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The $\gamma\to\nu\bar{\nu}$ and $\nu\to\gamma\nu$ reactions in strong magnetic fields

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The $\gamma\to\nu\bar{\nu}$ and $\nu\to\gamma\nu$ decay probabilities in a strong magnetic field are found by employing an effectively two-dimensional representation of the electron Green's function. The contribution of the $\gamma\to\nu\bar{\nu}$ photodecay to the neutrino luminosity of pulsars is estimated. The contributions of other diagrams with vacuum loops are discussed. Previous results obtained in the frequency range $\omega\gg m$, in which the crossed-field approximation is valid, are confirmed. In fields $\sim 10^{16}$ G the $\gamma\to\nu\bar{\nu}$ process competes with the $n+n\to n+p+e^-+\bar{\nu}$ reaction, so that vacuum polarization effects may influence the cooling of neutron stars in their initial evolution stage.

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In connection with the possible existence of ultrastrong magnetic fields $\sim B_0=4.41\times10^{13}$ G in the vicinity of a neutron star, calculations of various electrodynamic and weak processes in constant high-intensity electromagnetic fields have become quite timely. Thus, for example, in[1] they considered the processes $\gamma\to\nu\bar{\nu}$ and $\nu\to\gamma\nu$ in a strong crossed field $(\mathbf{E}\cdot\mathbf{B})=E^2-B^2=0$. Obviously, these calculations are of practical significance only in the energy region where the crossed-field approximation is equivalent to a constant and homogeneous magnetic field, since the possible realization of constant fields $\sim B_0$ occurs precisely in the case of a magnetic field.[1)] This is reached at photon and neutrino energies $\omega\gg m$. However, if the reactions $\gamma\to\nu\bar{\nu}$ and $\gamma\to\gamma\nu$ are considered in the sense of their contribution to the neutrino luminosity of pulsars, then it is the frequencies $\omega\lesssim m$ that are significant, since they receive the greater part of the energy radiated by the stars (with the exception of x-ray pulsars), and then the crossed field approximation is not suitable. It should be noted that in this region the reaction $\gamma\to\nu\bar{\nu}$ is suppressed in part, since a photon in a strong magnetic field acquires at $\omega\ll m$ an imaginary mass $(\omega<|\mathbf{q}|)$.[3] This effect, however, can be compensated for by the interaction of the radiation with a plasma, and at a sufficiently large electron density the photon will have a time-like momentum (see below), and the decay $\gamma\to\nu\bar{\nu}$ will be allowed. On the other hand, the difficulties in the calculation of diagrams with electron loops in a magnetic field, with exact allowance of the interaction with the field, were due to the absence of a convenient representation of the electron Green's function suitable for practical applications, so that it became necessary to use the crossed-field approximation.

In this paper we use the method developed by us to calculate diagrams in a strong magnetic field $B\sim B_0$ with the aid of an effectively two-dimensional represen-

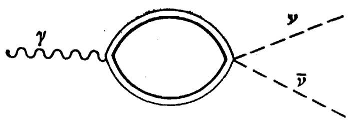

FIG. 1.

tation of the electron Green's function $B(x, y)$.[4] By calculating the Feyman integrals in two-dimensional momentum space we obtained expressions for the probabilities of the processes $\nu \to \gamma\nu$ and $\gamma \to \nu\bar{\nu}$ in a strong magnetic field, and in the region $\omega \gg m$ our results agreed with[1]. Estimates were made of the contributions of these processes to the neutrino luminosity of pulsars. The contributions of the competing processes $\gamma \to \gamma\nu\bar{\nu}$, $\gamma\gamma \to \nu\bar{\nu}$ and $\gamma + Ze \to \nu\bar{\nu} + Ze$ are discussed.

