[9]V. S. Boĭko, R. I. Garber, and L. F. Krivenko, Fiz. Tverd. Tela (Leningrad) 16, 1451 (1974) [Sov. Phys. Solid State 16, 930 (1974)].
[10]V. S. Boĭko, R. I. Garber, V. F. Kivshik, and L. F. Krivenko, Fiz. Tverd. Tela (Leningrad) 17, 1541 (1975) [Sov. Phys. Solid State 17, 1010 (1975)].
[11]V. S. Boĭko, R. I. Garber, and L. F. Krivenko, Fiz. Tverd. Tela (Leningrad) 16, 1233 (1974) [Sov. Phys. Solid State 16, 798 (1974)].
[12]L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) Nauka, 1965, Ch. 4.
[13]V. S. Boĭko, R. I. Garber, and V. F. Kivshik, Fiz. Tverd. Tela (Leningrad) 16, 591 (1974) [Sov. Phys. Solid State 16, 384 (1974)].
[14]A. M. Kosevich, Usp. Fiz. Nauk 84, 579 (1964) [Sov. Phys. Usp. 7, 837 (1965)].

Translated by R. T. Beyer

Theory of nonequilibrium phase transitions

V. F. Elesin, V. V. Kapaev, and Yu. V. Kopaev

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted February 23, 1976)
Zh. Eksp. Teor. Fiz. **71**, 714–726 (August 1976)

The character of phase transitions occurring on excitation of quasi-particles in the ordered phase by an external field is investigated for three models: an ordinary superconductor, an excitonic insulator (or structural transitions of the Peierls-transition type) and a superconductor with electron–electron repulsion. The structure of the ordered phase depends essentially on the form of the distribution function $n(\epsilon)$ of the nonequilibrium quasi-particles. If the magnitude of the gap Δ in the single-particle excitation spectrum is less than the energy of the Debye phonons, the distribution function $n(\epsilon)$ differs strongly from the quasi-Fermi function. For this reason, the uniform state of superconductors with pumping can be stable. In the opposite case of a large gap Δ, a quasi-Fermi distribution of the nonequilibrium excitations, with a nonzero chemical potential, is possible. If the ordered phase in this case is a consequence of attractive interaction, the uniform state in the ordered phase is unstable. For the example of an excitonic insulator the dependence of the period of the nonuniform state on the pumping intensity is found. But if the ordered phase is a consequence of repulsive interaction, so that its existence is possible only in conditions of pumping, for a quasi-Fermi (inverted) distribution of the excitations the uniform state is stable.

PACS numbers: 64.60.Bm, 74.20.Ef

1. INTRODUCTION

Great interest has recently developed in the study of the character of the phase transitions in systems situated in the field of an external source. Particular interest in this problem has arisen in connection with the search for possibilities of raising the critical temperature of the superconducting transition.

Most types of phase transition are connected with collective effects. As the temperature is raised, excitations (quasi-particles) appear, leading to a decrease of the degree of order (because of the collective character of the latter) and to the existence of a critical temperature.

The action of an external source can be reduced to an increase of the number of quasi-particles as compared with the equilibrium number, and to a change in the character of their energy distribution. The magnitude of the order parameter in nonequilibrium conditions can be determined by the equation for the equilibrium case, in which, in place of the equilibrium quasi-particle distribution function, we must substitute the solution of the kinetic equation.[1a] Usually, the electron-collision times over which the quasi-particle distribution function changes are long compared with the characteristic times of the variation of the order parameter.

In the kinetic equation the time dependence of the order parameter must be neglected, it being assumed that it has time to reach its stationary value for the particular quasi-particle distribution function $n(\varepsilon)$ at the given moment of time. The form of the function $n(\varepsilon)$ depends essentially on the magnitude of the order parameter Δ. The diagram technique of Keldysh[1b] turns out to be convenient for the description of such systems.

The properties of the system in the ordered phase turn out to be very sensitive to the form of the function $n(\varepsilon)$. For example, if $n(\varepsilon) > \frac{1}{2}$ in a certain energy interval, a superconducting state is found to be possible for a repulsive electron-electron interaction.[2] Instead of perfect diamagnetism, a system in a superconducting nonequilibrium state can possess perfect paramagnetism. In systems of the excitonic-insulator type,[3] in which the dielectric gap arises as a consequence of collective effects, magnetic ordering is possible on pumping.[4]

In the present paper we shall investigate the possibility of the appearance of nonuniform states on pumping, for three models: a normal superconductor (Sec. 2), an excitonic insulator (Sec. 3), and a superconductor with repulsive electron-electron interaction (Sec. 4). Similar problems have been investigated for a number of models in equilibrium conditions. For example, in superconductors in the presence of an exchange field the concentrations of electrons with opposite spins can be unequal. Therefore, in the superconducting state unpaired electrons remain present even at $T=0$. It was shown in[5,6] that in these conditions a nonuniform state of the superconductor arises. Analogously, in the case of a doped excitonic insulator, unpaired electrons exist above the gap even at $T=0$, and this also leads to the existence of charge-density or spin-density waves of large period.[7] We note that in both these cases[5–7] the distribution function of the unpaired particles is a Fermi function.

