$$\gamma(p,x)=\int_0^x e^{-t}t^{p-1}\,dt.$$

The expression for the distribution function of secondary ions (that is, ions produced by the beam) is only slightly changed. For slow ions Eq. (9) remains valid. For fast ions we must substitute $p_\perp^2+\mu M r^2$ for μMR^2 and t for t_0 in accordance with Eq. (A.2). The density of slow ions can be calculated by integration of their distribution function over $p_\perp$ up to $\mu M(R^2-r^2)$, and the density of fast ions can be calculated with this same integral taken to infinity. Outside the beam $\varkappa=0$ and the longitudinal momentum does not change. Since for $r > R$ we have $I_{st2}=0$, the value of the distribution function outside the beam is determined by its value at the boundary

$$f=\frac{n_a n_0 I_\tau t_0}{\pi^{3/2}\varepsilon_R}\left[\gamma\left(3/2;\frac{\varepsilon_\perp+\varepsilon_\parallel+\varepsilon_r}{\varepsilon_T}\right)-\gamma\left(3/2;\frac{\varepsilon_\perp+\varepsilon_\parallel+\varepsilon_r-\varepsilon_R}{\varepsilon_T}\right)\right],$$

$$\varepsilon_\perp=p_\perp^2,\quad \varepsilon_\parallel=(p_\parallel-\varkappa t_0)^2,\quad \varepsilon_r=\varepsilon_R+2\mu MR^2\ln(r/R),$$

where $\varepsilon_R=\mu MR^2$. The density of fast ions is

$$n_i=\frac{4(2MT)^{1/2}n_a n_0 I_\tau t_0}{\xi_1}\left[\frac{3}{2}\xi_1\rho^{3/2}-\frac{2^{5/2}}{3}\xi_1\xi_2^{3/2}\right.$$
$$\left.+\frac{1}{3}(\tau^{3/2}-2\xi_1^{3/2}-\sigma_1^{3/2}-\sigma_2^{3/2})-\frac{2}{15}(\tau^{5/2}-2\xi_1^{5/2}+\sigma_2^{5/2})\right],$$

$$\xi_1=\varepsilon_R/\varepsilon_T,\quad \xi_2=\varepsilon_r/\varepsilon_T,\quad \rho=1/2+\xi_2,\quad \tau=1/2+\xi_1-\xi_2,\quad \sigma_{1,2}=1/2-\xi_{1,2}.$$

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Translated by Clark S. Robinson

Stationary flows of a plasma through a magnetic barrier

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(Submitted March 25, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1342–1357 (October 1976)

The possibility of plasma flow through a region in which strong magnetic fields perpendicular to the direction of motion are produced by external currents is discussed. It is shown that stationary flow of a cold plasma is possible even if the (dynamic) plasma pressure is much lower than the pressure of the external magnetic field, i.e. $\beta\ll 1$. The passage of the plasma through the barrier is caused by the fact that the plasma generates a diamagnetic current that destroys the external field, and the plasma moves in fact in a very weak magnetic field. The field in the plasma is concentrated in a very thin skin layer with a thickness of the order of $\delta\approx c/\omega_0$ where ω_0 is the plasma electron frequency and c the velocity of light in vacuum. The value of β required for the existence of a stationary flow with practically constant velocity depends on the shape of the barrier and may be very small for a smoothly shaped barrier: $\beta\approx(\delta/\Delta)^2$ where Δ is the width of the transition region in which the external field drops to zero, $\delta\ll\Delta$. An exact solution is obtained for the case of the most difficult passage of singular external currents. The distributions of the fields, currents, charges, plasma density, and velocity are calculated. The feasibility of establishment of such stationary flows is discussed.

PACS numbers: 52.30.+r

1. INTRODUCTION

The possibility of the motion of a plasma stream across a strong[1)] magnetic field was observed experimentally more than ten years ago.[1] The investigation of the features of plasma injection into a strong-field region has since been pursued quite intensively, as is evidenced by the large number of experimental papers.[2–6] In the interpretation of the extensive experimental material, the passage of the plasma into the region of the strong field is attributed to the presence of an E_y component of the electric field and the subsequent drift of the plasma in the crossed fields E and B. The main cause of the appearance of the E_y component is assumed to be polarization of the plasmoid boundaries as the plasma enters into the magnetic-field region. It appears that the presence of E_y is necessary for the initial stage of the plasma motion in the region of the strong magnetic field.

We present an extremely simplified scheme of the interaction of the plasma stream with a magnetic barrier, in the following form: The plasma stream moves along the x axis from the region $x\to-\infty$ into the region $x\to+\infty$. The magnetic barrier is chosen for simplicity in the form of a rectangle of width $2a$ and field intensity B_0. The field is directed along the positive z axis (Fig. 1a).

It is easy to show that in this one-dimensional case $E_y(x)\equiv 0$. Indeed, for stationary flow curl $\mathbf{E}=0$, taking its z component and assuming $\mathbf{E}=\mathbf{E}(x)$, we obtain $E_y(x)\equiv 0$ if the boundary condition is $E_y\to 0$ as $x\to\infty$.

In the theoretical papers by Chapman and Ferraro[7,8] and in the unpublished papers of Rosenbluth and Garwin[9] (see also[10]), which have preceded the experiments, and in which there was no y-component of the electric field, the entire flux incident on the magnetic wall was fully reflected. In the cited papers they dealt with the one-dimensional problem of the equilibrium boundary between a cold plasma and a magnetic field, determined by the relation $\rho_0 U_0^2 = B_0^2/8\pi$. (Here $\mathbf{B}_0$ is the magnetic field intensity outside the volume occupied by the plasma).

The studies performed several decades ago[7,8] on the interaction of the plasma with a transverse magnetic field served as a stimulus for a large number of theoretical papers, in which the model considered in[7,8] was further developed. Account was taken of the thermal scattering in the plasma,[11] of the particles trapped by the field in the transition region,[12] and of the fact that the problem was not one-dimensional.[13] A detailed review of these studies was presented by Phelps.[14] One might thus conclude from the results of the theoretical papers that in the absence of E_y the plasma cannot move through a strong magnetic barrier.

In the present paper we show, within the framework of the one-dimensional model, that even in the absence of the E_y components there exist stationary flows of plasma through a magnetic barrier. It turns out that these flows are possible not only in the case when $\rho_0 U_0^2 > B_0^2/8\pi$, but also if the inequality is reversed, and that the parameter $\beta = 8\pi\rho_0 U_0^2/B_0^2$ can be very small. The possibility of plasma flow through an external magnetic field whose pressure exceeds the plasma pressure is due to the diamagnetism of the plasma. As we shall show, a diamagnetic current is produced in the plasma, and its field cancels out the external field. Thus, the plasma moves in a resultant field that can be much weaker than B_0. The foregoing pertains to a broad barrier. The difference between a narrow and broad magnetic barrier is determined by the value of the parameter $c/\omega_0 a$ (a is the width of the barrier) in comparison with unity. In the case of the broad barrier $c/\omega_0 a \ll 1$, the resultant field is localized in a narrow skin layer of approximate thickness c/ω_0 near the conductors and is negligibly small inside the barrier. The quantity $\delta = c/\omega_0$ is the usual inertial length that arises in many problems in which the interaction of the plasma with the magnetic field is considered.

