tion is negative). Simple calculations show that the angle of deflection is proportional to the cylinder radius a and to the angular velocity of rotation Ω. The reason for this deflection is the effect of entrainment of light by a moving medium: if the direction of propagation of the wave coincides with the direction of motion of the medium, then the phase velocity of the wave is

$$c/\sqrt{\varepsilon\mu}+u(1-1/\varepsilon\mu),$$

but if the velocities of the wave and of the medium are opposite, then the phase velocity is

$$c/\sqrt{\varepsilon\mu}-u(1-1/\varepsilon\mu).$$

In our case, u is in order of magnitude equal to $a\Omega$. An estimate with allowance for these relations leads to the following expression for the angle θ between the maximum of the scattering indicatrix and the initial direction of propagation of the wave:

$$\theta\approx\frac{u}{c}\sqrt{\varepsilon\mu}\left(1-\frac{1}{\varepsilon\mu}\right)=\frac{a\Omega}{c}\sqrt{\varepsilon\mu}\left(1-\frac{1}{\varepsilon\mu}\right). \quad (29)$$

From this result it follows that the field exerts on the rotating cylinder a force numerically equal to the change of the quantity of motion of the light per unit time in the scattering process. The direction of this force is opposite to the vector change of momentum of the wave during the scattering. This phenomenon may be regarded as an analog of the well-known Magnus effect in the mechanics of continuous media.

The effect considered occurs in the absence of absorption (conductivity). Allowance for conductivity presumably leads to a weakening of this effect.

The authors are grateful to V. L. Ginzburg, Ya. B. Zel'dovich, D. A. Kirzhnits, and S. I. Syrovatskiĭ for a number of valuable comments made during discussion.

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Translated by W. F. Brown, Jr.

The ponderomotive force of a high-frequency electromagnetic field in a dispersive medium

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(Submitted March 31, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1010–1016 (September 1976)

A general expression is obtained for the ponderomotive force of a high-frequency field with slowly varying amplitude in a transparent dispersive fluid and, in particular, in a plasma. For electromagnetic waves in an isotropic plasma, and also for irrotational oscillations, this expression coincides with that obtained earlier. In the general case, however, our expression contains time derivatives of the field amplitude; these may play a significant role, for example, in a magnetically active plasma.

PACS numbers: 52.40.Db

1. Since the formulation of the general problem of finding the time-average stress tensor and ponderomotive force acting on a dispersive medium in a high-frequency field ([1], §61), this problem has been the subject of a large number of papers. We cite first of all studies[2–5] in which this force was obtained for an isotropic collisionless plasma:

$$\mathbf{f}=-\frac{\omega_p^2}{16\pi\omega^2}\nabla|\mathbf{E}|^2. \quad (1)$$

Here $\mathbf{f}$ is the ponderomotive force acting on unit volume, $\omega_p=(4\pi ne^2/m)^{1/2}$ is the plasma frequency, and $\mathbf{E}$ is the amplitude of the high-frequency (hf) field:

$$\tilde{\mathbf{E}}={}^1/_2(\mathbf{E}e^{-i\omega t}+\text{c.c.}),\quad \tilde{\mathbf{H}}={}^1/_2(\mathbf{H}e^{-i\omega t}+\text{c.c.}). \quad (2)$$

A general phenomenological approach to the determination of the ponderomotive force of a hf field in a dispersive transparent medium was proposed by Pitaevskiĭ.[6] The expression obtained in[6], however, was limited by several conditions that will be discussed below. The one most important for us here is the assumption that the hf field is stationary (amplitude constant in time). In[7,8], on the basis of quasimicroscopic considerations, an expression was obtained for the ponderomotive force of a hf field in a magnetically active plasma. Here again the effects of nonstationarity of the hf field amplitudes were disregarded. (For a more detailed discussion of[7,8], see below.) These effects, however, may play an important role in investigation of nonlinear interaction of the hf field with the material and, in particular, in investigation of the dynamics of solitons in a plasma.

