing of the barrier.

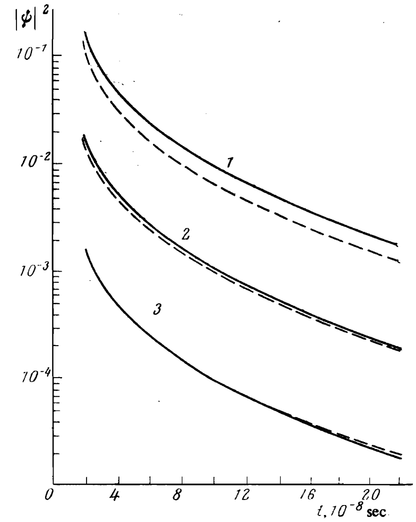

FIG. 2. Dependence of the square of the modulus of the wave function of UCN on the time t after the vanishing of the barrier, at a distance x from the barrier at a velocity $v=1$ m/sec: 1) $x=3\cdot10^{-6}$ cm; 2) $x=10^{-6}$ cm; 3) $x=3\cdot10^{-7}$ cm. The solid curves were calculated from the exact formula (24) and the dashed curves from the approximate formula (25).

3. INTERACTION OF UCN WITH A WEAKLY OSCILLATING ONE-DIMENSIONAL POTENTIAL

We consider the solution of the nonstationary Schrödinger equation in the case of a one-dimensional potential

$$i\frac{\partial\psi}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+[U_0(x)+U_1(x,t)]\psi, \qquad (28)$$

when the time-dependent part of the potential U_1 is small, so that we can confine ourselves to the first order of perturbation theory in U_1. We can then represent ψ in the approximate form

$$\psi=\psi_0(x)e^{-iE_0t}+(2\pi)^{-1/2}e^{-iE_0t}\int dx'\int d\Omega g_{E_0+\Omega}(x,x')e^{-i\Omega t}U_\Omega(x')\psi_0(x'),$$
$$U_\Omega(x)=(2\pi)^{-1/2}\int_{-\infty}^\infty dt\,e^{i\Omega t}U_1(x,t), \qquad (29)$$

where ψ_0 and E_0 are the wave function and energy of the unperturbed stationary state, and the function $g_E(x,x')$ is a solution of the equation (likewise stationary)

$$\left[E+\frac{1}{2}\frac{\partial^2}{\partial x^2}-U_0(x)\right]g_E(x,x')=\delta(x-x'). \qquad (30)$$

Let[2)] $U_0(x)\to0$ as $x\to-\infty$ and $U_0(x)>E_0$ as $x\to\infty$. Then $g_E(x,x')$ is conveniently represented in the form

$$g_E(x,x')=\frac{1}{ip}[\theta(x-x')\varphi_p(x)\psi_p(x')+\theta(x'-x)\varphi_p(x')\psi_p(x)],$$

where the functions ψ_p and φ_p are solutions of the homogeneous equations corresponding to (30),

$$\psi_p(x)\xrightarrow[x\to-\infty]{}e^{-ipx},\quad \varphi_p(x)\xrightarrow[x\to\infty]{}0,\quad \varphi_p(x)\xrightarrow[x\to-\infty]{}2\cos(px-\delta_p),\quad p=(2E)^{1/2}$$

(δ_p is a constant). We note that apart from normalization we have $\psi_0(x)=\varphi_{p0}(x)$. In particular, when $-x$ is large enough, so that $U_0(x)\ll p_0^2/2$, we have

$$\psi(x,t)=2\exp\left(-i\frac{p_0^2t}{2}\right)\cos(p_0x-\delta_{p_0})$$
$$+\frac{1}{i(2\pi)^{1/2}}\int_0^\infty dp\exp\left(-i\frac{p^2t}{2}-ipx\right)M(p,p_0),$$
$$M(p,p_0)=\int_{-\infty}^{\infty}dx\,U_\Omega(x)\varphi_p(x)\varphi_{p_0}(x),\quad \Omega={}^1/_2(p^2-p_0^2). \tag{31}$$

