Nonequilibrium stationary distributions of particles in a solid body plasma

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A direct calculation of the Boltzmann collision integral for power-law distribution functions $f(p) = \alpha|I|^{1/2}P^{2s}$ (I is the energy or particle flux in momentum space, p is the momentum, and α is a constant) shows that the integral possesses a first-order zero for exponents s that correspond to the solutions of the stationary homogeneous kinetic equation. An explicit expression is obtained for the energy (particle) flux and the direction of the flux in momentum space is determined. The analytic dependence of the coefficient α on s and n (n is the degree of homogeneity of the transition probability) is also determined. The regions of existence of power-law distribution functions in momentum space are found for particles the interaction between which can be described by a screened Coulomb potential. The experimental results on the current and energy of the electron emission induced by intense laser radiation in a metal foil are explained by invoking a nonequilibrium particle distribution. A comparison of the theoretical and experimental results on the current and its dependence on the retarding potential indicates that they are best explained by using nonequilibrium power-law distributions.

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1. INTRODUCTION

In investigations of non-equilibrium stationary particle-energy distributions, the non-equilibrium additive is usually regarded as small in comparison with the main equilibrium distribution. However, in Kolgomorov's paper on the theory of turbulence spectra it is shown that a stationary non-equilibrium distribution of waves in an inertial interval satisfies a power law and differs strongly from the equilibrium distribution. In the theory of weak turbulence, Zakharov[2] (see also[3–7]) obtained power-law spectra that lead to vanishing of the integral of the collisions between the waves. The question of the power-law spectra of weak and strong turbulence was considered in detail in a review by Kadomtsev and Kontorovich.[8] Kats *et al.*[9] have shown that for particles that can exist stationary power-law distributions $f = \alpha|I_i|^{1/2}p^{\beta s}$, which cause the vanishing of the Boltzmann collision integral

$$s_i = -\frac{n+9+\beta(i-1)}{2\beta}, \quad (i=0\ 1), \tag{1}$$

where $I_{1(0)}$ is the energy (particle) flux in momentum space, p is the momentum, and n is the degree of homogeneity of the transition probability. The simpler problem of the relaxation of a small fraction of electrons against the main background (in this case the collision integral can be linearized), when account is taken of the ionization and recombination processes, was approximately considered earlier[10] (see also the related problem of the distribution of neutrons in crystals[11]). In these cases, the distribution function can differ noticeably from Maxwellian, but its form depends strongly on the structure of the source and of the sink. To the contrary, in the paper of Kats *et al.*[9] and in the present paper the question considered is that of formation, in the presence of a source and sink, of a nonequilibrium stationary distribution of an arbitrary number of particles (in this case it is impossible to linearize the collision integral).[1)] Such distributions constitute an analog of the Kolgomorov turbulence spectrum. Usually in the analysis of power-law solutions of the kinetic equations for waves (particles) one uses the method of group symmetry in the space of the frequencies ω,[3–5] wave vectors k,[6,7] and particle momenta p.[9] But symmetry considerations do not make it possible to determine the constant α in the distribution, the flow direction, or the region of existence of the power-law distributions.

On the other hand, direct calculation[2] has yielded an exact expression for the collision integral for waves with a power-law distribution function as a function of the exponent s, and the integral of the exponent s at which the integral vanishes was identified. For acoustic turbulence, the energy flux in k-space was calculated directly.[3]

It is shown in the present paper that the Boltzmann collision integral for power-law distributions $f(p)$ can be represented in explicit form as a function of the momentum p and of the exponent s. It turns out that this approach can yield rather abundant information. Thus, in particular, it was found that the collision integral has a first-order zero for exponents s corresponding to solutions of the stationary homogeneous kinetic equation. Using this, it is possible to obtain an explicit expression for the energy (particle) flux in momentum space, its direction, and also the proportionality coefficient α between the flux and the normalization constant A that determines the particle density. It is important also that it is possible to find the regions of existence of the power-law solutions in problems with a screened Coulomb interaction.

We note that the question of the regions of homogeneous asymptotic forms of a transition probability that is not homogeneous in the momenta is closely connected with the question of the existence of power-law distributions of the type $f = Ap^{2s}$.[2)] It has been shown[9] that for distributions of particles with constant energy flux in momentum space, in the case of a Coulomb interaction

($s=-\frac{5}{4}$), the main contribution to the relaxation is made by particles with comparable momenta, i.e., in other words, there exists in inertia interval in which the influence of the source and of the sink is insignificant (this property, in analogy with turbulence theory is called locality). The natural assumption was made[9] that the Coulomb divergence is eliminated by the Debye screening. Our analysis of the regions of the existence of power-law distributions for the inhomogeneous transition probability confirms, in particular, this assumption (see Sec. 4).