The thickness of the transition layer between the plasma and the field[9] is equal to the hybrid Larmor radius $\rho_h = (\rho_e \rho_i)^{1/2}$, where $\rho_{e,i} = cU_0 m_{e,i}/eB_0$. It is easily seen that this quantity is equal to the inertial thickness c/ω_0 if the dynamic and magnetic pressures are equal. This explains immediately the condition for the passage of the barrier: to this end the barrier thickness must be less than the hybrid radius.[9]

For a broad barrier, the motion of the plasma depends essentially on its shape. It has turned out that the most difficult for the plasma to pass through are steep barriers ($\Delta \ll c/\omega_0$, where Δ is the characteristic region of variation of the barrier size), the limiting case of which is a barrier of rectangular shape (Fig. 1a). The plasma passes much "easier" through a broad barrier with a field that falls off smoothly ($\Delta \gg c/\omega_0$) towards the edges with a characteristic fall-off width $\sim\Delta$: at $\beta \gtrsim (\delta/\Delta)^2$ the plasma field velocity changes little.

In the present paper it is also shown that at $\beta < 1$ there can exist one more stationary solution with a reflection point inside the barrier. The depth of penetration of the plasma depends on the value of β. We shall discuss below the conditions under which a particular stationary solution is realized.

In Sec. 2 we discuss the formulation of the problem and write down a system of equations that makes it possible to describe the flow of a cold collisionless plasma (with a thermal scatter much smaller than the directional velocity) through a magnetic barrier. In Sec. 3, we obtain stationary solutions with passage of the plasma through the barriers by using for the sake of clarity, the "weak deceleration" approximation, i.e., we assume that the barrier does not perturb greatly the plasma parameters. We consider the flow of a cold plasma through magnetic barriers of various shapes.

An analysis of the plasma flow through magnetic barriers produced by rectangular currents is carried out in Sec. 4. The possibility of plasma flow through such barriers at arbitrarily small β is demonstrated. In Sec. 5, in the slow-deceleration approximation, we consider the flow of a plasma through a rectangular magnetic barrier. Assuming that the plasma currents cannot have the form of a Dirac δ-function, we obtain in Sec. 6 an exact solution of the problem of plasma flow

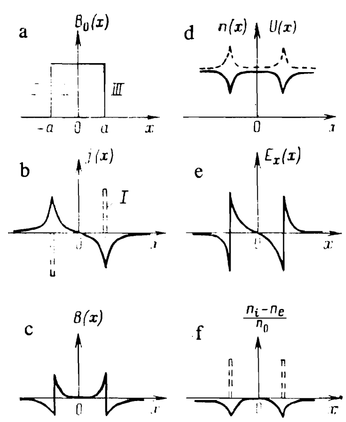

FIG. 1. Distribution of the parameters in the flow of a cold plasma through a broad ($a \gg \delta$) magnetic barrier. a) Distribution of the magnetic field of the barrier $B_0(x)$ in the absence of the plasma, b) Distribution of the diamagnetic current $j(x)$. The dashed lines show the surface current producing the magnetic field of the barrier, c) Distribution of the resultant magnetic field $B(x)$, d) Distribution of the plasma flow velocity $U(x)$. The dashed line shows the density distribution $n(x)$, e) Distribution of the electric field E_x. f) Distribution of the charge density $(n_i - n_e)/n_0$. The volume charge is negative throughout, and the positive charge, shown dashed, is concentrated in narrow layers at the barrier boundary.

through a rectangular magnetic barrier. In Sec. 7 we solve the problem of the reflection of a stream of cold collisionless plasma from a rectangular magnetic barrier. In the conclusion we discuss the feasibility of establishment of the obtained stationary solutions.

2. FORMULATION OF PROBLEM. FUNDAMENTAL EQUATIONS

To investigate the interaction of a stream of cold collisionless plasma with a magnetic barrier we use the equations of two-fluid hydrodynamics

$$n_e m_e V_{ex}\frac{d\mathbf{V}_e}{dx}=-n_e e\left\{\mathbf{E}+\frac{1}{c}[\mathbf{V}_e\times\mathbf{B}]\right\}-\nabla P_e-\tilde{m}n_e(\mathbf{V}_e-\mathbf{V}_i)\nu_{ei},$$

$$n_i m_i V_{ix}\frac{d\mathbf{V}_i}{dx}=n_i e\left\{\mathbf{E}+\frac{1}{c}[\mathbf{V}_i\times\mathbf{B}]\right\}-\nabla P_i-\tilde{m}n_i(\mathbf{V}_i-\mathbf{V}_e)\nu_{ei},$$

$$\operatorname{rot}\mathbf{E}=0,\quad \operatorname{rot}\mathbf{B}=\frac{4\pi}{c}(\mathbf{j}+\mathbf{I}_0),$$

$$\operatorname{div}\mathbf{B}=0,\quad \operatorname{div}\mathbf{E}=4\pi e(n_i-n_e),$$

$$\frac{d}{dx}(n_e V_{ex})=0,\quad \frac{d}{dx}(n_i V_{ix})=0,$$

where $\tilde{m}=m_e m_i/(m_e+m_i)$, $m_{i,e}$, $P_{i,e}$, $n_{i,e}$, and $\mathbf{V}_{i,e}$ are the mass, pressure, density, and ion and electron velocities, respectively; $\mathbf{E}$ is the electric field; $\mathbf{B}=\mathbf{B}_0+\tilde{\mathbf{B}}$ is the resultant magnetic field; $\mathbf{B}_0$ is the magnetic field of the barrier, i.e., the field produced by the currents $\mathbf{I}_0(x)$ flowing in the external conductors; $\tilde{\mathbf{B}}$ is the plasma magnetic self-field produced by the currents of the plasma itself; ν_{ei} is the frequency of the collisions between the ions and the electrons.

The problem is one-dimensional, so that all the functions depend only on x. The boundary conditions are assumed to be the following: in the region $x\to-\infty$ the electrons and ions have identical velocities $\mathbf{V}_e(-\infty)=\mathbf{V}_i(-\infty)=\mathbf{U}_0$, directed strictly along the x axis. The remaining components of the velocities, and of the fields in the region $x\to-\infty$ are assumed to be zero.