On the other hand, in an investigation of the self-focusing of transverse waves propagated along an external magnetic field B_0 (directed along the z axis) in a plasma, one of the authors (H. Washimi[9]), using the method of perturbation theory, obtained an expression for the ponderomotive force that can be reduced to the form

$$f_z=\frac{\varepsilon-1}{16\pi}\frac{\partial|\mathbf{E}|^2}{\partial z}+\frac{k}{16\pi\omega^2}\frac{\partial[\omega^2(\varepsilon-1)]}{\partial\omega}\frac{\partial|\mathbf{E}|^2}{\partial t}-\frac{\partial p}{\partial z},$$
$$f_r=-\frac{1}{16\pi}\frac{\partial[\omega(\varepsilon-1)]}{\partial\omega}\frac{\partial|\mathbf{E}|^2}{\partial r}-\frac{1}{4\pi}\frac{\partial B_z}{\partial r}B_0-\frac{\partial p}{\partial r}. \tag{3}$$

Here r is the distance from the z axis (cylindrical symmetry), $\varepsilon(\omega)=N^2(\omega)$, $N(\omega)$ is the refractive index for waves propagated along the magnetic field (for whistler waves, $\varepsilon=1-\omega_p^2/\omega(\omega-\omega_c)$; for Alfvén waves, $\varepsilon=c^2/V_A^2$, where V_A is the Alfvén velocity); $\mathbf{B}(r,z,t)$ is the total magnetic field (including, besides the external, also the induced slowly varying field). When $\omega_c\equiv eB_0/mc=0$, the expression (3) reduces to (1), if we neglect the force due to the kinetic pressure p. An important feature of (3) is the term containing a derivative with respect to "slow" time, on which, in general, the amplitude of the hf field depends in nonstationary nonlinear processes. Here the term containing $(k/\omega)\partial|\mathbf{E}|^2/\partial t$ may be comparable with $\partial|\mathbf{E}|^2/\partial z$ (if $N^2(\omega)$ is sufficiently large). The expression for $\mathbf{f}$ obtained in[6-8] and applied to the case considered in[9] does not agree with (3), differing by the absence of the term containing $\partial/\partial t$ in f_z. Furthermore, the expression for f_r calculated according to[6] also differs from (3).

In the present paper, a general expression will be obtained for the ponderomotive force in an arbitrary transparent medium with time dispersion, with allowance for the fact that the amplitude of the hf field may vary slowly with time. From this expression follow, in particular, (1) and (3), and also the results of[6-8], if the conditions are satisfied that were used in their derivation.

2. Since our arguments depend in an essential way on the work of Pitaevskiĭ,[6] we shall first of all discuss his basic results. The first of them is formulated as follows.

The variation of the free energy in an adiabatic (infinitely slow) change of the permittivity of the medium, and in the absence of an influx of electromagnetic energy into the medium from outside, is

$$\delta\mathcal{F}=\delta\mathcal{F}_0-\frac{1}{16\pi}\int E_i^*E_k\delta\varepsilon_{ik}\,dV, \tag{4}$$

where $\delta\mathcal{F}_0$ is the variation of the free energy in the absence of the high-frequency field. Although the derivation given in[6] was based on a certain simplified model, formula (4) has a quite general character. In particular, it can be shown that (4) is correct also for a collisionless plasma, provided the change process is adiabatic (infinitely slow).

The second result of Pitaevskiĭ that is important for us consists in the fact that if the medium is not only in a hf field but also in an external static magnetic field, with induction $\mathbf{B}^0$, then the dependence of the tensor ε_{ik} on $\mathbf{B}^0$ leads to the appearance of a magnetic moment of the medium, constant in time, induced by the hf field. This follows from the formula $\mathbf{H}^0=4\pi\partial F/\partial\mathbf{B}^0$,[1] where F is the free-energy density:

$$\mathcal{F}=\int F\,dV.$$

If $\mu=1$ without the hf field (that is, $4\pi\partial F_0/\partial\mathbf{B}^0=\mathbf{B}^0$), then by use of (4)[6]

$$\mathbf{H}^0=\mathbf{B}^0-\frac{1}{4}\frac{\partial\varepsilon_{ik}}{\partial\mathbf{B}^0}E_i^*E_k; \tag{5}$$

this determines the magnetic-moment density induced by the hf field: $4\pi\mathbf{M}=\mathbf{B}^0-\mathbf{H}^0$.