We note that in the present paper, by virtue of the method used by us to calculate the collision integral, it turned out to be convenient to investigate simultaneously the possibility of its divergences due to collisions with particles belonging to edges of the inertial interval, as well as those due to singularities of the transition probability. A number of important consequences that follow from such a radical change in the character of the distribution of the particles in energy and which have important practical significance (the influence exerted on the criterion of the positive energy yield in problems of controlled thermonuclear fusion, astrophysical applications, Landau damping, etc.) has been pointed out earlier.[9] The existence of such non-equilibrium stationary distributions presupposes the presence in momentum space of a source and sink of particles or of energy. This situation obtains in laser and microwave plasma heating, in the formation of secondary electrons by a powerful electron beam, when thermonuclear reactions take place in a plasma, etc. A plasma, however, is a highly unstable medium, in which collective processes play a particularly important role and can mask the action of collisions between particles. More attractive from this point of view are solids, where it is possible to control the disequilibrium of the system in the stable regime. In these substances, the energy (particle) source and sink can be produced by high-power laser radiation, by emission currents, or by natural or induced radioactivity.

One of the tasks of the present paper is to determine the non-equilibrium power-law distributions of electrons in solids and use them to explain certain features of the emission current from a metallic foil.

2. BOLTZMANN COLLISION INTEGRAL FOR ISOTROPIC POWER-LAW DISTRIBUTIONS

It is known (see, e.g., [12]) that the collision integral for particles of the same sort can be represented, in the case of classical statistics, in the form

$$\left(\frac{\partial f}{\partial t}\right)_{st}=\int d\mathbf{p}_1\,d\mathbf{p}_2\,d\mathbf{p}_3\,W(\mathbf{p},\mathbf{p}_1|\mathbf{p}_2,\mathbf{p}_3)\,[f_2f_3-ff_1]\times\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)\,\delta(E+E_1-E_2-E_3). \tag{2}$$

where $f_i=f(\mathbf{p}_i)$, $W(\mathbf{p},\mathbf{p}_1|\mathbf{p}_2,\mathbf{p}_3)$ is the probability of the transition as a result of collisions, $\mathbf{p}_i$ is the momentum, and $E_i=p_i^2/2m^*$ is the energy. We describe below a procedure for directly calculating the collision integral for an isotropic power-law distribution function $f=Ap^{2s}$. With the aid of a δ-function that expresses the momentum conservation law, we integrate (2) with respect to $\mathbf{p}_2$, and then introduce in place of $\mathbf{p}_1$ and $\mathbf{p}_3$ new variables $\mathbf{p}_1$ and $\mathbf{q}$, and reduce the collision integral to the form

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-m^*A^2\int d\mathbf{q}\,d\mathbf{p}_1\,W(\mathbf{p},\mathbf{p}_1|\mathbf{p}+\mathbf{q},\mathbf{p}_1-\mathbf{q})\times[|\mathbf{p}+\mathbf{q}|^{2s}|\mathbf{p}_1-\mathbf{q}|^{2s}-p^{2s}p_1^{2s}]\,\delta(q(p_1-p-q)), \tag{3}$$

where $\mathbf{q}=\mathbf{p}_1-\mathbf{p}_3$. The argument of the function in (3) can vanish at $\mathbf{p}_1-\mathbf{p}-\mathbf{q}=0$, $\mathbf{q}=0$ and $\mathbf{q}(\mathbf{p}_1-\mathbf{p}-\mathbf{q})=0$. The first case is of no interest, since it corresponds merely to the interchange of the positions of the particles as a result of collisions, so that the square bracket vanishes, meaning also $(\partial f/\partial t)_{st}$. By introducing the angles θ and θ_1 between the vector $\mathbf{q}$ and the vectors $\mathbf{p}$ and $\mathbf{p}_1$, respectively, changing over to spherical coordinates in (3) for $\mathbf{p}_1$ and $\mathbf{q}$, integrating with the aid of the δ-function with respect to p_1, and changing to the dimensionless variable $\tilde{q}(\tilde{q}=q/p)$, we obtain

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-m^*A^2p^{4s+r+1}\int_0^\infty d\tilde{q}\,\tilde{q}\int do\int do_1\,W\frac{1}{|\cos\theta_1|}\left(\frac{\cos\theta+\tilde{q}}{\cos\theta_1}\right)^2\times\left[|1+2\tilde{q}\cos\theta+\tilde{q}^2|^s\left|\left(\frac{\cos\theta+\tilde{q}}{\cos\theta_1}\right)^2-2\tilde{q}(\cos\theta+\tilde{q})+\tilde{q}^2\right|^s-\left|\frac{\cos\theta+\tilde{q}}{\cos\theta_1}\right|^{2s}\right], \tag{4}$$

where r is the dimensionality of the transition probability and $do_i=\sin\theta_i d\theta_i d\varphi_i$.