We assume that the directional velocity of the plasma is much lower than the thermal velocities of the electrons and ions, so that we can neglect in the initial system of equations the forces connected with the presence of gradients of the electron and ion pressures. Using the boundary conditions and taking into account the one-dimensional character of the problem, it is easy to show that $E_y(x)=E_z(x)\equiv 0$, $B_x(x)=B_y(x)\equiv 0$, $V_{ez}=V_{iz}\equiv 0$. The electric-field component E_x is produced in the plasma as a result of the difference between the inertia of the electrons and the ions. The plasma is assumed to be quasineutral ($n_i\approx n_e\approx n(x)$). The quasineutrality assumption means in this case that the relative charge separation $|(n_i-n_e)/n\ll 1$, but the electric field due to this separation is such that the ion motion is determined mainly only by this field. We shall derive below the condition for the applicability of this assumption.

From the continuity equation for the electrons and ions and from the quasineutraility of the plasma it follows that $V_{ex}=V_{ix}\equiv U(x)$, so that the force of friction between the electrons and ions has only a y-component. The condition under which the term that takes into account the friction between the electrons and ions can be neglected can be easily derived from the initial system of equations. For this purpose we must have $\nu_{ei}V_{ey}\ll U dV_{ey}/dx$, or, recognizing that the characteristic scale of variation of the quantities is c/ω_0, we must have $c/\omega_0\ll U/\nu_{ei}$ i.e., the thickness of the inertial skin layer is much less than the electron mean free path.

The system of equations of two-fluid hydrodynamics, which describes the interaction of a stream of cold collisionless plasma with a magnetic barrier, under conditions that terms of order m_e/m_i are neglected and quasineutrality is taken into account, can be written in the form

$$m_i U\frac{dU}{dx}=eE_x,\quad E_x+\frac{V_y}{c}B=0,$$

$$m_e U\frac{dV_y}{dx}=\frac{e}{c}BU,\quad \frac{dB}{dx}=\frac{4\pi}{c}enV_y,\quad nU=n_0U_0, \tag{1}$$

where V_y is the electron velocity component along the y axis.

3. APPROXIMATION OF "WEAK DECELERATION" OF COLD COLLISIONLESS PLASMA

1. *Barrier with linear growth and decrease of the field.* We consider the flow of a cold collisionless plasma through a magnetic barrier produced by rectangular currents

$$B_0(x)=\begin{cases}0 & \text{(I)},\\ (-B_0/2\Delta)(x+a+\Delta) & \text{(II)},\\ -B_0 & \text{(III and IV)},\\ (B_0/2\Delta)(x-a-\Delta) & \text{(V)},\\ 0 & \text{(VI)}.\end{cases} \tag{2}$$

The regions I, II, III, IV, V, VI are respectively the regions $x<-(a+\Delta)$, $|x+a|<\Delta$, $(-a+\Delta)<x<0$, $0<x<(a-\Delta)$, $|x-a|<\Delta$, $x>(a+\Delta)$.

From (1) we easily obtain equations for the determination of the vector potential $A\equiv A_y(B=dA/dx)$ and the rate of flow of the plasma $U(x)$

$$\frac{d^2A}{dx^2}-\frac{\omega_0^2}{c^2}A=-\frac{4\pi}{c}I_0(x), \tag{3}$$

$$U(x)=U_0(1-\alpha^2A^2)^{1/2}, \tag{4}$$

$$\alpha^2=e^2/m_i m_e c^2 U_0,\quad I_0(x)=-(c/4\pi)\,dB_0/dx.$$

The weak-deceleration approximation means that

$$\theta=\max\frac{|U(x)-U_0|}{U_0}\ll 1.$$

A solution of (3) is the odd function $A(x)$, so that we shall write it down only in the regions I-III. The solution of (3) in the case of a broad and smoothly varying barrier

$$a\gg\Delta\gg c/\omega_0=\delta \tag{5}$$

takes the form

$$A(x)=\begin{cases}(B_0\delta^2/4\Delta)\,e^{(a+x+\Delta)/\delta} & \text{(I)},\\ (-B_0\delta^2/2\Delta)\,[e^{-\Delta/\delta}\operatorname{ch}[(a+x)/\delta]-1] & \text{(II)},\\ (-B_0\delta^2/2\Delta)\,e^{-(a-\Delta)/\delta}\operatorname{sh}(x/\delta) & \text{(III)}.\end{cases} \tag{6}$$

if from the equations above we easily obtain

$$|A|_{max}\sim B_0\delta^2/\Delta,\quad B_{max}\sim B_0\delta/\Delta,\quad E_{max}\sim eB_0^2\delta^3/mc^2\Delta^2.$$

From (4) and (6) we have $\theta\approx\delta^2/\beta\Delta^2$ at $\theta\ll 1$, whence $\beta\gg(\delta/\Delta)^2$. Since by definition $\delta/\Delta\ll 1$, a plasma can move across a magnetic field with slow deceleration also when $\beta\ll 1$. We note that

$$\max|(n_i-n_e)/n|\sim\omega_B^2\delta^2/\omega_0^2\Delta^2$$

and the quasi-neutrality of the plasma is satisfied if $\omega_B^2\delta^2/\omega_0^2\Delta^2\ll 1$. Here $\omega_B=eB_0/m_ec$ is the electron cyclotron frequency.

2. *Exponential barrier.*

$$B_0(x)=\begin{cases}B_0e^{[(x+a)/\Delta]} & \text{(I)},\\ B_0 & \text{(II)},\\ B_0e^{[-(x-a)/\Delta]} & \text{(III)}.\end{cases}\qquad(7)$$

Regions I, II, III are respectively the regions $x<-a$, $|x|<a$, $x>a$. In the case (5), the solution of (3) is

$$A(x)=\begin{cases}(B_0\delta^2/\Delta)\,[{}^1/_2e^{(x+a)/\delta}-e^{(x+a)/\Delta}] & \text{(I)},\\ (B_0\delta^2/\Delta)\,e^{-a/\delta}\,\mathrm{sh}\,(x/\delta) & \text{(II)},\\ (B_0\delta^2/\Delta)\,[e^{-(x-a)/\Delta}-{}^1/_2e^{-(x-a)/\delta}] & \text{(III)}.\end{cases}\qquad(8)$$

In analogy with the preceding case we have

$$|A|_{max}\sim B_0\delta^2/\Delta,\quad B_{max}\sim B_0\delta/\Delta,\quad E_{max}\approx eB_0^2\delta^3/mc^2\Delta.$$

Flow with slow deceleration, $\theta\ll 1$, is realized at $\beta\gg(\delta/\Delta)^2$, a condition satisfied also at $\beta\ll 1$ if $\delta/\Delta\ll 1$. The quasilinearity condition is the same as in the case considered above.