Assuming that

$$\operatorname{rot}\mathbf{H}^0=0, \tag{6}$$

Pitaevskiĭ obtains from (4) and (5) the following expression for the ponderomotive-force density of the hf field:

$$\mathbf{f}^{(p)}=-\nabla p+\frac{1}{16\pi}\left[\nabla\left(E_i^*E_k\frac{\partial\varepsilon_{ik}}{\partial\rho}\rho\right)-E_i^*E_k\nabla\varepsilon_{ik}+\frac{\partial\varepsilon_{ik}}{\partial B_j^0}\nabla B_j^0E_i^*E_k\right]. \tag{7}$$

For a gas and for a plasma, where the tensor ε_{ik} is proportional to ρ, $\rho\partial\varepsilon_{ik}/\partial\rho=\varepsilon_{ik}-\delta_{ik}$, so that

$$\nabla\left(E_i^*E_k\frac{\partial\varepsilon_{ik}}{\partial\rho}\rho\right)-E_i^*E_k\nabla\varepsilon_{ik}=(\varepsilon_{ik}-\delta_{ik})\nabla(E_i^*E_k). \tag{8}$$

From (7) and (8), in particular, expression (1) follows if we set $\varepsilon_{ik}=\delta_{ik}(1-\omega_p^2/\omega^2)$. Furthermore, from (7) and the relation $f_i^{(p)}=\partial\sigma_{ik}^{(p)}/\partial x_k$ Pitaevskiĭ finds the stress tensor $\sigma_{ik}^{(p)}$:

$$\sigma_{ik}^{(p)}=-p\delta_{ik}+\frac{1}{16\pi}E_l^*E_m\left(\rho\frac{\partial\varepsilon_{lm}}{\partial\rho}-\varepsilon_{lm}\right)\delta_{ik}+\frac{1}{16\pi}(E_i^*D_k+\text{c.c.})$$
$$-\frac{1}{8\pi}(H^0)^2\delta_{ik}+\frac{1}{4\pi}H_i^0B_k^0. \tag{9}$$

Here he assumes that the hf field is irrotational; that is,

$$\operatorname{rot}\tilde{\mathbf{E}}=0. \tag{10}$$

Thus the results (4), (7), and (9) are obtained under the following assumptions: a) the variation of the state of the medium in (4) is adiabatic (infinitely slow); b) the hf field satisfies the condition (10); c) the constant magnetic field satisfies the condition (6).

3. We shall now see how the results given above are altered by abandonment of these restrictions.

We shall first abandon restrictions b) and c) but keep the condition a). Let ε_{ik} depend only on the density of the medium and on the stationary magnetic field. Then

$$\delta\varepsilon_{ik}=\frac{\partial\varepsilon_{ik}}{\partial\rho}\delta\rho+\frac{\partial\varepsilon_{ik}}{\partial\mathbf{B}^0}\delta\mathbf{B}^0. \tag{11}$$

In order to find expressions for $\delta\rho$ and $\delta\mathbf{B}^0$ during a deformation, it is necessary to make definite assumptions about the character of the medium. We shall suppose that it is a fluid; that is, $\delta\rho$ is determined by the equation of continuity $\partial\rho/\partial t + \operatorname{div}(\rho\mathbf{v}) = 0$. On setting $\delta\rho = \Delta t\, \partial\rho/\partial t$ and introducing the (infinitely small) displacement vector of a point, $\boldsymbol{\xi}(\mathbf{r}) = \mathbf{v}\Delta t$, we get

$$\delta\rho = -\operatorname{div}\{\rho(\mathbf{r})\boldsymbol{\xi}(\mathbf{r})\}. \tag{12}$$

As regards $\delta\mathbf{B}^0$, upon change of $\mathbf{B}^0$, in general, induced electric fields occur, which may produce corresponding quasistatic conduction currents. If the medium is an ideal dielectric (with respect to constant currents), then by virtue of Maxwell's equations the relation (6) used by Pitaevskiĭ holds. We shall here consider the opposite limiting case, in which the resistance to a constant current is zero (for example, in a collisionless plasma). Then the induced electric field $\mathbf{E}^0$ will be perpendicular to the motion; that is, $\mathbf{E}^0 = -[\mathbf{v}\times\mathbf{B}^0]/c$. On using the fact that $\delta\mathbf{B}^0 = \Delta t\, \partial\mathbf{B}^0/\partial t = -c\Delta t \operatorname{curl}\mathbf{E}^0$, we get

$$\delta\mathbf{B}^0 = \operatorname{rot}[\boldsymbol{\xi}(\mathbf{r})\times\mathbf{B}^0]. \tag{13}$$