For the sake of argument we consider first the process $\gamma \to \nu\bar{\nu}$ (see Fig. 1); the matrix element of the reaction $\nu \to \gamma\nu$ is obtained by a suitable crossing transformation. According to the result of[4], the electron Green's function in a strong magnetic field takes the form

$$G(x,y)=\varphi(x,y)G(x-y), \quad \varphi(x,y)=\exp\left\{-\frac{i\gamma}{2}(x_1+y_1)(x_2-y_2)\right\},$$
$$G(x-y)=-\frac{\gamma}{2(2\pi)^3}\exp\left\{-\frac{\gamma}{4}[(x_1-y_1)^2+(x_2-y_2)^2]\right\}$$
$$\times(1-i\gamma_1\gamma_2)\int d^2k\, e^{-ik(x-y)}\frac{\hat{k}+m}{k^2-m^2}, \tag{1}$$

where $\gamma = |eB|$, $d^2k = dk_0 dk_3$, $\hat{k} = \gamma_0 k_0 - \gamma_3 k_3$, $k^2 = k_0^2 - k_3^2$, and the formula is valid at $|k^2 - m^2|/\gamma \ll 1$, which imposes definite limitations on the momenta of the external lines of the diagram that includes $G(x, y)$. The calculations in[1] were carried out by Adler's method,[5] in which the $V - A$ vertex was replaced by a pseudoscalar γ^5 plus an anomalous Adler term. For our purposes it is simpler to use a direct calculation by the Rosenberg method,[6] wherein the final result is obtained from gauge-invariance considerations. The two methods are actually identical and lead to the same result.

Recognizing that the factor $\varphi(x, y)$ is cancelled out because of the even number of vertices in the loop, and that the momentum is conserved, we obtain for the matrix element

$$M=-eG(2\pi)^{1/2}[\bar{u}_\nu\gamma^\mu(1+\gamma^5)u_{\bar{\nu}}]e^{*\sigma}\int d^4z\, e^{-i(qz)}\,\mathrm{Sp}\{G(z)\gamma_\mu G(-z)\gamma_\sigma$$
$$+G(z)\gamma_\mu\gamma^5 G(-z)\gamma_\sigma\}, \tag{2}$$

where q is the photon momentum and e^σ is the polarization vector. The first term corresponds to the contribution of the vector current and yields zero upon convolution with the linear bracket, since this bracket is proportional to q because all three momenta are parallel (this follows from the kinematics). The matrix element can then be written in the form

$$M=-\frac{eG\gamma}{2i(2\pi)^{5/2}}\exp\left(-\frac{q_1^2+q_2^2}{2\gamma}\right)[\bar{u}_\nu\gamma^\mu(1+\gamma^5)u_{\bar{\nu}}]e^{*\sigma}I_{\sigma\mu}, \tag{3}$$

$$I_{\sigma\mu}=\int d^2k\,\mathrm{Sp}\left\{\gamma_1\gamma_2\gamma_\sigma\frac{\hat{k}+m}{k^2-m^2}\gamma_\mu\gamma^5\frac{\hat{k}+\hat{q}+m}{(k+q)^2-m^2}\right\}, \quad \sigma,\mu=0.3;$$
$$I_{\sigma\mu}=0, \quad \sigma,\mu=1.2. \tag{4}$$

The general expression for the two-dimensional pseudotensor of the second rank is[2)]

$$I_{\sigma\mu}=Ce_{\sigma\mu}+Ae_{\sigma\tau}q^\tau q_\mu,$$
$$e_{03}=-e_{30}=1, \quad e_{33}=e_{00}=0. \tag{5}$$

Since we should have $I_{\sigma\mu}q^\sigma = 0$, we get $C = 0$ from gauge-invariance considerations, and the coefficient A is given by a converging integral

$$A=8i\int_0^1 dx(1-x)x\int\frac{d^2k}{(k^2-\alpha^2)^2}, \quad \alpha^2=m^2-q^2x(1-x). \tag{6}$$

The integrals can be easily evaluated in the region of space-like momenta $q^2 < 0$ followed by analytic continuation into the region $q^2 > 0$. As a result we get

$$A=\frac{8\pi}{q^2}\left(1-\frac{\varphi}{\sin\varphi}\right), \quad 0\leqslant q^2<4m^2, \tag{7}$$

$$A=\frac{16\pi}{q^2}\left(\frac{1}{2}-\frac{\xi\ln\xi}{1-\xi^2}\right)-i\frac{16\pi^2}{q^2}\frac{\xi}{1-\xi^2}, \quad q^2>4m^2, \tag{8}$$
$$\sin^2\frac{\varphi}{2}=\frac{q^2}{4m^2}, \quad \frac{q^2}{m^2}=\frac{(1+\xi)^2}{\xi}.$$