3. *Power law-barrier*

$$B_0(x)=\begin{cases}0 & -\infty<x<-(a+\Delta)\\ (B_0/\Delta^n)\,(x+a+\Delta)^n & -(a+\Delta)<x<-a\\ B_0 & |x|<a\\ (B_0/\Delta^n)\,(x-a-\Delta)^n & a<x<a+\Delta\\ 0 & x>a+\Delta\end{cases}\qquad(9)$$

$$(n=1,\,2,\,3,\ldots).$$

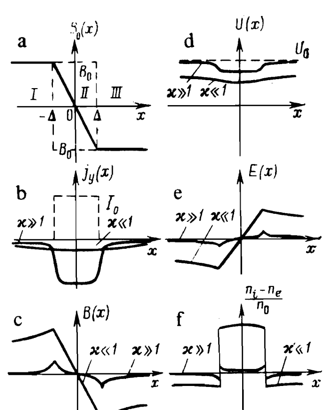

FIG. 2. Distribution of the parameters of a cold plasma flowing through a magnetic barrier produced by a rectangular current: a) Distribution of the magnetic field $B_0(x)$ of the barrier in the absence of a plasma, b) Distribution of diamagnetic current $j_y(x)$. The dashed line shows the external current producing the barrier field, c) Distribution of the resultant magnetic field $B(x)$. d) Distribution of the plasma flow velocity $U(x)$. e) Distribution of electric field E_x. f) Distribution of the charge density $(n_i-n_e)/n_0$.

Under the condition (5), in analogy with the preceding cases, the plasma can move across the magnetic field with practically no change of velocity even if $\beta\ll 1$.

In this section we presented several examples of plasma motion to smoothly varying ($\Delta\gg\delta=c/\omega_0$) magnetic barriers. It is impossible to prove in similar fashion the possibility of plasma motion through steep ($\Delta\ll\delta=c/\omega_0$) magnetic barriers at $\beta\ll 1$, since the weak-deceleration approximation used in the present section is not valid. Indeed, as shown by a subsequent analysis (see Sec. 4), steep magnetic barriers slow down the plasma appreciably.

In the next section we demonstrate by a standard method the feasibility of the motion of a plasma with $\beta\ll 1$ also through steep barriers.

4. PROOF OF THE FEASIBILITY OF PLASMA MOTION ACROSS A MAGNETIC BARRIER WITH $\beta\ll 1$ AT ARBITRARY RATIO δ/Δ

We consider a magnetic barrier produced by a rectangular current (Fig. 2). We change over in Eqs. (3) and (4) to dimensionless variables. Let the length be measured in units of $\delta=c/\omega_0$, the vector potential in units of $1/\alpha$, and the velocity in units of U_0. We introduce the notation

$$W(x)=U(x)/U_0,\quad \varkappa=\Delta/\delta=\Delta/(c/\omega_0),\quad I_0=1/(2\beta)^{1/2}\varkappa.\qquad(10)$$

Equations (4) and (3) then take the form

$$W=(1-A^2)^{1/2},\qquad(11)$$

$$\frac{d^2A}{dx^2}=\begin{cases}A/(1-A^2)^{1/2} & x<-\varkappa,\\ A/(1-A^2)^{1/2}-I_0 & |x|<\varkappa,\\ A/(1-A^2)^{1/2} & x>\varkappa.\end{cases}\qquad(12)$$

From symmetry considerations, the solutions of (11) and (12) are even functions $W(x)$ and $A(x)$. From $A(x)=A(-x)$ it follows that $B(0)=0$. Since $W(x)$ and $A(x)$ are even, it suffices to solve Eqs. (11) and (12) at $-\infty<x\leq 0$.

Multiplying the right- and left-hand sides of (12) by dA/dx and integrating, we lower its order by unity. The result can be conveniently written in the form

$$\frac{1}{2}\frac{dA}{dx}+\mathcal{U}(A)=0,\qquad(13)$$

and

$$\mathcal{U}(A)=\begin{cases}(1-A^2)^{1/2}-1 & -\infty<x<-\varkappa,\\ (1-A^2)^{1/2}-(1-A^2(0))^{1/2}+I_0(A-A(0)) & -\varkappa<x\leq 0.\end{cases}\qquad(14)$$

Formally, Eq. (13) can be regarded as describing one-dimensional motion of a fictitious particle of unit mass with zero total energy in a potential well $\mathcal{U}(A)$. A plot of $\mathcal{U}(A)$ is shown in Fig. 3. Solutions with $B(0)=0$ exist if $A(0)\leq A$. The quantity A is determined from the condition $(d\mathcal{U}/dA)_{A=A^*}=0$. Calculations yield

$$A_*=I_0/(1+I_0^2)^{1/2}.$$

Since $A(0)\leq A_*$, we have

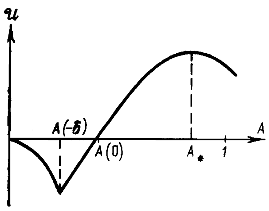

FIG. 3. Schematic form of the function $\mathcal{U}(A)$.

$$U_{min}=U_0(1-A^2(0))^{1/2}\geqslant U_0/(1+\tilde{I}_0^2)^{1/2}.$$

It is seen that the minimum value of the velocity can tend to zero only as $\tilde{I}_0\to\infty$. For finite $\tilde{I}_0$, the minimum value of the plasma velocity need not necessarily be small. Since $\tilde{I}_0=1/(2\beta)^{1/2}\varkappa$, each fixed I_c in the (β, $\varkappa$) plane corresponds to a definite hyperbola. When $\tilde{I}_0$ varies from zero to infinity, the entire region of possible β and ν is covered.

From (13) we easily obtain the connection between $\varkappa$, β and $A(0)$. It can be written in the form

$$\varkappa=\int\limits_{A(-\varkappa)}^{A(0)}\frac{dA}{[1-A^2(0)]^{1/2}-[1-A^2]^{1/2}+\tilde{I}_0(A(0)-A)},\qquad (15)$$

$$A(-\varkappa)=A(0)-[1-(1-A^2(0))^{1/2}]/\tilde{I}_0.$$

At fixed $\tilde{I}_0$, Eq. (15) determines $\varkappa$ as a function of $A(0)$, with $\varkappa\to\infty$ as $A(0)\to A_*$ and $\varkappa\to 0$ as $A(0)\to 0$. At all other values of $A(0)$, the function $\varkappa(A(0))$ is monotonic. Therefore, at an arbitrary fixed $\tilde{I}_0$, when $A(0)$ changes from zero to A_*, the quantity $\varkappa$ runs through all the values from zero to infinity. This means that at arbitrary β and $\varkappa$ there can exist solutions describing plasma flow through a magnetic field produced by a rectangular field.

Let now the magnetic field $B_0(x)$ be produced by two rectangular currents and let it have the shape (2). By virtue of the symmetry of the problem with respect to the plane $x=0$, it suffices to obtain the solutions for $-\infty<x\leqslant 0$. The equation for the vector potential is written in a form analogous to (12).