Let us find the particle flux I_0 and the energy flux I_1 in momentum space, recognizing that in this case the fluxes are expressed in terms of the collision integral in the following manner

$$\operatorname{div}(j_i(p)\mathbf{p}/p)=-E^i(\partial f/\partial t)_{st}, \tag{5}$$

where $I_i=4\pi p^2 j_i$. For W, which is a homogeneous function of degree n in the momenta, namely $W=c_1q^n$, where n is a real number (and $n=r$), a particular solution of (5) with allowance for (4) is (see the Appendix)[3)]

$$I_i=A^2(2m^*)^{1-i}p^{4s+n-7-2i}\frac{R(s,n)}{4s-n-7-2i}. \tag{6}$$

As seen from (5), if s_i satisfies the condition

$$\gamma=4s-n-7-2i=0^{4)}, \tag{7}$$

then the flux I_i differs from zero and is constant in momentum space, while the other flux is equal to zero if $R(s,n)$ has a first-order of zero at the point s_i (in this case the collision integral (4) is equal to zero). Consequently the distribution function $f=Ap^{2s}$ corresponds to a nonequilibrium stationary situation with a constant flux of energy or particles. The direction of the flux is determined by the sign of the derivative $dR/d\gamma$ at the point $\gamma=0$, and the quantity A is connected with the flux intensity by the expression

$$A=\alpha|I|^{1/2},\quad \alpha_i^2=(2m^*)^{i-1}\lim_{\gamma\to0}|dR/d\gamma|^{-1}. \tag{8}$$

It should be borne in mind (see (6)) that the direction of the particle flux (energy flux) depends essentially on the form of the transition probability W, and these fluxes can be either mutually opposite in momentum space, or of the same direction. As follows from the expression for $R(s,n)$ (see the Appendix), it consists in the general case of terms that contain a first-order zero at the point $\gamma = 0$, and also of divergent terms (for the conditions under which there are no such terms, see below). The nature of the diverging term is twofold. First, it can be connected in a number of cases with the influence of the collisions between the considered particles and the particles that have small ($\mathbf{p}_i \to 0$) and large ($\mathbf{p}_i \to \infty$) momenta. Second, the divergence can be due to singularities of the transition probability (in Sec. 2 are indicated the inhomogeneity exponents of the transition probability, at which divergences of the first or second type take place). In the present paper (see Sec. 4), using as a concrete example a screened Coulomb interaction, we show how to eliminate divergences of the second type, thus indicating that the earlier assumption[9] concerning the role of the Debye screening is natural; we emphasize that, for example, a distribution with a constant energy flux $s_1 = -\frac{5}{4}$ has no divergences of the first type. [9]

It should be noted that we are as yet unable to propose a regular procedure for the elimination of the divergences of the first type, so that the question of the existence of distributions that are nonlocal remains open in this sense. Equation (4) leads directly to convergence conditions that ensure locality of the power-law distribution, and these conditions coincide with those obtained earlier.[9] For convenience, we present these conditions expressed in terms of the degree of homogeneity of the transition probability: in collisions with immobile particles, or else when the particles are stopped after the collisions ($\theta \to \pi/2$) we have

$$n<-1, \quad n<-3.$$

In collisions with particles having very large momenta ($\theta_1 \to \pi/2$) we have

$$n>-3, \quad n>-5$$

(here, and below, the first of the inequalities pertains to the distributions with constant I_0, and the second to those with constant I_1).

In addition, in order for (4) to be convergent it is necessary to stipulate also satisfaction of additional conditions connected with singularities of the transition probability:

at small momentum transfers ($\tilde{q} \to 0$)

$$n>-4, \quad n>-4,$$

at very large momentum transfers ($\tilde{q} \to \infty$)

$$n<3, \quad n<5.$$

Thus, the presented inequalities show that for a transition probability that is a power-law function of the momentum transfer ($W = c_1 q^n$), the collision integral converges for power-law distributions corresponding to W with

$$-3<n<-1 \quad (I_0=\text{const}),$$

$$-4<n<-3 \quad (I_1=\text{const}), \tag{9}$$

which according to (1) corresponds to exponents s in the intervals

$$-{}^3/_2<s_0<-1, \quad -{}^3/_2<s_1<-{}^5/_4. \tag{10}$$

In the important case of Coulomb interactions, as seen from the foregoing inequalities, the collision integral diverges at small momentum transfers, but this divergence can be eliminated by Debye screening. Consequently, the restriction connected with the divergence of the integral as a result of the transition probability is not encountered, and a local power-law distribution can exist with a constant energy flux in momentum space. In Sec. 4 below we shall consider this question in greater detail.

We note that the existence of a stationary power-law distribution function corresponds to the presence of a constant non-zero flux in momentum space, the intensity of which (in accordance with (8)) determines the particle density in this distribution. On the other hand, the conservative character of the flux is ensured by the source and the sink, the locations of which should be made consistent with the flow direction.

3. ELECTRON-ELECTRON COLLISION INTEGRAL

When considering normal electron-electron collisions in solids in the free-electron approximation, one uses a collision integral of the type of (2), in which the expression customarily used for the transition probability W is based on the "jellium" model,[13] namely

$$W=2e^4/(q^2+a_1^2+\Omega^2 q^2 \omega^{-2})^2, \tag{11}$$

$$\hbar\omega=E_2-E_3, \quad a_1^2=3m\hbar^2\omega_e^2/2E_F, \quad \Omega^2=z^2(m/M)\omega_e^2, \quad \omega_e^2=4\pi e^2 n_e/m.$$

n_e is the electron density, z is the valence, M is the ion mass. When the collision integral takes the form (4), we obtain for (11) the expression

$$W=2e^4\left\{\tilde{q}^2+\frac{a_1^2}{p^2}\left[1+\frac{4}{3}z^2\left(\frac{m}{M}\right)\frac{p_F^2}{p^2(\tilde{q}+2\cos\theta)^2}\right]\right\}^{-2}. \tag{12}$$

The dimensionality r, which enters in (4) is here equal to -4. However, it should be noted that here W is not a homogeneous function of the momenta and therefore a direct application of the results of (6)–(8) is impossible.