On substituting (11)–(13) in (4), we get, after simple transformations, the variation $\delta\mathcal{F}$ in the form $\delta\mathcal{F} = -\int \mathbf{f}^{(s)}(\mathbf{r})\xi(\mathbf{r})dV$, where $\mathbf{f}^{(s)}$ is defined by the expression

$$\mathbf{f}^{(s)}(\mathbf{r}) = \mathbf{f}^{(p)}(\mathbf{r}) - \frac{1}{4\pi}[\mathbf{B}^0\times\operatorname{rot}\mathbf{H}^0]. \tag{14}$$

In other words, upon removal of the restrictions b) and c) an expression for the density $\mathbf{f}^{(s)}$ of ponderomotive forces is obtained that differs from Pitaevskiĭ's expression (7) by the second term in (14); it has the meaning of a Lorentz force acting on the stationary conduction currents in the medium.[2)] On determining the tensor $\sigma_{ik}^{(s)}$ from the relation $f_i^{(s)} = \partial\sigma_{ik}^{(s)}/\partial x_k$ and using Maxwell's equations for the high-frequency field,

$$\operatorname{rot}\tilde{\mathbf{E}} = -\frac{1}{c}\frac{\partial\tilde{\mathbf{H}}}{\partial t}, \qquad \operatorname{rot}\tilde{\mathbf{H}} = \frac{1}{c}\frac{\partial\tilde{\mathbf{D}}}{\partial t}, \tag{15}$$

$$\frac{\partial\tilde{D}_i}{\partial t} = -\frac{i\omega}{2}\varepsilon_{ik}E_k e^{-i\omega t} + \text{c.c.}, \tag{16}$$

we get

$$\begin{aligned}\sigma_{ik}^{(s)} &= \sigma_{ik}^{(p)} - \frac{1}{8\pi}\langle(\tilde{H})^2\rangle\delta_{ik} + \frac{1}{4\pi}\langle\tilde{H}_i\tilde{H}_k\rangle = -p\delta_{ik} \\ &+ \frac{1}{16\pi}\left[E_l^* E_m\left(\rho\frac{\partial\varepsilon_{lm}}{\partial\rho} - \varepsilon_{lm}\right) - H_l^* H_l\right]\delta_{ik} \\ &+ \frac{1}{16\pi}(E_i^* D_k + H_i^* H_k + \text{c.c.}) - \frac{1}{8\pi}(H^0)^2\delta_{ik} + \frac{1}{4\pi}H_i^0 B_k^0.\end{aligned} \tag{17}$$

Thus in the stress tensor $\sigma_{ik}^{(s)}$ there appear additional terms, quadratic in the $\tilde{H}_i$; but the terms dependent on the stationary magnetic field remain the same as in Pitaevskiĭ's tensor $\sigma_{ik}^{(p)}$.

The expression (14) coincides with the ponderomotive force obtained from the two-fluid equations for a plasma in Refs. 7, 8, and (17) coincides with the corresponding stress tensor obtained in[8]. The expressions for $\mathbf{f}^{(s)}$, when applied to the special case considered in[9], differ from (3) by absence of the time derivative.

4. We now consider the general case, in which the amplitudes of the hf fields $\mathbf{E}$ and $\mathbf{H}$ are slow functions of time, so that instead of (16) we have

$$\begin{aligned}\frac{\partial\tilde{D}_j}{\partial t} &= \frac{1}{2}\left[-i\omega\varepsilon_{jk}E_k + \frac{\partial(\omega\varepsilon_{jk})}{\partial\omega}\frac{\partial E_k}{\partial t}\right]e^{-i\omega t} + \text{c.c.}, \\ \frac{\partial\tilde{\mathbf{H}}}{\partial t} &= \frac{1}{2}\left(-i\omega\mathbf{H} + \frac{\partial\mathbf{H}}{\partial t}\right)e^{-i\omega t} + \text{c.c.}\end{aligned} \tag{18}$$