Unpaired particles also arise under the action of a pumping source, both in a superconductor and in an excitonic insulator. But, unlike in the above-mentioned equilibrium cases, here quasi-electron and quasi-hole excitations already exist simultaneously at $T=0$. The form of their distribution functions can differ substantially from the Fermi form, depending both on the magnitude of the gap in the electron spectrum and on the pumping intensity.

For normal superconductors, in view of the small magnitude of the superconducting gap as compared with the Debye energy of the phonons, $n(\varepsilon)<\frac{1}{2}$ for any pumping intensity.[8] The deviation of $n(\varepsilon)$ from the quasi-Fermi function leads to the result that, even with pumping, the uniform state of the superconductors turns out to be stable (cf. Sec. 2).

In the case of excitonic insulators (which include systems in which the dielectric gap is a consequence of the appearance of charge- or spin-density waves), the magnitude of the gap can be greater than the Debye energy of the phonons. Therefore, the distribution function of the quasi-electrons and quasi-holes can be of the Fermi type. Then the uniform state is found to be unstable, and long-wavelength density modulations should exist in the dielectric phase (cf. Sec. 3).

In the case of superconductivity with repulsive interaction (cf. Sec. 4), the superconducting state arises only when $n(\varepsilon)>\frac{1}{2}$, and this is possible only for a large magnitude of the gap Δ.[2] Below it will be shown that the uniform state is stable, despite the Fermi character of the distribution of the quasi-particles excited by the pumping. Formally, the mathematical description of all the models considered below is the same. We write the Hamiltonian of the system in the following form:

$$H=H_0+g\int\Psi_{i\alpha}^+(\mathbf{r})\Psi_{j\beta}^+(\mathbf{r})\Psi_{j\beta}(\mathbf{r})\Psi_{i\alpha}(\mathbf{r})\,d\mathbf{r}, \tag{1}$$

where H_0 is the Hamiltonian of the noninteracting particles; i, j are band indices.

For an excitonic insulator the scattering of an electron from one band by a hole from another band possesses a singularity, and therefore in the Hamiltonian (1) we keep the term with $i\neq j$ and $g>0$, which corresponds to electron-hole attraction.

We shall treat superconductivity in a one-band model ($i=j$), both for the case of electron-electron attraction ($g<0$) and for the case of repulsion ($g>0$) between the electrons. Since we shall be interested in a nonequilibrium state of the system, it is necessary to write down a system of equations for the normal (G) and anomalous (F) Green functions, using the Keldysh technique for nonequilibrium processes.[1b] The solution of the system for the nonuniform case, when

$$\Delta(\mathbf{r})=\pm igF^+(\mathbf{r},\mathbf{r})=\Delta e^{2i\mathbf{q}\mathbf{r}} \tag{2}$$

(the plus sign in (2) corresponds to a superconductor and the minus sign to an excitonic insulator), has the form

$$G(\mathbf{p},\omega)=\frac{u_p^2}{\omega+\mathbf{pq}/m-E+i\delta}+\frac{v_p^2}{\omega+\mathbf{pq}/m+E-i\delta}$$
$$+2\pi i\left\{n_p u_p^2\delta\left(\omega+\frac{\mathbf{pq}}{m}-E\right)-n_{-p}v_p^2\delta\left(\omega+\frac{\mathbf{pq}}{m}+E\right)\right\}, \tag{3}$$
$$F^+(\mathbf{p},\omega)=\frac{-\Delta}{(\omega+\mathbf{pq}/m-E+i\delta)(\omega+\mathbf{pq}/m+E-i\delta)}-$$
$$-\frac{\pi\Delta i}{E}\left\{n_p\delta\left(\omega+\frac{\mathbf{pq}}{m}-E\right)+n_{-p}\delta\left(\omega+\frac{\mathbf{pq}}{m}+E\right)\right\}, \tag{4}$$

where

$$E=\sqrt{\xi^2+\Delta^2},\quad u_p^2,\ v_p^2={}^1/_2(1\pm\xi/E),$$

and n_p is the quasi-particle distribution function, obeying the kinetic equation.[1a,8]

Substituting the expression for F^+ from (4) into (2) we obtain the equation for Δ:

$$1=\pm gN(0)\int_{-1}^{1}\frac{dx}{2}\int_0^{\tilde\omega}\frac{d\xi}{(\xi^2+\Delta^2)^{1/2}}\left[1-2n\left(E+\frac{qpx}{m}\right)\right]. \tag{5}$$

The plus sign in the equation corresponds to the excitonic insulator and the minus sign to the superconductor, $N(0)$ is the density of states at the Fermi level, and $\tilde\omega$ is the characteristic cutoff energy, equal to the Debye energy $\hbar\omega_D$ of the phonons in the case of a normal superconductor and of the order of the energy of the plasma oscillations in the cases of the excitonic insulator and the superconductor with repulsion.