As follows from (6), the condition for the applicability of (1) for the Green's function takes in this case the form $\gamma \gg q^2$, $\gamma \gg m^2$. In the same approximation we can neglect the exponential factor in (3). Taking (3)–(8) into account we obtain a general expression for the probability of the $\gamma \to \nu\bar{\nu}$ decay per unit time.

$$W_\perp=\frac{e^2G^2\gamma^2}{12(2\pi)^6}\omega^5\sin^6\theta|A|^2,$$
$$\omega=q_0=|\mathbf{q}|, \quad \theta=\widehat{\mathbf{qB}}, \quad q^2=\omega^2\sin\theta, \tag{9}$$

where the symbol $\perp$ denotes that the photon is polarized in the plane of the vectors $\mathbf{B}$ and $\mathbf{q}$. The decay probability of a photon with orthogonal polarization vanishes in our approximation.

In the limiting cases $q^2/m^2 \ll 1$ and $q^2/m^2 \gg 1$ we have $A = -4\pi/3m^2$ and $A = 8\pi/q^2$, respectively. In the second case, after averaging over the polarizations, the result (9) coincides with that of[1].

We note that since all three momenta are collinear the phase volume of the decay of a massless particle into two massless particles is subject in general to an uncertainty. The usual method of getting around this difficulty is to ascribe to the initial particle a bare mass $\mu^2 > 0$, which is made to tend to zero after calculating the phase volume. In the case of a crossed field, for example, the introduction of the bare mass is physically justified by the fact that the polarization operator of the proton has at

$$\chi^2=(F_{\mu\nu}q^\nu)^2/B_0^2m^2\gg1$$

the corresponding "required" sign. As already noted, in a purely magnetic field $\gtrsim B_0$ the contribution of the vacuum polarization leads to the appearance of an imaginary mass $\mu^2 < 0$.[3] The situation can be saved by taking into account the interaction with the plasma,[7] which makes the required contribution to the mass. Namely, the hindrance with respect to the magnetic field is listed upon satisfaction of the condition

$$4\pi(n_e\lambda_C^3)\left(\frac{m}{\omega}\right)^2 > \frac{1}{6\pi}\left(\frac{B}{B_0}\right)\sin^2\theta, \tag{10}$$

which is valid if the electron gas has a sufficiently high density n_e or if the photon frequency ω is low enough (λ_C is the Compton wavelength of the electron). We have taken into account here the fact that the influence of the field $B \sim B_0$ on a nonrelativistic ionized plasma reduces to the fact that its motion becomes one-dimensional. Therefore, strictly speaking, the left-hand side of the inequality is written out for the case when the photon polarization vector is directed along the field. It is clear from the foregoing that when the condition (10) is satisfied the considered photon decay mechanism no longer depends on the plasma density.

Assuming that the decay is allowed, we obtain the power of the neutrino radiation from a unit volume (from the Planck distribution of the equilibrium radiation field)

$$I_\nu = \frac{\alpha G^2\gamma^2}{3(\pi/2)^6}\left(\frac{kT}{mc^2}\right)^9 \frac{mc^2}{\lambda_C^3(\lambda_C/c)}. \tag{11}$$

Arguments analogous to the preceding ones in the case of the crossing process $\nu \to \nu\gamma$ show that for a correct calculation of the phase volume the square of the bare mass of the photon should be negative, a fact ensured by the contribution of the magnetic field, (we assume that $m_\nu = 0$). In the presence of a plasma, on the other hand, an inequality opposite to (10) should be satisfied. If the $\nu \to \nu\gamma$ decay is allowed, then we obtain from (3), (5), (7), and (8) the following expression for the probability of the crossing process:

$$W_\nu = \frac{e^2G^2\gamma^2}{2(2\pi)^7}\frac{\sin^2\theta}{k_0}\int (kq)\frac{|A|^2}{q^2}\delta^{(4)}(k-q-k')\,d^3k'\,d^3q, \tag{12}$$

where k and k' are the momenta of the initial and final neutrinos, and θ is the angle between **k** and **B**. At $k^2 \gg m^2$ and $(q^2)_{eff} \gg m^2$, when the crossed-field approximation can be used, we obtain the result of[1], and at $k^2 \ll m^2$ we have

$$W_\nu = \frac{e^2G^2\gamma^2}{5\cdot 27(2\pi)^4}\left(\frac{\omega}{m}\right)^5 m\sin^6\theta, \quad \omega \equiv k_0 \tag{13}$$

Thus, $W_\perp/W_\nu = 5$ and, at any rate, the photodecay predominates if the condition (10) is satisfied.