It will be shown in Sec. 4 that in definite regions of momentum space, the limits of which depend on the external physical conditions, there can exist power-law functions of the particle distribution. Their degrees correspond to different asymptotic forms of the transition probability W, which are already homogeneous functions of the momenta. Let us consider different asymptotic expressions for $\bar{W}$, represented by expression (12), and obtain the corresponding degrees.

For small energy transfers (low temperatures) we have

$$W=\frac{9e^4}{8(m/M)^2(a_1 z p_F)^4}p^8(\tilde{q}+2\cos\theta)^4, \quad \text{i.e.,} \quad n=4.$$

From condition (6) we get $s_0 = -\frac{11}{4}$ and $s_1 = -\frac{13}{4}$, in accordance with (1). Here and below, the subscripts 0 (1) correspond to the constant particle (energy) flux.

At appreciable energy transfers (high temperatures) we have

$$\tilde{W}=\frac{2e^4}{(\tilde{q}^2+a_1^2/p^2)^2}. \tag{13}$$

As will be shown below (see Sec. 4), in the region of small $a=a_1/p$ ($a\ll 1$) there can be formed a local power-law distribution $f=Ap^{2s}$, corresponding to the asymptotic form of the transition probability describing the pure Coulomb interaction (exponent $s_1=-\frac{5}{4}$),[9] and corresponding to the energy flux in momentum space. As to large a ($a\gg 1$, with $n=0$) then, as follows from the inequalities (9), in this case a local power-law distribution cannot exist, so that we shall not analyze $(\partial f/\partial t)_{st}$ in detail for $a\gg 1$. We shall illustrate a typical analysis of the regions of momentum space in which the parameter a is small ($a\ll 1$) by using as examples three media: a metal, a semiconductor, and a plasma. Thus, in the case of metals $a^2=a_1^2/p^2$ is of the order of unity, for momenta $p\sim p_F$, and a power-law non-equilibrium distribution function can be formed at energies exceeding E_F, inasmuch as the parameter a is small for such energies. For semiconductors, a^2 is small even for momenta $p\sim p_F$, so that the region of the existence of the power-law non-equilibrium distribution function includes energies on the order of the Fermi energy. In the case of a laboratory plasma, the parameter a, defined by a somewhat different expression

$$a^2=4\pi e^2 n_e \hbar^2/\bar{p}^2 p^2.$$

where $\bar{p}$ is the mean value of the momentum, turns out to be much smaller than unity for almost all the momenta, so that a power-law distribution function can exist in practically the entire energy interval. This fact can be established also by a somewhat different method. It is well known that in the case of a plasma a collision integral describing the interaction of charged particles can be written in the Landau form[12]

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-\operatorname{div}\mathbf{j}_0,\quad j_{0i}=\pi e^4\lambda\int\frac{u^2\delta_{ik}-u_iu_k}{u^3}\left[f\frac{\partial f'}{\partial p_k'}-f'\frac{\partial f}{\partial p_k}\right]d\mathbf{p}'. \tag{14}$$

where $\mathbf{u}=\mathbf{v}-\mathbf{v}'$ and λ is the Coulomb logarithm. By substituting in (14) an isotropic power-law distribution function we can obtain by rather simple calculations[9] the expression

$$\left(\frac{\partial f}{\partial t}\right)_{st}=16\pi^2 m^* e^4\lambda A^2 p^{4s}\frac{(4s+3)(4s+5)}{(s+1)(2s+3)(2s+5)}+\frac{16\pi^2 m^* e^4\lambda A^2}{3}p^{4s}\lim_{\substack{p_1\to 0\\ p_2\to\infty}}\left\{\frac{2s^2}{2s+3}\left(\frac{p_1}{p}\right)^{2s+3}+\frac{(2s+1)s}{2s+2}\left(\frac{p_2}{p}\right)^{2s+2}-\frac{(2s+3)}{2}\left(\frac{p_2}{p}\right)^{2s}-\frac{(2s-2)s}{(2s+5)}\left(\frac{p_1}{p}\right)^{2s+5}\right\}. \tag{15}$$

4. REGIONS OF EXISTENCE OF POWER-LAW DISTRIBUTIONS

The purpose of the present section is to consider the collision integral (4) with a transition probability W that is not a homogeneous function of the momenta. For simplicity we express the transition probability W in the form (13), corresponding to a screened Coulomb potential. Inasmuch as in this case there is not a single power-law function that causes the collision integral to vanish in the entire momentum space, and different asymptotic forms of W correspond each to two pairs of different powers (each pair corresponds to constancy of one of the fluxes, I_0 or I_1), it is necessary to determine the regions of the existence of the power-law distribution functions.