Multiplying the right-hand and left-hand sides of this equations by dA/dx, and integrating, we decrease its order by unity. It is convenient to recast the result in the form (13), where

$$\mathcal{U}(A)=\begin{cases}(1-A^2)^{1/2}-1,\\ (1-A^2)^{1/2}-1+\tilde{I}_0(A-A_1),\\ (1-A^2)^{1/2}-1+\tilde{I}_0(A_2-A_1),\end{cases}\qquad (16)$$

$$A_1=A(-a-\varkappa),\quad A_2=A(-a+\varkappa).$$

We consider the flow of a plasma through a barrier with $a\to\infty$. It is obvious that solutions with plasma passage can exist in this case if $B(0)\to 0$. But then we have $A_1\to A_2$ in (16). As $A_1\to A_2$, the form of the potential well becomes the same as for the flow of a plasma through one current, and consequently the conditions for the flow of plasma through the magnetic barrier turn out to be similar. This means that at arbitrary β and $\varkappa$ there can exist solutions describing the flow of a cold collisionless plasma through a magnetic barrier produced by sufficiently widely separated currents of rectangular form.

The foregoing proof of the existence of the solution with passage of a plasma having $\beta\ll 1$ through an infinitely broad barrier proves all the more the possibility of passage of a plasma with $\beta\ll 1$ through a barrier of finite width.

In the case of sufficiently thin conductors, the external current can be regarded as a δ-function (in the sense of Dirac) and the natural question is whether it can be cancelled out by the diamagnetic current of the plasma. The ratio of the total external current to the total plasma current flowing in the layer $|x|<\varkappa$ is

$$2\tilde{I}_0\varkappa\Big/\int\limits_{-\varkappa}^{+\varkappa}A(1-A^2)^{-1/2}\,dx.$$

If we assume $2\tilde{I}_0\varkappa$ constant, then

$$\lim_{\varkappa\to 0}2\tilde{I}_0\varkappa\Big/\int\limits_{-\varkappa}^{+\varkappa}A(1-A^2)^{-1/2}\,dx=\frac{A_*(1-A^2(0))^{1/2}}{A(0)(1-A_*^2)^{1/2}}.$$

This ratio is ~ 1 at $A(0)\to A_*$, this proving the existence in the plasma of narrow currents that cancel out the external singular currents. It is obviously of interest to consider the case when for some reason, unaccounted for in the described model, the plasma current cannot be made narrow enough to compensate for the external singular current. Is the motion of the plasma across the magnetic field at $\beta<1$ possible in this case? This question is answered in Sec. 6.

5. FLOW OF PLASMA THROUGH A MAGNETIC BARRIER PRODUCED BY SINGULAR EXTERNAL CURRENTS (WEAK DECELERATION APPROXIMATION)

Let the barrier take the form of a step (Fig. 1a): $B_0(x)=B_0$ at $|x|<a$ and $B_0(x)=0$ at $|x|>a$. This field distribution is produced by two infinite plates located at $x=\pm a$, through which a planar current flows in the y direction (marked by the dashed lines in Fig. 1b). The plates are assumed permeable to the plasma: this may be, for example, a rectangular solenoid whose dimensions in the y and z directions are large in comparison with a.

We consider first the case of weak deceleration, $\max|(U(x)-U_0)/U_0|\ll 1$. Under this assumption we obtain from (1)

$$\frac{d^2V_y}{dx^2}-\frac{\omega_0^2}{c^2}V_y=\omega_B\{\delta(x+a)-\delta(x-a)\},\qquad (17)$$

where $\delta(x)$ is the Dirac δ-function.

The solution of (17) is obtained in the form

$$\frac{V_y}{c}=\begin{cases}-(\omega_B/\omega_0)e^{\xi}\,\mathrm{sh}\,\xi_0 & \text{(I)},\\ (\omega_B/\omega_0)e^{-\xi_0}\,\mathrm{sh}\,\xi & \text{(II)},\\ (\omega_B/\omega_0)e^{-\xi}\,\mathrm{sh}\,\xi_0 & \text{(III)}.\end{cases}\qquad (18)$$

$$\xi=x/\delta,\quad \xi_0=a/\delta.$$

Regions I, II, and III are respectively the regions $x<-a$, $|x|<a$, $x>a$.

As follows from (18), the value of V_y at the points $x=\pm a$ depends essentially on the value of the parameter ξ_0 in comparison with unity. It is easy to clarify the physical meaning of the parameter ξ_0. To this end we compare the plasma current with the value of the external field flowing over the plates. The external current is equal to

$$I(x)=-\frac{c}{4\pi}B_0\{\delta(x+a)-\delta(x-a)\}.$$

The total external current flowing in one direction is

$$\int_{-\infty}^{0} I(x)\,dx=-cB_0/4\pi.$$

The plasma self-current flowing in one direction is

$$\int_{-\infty}^{0} j_y(x)\,dx=\left(\frac{cB_0}{4\pi}\right)\left[1-\exp\left(-\frac{a}{\delta}\right)\right].$$

At $\xi_0=a/\delta\gg 1$ we find that the diamagnetic current in the plasma produces a magnetic field of the order of the external field $B_0(x)$. At $a/\delta\ll 1$ we have

$$\int_{-\infty}^{0} j_y(x)\,dx\Big/\int_{-\infty}^{0} I(x)\,dx=-a/\delta.$$

Thus, the quantity $\xi_0=a/\delta$ is a measure of the diamagnetism of the plasma as it passes through the magnetic barrier.

We use (18) to calculate the resultant field $B(x)$ (Fig. 1c),

$$B(x)=\begin{cases}-B_0e^{\xi}\operatorname{sh}\xi_0 & \text{(I)},\\ B_0e^{-\xi_0}\operatorname{ch}\xi & \text{(II)},\\ -B_0e^{-\xi}\operatorname{sh}\xi_0 & \text{(III)}.\end{cases} \tag{19}$$

It is seen from (19) that if $a\gg\delta$, then the resultant field differs substantially from zero only in a narrow vicinity of thickness δ near the plate, i.e., the field of the external current is confined to a skin layer by the currents of the plasma and does not penetrate into the interior of the plasma. The maximum absolute value of the field is $B_0/2$, and the field has a discontinuity from $-B_0/2$ to $B_0/2$ on going from one side of the plate to the other (Fig. 1c).

For a plasma of lower density, for which $a\ll\delta$, the profile of the field $B(x)$ is close to the profile of the external field $B_0(x)$, since the manifestation of the diamagnetism is weak in this case. The distribution of the velocity $U(x)$ in the case of weak slowing down is

$$U(x)\approx U_0[1-(m_e/2m_i)(V_y^2/U_0^2)]. \tag{20}$$

From (20), using (18), we obtain (Fig. 1d)

$$U(x)=\begin{cases}U_0(1-\beta^{-1}e^{2\xi}\operatorname{sh}^2\xi_0) & \text{(I)},\\ U_0(1-\beta^{-1}e^{-2\xi_0}\operatorname{sh}^2\xi_0) & \text{(II)},\\ U_0(1-\beta^{-1}e^{-2\xi}\operatorname{sh}^2\xi_0) & \text{(III)}.\end{cases} \tag{21}$$