2. INVESTIGATION OF THE STABILITY OF THE UNIFORM STATE OF A SUPERCONDUCTOR WITH PUMPING

On irradiation of a superconductor with a light of frequency Ω greater than the magnitude of the gap Δ ($\hbar\Omega>2\Delta$), breaking of the Cooper pairs occurs, as a result of which quasi-particles in excess of the equilibrium number appear above the gap. Together with the usual heating of the system, this should lead to suppression of the superconductivity, as was indeed observed in the experiments of[9,10]. To explain these experiments, a model was proposed in[11] in which the distribution function of the nonequilibrium quasi-particles was assumed to be a quasi-equilibrium function with a nonzero chemical potential μ:

$$n(\varepsilon)=[e^{(\varepsilon-\mu)/T}+1]^{-1}. \tag{6}$$

In this case, the phase transition to the normal state with increase of pumping intensity I turns out to be a first-order transition. However, in[8] it was shown that the distribution function of the nonequilibrium quasi-particles differs substantially from the Fermi form (6) and cannot exceed the value $\frac{1}{2}$ for any energy ε.

The reason for this restriction is that, for $\Delta \ll \hbar\omega_D$ (which is fulfilled for all known superconductors), the processes of scattering and annihilation of quasi-particles proceed at an approximately equal rate (if we do not assume that $n(\varepsilon) \ll 1$), since they are due to one-phonon processes. As a result, with increase of the pumping intensity the gap Δ vanishes when $\bar{n} = \frac{1}{2}$.[8] The parameter β_c characterizing the critical pumping intensity I is defined by the following expression:

$$\beta_c = \frac{4e^2}{\pi\Omega\Delta_0^2 g\tau} E_c^2,$$

where E_c is the amplitude of the field with frequency Ω, τ is the electron-momentum relaxation time, and Δ_0 is the gap in the absence of pumping. Precisely such a smooth transition to the normal state was observed, apparently, in the experiments of [12,13].

In the experimental work of [12-14], a smooth increase of resistivity with increase of pumping intensity was observed at sufficiently high pumping intensities. On this basis, the hypothesis that a nonequilibrium intermediate state was being observed in the experiments was put forward.

In the theoretical papers[15,16] it was stated that instability of the uniform state of a superconductor with pumping is possible under the condition

$$\bar{n} > 2T/\Delta_0, \tag{7}$$

where $\bar{n}$ is the dimensionless quasi-particle concentration, defined below. Again, the quasi-Fermi function (6), which goes over for small pumping to the Boltzmann function

$$n(\varepsilon) \approx \exp\left(\frac{\mu-\varepsilon}{T}\right). \tag{8}$$

was used for the quasi-particle distribution function $n(\varepsilon)$. Starting from a comparison of the terms describing scattering and annihilation of quasi-particles in the kinetic equation, one can show that the function (8) is valid if the condition

$$\bar{n} \ll {}^1/_{28}(T/\Delta_0)^{1/2}, \tag{9}$$

which contradicts the condition (7), is fulfilled. Therefore, the question of the stability of the uniform state of a superconductor with pumping must be solved using the distribution functions obtained from the solution of the kinetic equation. For small pumping levels, the form of the function $n(\varepsilon)$ was obtained in[17].

As was noted above, the uniform superconducting state goes over into the normal state at $\bar{n}_c = \frac{1}{2}$, which corresponds to a certain value of the dimensionless pumping-field amplitude $\sqrt{\beta_c}$. To solve the question of the stability of the uniform state (with total pair momentum $q = 0$) relative to states with $q \neq 0$, it is necessary to investigate the dependence of the quantity $\sqrt{\beta_c}$ on the wave vector q.[6] If a state with $q \neq 0$ is more favorable, the magnitude of β_c should increase with q, reaching a maximum value at a certain q_0.

To find $\beta_c(q)$ we shall make use of Eq. (5) with $\Delta = 0$, and for the quasi-particle distribution function we shall use the expression[8]

$$n(\varepsilon) = \frac{1}{2}\left(1 - \frac{\varepsilon}{(\varepsilon^2 + 4\Delta_0^2\bar{n}^2)^{1/2}}\right), \quad \bar{n} = \frac{1}{\Delta_0}\int_0^\infty n(\varepsilon)\,d\varepsilon \tag{10}$$

and the condition

$$\frac{1}{2\Delta_0^2}\int_{-1}^{1} dx \int_0^\infty d\varepsilon \int_0^\infty d\varepsilon'\, n(\varepsilon + qvx)\, n(\varepsilon' - qvx) = \beta_c, \tag{11}$$

$$\bar{n}^2 = \beta_c. \tag{12}$$

The condition (11) is obtained by integrating the kinetic equation and describes the balance of the numbers of quasi-particles being created by the source and being annihilated. Substituting (10) into (11) gives (12).