We note that the matrix element $M(p, p_0)$ is symmetrical with respect to the substitution $p\rightarrow p_0$, as it should be in accordance with the detailed-balancing principle. In addition, since the reflected waves with different p do not interfere with one another (as $x\rightarrow-\infty$), the probabilities of reflection of UCN with different momenta can be considered independently. Accordingly, without loss of generality, we can assume that $U_1(x,t)$ depends harmonically on the time:

$$U_1(x,t)=\tilde{U}_1(x)\cos\omega t. \tag{32}$$

The relations (31) then take the form

$$\psi=\varphi_{p_0}(x)\exp(-ip_0^2t/2)+\tilde{M}(p_+,p_0)\exp(-ip_+^2t/2-ip_+x)$$
$$+\tilde{M}(p_-,p_0)\exp(-ip_-^2t/2-ip_-x),$$
$$\tilde{M}(p,p_0)=\frac{1}{2p}\int_{-\infty}^{\infty}dx\,\tilde{U}_1(x)\varphi_p(x)\varphi_{p_0}(x),\quad p_\pm=(p_0^2\pm2\omega)^{1/2}. \tag{33}$$

If the potential varies sufficiently slowly with the coordinate, so that $|\partial U_0/\partial x|\ll p_0^2$, then $\tilde{M}(p,p_0)$ can be calculated in the quasiclassical approximation (see, e.g., [3]):

$$\tilde{M}(p,p_0)=-2\,\mathrm{Im}\int_C dx\,U_1(x)\left[\frac{2U_0(x)}{p_0^2}-1\right]^{-1/4}\left[\frac{2U_0(x)}{p^2}-1\right]^{-1/4}$$
$$\times\exp\left\{\int_x^{a_1}(2U_0(x')-p_1^2)^{1/2}dx'-\int_x^{a_2}(2U_0(x')-p_2^2)^{1/2}dx'\right\}, \tag{34}$$

where a_i is the root of the equation $2U_0(a_i)=p_i^2$, with p_2^2 and p_1^2 respectively the larger and smaller values of p_0^2 and p^2, while the contour C passes in the upper half-plane below all the singular points of the potentials U_0 and U_1 (it is assumed that neither U_0 nor U_1 has singular points on the real axis at finite values of x). In particular, if near the turning point

$$|p-p_0|\ll\frac{1}{U_0}\left|\frac{\partial U_0}{\partial x}\right|, \tag{35}$$

then

$$\tilde{M}(p,p_0)\approx2\int_{-\infty}^{a}dx\,U_1(x)[1-2U_0(x)p_0^{-2}]^{-1/2}. \tag{36}$$

We note that, as seen from this formula, $\tilde{M}(p,p_0)$ depends essentially on the behavior of $U_0(x)$ and $U_1(x)$ in all of space, and not only near the turning point of the classical motion, as might appear at first glance.

For certain concrete dependences of U_0 and U_1 on x, the matrix element $\tilde{M}(p,p_0)$ can be calculated exactly (see, e.g., [4]). In particular, in the case of practical importance[3)]

$$U_0(x)={}^1/_2q^2e^{x/a},\quad \tilde{U}_1(x)=\lambda U_0(x) \tag{37}$$

(q, a, and λ are constants), elementary calculations yield

$$\varphi_p(x)=2\left[\frac{2}{\pi}pa\,\mathrm{sh}(2\pi pa)\right]^{1/2}K_{2ipa}(2qae^{x/2a}),$$
$$\tilde{M}(p_\pm,p_0)=\lambda\left[\frac{p_0\,\mathrm{sh}(2\pi p_\pm a)\,\mathrm{sh}(2\pi p_0a)}{p_\pm\,\mathrm{sh}^2[\pi a(p_\pm+p_0)]}\right]^{1/2}\frac{p_\pm+p_0}{2}a\frac{\pi(p_\pm-p_0)a}{\mathrm{sh}[\pi a(p_\pm-p_0)]}. \tag{38}$$

$K_\nu(z)$ are Macdonald functions. Hence, at $\omega\ll p_0^2$ and $a\omega\ll p_0$ (i.e., if the condition (35) is satisfied we have $\tilde{M}(p_\pm,p_0)=\lambda p_0a$, which agrees with calculation by formula (36).