The distribution of the electric field in the case of weak slowing down is

$$\frac{E_x(x)}{B_0}=\begin{cases}(-\omega_B/\omega_0)e^{2\xi}\operatorname{sh}^2\xi_0 & \text{(I)},\\ (-\omega_B/\omega_0)e^{-2\xi_0}\operatorname{sh}2\xi & \text{(II)},\\ (\omega_B/\omega_0)e^{-2\xi}\operatorname{sh}^2\xi_0 & \text{(III)}.\end{cases} \tag{22}$$

As seen from Fig. 1e, the field E_x has a discontinuity at the points $x=\pm a$, corresponding to a surface charge density

$$\sigma=(B_0/4\pi)(\omega_B/\omega_0)e^{-\xi_0}\operatorname{sh}\xi_0. \tag{23}$$

For the volume charge density we obtain (Fig. 1f)

$$\frac{n_i-n_e}{n_0}=\begin{cases}-2(\omega_B/\omega_0)^2e^{2\xi}\operatorname{sh}^2\xi_0 & \text{(I)},\\ -(\omega_B/\omega_0)^2e^{-2\xi_0}\operatorname{ch}2\xi & \text{(II)},\\ -2(\omega_B/\omega_0)^2e^{-2\xi}\operatorname{sh}^2\xi_0 & \text{(III)}.\end{cases} \tag{24}$$

The quasineutrality condition, as seen from (24), is equivalent to the requirement

$$\omega_B^2/\omega_0^2\ll 1. \tag{25}$$

In problems concerning the structure of the boundary layer between the plasma and the field one usually obtains an analogous quasineutrality condition[10]: $m_iU_0^2\ll m_ec^2$. Actually, $\omega_B^2/\omega_0^2\sim m_iU_0^2/m_ec^2$, since in the next section it will be shown that the flow of cold collisionless plasma through a rectangular magnetic barrier is possible only if $\beta>\frac{1}{4}$. From (21) we obtain the condition for the applicability of the weak slowing-down approximation

$$\max|(U(x)-U_0)/U_0|=\beta^{-1}e^{-2\xi_0}\operatorname{sh}^2\xi_0\ll 1, \tag{26}$$

which in the limiting case $\xi_0\ll 1$ yields $\beta\gg\xi_0^2$, or $a^2\ll 2\rho_e\rho_i$, where ρ_e and ρ_i are the Larmor radii of the electrons and ions, respectively, calculated from the velocity U_0: $\rho_{e,i}=U_0/\omega_{Be,i}$.

6. EXACT SOLUTION OF THE PROBLEM OF THE PASSAGE OF A COLD COLLISIONLESS PLASMA THROUGH A RECTANGULAR MAGNETIC BARRIER

We shall show that the solutions obtained in Sec. 5 in the weak-slowing-down approximation and describing the flow of a plasma through a rectangular barrier differ little from the exact solutions. Thus, we shall not assume that the parameter $m_eV_y^2/m_iU_0^2$ is small. Then instead of (17) we obtain the equation

$$\frac{d^2V_y}{dx^2}-\frac{\omega_0^2}{c^2}V_y\left(1-\frac{m_eV_y^2}{m_iU_0^2}\right)^{-1/2}=\omega_B\{\delta(x+a)-\delta(x-a)\}. \tag{27}$$

We solve this equation separately in regions I, II, and III with the following matching conditions:

$$V_y^{\rm I}(-a)=V_y^{\rm II}(-a),\quad V_y^{\rm II}(a)=V_y^{\rm III}(a), \tag{28a}$$

$$\frac{dV_y^{\rm II}}{dx}(-a)-\frac{dV_y^{\rm I}}{dx}(-a)=\omega_B,\quad \frac{dV_y^{\rm III}(a)}{dx}-\frac{dV_y^{\rm II}}{dx}(a)=-\omega_B. \tag{28b}$$

In the three regions we obtain respectively

$$\frac{dV_y}{dx}=-2^{1/2}\delta U_0\left(\frac{m_i}{m_e}\right)^{1/2}\left[1-\left(1-\frac{m_eV_y^2}{m_iU_0^2}\right)^{1/2}\right]^{1/2} \qquad \text{(I)},$$
$$\frac{dV_y}{dx}=2^{1/2}\delta U_0\left(\frac{m_i}{m_e}\right)^{1/2}\left[P^2-\left(1-\frac{m_eV_y^2}{m_iU_0^2}\right)^{1/2}\right]^{1/2} \qquad \text{(II)} \qquad (29)$$
$$\frac{dV_y}{dx}=-2^{1/2}\delta U_0\left(\frac{m_i}{m_e}\right)^{1/2}\left[1-\left(1-\frac{m_eV_y^2}{m_iU_0^2}\right)^{1/2}\right]^{1/2} \qquad \text{(III)}.$$

Here P^2 is the still unknown integration constant. The solution of equations (29) can be obtained in parametric form. We put $V_y=-U_0(m_i/m_e)^{1/2}\sin 2\varphi$. The parameter φ runs through the following values: $0\leqslant\varphi<\varphi_1$ in region I, $\varphi_2\leqslant\varphi\leqslant\varphi_3$ in region II, and $\varphi_4\leqslant\varphi\leqslant\pi$ in region III. Here $0<\varphi_1<\frac{1}{4}\pi$, $\frac{1}{4}\pi<\varphi_2<\frac{1}{2}\pi$, $\frac{1}{2}\pi<\varphi_3<\frac{3}{4}\pi$, $\frac{3}{4}\pi<\varphi_4<\pi$. The values of φ_1 and φ_2 are obtained from the joining conditions: $\varphi_4=\pi-\varphi_1$, $\varphi_3=\pi-\varphi_2$.

We write down the solution in the three regions.

In region I:

$$V_y=-U_0(m_i/m_e)^{1/2}\sin 2\varphi,$$
$$dV_y/dx=-2\delta U_0(m_i/m_e)^{1/2}\sin\varphi, \qquad (30)$$
$$x/\delta=\ln|\operatorname{tg}\varphi/2|+2\cos\varphi+C_1.$$

In region II

$$V_y=-U_0(m_i/m_e)^{1/2}\sin 2\varphi,$$
$$\frac{dV_y}{dx}=2^{1/2}\delta U_0(m_i/m_e)^{1/2}(P^2+\cos 2\varphi)^{1/2}, \qquad (31)$$
$$\frac{x}{\delta}=\left(k-\frac{2}{k}\right)\{K(k)-F(k,\varphi)\}+\frac{2}{k}\{E(k)-E(k,\varphi)\}.$$

In (31) we used the notation $k^2=2/(P^2+1)$. $F(k,\varphi)$, $E(k,\varphi)$, are incomplete elliptic integrals of the first and second kind, respectively, $K(k)=F(k,\frac{1}{2}\pi)$ and $E(k)=E(k,\frac{1}{2}\pi)$. The integration constant is chosen from the condition $x=0$ at $\varphi=\pi/2$, i.e., $V_y=0$ at the center of the barrier. In region III the expressions are analogous to expressions (30).