Now we may no longer start from formula (4), whose validity is restricted by condition a) (the time derivatives of the amplitudes, though small, are nevertheless finite). To the expression for the stationary ponderomotive force $\mathbf{f}^{(s)}$ there may now be added, in general, terms proportional to $\partial|\mathbf{E}|^2/\partial t$, which, as has already been mentioned, may be comparable with the spatial derivatives. Similar terms may also appear in the stress tensor σ_{ik}. But since $\sigma_{ik}^{(s)}$ contains no derivatives at all, the corrections to $\sigma_{ik}^{(s)}$ that are proportional to the small time derivatives may be supposed small. Neglecting them, that is setting

$$\sigma_{ik} = \sigma_{ik}^{(s)}, \tag{19}$$

we may now calculate the ponderomotive force by use of the stress tensor. Here, however, it is necessary to use for $\mathbf{f}$ the four-dimensional relation (cf.[1,10])

$$f_i = \frac{\partial T_{i\alpha}}{\partial x_\alpha} = \frac{\partial T_{ik}}{\partial x_k} - \frac{\partial T_{i0}}{\partial x_0}. \tag{20}$$

Here $T_{\alpha\beta}$ is the energy-momentum tensor; $\alpha, \beta = 1, 2, 3, 4$; $i, k = 1, 2, 3$; $x_0 = ct$. Then $T_{ik} = \sigma_{ik} = \sigma_{ik}^{(s)}$, $T_{00} = w$ (the energy density), and the elements T_{0i} are proportional to the components of mean energy flow: $T_{0i} = (1/4\pi)\langle|\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}|_i\rangle$. As regards the components T_{i0} that enter in (20), since the tensor $T_{\alpha\beta}$ must be symmetric, $T_{i0} = T_{0i}$.[13)] When the amplitude of the hf field is independent of time, (20) reduces to the formula used above, $f_i = \partial\sigma_{ik}/\partial x_k$. We note also that the above expressions for $T_{\alpha\beta}$ agree with the choice of a tensor that generalizes to the case of a dispersive medium the well-known energy-momentum tensor of Abraham.[4)]

Thus, substituting the above components of $T_{\alpha\beta}$ in (20) and using (19), (15), and (18), we get

$$\mathbf{f} = \mathbf{f}^{(s)} + \frac{1}{16\pi c}\left(\frac{\partial}{\partial t}[\{(\hat{\varepsilon} - \hat{I})\mathbf{E}\}\mathbf{H}^*] + \omega\left[\left\{\frac{\partial\hat{\varepsilon}}{\partial\omega}\frac{\partial\mathbf{E}}{\partial t}\right\}\mathbf{H}^*\right] + \text{c.c.}\right), \tag{21}$$

where $\mathbf{f}^{(s)}$ is given by expression (14) and where $\hat{I}$ is the unit tensor. It is seen from (21) that the correction for nonstationarity of the amplitudes of the hf field vanishes for irrotational hf waves, since in this case $\mathbf{H} = 0$. In a nondispersive medium, the expression (21) agrees with the mean (over the hf oscillations) value of the ponderomotive force with allowance for the Abraham term (see, for example,[1,10,11]). If the field $\tilde{\mathbf{E}}$, $\tilde{\mathbf{H}}$ is a quasiplane wave with wave vector $\mathbf{k}$, then, neglecting second time derivatives, one can substitute in (21)

$$\mathbf{H} = \frac{c}{\omega}[\mathbf{k}\times\mathbf{E}]. \tag{22}$$

In particular, in an isotropic medium, where $\varepsilon_{ik} = \varepsilon\delta_{ik}$, the expression (21) takes the form

$$\mathbf{f} = \mathbf{f}^{(s)} + \frac{\mathbf{k}}{16\pi\omega^2}\frac{\partial[\omega^2(\varepsilon-1)]}{\partial\omega}\frac{\partial|\mathbf{E}|^2}{\partial t}, \tag{23}$$

where we have taken into account that in an isotropic medium it may be supposed that $\mathbf{E}\perp\mathbf{k}$. Hence it follows that for a plasma without a magnetic field, where $\varepsilon = 1 - \omega_p^2/\omega^2$, the term proportional to the time derivative disappears, so that the force is determined by the expression (1) even for nonstationary amplitude of the hf field.