After integration, we find from (5)

$$\frac{1}{2}\ln\psi + \ln\frac{vq}{\Delta_0} = 1 - \frac{1}{2}(\psi+1)^{1/2}\ln\frac{(\psi+1)^{1/2}+1}{(\psi+1)^{1/2}-1}, \tag{13}$$

$$\psi = \psi(q) = \left(\frac{2\Delta_0\bar{n}}{vq}\right)^2. \tag{14}$$

In the limit of small q ($\psi \to \infty$), from (13) and (12) we obtain

$$2\sqrt{\beta_c} = 1 - \frac{1}{3}\left(\frac{vq}{\Delta_0}\right)^2,$$

i.e., the quantity $\sqrt{\beta_c}$ decreases with increasing q ($d\sqrt{\beta_c}/dq < 0$). If we can show that the expression $d\sqrt{\beta_c}/dq$ does not have extrema for any q, then this will mean that $d\sqrt{\beta_c}/dq$ is negative everywhere and a state with $q \neq 0$ is unfavorable.

Calculating the derivative from (13) and equating it to zero, we obtain the following equation:

$$\frac{\psi}{2}\ln\frac{(\psi+1)^{1/2}+1}{(\psi+1)^{1/2}-1} - (\psi+1)^{1/2} = 0. \tag{15}$$

It is not difficult to show that for $0 < q < \infty$ ($0 < \psi < \infty$) this equation has no solutions. Consequently, $d\sqrt{\beta_c}/dq < 0$, i.e., the uniform state is stable for $0 < \beta < \beta_c$.

The analysis performed shows the sensitivity of the criterion for stability of the uniform state to the form of the distribution function of the nonequilibrium quasi-particles. According to the criterion (7) for instability of the uniform state, obtained in[15,16] for a quasi-Fermi excitation distribution function, a nonuniform state at $T = 0$ should arise at arbitrarily low pumping levels. However, the investigation we have performed at $T = 0$ with a distribution function that is the self-consistent solution of the kinetic equation shows that the uniform state of the superconductor is stable even at high pumping levels.

It is possible that, by virtue of the specific conditions of the experiments of [13,14], the function $n(\varepsilon)$ differs from the function (10) investigated by us. It is difficult, therefore, to reach a definite conclusion about the character of the superconducting state investigated in the experiments of [13,14]. A nonuniform state of a superconductor with pumping could, in particular, be a consequence of inhomogeneity of properties of the samples investigated.

3. LONG-WAVELENGTH MODULATIONS IN THE NONEQUILIBRIUM STATE OF AN EXCITONIC INSULATOR

In an excitonic insulator, the plasma frequency, which is of the order of the Fermi energy ε_F, appears (in place of ω_D for the superconductor) in the expression for Δ, and therefore the quantity 2Δ can be greater than the Debye energy $\hbar\omega_D$ of the phonons. Then there are no one-phonon annihilation terms in the kinetic equation and the quasi-particle distribution function can be described by the quasi-Fermi function (6). At $T=0$ this is a step, i.e., the states in the conduction band of the excitonic insulator are filled by quasi-particles up to a certain level μ, and the states in the valence band are filled by quasi-holes to the level $-\mu$.

If, however, $2\Delta<\hbar\omega_D$, all the results of the preceding section are applicable to the excitonic insulator, i.e., a uniform state ($q=0$) is realized in this case.

We shall assume that $2\Delta>\hbar\omega_D$; then, for $q=0$, the results for the excitonic insulator[4] coincide with the results[11] for a superconductor with a quasi-Fermi distribution function. The dependence of the quantity Δ on the concentration n of nonequilibrium excitations is shown in Fig. 1 (curve 1). The phase transition with respect to the pumping intensity is a first-order transition.

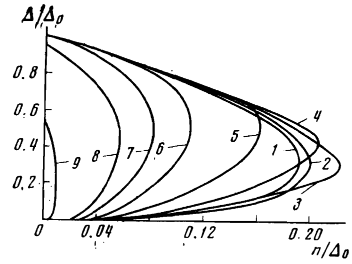

FIG. 1.

As was shown in[15], when the excitations have a quasi-Fermi distribution function, at a sufficiently low temperature the uniform state of the system is unstable against spatial fluctuations of the concentration of excitations. The criterion for instability of the system is the relationship[15]

$$\partial\mu/\partial n<0. \tag{16}$$

It is easy to show that at $T=0$

$$\frac{\partial\mu}{\partial n}=\frac{\Delta_0+3\Delta}{\Delta_0-3\Delta}$$

(Δ_0 is the magnitude of the gap in the absence of pumping), and, since on the stable branch of the curve $\Delta(n)$ the quantity $\Delta>\frac{1}{3}\Delta_0$, the relationship (16) is fulfilled for the whole of the stable branch.