We consider the collision integral (4) with the transition probability[5)]

$$\tilde{W}=2e^4/(\tilde{q}^2+a^2)^2,\quad a^2=a_1^2/p^2.$$

Integrating with respect to the angles θ_1, φ, and φ_1 and making the substitution $x\cos\theta$, we obtain the collision integral in the form

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-\frac{16\pi^2 m^* e^4 A^2}{(2s+2)}p^{4s}\int_{-1}^{1}dx\int_0^\infty\frac{d\tilde{q}\tilde{q}}{(\tilde{q}^2+a^2)^2}\{|1+2\tilde{q}x+\tilde{q}^2|^s|x|^{2s+2}-|x+q|^{2s+2}\}. \tag{16}$$

Using standard integrals,[14] we transform (16) into a sum of two integrals

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-\frac{8\pi^2 m^* e^4 A^2}{(s+1)(2s+3)}p^{4s}\left\{\int_0^1\frac{d\tilde{q}\tilde{q}}{(\tilde{q}^2+a^2)^2}\left[(1+\tilde{q}^2)^s\left(F\left(-s,2s+3;2s+4;-\frac{2\tilde{q}}{1+\tilde{q}^2}\right)+F\left(-s,2s+3;2s+4;\frac{2\tilde{q}}{1+\tilde{q}^2}\right)\right)-(1+\tilde{q})^{2s+3}-(1-\tilde{q})^{2s+3}\right]+\int_0^1\frac{d\tilde{q}\tilde{q}^{-2s-2}}{(1+a^2\tilde{q}^2)^2}\left[\tilde{q}^3(1+\tilde{q}^2)^s\left(F\left(-s,2s+3;2s+4;-\frac{2\tilde{q}}{1+\tilde{q}^2}\right)+F\left(-s,2s+3;2s+4;\frac{2\tilde{q}}{1+\tilde{q}^2}\right)\right)-(1+\tilde{q})^{2s+3}+(1-\tilde{q})^{2s+3}\right]\right\}. \tag{17}$$

To obtain the regions of the existence of the power-law distribution functions corresponding to different asymptotic forms of the transition probability $\tilde{W}$, it is necessary to consider the collision integral for two limiting cases ($a\ll 1$ and $a\gg 1$).

In the case of small a ($a\ll 1$), the dependence of the collision integral on a is connected only with the first integral in (17), the value of which is determined by a small vicinity of the point $\tilde{q}=0$, or the order of a, and is obtained by expanding the integrands in powers of $\tilde{q}$. The final form of the collision integral for small a is

$$\left(\frac{\partial f}{\partial t}\right)_{st}=8\pi^2 m^* e^4 A^2 p^{4s}\left\{\frac{(4s+3)(4s+5)}{(s+1)(2s+3)(2s+5)}\left[\ln\frac{1}{a^2}+\frac{(2s+1)(2s+3)(2s+5)\pi^2\Gamma(s+2)}{2^{2s+5}\Gamma(-2s)\Gamma\left(\frac{2s+7}{2}\right)\Gamma\left(\frac{4s+7}{2}\right)\sin^2\left(\pi\frac{2s-1}{2}\right)}\right]+O(a^2\ln a^2)\right\}. \tag{18}$$

Since the screening was not treated in a self-consistent manner (a_1 was assumed to be a certain specified parameter corresponding to the distribution function in the entire momentum region), it follows that, as usual (see, e.g., the derivation of the Landau and Lenard-Balescu equation[15]), the weak dependence of the "telling" momentum p, which enters under the logarithm sign, must not be taken into account, since this corresponds to an exaggeration of the calculation accuracy. Thus, the quantity a under the logarithm sign in (18) corresponds to a certain characteristic momentum of the power-law section of the distribution function.

Expression (18) vanishes for $s = -\frac{5}{4}$ accurate to terms of order $a^2 \ln a^2$.[6] Consequently, at small values of a the distribution function has a power-law form and corresponds to an asymptotic form of W with exponent $n = -4$, describing the direct Coulomb interaction between the electrons. The foregoing analysis shows therefore that in the momentum region indicated for the transition probability (13), on the one hand, the Debye screening of the Coulomb interaction eliminates the "Coulomb divergence," and on the other hand it does not influence the exponent of the non-equilibrium stationary distribution function of the particles.

Expressions (15) and (18) lead to the same degree for the non-equilibrium distribution function. However, expression (18) yields more complete information, since, first, it indicates the region of the existence of the power-law function, $a \ll 1$, and second, it indicates that in a certain region of momentum space the direction of the energy flux is opposite (positive) to its direction at large momenta.[7] The limits of this region are determined by the ratio of the logarithmic term to the second term in the square bracket of (18).