Besides the process $\gamma \to \nu\bar{\nu}$, an appreciable contribution to the neutrino luminosity of pulses can be made also by other reactions with the vacuum loops (in a magnetic field) $\gamma \to \gamma\nu\bar{\nu}$, $\gamma\gamma \to \nu\bar{\nu}$, and $\gamma + Ze \to \nu\bar{\nu} + Ze$. The first and second can dominate in the case when the condition (10) is satisfied, the third makes the contribution at a sufficiently high density of the plasma in the magnetosphere. We shall show that in fields $B \gtrsim B_0$ the matrix elements of these reactions do not depend on the field. Integrating over the coordinates in the corresponding matrix elements, we obtain

$$M \sim -i\alpha C\gamma[\bar{u}\gamma^\sigma(1+\gamma^5)u]e^\mu e^\nu I_{\mu\nu\sigma}, \tag{14}$$

$$I_{\mu\nu\sigma} = \int d^2k\,\mathrm{Sp}\left\{(1-i\gamma_1\gamma_2)\frac{1}{\hat{q}+\hat{p}-m}\gamma_\nu(1-i\gamma_1\gamma_2)\right.$$
$$\left.\times\frac{1}{\hat{q}-m}\gamma_\mu(1-i\gamma_1\gamma_2)\frac{1}{\hat{q}-\hat{p}'-m}\gamma_\sigma(1+\gamma^5)\right\} + I'_{\mu\nu\sigma}, \tag{15}$$

where $I'_{\mu\nu\sigma}$ differs from the first term in that the momenta of the internal lines are of opposite sign, and p and p' are the photon momenta. Owing to the presence of the fact that $(1 - i\gamma_1\gamma_2)$, the tensor $I_{\mu\nu\sigma}$ can be different from zero only at μ, ν, $\sigma = 0$ or 3 (just as in the preceding case, this means that the photons have a polarization of the type $\perp$). In "four-dimensional" electrodynamics, the addition of $I'_{\mu\nu\sigma}$ results in only the "pseudotensor" contribution remaining (the Furry theorem). In this case, as can be seen, the pseudocontribution to $I_{\mu\nu\sigma} \equiv 0$ also vanishes. It follows therefore that it is necessary to retain the next term of the expansion of $G(x, y)$ in powers of $(k^2 - m^2)/\gamma$, and first nonzero term of the expansion of M will not depend on the field.

Thus, the relative contribution to the neutrino luminosity of each of the vacuum diagrams is determined not only by the order of the expansion in the electron charge, but depends significantly also on the concentration of the plasma and on the field strength. As noted by Adler,[8] an analogous situation obtains also in the splitting of a photon in a magnetic field.

We note in conclusion that the process $\gamma \to \nu\bar{\nu}$ can exert an influence on the cooling of a neutron star during the initial stage of its evolution, when the field can reach $\sim 10^{16}$ G. Taking by way of estimates typical values $\sim 10^{19}$ cm^3 for the volume and $\sim 10^9$ °K for the temperature, we obtain from (11) a neutrino luminosity due to photodecay $\sim 10^{37-38}$ erg/sec, which is comparable with the contribution of the passes $n + n \to n + p + e^- + \bar{\nu}$, which is assumed to be the principal one[9] (the photon luminosity from the volume is suppressed by the large absorption). We arrive at the conclusion that the effects of the polarization of the vacuum are significant for the evolution of macroscopic objects such as neutron stars.

1) According to the results of Ritus and Nikishov[1] the crossed-field approximation is valid if the inequalities $|(F_{\mu\nu}q^\nu)^2| \gg m^2|F_{\mu\nu}F^{\mu\nu}|$ and $m^2|F^*_{\mu\nu}F^{\mu\nu}|$, are satisfied, where $F_{\mu\nu}$ is the tensor of the constant external field and $F^*_{\mu\nu}$ is the dual tensor.

2) We note that $e_{\mu\tau}q^\tau q_\sigma = e_{\sigma\tau}q^\tau q_\mu - q^2 e_{\sigma\mu}$.

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