Thus, at $T=0$ it is favorable for the system to go over to a nonuniform state. These nonuniformities are in a certain sense analogous to the electron-hole droplets in an ordinary semiconductor, in which the magnitude of the gap E_g is assumed to be independent of the concentration of excitations, and the interaction of electrons and holes is responsible for the appearance of the droplets. But in the case of an excitonic insulator with pumping the nonuniform state arises because of the effect of the excitations on the magnitude of the gap Δ.

We can convince ourselves that the relationship (16) is also fulfilled in the equilibrium state of an excitonic insulator in the presence of doping—to be precise, $\partial\mu/\partial n=-1$. Therefore, in this case too, it is favorable for the system to go over to a nonuniform state. An analogous conclusion was reached in[7] by solving the equation for Δ with $q\neq 0$ and finding the minimum of the thermodynamic potential Ω.

We assume that in the doping the positive charge of the ions is distributed uniformly and, therefore, the Coulomb interaction between the impurity ions and the electrons hinders the appearance of a nonuniform distribution of electrons. In the presence of pumping, however, the number of electrons is equal to the number of holes. If the electron concentration is increased in a certain region, then the hole concentration is also increased in the same region, and, consequently, the local electroneutrality condition is preserved.

The criterion (16) for instability of the uniform state of the system can be obtained starting from the following considerations. As is well known, the Debye screening radius r_D is given by

$$r_D^{-2}=\frac{4\pi e^2}{\varepsilon_0}\frac{\partial n}{\partial\mu}$$

and for $\partial\mu/\partial n<0$ the quantity r_D becomes purely imaginary. This indicates that the Coulomb potential of the charge is oscillating. Thus, a uniform distribution of charge is unstable.

We turn now to the quantitative description of the nonuniform state for an excitonic insulator with pumping To determine the dependence of the period of the nonuniform state on the concentration of excitations we shall seek solutions for Δ in the form (2). In the presence of pumping, coexistence of singlet and triplet pairings is possible, and this leads to magnetic ordering in the system.[4] We shall assume that the singlet electron-hole interaction constant $g_s>0$, and the triplet constant $g_t<0$. In this case stable compatible solutions for the singlet and triplet order parameters do not exist. Moreover, just as for an excitonic insulator with doping,[19] superconducting pairing of excitations is possible if the effective superconducting interaction corresponds to an attraction. We shall assume the opposite situation, i.e., we shall assume that superconducting pairing of excitations is impossible.

Thus, our system will be described by the Green functions (3), (4). The energy spectrum of the excitations, which is determined by the poles of the Green function, has the form

$$\omega_{1,2}=\mathbf{pq}/m\pm\sqrt{\xi^2+\Delta^2}.$$

Consequently, although the absolute value $|\Delta(\mathbf{r})|$ of the gap is a uniform and isotropic quantity, the excitation spectrum of the system becomes anisotropic. For q greater than a certain critical value, this anisotropy leads to "overlap" of the bands of the excitonic insulator and to redistribution of the electrons between the valence band and the conduction band.

Substituting

$$n(\varepsilon)=\begin{cases}0 & \text{for } \varepsilon>\mu\\ 1 & \text{for } \varepsilon\leqslant\mu\end{cases}$$

into Eq. (5) and performing the integration, we obtain an equation for Δ:

$$-\ln\frac{\Delta}{\Delta_0}=\frac{\Delta}{Q}\{G(r_+)-G(r_-)\}, \tag{17}$$

where

$$r_\pm=\frac{\mu\pm Q}{\Delta},\quad G(x)=\theta(x-1)\{x\ln(x+\sqrt{x^2-1})-\sqrt{x^2-1}\},$$

$$\theta(x)=\begin{cases}1 & \text{for } x>0\\ 0 & \text{for } x\leqslant 0\end{cases},$$

$\Delta_0=2\tilde{\omega}e^{-1/gN(0)}$ is the dielectric gap in the absence of pumping for $q=0$, and $Q=p_Fq/m$.

Equation (17) differs from the corresponding equation for the case of a doped excitonic insulator[7] by the absence of the factor 2 in front of $\ln(\Delta/\Delta_0)$ and by a slightly different definition of the function $G(x)$. (In[7] the factor $\theta(|x|-1)$, as distinct from $\theta(x-1)$ in our case, appears in the definition of $G(x)$.) These differences arise from the fact that in our problem we have, at the same time, quasi-Fermi levels for the electrons ($+\mu$) and for the holes ($-\mu$), whereas in the case of doping there is a single Fermi level μ.