5. LASER-INDUCED EMISSION CURRENT

It is known[16–18] that when a metallic foil is bombarded by a pulse from a high-power laser with $Q = 10^4$ erg-cm^{-2} sec^{-1}, two peaks of the emission current are observed. The first, almost synchronous with the laser pulse, contains a large number of "fast electrons" (the maximum energy for tungsten is 14.5 eV). The second peak, which follows with a delay $\tau \sim 10^{-7}$–10^{-8} sec relative to the first, contains electrons with energies not exceeding 2 eV.

A satisfactory explanation of the appearance of fast electrons as being due to the Maxwellian distribution function is impossible,[16] since the experimental results[17] would then correspond to a temperature $T_e = 30\,000\,^\circ$K, which exceeds by one order of magnitude the melting temperature of tungsten. As to the emission current, there are two well known mechanisms for its appearance: a multiquantum photon effect and thermionic emission. These yield respectively the following expressions for the emission current density[16]:

$$i_{\rm ph} = 2^{-3n}(em\omega_L^2/\hbar)\, n^{1/2} (8\pi e^2 Q/mc\varphi\omega_L^2)^n,$$
$$i_{\rm te} = c_2 (T_e^3/\varphi) \exp(-\varphi/kT_e). \qquad (19)$$

where φ is the work function, $n = \mathrm{ent}[1 + \varphi/\hbar\omega_L]$, ω_L is the laser emission frequency, and c_2 is a factor that depends on the distribution of the illumination in the spot. Let us compare the values of the emission currents corresponding to these mechanisms with the experimental value of the current $i_{\rm exp}$ for a tungsten foil ($\varphi = 4.5$ eV; $m^* = 0.5m$; $E_F = 5$ eV; $\omega_L = 10^{15}$ sec^{-1}, $E_{\max} = 24$ eV is the maximum energy in the distribution). The numerical values were taken from Knecht's paper[17]

$$I_{\rm te} = 0.33 \cdot 10^{-14}, \quad I_{\rm ph} = 10^{-5}\, T_e^3 \exp(-5.22 \cdot 10^4/T_e), \qquad (20)$$

where $I = i/i_{\rm exp}$. According to the theoretical and experimental data[16] the quantity T_e in $I_{\rm te}$ does not exceed 1800 °K, from which we get $I_{\rm te} \sim 10^{-6}$.

Thus, the emission currents calculated on the basis of the multiquantum photoeffect and thermionic emission are both underestimated. In this section we therefore propose a mechanism that makes it possible to explain the available experimental data on the emission current. It seems to us that the considered situation corresponds to an essentially non-equilibrium situation connected with the presence of a source (high-power laser radiation) and a sink (emission current) in momentum space. It is intuitively clear[8] that a power-law distribution can be established in an energy interval in which the particle density produced by the source exceeds the density in the equilibrium distribution. In our case the start of this interval is the level to which the particles are transferred from the Fermi sphere by the two-quantum photoeffect. In addition to this source, particles connected with three and more quantum transitions will also land in this interval, but the densities of the particles produced by them will be much smaller and will be disregarded. The formation of the non-equilibrium distribution in this energy interval will be due to electron collisions, since electrons with such energies cannot interact effectively with phonons.

Let us compare the times of the electron-energy relaxation as a result of electron-electron and electron-phonon collisions. According to[19], in a high-frequency electromagnetic field $\hbar\omega \gg kT$ at high temperatures $T \gg \Theta_D$ (Θ_D is the Debye temperature) the frequency of the electron-electron collisions is determined by the expression

$$\gamma^{ee}(\omega, T) = \gamma_0^{ee}(T)\,[1 + (\hbar\omega/kT)^2], \qquad (21)$$

where $\gamma_0^{ee}(T)$ is the classical high-temperature frequency of the electron-electron collisions and is proportional to T^2 (see, e.g.,[20]). On the other hand, the frequency of the electron-phonon collisions under the same conditions takes the form[20]

$$\gamma^{ef} = f(\Theta_D)\, T/\Theta_D, \qquad (22)$$

where $f(\Theta_D)$ is the classical high-temperature frequency of electron collisions with phonons at $T = \Theta_D$.

For the purpose considered in the present section, the conditions necessary for (21) and (22) to be valid are satisfied since $\omega_L \approx 10^{15}$ sec^{-1}, $T_e = 1800$ K, $\Theta_D = 315$ K.[20] We note that the reciprocals of the collision frequencies, defined in accordance with (21) and (22), do not coincide in the general case with the electron-energy relaxation times, since it is necessary to take additional account of the number of collisions needed for the particle to lose its energy E, i.e., the factor $\eta = E/E_1$ (E_1 is the energy lost by the electron in one collision). According to[20,21], in our case we have $\gamma^{ef} \approx \gamma_0^{ee}(T)$, $\gamma^{ee} > \gamma_0^{ee}(T)$, while the factor η can be of the order of unity for electron-electron collisions, while for electron-phonon collisions we have $E/k\Theta_D \sim 10^2$ ($E \sim \hbar\omega$). Thus, in our case the time of electron-energy relaxation as a result of electron-electron collisions is much shorter than the time of relaxation due to electron-phonon processes.