The previously considered[1] oscillating flat wall, i.e., the potential of the type

$$U(x,t)={}^1/_2q^2\theta(x-a\cos\omega t) \tag{39}$$

can also be described in first-order perturbation theory in a by an expression of the type (33), where

$$\varphi_p=2\cos(px+\delta_p)\theta(-x)+\frac{2p}{q}e^{-\varkappa x}\theta(x);$$
$$\varkappa=(q^2-p^2)^{1/2},\quad \mathrm{tg}\,\delta_p=\varkappa/p,\quad \tilde{M}(p_\pm,p_0)=p_0a. \tag{40}$$

4. DISCUSSION OF RESULTS AND CONCLUSIONS

Let us examine certain possibilities of experimentally investigating the behavior of UCN in a nonstationary force field. First, taking into account the nonstationary character of the wave function of the UCN, it is necessary to refine the principles of the methods used for their detection. We shall assume that the experimentor has at his disposal the following two types of ideal detectors: 1) a detector of the UCN density,[4)] which makes it possible to measure $|\psi(x,t)|^2$; 2) a detector of the momentum distribution of the UCN,[5)] which makes it possible to measure the square of the modulus of the Fourier transform of the form of the wave function.

In experiments in which a high potential barrier appears and vanishes, both types of detectors can be used in principle. We consider first the case of registration of $|\psi(x,t)|^2$.

As seen from formulas (7)–(13) and (17)–(19), one of the most interesting experiments is the investigation of $|\psi(x,t)|^2$ at a sufficiently large distance from the barrier at relatively short times after its vanishing, when[6)] $x-a-v_0t\gtrsim(2\hbar t/m)^{1/2}$. In this region, according to the laws of classical mechanics, there should be no UCN at all.

Formulas (7)–(13) and (17)–(19) were derived under the assumption that the initial wave function (at $t\leqslant0$) is stationary (it takes respectively the form (4) and (15)). If at $t\leqslant0$ the wave functions of the UCN are superpositions of stationary wave functions, a simple generalization of the results for $|\psi(x,t)|^2$ (averaging the plots of Fig. 1 or the data of Table I over the initial distribution) is possible only in the case when the relative phase shifts of all the wave functions constituting the initial state are statistically independent. It is relatively easy to realize such an initial state in experiments with two barriers. It is necessary for this pur-

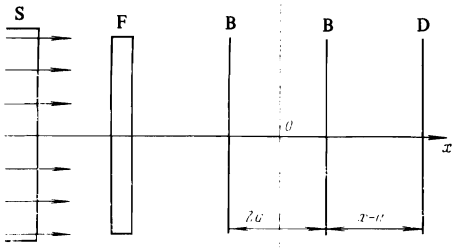

FIG. 3. Diagram of a possible experiment with vanishing of two barriers.

pose that the UCN stay between the barriers for a time $\tau_0 \gtrsim 4a/\bar{v}$, where $\bar{v}$ is the average value of the neutron velocity component normal to the barrier.

Thus, it is natural to propose the following experimental setup (Fig. 3). The stream of UCN from the source S passes through a system of filters[7] F, which cuts out a line which is narrow with respect to the normal velocity component in the interval Δv, and also with respect to time (in order to decrease the background), and falls into the gap between the barriers BB at the instant when the gaps are produced. The UCN trapped in this space are confined there for a time τ_0, and then, at the instant $t=0$, the barriers BB vanish and the time dependence of the function $|\psi|^2$ at the location of the detector D is registered. (It is possible to use for this purpose, a set of different detectors located at different x.) It is necessary here to satisfy the following conditions. The thickness of the detector D should be low enough so that the averaging of $|\psi(x, t)|^2$ over the volume of the detector does not affect strongly the time dependence of $|\psi|^2$ shown in Fig. 1a. It is obvious that the second barrier and the detector must be parallel with the same accuracy. For this reason, velocity scatter Δv should be small. Figure 1b shows the results of averaging of the plot of Fig. 1a for the case $\bar{v}=3$ m/sec and $x=3a=1.5\times10^{-3}$ cm under the assumption that the detector thickness is 3×10^{-5} cm and that the velocities have a Gaussian distribution with mean square deviations $\Delta v/v$ respectively equal to 0.01, 0.03, and 0.06.