We note also that if the hf field is a circularly polarized wave, propagated along the magnetic field (whose direction has been taken as the z axis), then on introducing the notation

$$\varepsilon = \varepsilon_{xx} \pm i\varepsilon_{xy} \tag{24}$$

(the signs $\mp$ correspond to right- and left-polarized waves), we can after simple calculations express the ponderomotive force again in the form (23), which leads to the expression (3).

5. Finally, we shall consider how one can obtain the expression (21) for a plasma by starting from the averaged equations of two-fluid hydrodynamics used by Klíma[7] and also in[9]. In fact, from these equations it follows directly that the ponderomotive force of the hf field can be written in the form

$$\mathbf{f} = \frac{1}{c}\langle[\tilde{\mathbf{j}}\times\tilde{\mathbf{H}}]\rangle + \frac{1}{c}[\mathbf{j}^0\times\mathbf{B}^0] - nm_e\langle(\tilde{\mathbf{v}}_e\nabla)\tilde{\mathbf{v}}_e\rangle - nm_i\langle(\tilde{\mathbf{v}}_i\nabla)\tilde{\mathbf{v}}_i\rangle, \tag{25}$$

where $\tilde{\mathbf{v}}$ is the oscillatory component of the velocity ($\langle\tilde{\mathbf{v}}\rangle = 0$), $\tilde{\mathbf{j}}$ is the current produced by the hf field, $\mathbf{j}^0 = (c/4\pi)\,\mathrm{curl}\mathbf{H}^0$, and $\mathbf{H}^0$ is defined in (5); the indices e and i refer to the electrons and to the ions. On using

$$\tilde{\mathbf{j}} = \frac{1}{4\pi}\frac{\partial}{\partial t}(\tilde{\mathbf{D}} - \tilde{\mathbf{E}}), \tag{26}$$

we get

$$\langle[\tilde{\mathbf{j}}\times\tilde{\mathbf{H}}]\rangle = \frac{1}{16\pi}\left\{\left[\left(\frac{\partial(\omega\hat{\varepsilon})}{\partial\omega} - \hat{I}\right)\frac{\partial\mathbf{E}}{\partial t}\times\mathbf{H}^*\right] - i\omega[(\hat{\varepsilon} - \hat{I})\mathbf{E}\times\mathbf{H}^*] + \text{c.c.}\right\}. \tag{27}$$

By substituting in the last term

$$\mathbf{H}^* = \frac{i}{\omega}\left(c\,\mathrm{rot}\,\mathbf{E}^* + \frac{\partial\mathbf{H}^*}{\partial t}\right),$$

we can rewrite (25) in the form

$$\mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2,$$

$$\mathbf{f}_1 = \frac{1}{16\pi c}\left\{\left[\left(\frac{\partial(\omega\hat{\varepsilon})}{\partial\omega} - \hat{I}\right)\frac{\partial\mathbf{E}}{\partial t}\times\mathbf{H}^*\right] + \left[(\hat{\varepsilon} - \hat{I})\mathbf{E}\times\frac{\partial\mathbf{H}^*}{\partial t}\right] + \text{c.c.}\right\}, \tag{28}$$

$$\mathbf{f}_2 = \frac{\hat{\varepsilon} - \hat{I}}{16\pi}[\mathbf{E}\times\mathrm{rot}\,\mathbf{E}^*] - \frac{n}{4}\sum_{\alpha=i,e} m_\alpha(\mathbf{v}_\alpha\nabla)\mathbf{v}_\alpha^* + \text{c.c.} + \frac{1}{c}[\mathbf{j}^0\times\mathbf{B}^0]. \tag{29}$$

The expression (28) coincides with the term containing $\partial/\partial t$ in (21); (29), it can be shown, leads to the expression for f that was obtained in[7,8] and that agrees with $\mathbf{f}^{(s)}$ in (14).

The authors are grateful to V. L. Ginzburg and L. P. Pitaevskiĭ for discussion of the work and for helpful comments.

One of us (H.W.) expresses his thanks to the Academy of Sciences, USSR, for arranging his stay in the USSR, and to the Japan Society for the Promotion of Science for financial support.

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[2)] Terms dependent on the amplitudes of the high-frequency magnetic field intensity of course do not enter in the ponderomotive force, since we are assuming that the high-frequency magnetic permeability is unity; that is, $\tilde{H}_i = \tilde{B}_i$.

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Translated by W. F. Brown, Jr.