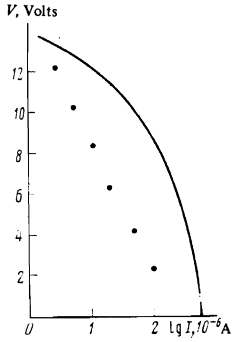

FIG. 1. Dependence of the logarithm of the emission current $\log I$ on the starting potential V (solid line); the dark circles show the experimental points).

A comparison of the laser pulse duration and of the relaxation times shows that the electron distribution function in our case is quasi-stationary and is determined mainly by electron-electron collisions. Consequently, it can be obtained from the condition that the collision integrals (17) vanish. From the analysis carried out in Sec. 4 it is seen that for metals in the energy region $E-E_F>\hbar\omega_L$ there can exist a power-law distribution corresponding to a constant energy flux $s_1=-\frac{5}{4}$ in momentum space. It should be noted that the distribution will be formed both on account of collisions with electrons belonging to the interval $E-E_F>\hbar\omega_L$, and on account of collisions with electrons of the fundamental background (equilibrium). However, since relaxation on the fundamental background leads in similar problems also to a power-law-like electron distribution,[10] and the result, as will be shown later, depends rather weakly on the degree s of the distribution, we shall use for estimates the degree $s_1=-\frac{5}{4}$. The emission current density for such a distribution function, $f=Ap^{2s}$ is obtained in analogy with the Richardson formula[16] and is determined by the expression

$$i_s=\frac{\pi A E_{max}^{s+2}(2m^*)^{s+1}e}{(s+1)(s+2)}\left[s+1-\frac{E_F+\varphi}{E_{max}}(s+2)+\left(\frac{E_F+\varphi}{E_{max}}\right)^{s+2}\right], \quad (23)$$

where E_{max} is the maximum energy in the power-law distribution. To estimate the value of A we equate the particle density produced by the two-quantum photoeffect to the particle density in the power-law distribution $s_1=-\frac{5}{4}$, the lower limit of which is the level to which the photoeffect transfers the particles from the Fermi sphere, and the upper level is E_{max}. Then

$$A\approx 10^{18}\ \text{g}^{1/2}\,\text{cm}^{-7/2}\,\text{sec}^{1/2},\quad i_s=2.4\cdot 10^{-18}\,Ai_{exp}=2.4\,i_{exp}. \quad (24)$$

The figure compares the experimental data of[17] with the dependence of the emission current density on the retarding potential V, as described by formula (23), in which φ is replaced by the effective work function $\varphi+eV$.

Thus, the use of the proposed mechanism yields an acceptable value of the emission current and of its dependence (fast peak) on the retarding potential. As to the slow peak, on its initial section the emission current receives contributions not only from the equilibrium distribution (thermionic emission), but also a nonequilibrium stationary increment due to the "disintegration" of the power-law distribution.

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APPENDIX

Let the transition probability $W=c_1q^n$ (n is any real number), and then the collision integral (4) is easily integrated with respect to θ_1, φ, and φ_1 and reduces to the form

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-\frac{2\pi^2 m^* c_1 A^2}{s+1}p^{4s+n+4}[I^{(1)}-I^{(2)}], \quad (A1)$$

$$I^{(1)}=\int_{-1}^{1}dx\int_0^\infty d\tilde q\,\tilde q^{n+1}|1+2\tilde q x+\tilde q^2|^s|x|^{2s+2},$$

$$I^{(2)}=\int_{-1}^{1}dx\int_0^\infty d\tilde q\,\tilde q^{n+1}|x+q|^{2s+2},\quad x=\cos\theta.$$

The calculation of the integral $I^{(2)}$ is elementary, and we write down directly the answer, expressed in terms of the beta function $B(x,y)$

$$I^{(2)}=\frac{1}{2s+3}[\mathrm{B}(2s+4,n+2)-\mathrm{B}(-2s-n-5,2s+4)+\mathrm{B}(n+2,-2s-n-5)]. \quad (A2)$$

In the calculation of $I^{(1)}$ it is convenient to integrate first with respect to q (see, e.g., [14])

$$I^{(1)}=2^{-(2s+1)/2}\Gamma\left(\frac{1-2s}{2}\right)\mathrm{B}(n+2,-n-2s-2)\int_0^1 dx\,x^{2s+2}(1-x^2)^{(2s+1)/4}\times[P^{(2s+1)/2}_{(2n+3+2s)/2}(-x)+P^{(2s+1)/2}_{(2n+3+2s)/2}(x)],$$

where $P_\nu^\mu(x)$ is a spherical function. Using the properties of solid spherical harmonics

$$P_\nu^\mu(-x)=-\frac{\sin(\pi\nu)}{\sin(\pi\mu)}P_\nu^\mu(x)+\frac{\sin(\pi(\nu+\mu))\Gamma(\nu+\mu+1)}{\sin(\pi\mu)\Gamma(\nu-\mu+1)}P_\nu^{-\mu}(x),$$