The corresponding experiment can be carried out in a cyclic regime with frequency $\nu=1(\tau_0+\tau_1)$, where τ_1 is the time during which there are no barriers.

By way of example we consider the following case: $2a=10^{-3}$ cm, $x-a=10^{-3}$ cm, $\tau_0=\tau_1=5\times10^{-6}$ sec (i.e., $\nu=10^5$ Hz), $v=300$ cm/sec, $\Delta v/\bar{v}=0.03$, and the detector thicknesses $\Delta x=3\times10^{-5}$ cm. Then, at a total UCN flux to the filters $F=100$ neut/cm^2 sec, a minimum cross section area of the system $S=50$ cm^2, and a neutron-detector registration efficiency $\mu=0.1$, the total number of detector counts per second is

$$N=baFS\nu\mu\Delta v/\bar{v}^2,$$

where b is a numerical factor on the order of unity, which depends on the form of the UCN spectrum, and takes also into account the UCN losses in the course of formation of barriers and in the passage of the filters. Therefore, putting $b\approx0.4$, we obtain $N=0.1$ neut/sec.

This experiment can be modified by using a beam of cold or even thermal neutrons that are well collimated in one direction and are at a grazing angle to the installation considered above. It is then possible to obtain a much higher neutron density accumulated between the barriers, owing to the appreciable broadening of the admissible region of variation of the neutron velocities in the plane of the barriers.

Another type of experiment with registration of the UCN density can consist of measuring $|\psi(x, t)|^2$ as a function of the time when one barrier is produced. However, as seen from Fig. 2, the effects of interest to us, come into play at very short distances from the barrier ($\sim10^{-6}$ cm) and at very short times after the instant of the production of the barrier ($\sim10^{-7}$ sec),[8] so that this type of experiment (at least at first glance) seems more difficult to realize than the above-considered experiment with vanishing of two barriers.

We consider now the case of registration of the momentum distribution of the UCN. As seen from (16), the momentum distribution of ($|A(p)|^2$) in an experiment in which two barriers vanish oscillates rapidly (with a relative period $\Delta p/p\sim1/n_0=\pi\hbar/p_0a$, which amounts to 6.6×10^{-3} at $\bar{v}=300$ cm/sec and $a=10^{-3}$ cm), and that these oscillations are difficult to observe. However, the momentum distribution averaged over these oscillations, which takes the form

$$\overline{|A(p)|^2}=1/4\pi a(p-p_0)^2,$$

seems to be readily amenable to experimental analysis.

Finally, we consider the possibility of experimentally observing the interaction of UCN with a weakly oscillating field. The problem considered in Sec. 3 corresponds to the case of registration of neutrons at a large distance from the reflecting potential. Therefore observation of the time dependence of the UCN density (for example, the shift of its phase relative to the phase of the oscillation of the potential) is of no interest under such conditions.

The action of an oscillating potential on the pulsed distribution is easiest to investigate in experiments on the reflection of UCN from a flat oscillating wall. To increase the intensity we can use a beam of high-velocity neutrons incident on the reflecting wall at a grazing angle. Thus, at a grazing angle $\sim1°$ and a neutron velocity $v\sim50$ m/sec (i.e., at $v_\perp\sim1$ m/sec), one should expect a noticeable effect at $\omega\sim10^7$ sec^{-1} and $a\sim10^{-6}$ cm.

It is interesting to trace in similar experiments the transition from the pure quantum case $pa\ll\hbar$ to the classical case $pa\gg\hbar$. In the former case the scattering-induced change in the neutron energy is $\pm\hbar\omega$ (in first-order perturbation theory); with further increase, the divergence of the perturbation theory becomes worse; finally, in the limit $pa\gg\hbar$, the change of the neutron energy is determined by the velocity of the wall at the instant of scattering.