We shall investigate Eq. (17) for a given concentration n of excitations. For this it is necessary to have a relation connecting the position of the quasi-Fermi level μ with the excitation concentration n.

From the function G_{11}, defined by formula (3), and G_{22}, which has a form analogous to G_{11}, we find the number of electrons in the conduction band of the excitonic insulator:

$$\frac{N}{V}=\int d\omega\, d\mathbf{p}\{G_{11}(\mathbf{p},\omega)-G_{11}^0(\mathbf{p},\omega)+G_{22}(\mathbf{p},\omega)-G_{22}^0(\mathbf{p},\omega)\}, \tag{18}$$

where V is the volume of the system and G_{11}^0 and G_{22}^0 are the Green functions in the absence of pumping. Substituting the expressions for G_{11} and G_{22} into (18), we obtain

$$n_0=\frac{N}{4N(0)V\Delta_0}=\frac{\Delta^2}{4Q}[\gamma(r_+)-\gamma(r_-)], \tag{19}$$

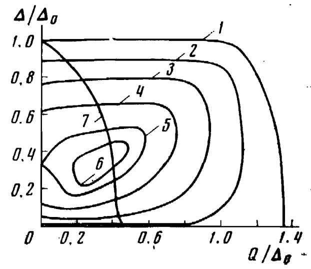

FIG. 2.

where $N(0)$ is the density of states at the Fermi level, and

$$\gamma(x)=\theta(x-1)\{x\sqrt{x^2-1}-\ln(x+\sqrt{x^2-1})\}.$$

For $Q>\Delta$ the overlap of the bands leads to redistribution of the electrons. The number N_1 of electrons in the conduction band as a result of the overlap is given by

$$n_1=\frac{N_1}{4N(0)V\Delta_0}=\frac{\Delta^2}{4Q}\gamma\left(\frac{Q}{\Delta}\right). \tag{20}$$

The number of electrons thrown into the conduction band by the external source is equal to the difference between the total number of electrons in the band and the number due to the overlap. Thus, the relationship of interest to us between n and μ has the form

$$n\equiv n_0-n_1=\frac{\Delta^2}{4Q}\left\{\gamma(r_+)-\gamma(r_-)-\gamma\left(\frac{Q}{\Delta}\right)\right\}. \tag{21}$$

We note that the concentration of excitations, like $|\Delta(\mathbf{r})|$, is uniform over the sample. The system of equations (17), (21) determines the dependence of Δ on n for different values of the parameter Q. This dependence is depicted in Fig. 1 (curve 1 corresponds to the uniform state, i.e., $Q=0$; for curves 2–9 the parameter Q/Δ_0 is respectively equal to: 2) 0.1; 3) 0.3; 4) 0.5; 5) 0.7; 6) 0.9; 7) 1.0; 8) 1.1; 9) 1.3). As can be seen from Fig. 1, for small n the nonuniformity has a weak effect on the dielectric pairing so long as $Q<\Delta_0$. An interesting feature of the nonuniform solutions is the fact that there exists a region of concentrations of excitations ($0.19<n/\Delta_0<0.22$) in which there are nonuniform solutions while the uniform state is no longer possible. In analogy with the case $Q=0$, the lower branches of the curves $\Delta(n)$ for $Q\neq 0$ correspond to unstable states.

Figure 2 shows the dependence of the quantity Δ on Q for different excitation concentrations n. (The parameter n/Δ_0 is respectively equal to: for curve 1) 0; 2) 0.05; 3) 0.10; 4) 0.15; 5) 0.19; 6) 0.21.) The lower branches of the curves $\Delta(Q)$ correspond to unstable states. As can be seen from the figure, for small Q the quantity Δ increases with increase of Q, and the rate of increase increases with increase of n. At a certain Q the quantity Δ reaches a maximum (at the point of the maximum, $\Delta\sim Q$) and then decreases, dropping to zero from finite values of Δ. For $n>0.19\Delta_0$ (such concentrations of excitations suppress dielectric pairing in the uniform state), the solutions for Δ start out from finite values of Q. Thus, nonuniformity favors pairing.

From the whole region of Q in which solutions for Δ exist for a given n, the state that is realized is that with $Q = Q_{opt}$, corresponding to the minimum of the free energy F.

To find this value of Q we shall calculate the difference of the free energies in the dielectric and metallic phases from the formula

$$\Delta F = F_D - F_M = -\int_0^\Delta \Delta^2 \frac{d(1/g)}{d\Delta} d\Delta.$$

Minimizing ΔF with respect to Q, we obtain an equation for the optimal value Q_{opt}:

$$r_-^2\left(r_- \frac{\Delta}{Q}+3\right)\theta(r_-) - r_+^2\left(r_+ \frac{\Delta}{Q}-3\right) - 3[\gamma(r_+)+\gamma(r_-)] + 2\frac{\Delta}{Q}[\delta(r_+)-\delta(r_-)] = 0, \tag{22}$$

where $\gamma(x)$ is defined in (19) and

$$\delta(x) = \theta(x-1)[{}^3/_2 x\gamma(x) - (x^2-1)^{3/2}].$$

Equation (22) differs from the corresponding equation in[7] by the presence of the factor $\theta(r_-)$ in the first term and by a slightly different definition of the functions $\gamma(x)$ and $\delta(x)$.