We now determine the constants φ_1, φ_2, C_1, and P^2. From (28a) we obtain $\sin 2\varphi_1=\sin 2\varphi_2$, i.e.,

$$\varphi_2={}^1/_4\pi-\varphi_1. \qquad (32)$$

From (28b) we have

$$\delta U_0(m_i/m_e)^{1/2}\{2^{1/2}(P^2+\cos 2\varphi_2)^{1/2}+\sin\varphi_1\}=\omega_B. \qquad (33)$$

Equations (32) and (33) yield the expressions $\varphi_1=\varphi_1(P)$ and $\varphi_2=\varphi_2(P)$. The value of $C_1(P)$ we obtain from (30)

$$-a/\delta=\ln|\operatorname{tg}\varphi_1(P)/2|+2\cos\varphi_1(P)+C_1(P). \qquad (34)$$

We can now calculate P^2 from (31)

$$-a/\delta=(k-2/k)\{K(k)-F(k,\varphi_2(P))\}+(2/k)\{E(k)-E(k,\varphi_2(P))\}. \qquad (35)$$

Expressions (30) and (31) jointly with the expressions for the constants (32)–(35) give the exact solution of the problem in parametric form.

We now obtain the limit of the existence of the obtained solution. The sought limit is determined from the condition $m_eV_y^2=m_iU_0^2$, with the velocity $U(x)$ vanishing at the points $x=\pm a$. From (30) and (31) we then have $\varphi_1=\varphi_2=\pi/4$. Equation (33) yields the connection between P^2 and β:

$$P=-1+\beta^{-1/2}. \qquad (36)$$

From (35) we obtain

$$-\frac{a}{\delta}=\left(k-\frac{2}{k}\right)\left\{K(k)-F\left(k,\frac{\pi}{4}\right)\right\}+\frac{2}{k}\left\{E(k)-E\left(k,\frac{\pi}{2}\right)\right\}. \qquad (37)$$

Expressions (36) and (37) determine the connection between the quantity β_{cr} (the obtained solutions exist only at $\beta>\beta_{cr}$) and the parameter a/δ.

We consider two limiting cases:

a) $a/\delta\ll 1$. Using the asymptotic expressions for elliptic integrals as $k\to 0$ ($\beta\to 0$ from (36)), we obtain

$$\beta_{cr}=2(a/\delta)^2, \quad \text{or} \quad a\leqslant(\rho_e\rho_i)^{1/2}.$$

b) $a/\delta\gg 1$. It is seen from (37) that in this case it it is necessary that $k\to 1$, i.e., $\beta\to\frac{1}{4}$. Again using the asymptotic form of the elliptic integrals as $k\to 1$, we get

$$\beta_{cr}\approx{}^1/_4(1-0.8\exp(-a/\delta))\approx{}^1/_4.$$

We note once more with respect to the obtained solutions that validity depends essentially on the assumption that the plasma currents cannot be of the δ-function type, in contrast to the external currents.

7. REFLECTION OF A FLUX OF COLD PLASMA FROM A MAGNETIC BARRIER

In the preceding section we have considered the case of passage of a plasma through a magnetic barrier, i.e., we have assumed that in the region $x\to-\infty$ there is no flux of reflected plasma. From physical considerations it is clear that at $\beta<1$ there can exist also a different type of solution, in which the plasma is reflected from the magnetic barrier. Let us see how the picture of the interaction of the plasma with the rectangular barrier appears in the presence of a reflected flux.

Let x_0 be the coordinate of the point at which the velocity of the flux vanishes. We write down the equation for the vector potential A

$$\frac{d^2A}{dx^2}=2\frac{\omega_0^2}{c^2}A\left[1-\frac{e^2A^2}{m_em_iU_0^2c^2}\right]^{1/2}+B_0\delta(x+a), \qquad x<x_0,$$
$$\frac{d^2A}{dx^2}=B_0\delta(x-a), \qquad x>x_0. \qquad (38)$$

Just as in Sec. 6, we seek the solution in parametric form:

$$A=U_0(m_i/m_e)^{1/2}(m_ec/e)\sin 2\varphi. \qquad (39)$$

In region I ($x<-a$) we obtain

$$\frac{dA}{dx}=-2^{3/2}\delta U_0\left(\frac{m_i}{m_e}\right)^{1/2}\frac{m_ec}{e}\sin\varphi, \qquad 0<\varphi<{}^1/_4\pi. \qquad (40)$$

In region II ($-a<x<x_0$)

$$\frac{dA}{dx}=2\delta U_0\left(\frac{m_i}{m_e}\right)^{1/2}\frac{m_ec}{e}(P^2+\cos 2\varphi)^{1/2}, \qquad \varphi_2<\varphi<{}^3/_4\pi. \qquad (41)$$

The condition for matching at the point $x = x_0$ corresponding to the value $\varphi = \frac{3}{4}\pi$, yields $B(x_0) = B_0$, or:

$$2\delta U_0 (m_i/m_e)^{1/2} m_e c P/e = B_0,$$

from which we obtain the connection between P and $\beta' = 8\pi m_i n_0 U_0^2 / B_0^2$

$$P = [2\beta']^{-1/2}. \tag{42}$$

Let us find $x(\varphi)$ in region II (the expressions for $x(\varphi)$ in region I coincide with (30), where it is necessary to replace ω_0 by $\omega_0/2^{1/2}$)

$$\frac{x}{\delta} = -\int_0^{\varphi} \frac{\cos 2\alpha \, d\alpha}{[P^2 + \cos 2\alpha]^{1/2}} + C_1. \tag{43}$$

The constant C_1 is determined from the condition $x = -a$ at $\varphi = \varphi_2 = \frac{1}{2}\pi - \varphi_1$:

$$\frac{x+a}{\delta} = -\frac{k}{2^{1/2}} \int_{\varphi_2}^{\varphi} \frac{1 - 2\sin^2\alpha}{(1 - k^2 \sin^2 \alpha)^{1/2}} d\alpha, \quad k^2 = \frac{2}{P^2 + 1}, \tag{44}$$

Formulas (39)–(44) determine the distribution of the vector potential A in parametric form.

From (44) we can easily determine the depth of penetration of the plasma $h = x_0 + a$ into the barrier for a given β'

$$h/\delta = -2^{-1/2} \{ (k - 2/k) [2K(k) - F(k, {}^1/_4\pi) - F(k, \varphi_2)] + (2/k) [2E(k) - E(k, {}^1/_4\pi) - E(k, \varphi_2)] \}. \tag{45}$$

In two limiting cases we obtain from (45) the following: a) at $\beta' \ll 1$ we have $h \sim \delta(\beta')^{1/2}/2 = (\rho_e \rho_i)^{1/2}$, i.e., the depth of penetration is equal to the hybrid Larmor radius; b) as $\beta' \to \frac{1}{2}$ we have $h/\delta \to \infty$.

If the depth of penetration h becomes larger than the width of the barrier $2a$, then solutions with reflection do not exist and the pass-through regime considered above is realized. At $\beta' > \frac{1}{2}$ flow will be realized with passage of the plasma through any rectangular barrier.