For potentials that decrease slowly with distance, their weak oscillations lead to an essentially lower probability of the inelastic scattering than in the case of a steplike potential,[9] as seen, for example, from formula (38). The effect then decreases exponentially with increasing frequency and in real cases it can be noted only at $\omega \sim 10^3$ sec^{-1}, corresponding to small changes of the energy. Therefore the experimental observation of the inelastic interaction of UCN with such potentials is much more complicated. Nonetheless, the effect of heating of the UCN when reflected from an oscillating (as a result of current pulsations) magnetic wall must be taken into account when designing magnetic traps. As follows from the foregoing, the most dangerous here are the low frequencies.

In conclusion, the authors are sincerely grateful to Yu. G. Abov, B. G. Erozolimskiĭ, V. K. Ignatovich, L. A. Kirzhnits, P. A. Krupchitskiĭ, O. A. Langer, V. M. Lobashev, V. I. Lushchikov, V. G. Nosov, M. I. Padgoretskiĭ, A. V. Stepanov, A. V. Strelkov, and I. M. Frank for interest in the work and for stimulating discussions.

1)Such a barrier can be realized, for example, by placing a ferromagnetic foil in an alternating magnetic field parallel to the surface of the foil.

2)A particular case of such a potential, namely a rectangular potential threshold, was discussed by us earlier in an analysis of the heating of UCN by reflection from a weakly oscillating wall of a trap.[1] A more general dependence of U_0 on x can be produced for example, with the aid of a magnetic field from specially arranged current-carrying conductors.[2]

3)Vladimirskiĭ has shown[2] that such a potential can be realized in actual constructions of magnetic traps.

4)For this purpose, for example, we can register the absorption of the UCN in a thin foil that perturbs weakly the wave function of the neutron. In order for the registration efficiency to be independent of the UCN momentum, the foil material must have a near-zero coherent neutron scattering length.

5)Inasmuch as in all the examples considered above the momentum distribution of the UCN that are acted upon by the nonstationary potential is subsequently independent of the time, it is possible to use for these measurements, for example, the method of spatial separation of UCN beams moving with different momentum components perpendicular to an equipotential plane and having equal but nonzero momentum components in this plane. It is convenient to direct the primary UCN beam at a grazing angle with the equipotential plane.

6)We use here the customary units. Accordingly, p_0 is replaced by the UCN velocity $v_0 = p_0/m$, and z is multiplied by $(\hbar/m)^{1/2}$.

7)For example, interference filters.

8)Therefore, in particular, the major role may be assumed by effects connected with the non-ideal character of the barrier (finite height and width) and the specific features of its real structure (in the case of a magnetic barrier, for example, by the character of the behavior of the magnetic field near the barrier).

9)This is natural, for the effective time of interaction of the UCN with the wall increases in this case.

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Translated by J. G. Adashko

A canonical description of multiquantum resonance interactions of radiation with matter

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(Submitted March 10, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1712–1725 (November 1976)

It is shown that the response of matter in multiphoton resonance interactions with light involving an arbitrary number of energy levels can be described by the Neumann equation with a certain effective Hamiltonian $\mathscr{V}$ in the right-hand side, and with off-diagonal matrix elements that do not vanish for resonant transitions. The explicit form of this effective Hamiltonian is found. The polarization of the matter can be determined, with allowance for the effects of saturation and the Stark shifts of the levels, with the aid of $\mathscr{V}$ and a generalized dipole moment D that depends on the amplitudes and phases of the interacting fields. As a result, the description of complex resonance interactions of matter with strong fields, including coherent processes, is much simplified. The following matters are treated as examples: induced transition probabilities for an arbitrary number of resonances, the stationary nonlinear susceptibility of a molecule in incoherent and coherent multiphoton processes, and the nonlinear dielectric constant that arises in q-photon scattering of an ultrashort pulse.

PACS numbers: 42.65.—k

Among the multitude of nonlinear optical phenomena observed in recent years, one can distinguish a large group of resonance processes that take place when conditions of the following type are satisfied:

$$\sum_j n_j\omega_j = \omega_{mn} + \nu_s, \tag{1}$$

where ω_{mn} are the resonance frequencies of the matter,