Solving Eqs. (17), (21) and (22) jointly for each n, we find Q_{opt}. In the (Δ, Q)-plane the values of Q_{opt} lie on the line 7 in Fig. 2. It can be seen from the figure that the quantity Δ for $Q = Q_{opt}$ is greater than Δ for $Q = 0$ and the same level of pumping. With increase of concentration of excitations, the quantity Q_{opt} increases. Throughout, solutions of the form $\Delta(\mathbf{r}) = \Delta e^{2i\mathbf{q}\cdot\mathbf{r}}$ (2) were investigated above. This corresponds to a spatially uniform distribution of electron density. As was shown in[6], for a superconductor in an exchange field the free-energy minimum corresponds to a solution for Δ in the form of a combination of solutions of the type (2), corresponding to a three-dimensional periodic lattice. In the case of an excitonic insulator, such a three-dimensional lattice will correspond to a long-wavelength modulation of the original periodic lattice. The size of the period of the modulation is determined by the value of $1/q_{opt}$ and depends on the pumping intensity in accordance with Fig. 2 (curve 7).

The question of the possibility of an excitonic mechanism for superconductivity, in which the characteristic electron energy should play the role of ω_D in the expression for Δ, has recently been discussed (cf., e.g., [20]). In this case the relationship $2\Delta > \omega_D$ can be fulfilled and the excitation distribution function can be of the type (6). All the results of this section will be applicable to such superconductors.

Up to now we have considered the case of an excitonic insulator formed from a semimetal with band edges coinciding in momentum space. In this case, in the absence of pumping, the crystal structure does not change in the metal-insulator phase transition. But if the band edges are spaced by a vector $\mathbf{w}$, then, in the absence of pumping, in the metal-insulator phase transition a new period, the size of which is inversely proportional to $\mathbf{w}$, appears in the system.

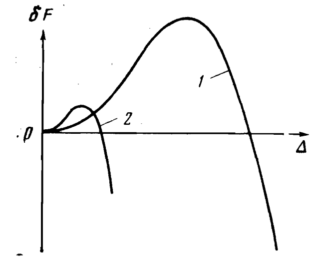

FIG. 3.

In this case, in the presence of pumping, in place of (2) the solution for Δ will be $\Delta(\mathbf{r}) = \Delta \exp\{i(2\mathbf{q}+\mathbf{w})\cdot\mathbf{r}\}$, and the quantity Q_{opt} will again be determined by the curve 7 in Fig. 2. In the absence of pumping, instead of a phase transition of the type in which the period is doubled, a long-wavelength modulation will be superimposed on the doubling.

4. SUPERCONDUCTIVITY IN SYSTEMS WITH ELECTRON-ELECTRON REPULSION

As was shown in[2,21,22], for $n(\varepsilon) > \frac{1}{2}$ a superconducting state is possible for systems with a repulsive electron-electron interaction. In[21] a model of a semiconductor was considered in which, in order to realize a regime of inverted population on pumping, the gap width E_g was greater than the energy of the Debye phonons. The possibility of realizing such a state in a metallic model was considered in[22].

In this section we shall investigate the stability of the uniform superconducting state for the example of the metallic model.

We recall that the metallic state with an inverted distribution of electrons in a layer 2μ about the Fermi level is unstable.[2] The magnitude of the gap Δ in the superconducting state is determined by the expression

$$\Delta = \frac{2\mu^2}{\tilde{\omega}_p} \exp\left\{-\frac{1}{gN(0)}\right\}, \tag{23}$$

where $\tilde{\omega}_p$ is of the order of the energy of the plasma oscillations and g is the effective inter-electron repulsion constant ($g > 0$).

The difference δF of the free energies of the superconducting and normal states for inverted occupation is equal to $+\Delta^2/2$, i.e., the maximum of the function δF corresponds to the superconducting state (see Fig. 3, curve 1) and the minimum corresponds to the state with $\Delta = 0$. It was shown in[2] that this state with the minimum $\delta F(\Delta)$ is unstable. But the state with the maximum $\delta F(\Delta)$ is stable for $g > 0$.[23] Using the approach developed in this paper, we obtain for the boundary of stability in the limit $Q \to 0$, $\omega/Q \to 0$:

$$\int \left(\frac{1-2n(E)}{E^3} + \frac{2}{E^2}\frac{dn}{dE}\right) d^3\mathbf{p} = 0. \tag{24}$$

Substituting the function $n(\varepsilon)$ from (6) into (24), it is easy to convince onself that the state at the maximum of

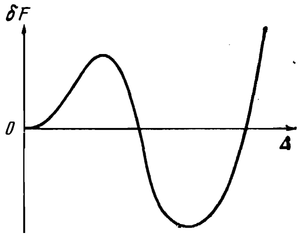

FIG. 4.