and calculating the integrals, we obtain

$$I^{(1)}=\mathrm{B}(n+2,-n-2s-2)\left[\left(1-\frac{\sin[\pi(2s+3+2n)/2]}{\sin[\pi(2s+1)/2]}\right)\frac{1}{2s+3}\,{}_3F_2\left(\frac{n+2}{2},-\frac{n+2s+2}{2},1;\frac{1-2s}{2},\frac{2s+5}{2};1\right)\right.$$
$$\left.+\frac{\Gamma(2s+n+3)\Gamma((1-2s)/2)\Gamma((2s+3)/2)\Gamma(s+2)}{2^{2s+2}\Gamma(n+2)\Gamma((4s+n+7)/2)\Gamma((2s-n+3)/2)}\frac{\sin(\pi(2s+n+2))}{\sin(\pi(2s+1)/2)}\right], \quad (A3)$$

where ${}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z)$ is a hypergeometric function. Thus, the expression for $R(s,n)$, which enters in formula (6), will be for the flux in momentum space

$$R(s,n)=\frac{4\pi^3c_1}{(s+1)(2s+3)}\left\{\mathrm{B}(n+2,-2s-n-2)\left(1-\frac{\sin(\pi(2s+2n+3)/2)}{\sin(\pi(2s+1)/2)}\right)\right.$$
$$\times\,{}_3F_2\left(\frac{n+2}{2},-\frac{n+2s+2}{2},1;\frac{1-2s}{2},\frac{2s+5}{2};1\right)-\mathrm{B}(2s+4,n+2)$$
$$+\mathrm{B}(-2s-n-5,2s+4)-\mathrm{B}(n+2,-2s-n-5)$$
$$\left.-\frac{\pi^2(2s+1)(2s+3)(4s+n+7)(4s+n+9)\Gamma(s+2)}{2^{2s+5}\Gamma(-2s)\Gamma((2s-n+3)/2)\Gamma((4s+n+11)/2)\sin^2(\pi(2s+1)/2)}\right\}. \quad (A4)$$

We have considered in detail the case of a quadratic dispersion law. Consideration of an arbitrary dispersion law[9)] $E=p^\beta/\alpha_1$ (β and α are certain constants) leads to complications only in the calculation of the expression for $R(s,\beta,n)$. On the other hand, to find the exponents corresponding to the nonequilibrium distribution functions that conserve the particle flux (energy flux) in momentum space is a rather simple matter for a transition probability W which is homogeneous in the momentum. It is convenient for this purpose to introduce the dimensionless variable $\mathbf{p}_i/p$ and then the collision integral (2) reduces to an integral that is independent of p, and to a factor $p^{2\beta s+n+6-\beta}$. Calculating the fluxes in momentum space in analogy with (5)

$$\operatorname{div}\left(j_i \frac{\mathbf{p}}{p}\right) = -\left(\frac{p^{\beta}}{\alpha_1}\right)\left(\frac{\partial f}{\partial t}\right)_{st},$$

$$I_i = 4\pi\alpha_1^{1-i} p^{2\beta s-n-9-\beta(i-1)} \frac{R(s,\beta,n)}{2\beta s-n-9-\beta(i-1)} A^2,$$

we see that conservation of the flux I_i corresponds to a value of s_i that causes the exponent of p to vanish under the condition that $R(s,\beta,n)$ has a first-order zero for this value of s_i. Consequently

$$s_i = -\frac{n-9-\beta(i-1)}{2\beta},$$

which agrees with expression (1) obtained earlier[9] by a group-theory method.

1)With the exception of Sec. 5, where we deal with the nonequilibrium distribution of fast electrons produced under the influence of laser radiation.

2)On the basis of an analysis of the behavior of the characteristic collision frequencies for the scattering of plasmons by particles and by one another, regions of the existence of different spectra have been found[7] as functions of the values of the parameter kr_D (r_D is the Debye radius).

3)We note that the expression for $R(s,n)$ in the case of an inhomogeneous transition probability follows from (18) (see below).

4)This demonstrates the possibility of determining the powers s_i (which coincide with (1)) by a method differing from that used by Kats *et al.*[9]

5)At this transition probability, only the divergence of $(\partial f/\partial t)_{st}$, corresponding to collisions with small momentum transfer, is eliminated.

6)It should be noted that the vanishing of expression (18) at $s=-3/4$ does not correspond to the solution of the kinetic equation, since the distribution with $s=-3/4$ is nonlocal (see Sec. 2), i.e., at $s=-3/4$ the expressions for $(\partial f/\partial t)_{st}$ contain divergent terms in addition to (18).

7)In this connection, we call attention once more to the need for making the locations of the source and sink consistent with the flow direction.

8)Strictly speaking, the question of the source power needed to establish a stable power-law distribution is quite complicated, and its solution, likely, cannot be based on the thermodynamics of nonequilibrium processes, which is valid only at small deviations from the equilibrium distribution function, but must apparently be solved in the spirit of the Lyapunov theory.

9)A Boltzmann collision integral with a transition probability that is homogeneous in the momenta and with an arbitrary power-law dispersion for the particles was considered earlier[9] by a group-theory method.

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