We call attention to the fact that in this section we have designated the ratio of $\rho_0 U_0^2$ to $B_0^2/8\pi$ by β' and not by β. The point is that the plasma pressure on the barrier is in this case equal to $2\rho_0 U_0^2$, and the ratio of the plasma pressure to the magnetic-field pressure is equal to

$$\beta = \frac{2\rho_0 U_0^2}{B_0^2/8\pi} = 2\beta'.$$

Thus, the reflection from a broad barrier can take place under the natural condition $\beta < 1$.

The distribution of the resultant field $B(x)$, of the plasma current, and of the plasma velocity are shown in Fig. 4. The depencence of the penetration depth h on β, given by expressions (42), (44), and (45), is shown in Fig. 5.

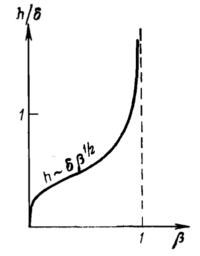

FIG. 5. Dependence of the depth of penetration h (in units of δ) of the plasma into the magnetic barrier (for a stationary solution with reflection) on the value of β.

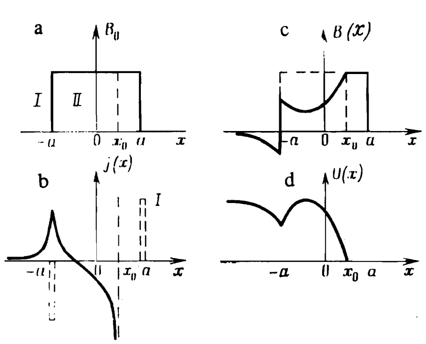

FIG. 4. Parameter distribution in the reflection of a cold plasma from a rectangular barrier. a) Distribution of the barrier field $B_0(x)$ in the absence of plasma (x_0 is the point where the flux is reflected), b) Distribution of diamagnetic current $j(x)$, c) Distribution of the resultant magnetic field $B(x)$, d) Distribution of the velocity $U(x)$ of the incident plasma flux.

8. CONCLUSION

Thus, as shown earlier, the plasma can move perpendicular to the magnetic field at $\beta \ll 1$. An exception is the case given in Secs. 5 and 6, when the currents of the external conductors are singular, and the diamagnetic currents in the plasma cannot cancel them. A corresponding similar situation is produced under conditions when the region of localization of the diamagnetic currents turns out to be much larger than the region occupied by the external conductors, so that the cancellation of the external currents does not take place and the plasma cannot move across a magnetic field with $\beta < \frac{1}{4}$. We note, however, that in this case the plasma motion across the magnetic field can occur at $\beta < 1$ (but $\beta > \frac{1}{4}$). It must be borne in mind, however, that the assumption that the diamagnetic currents of the plasma cannot be singular is outside the framework of the model considered above. Indeed, it is easy to understand the reason why in our model the magnetic barrier produced by so arbitrarily concentrated currents is incapable of confining a plasma flux with $\beta \ll 1$. In regions where the plasma is stopped by the magnetic barrier, $U(x) \to 0$, it follows also from the continuity equation that $n(x) \to \infty$, so that the y component of the diamagnetic current j_y can become arbitrarily large. This is precisely the gist of the cancellation of the singular external currents by the plasma currents; this effect explains the possibility of plasma flow with $\beta < 1$ through the magnetic barrier made up by singular currents.

A computer calculation of the dependence of $U_{\min}$ on β, carried out by formula (15) at different $\varkappa = \Delta/(c/\omega_0)$, shows that this dependence depends essentially on $\varkappa$ (Fig. 6). It is seen that at $\varkappa \gg 1$ strong deceleration of the plasma takes place only at $\beta \ll 1$. On the other hand

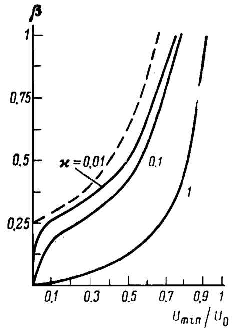

FIG. 6. Dependence of the minimal velocity of the plasma flux, in flow through a rectangular current, on β at various $\varkappa = \Delta/\delta$.

if $\varkappa \ll 1$, then a decrease in the plasma flux to practically zero takes place already at $\beta \sim \frac{1}{4}$. The singular distribution of the external currents corresponds in Fig. 6 to the dashed curve that shows that only at $\beta > \frac{1}{4}$ can the plasma move through the singular external currents. At an arbitrarily small broadening of the currents, the plot of $U_{\min}$ against β reaches the origin, corresponding to a possibility of plasma motion through arbitrarily narrow currents at arbitrary β. In this sense, the results of the general analysis of plasma flow, which demonstrates the possibility of flow through arbitrarily narrow currents at arbitrary β, become matched with the exact solution for the plasma flows (only at $\beta > \frac{1}{4}$) through singular currents.

The results obtained above do not contradict in any way the classical results, according to which the plasma can move through a transverse magnetic field only under the condition $\beta > 1$. Indeed, the ratio of the plasma pressure to the pressure of the resultant field in the obtained solutions exceeds unity. This means that if the plasma penetrates once in some manner the barrier and annihilates the field in it, its subsequent motion is possible even when $\beta < 1$.

The question arises as to how the obtained stationary solutions can be established. In fact, as shown in Sec. 7, at $\beta < 1$ there exists, besides the obtained solutions, also solutions in which the plasma is reflected from the barrier, i.e., incident and reflected fluxes exist as $x \to -\infty$. The depth of penetration of the plasma into the barrier then depends on the value of β. It is obvious that if at the initial instant of time we begin to inject from $x \to -\infty$ a plasma with $\beta < 1$ into the barrier, then reflection takes place, and ultimately a stationary flow of the type obtained in Sec. 7 is established, in which the plasma does not go into the region $x \to \infty$. One can imagine, however, also other possibilities. Assume that we have a plasma flux in all of space and, starting with a certain instant of time, we begin to "turn on" slowly an external field. It is clear that in this case we arrive at solutions with through transmission. The magnetic field cannot enter into the interior of the plasma volume and will be localized near the conductors.

One other method of establishing the obtained flows is to inject at the initial instant into the barrier a plasma flux with $\beta > 1$. It will pass through the magnetic field and will decrease the latter via its diamagnetism. We then can, gradually decreasing $\rho_0 U_0^2$, bring β down to a value much less than unity.

We are grateful to G. I. Kichigin for useful discussions.

[1)] We shall say that the field is strong if the plasma pressure (dynamic plus thermal) is less than the magnetic-field pressure $B^2/8$ as measured in vacuum

$$8\pi(\rho_0 U_0^2 + n_0 T)/B_0^2 = \beta < 1.$$

Here $\rho_0 = n_0 m_i$; n_0 and U_0 are respectively the concentration and velocity of the plasma far from the magnetic barrier, m_i is the ion mass, and T is the plasma temperature.

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Translated by J. G. Adashko