$\delta F(\Delta)$ is stable for $g > 0$. Therefore, despite the fact that this state is energetically unfavorable compared with the state $\Delta = 0$, it is this state which should exist as a stationary state, since the state $\Delta = 0$ is unstable.

We note that, in systems with attraction, e.g., in the case of the excitonic insulator considered in Sec. 3 (cf. Fig. 4), for $n(\varepsilon) > \frac{1}{2}$ an unstable state corresponds to the maximum of $\delta F(\Delta)$. The state $\Delta = 0$ with the minimum $\delta F(\Delta)$ is stable. At low pumping intensities the state at the minimum of $\delta F(\Delta)$, with $\Delta \neq 0$ is realized, since it is energetically more favorable than the state with $\Delta = 0$.

As was shown in[22], the superconducting state for repulsive interaction with pumping possesses perfect paramagnetism, in place of the perfect diamagnetism for ordinary superconductors. This should lead to an oscillatory character of the penetration of a magnetic field into such a superconductor. A similar picture should obtain in the nonuniform state of a superconductor in a strong exchange field.[5]

Starting just from the perfect paramagnetism of superconductors with repulsion, we can conclude on the basis of a Maxwell equation that such a state is magnetically unstable even in the absence of an external field. In fact, to solve the question of the stability of the uniform state of such superconductors it is necessary to solve Eq. (5) for Δ jointly with the equation obtained by minimizing the free energy with respect to q, as we did in the preceding section.

Before proceeding to this question, we shall find the amplitude $\Gamma(\omega, q)$ for electron-electron scattering with nonzero total momentum q for a metal with inverted occupation. Carrying out calculations analogous to those for the case of an equilibrium metal with electron-electron attraction,[24] for the pole of $\Gamma(\omega_0, q)$ we obtain, for $qv/\Omega \ll 1$, the following expression:

$$\omega_0 = i\Omega(1 - v^2|\mathbf{q}|^2/6\Omega^2), \tag{25}$$

where Ω is determined by the expression (23) for Δ. Thus, a system with inverted occupation possesses maximum instability with respect to pairing of electrons with zero total momentum. If pairing with $q \neq 0$ occurs, such a state will be unstable with respect to pairing with $q = 0$. This is true at least for small q.

The analogous conclusion concerning the stability of the uniform superconducting state against collective excitations with small q follows from the condition (24).

We shall make use of a qualitative criterion for the stability of the uniform state, proposed in[15] and used by us in Sec. 3 for the excitonic insulator. A breakdown of the system into regions, in one of which the concentration n of excitations is higher than in the uniform case while in the other it is lower, is favorable if the derivative of the quasi-Fermi level μ with respect to n is negative. Using for μ the expression[2]

$$\mu = \sqrt{\Delta^2 + n^2},$$

and for Δ the expression (23), it is easy to see that $\partial\mu/\partial n > 0$. That a nonuniform solution is unfavorable for superconductivity with repulsion is connected with the fact that the gap Δ increases with increase of the pumping intensity, whereas for systems with attraction Δ falls with increase of n (cf. Sec. 3). It is for precisely this reason that a nonuniform state is more favorable in superconductors with an exchange magnetic field[5,6] or in excitonic insulators with doping.[7]

If, in a superconductor with repulsion, the gap Δ has decreased at a certain place on account of a fluctuation, the concentration of excitations in this place increases as a result of diffusion from neighboring regions. But with increase of n the gap Δ in this region should, according to (23), increase, i.e., the fluctuation does not grow in time, but attenuates.

We shall now consider this question in more detail. If we introduce the parameter $\Delta = 2\tilde{\omega}\exp(1/gN(0))$, the system of equations for Δ and Q_{opt} coincides with (17), (22) for the excitonic insulator. The difference in the solutions for Δ in these cases consists in the fact that for the superconductor we must consider the solutions corresponding to the maximum of the free energy (the lower branches of the curves in Fig. 2). Numerical calculations (cf. Fig. 2) show that the solutions of the system (17), (22) for a superconductor with repulsion correspond to the region $\mu < Q$ and $\Delta \ll Q$. In this case, neglecting terms of order Δ^2/Q^2 in the right-hand side of Eq. (17), after simple transformations we obtain

$$\ln\frac{\Delta}{\Delta_0} = \frac{\mu + Q}{\mu}\ln\frac{2(\mu + Q)}{e\Delta_0}. \tag{26}$$

Formula (26) determines Δ as a function of Q for given μ. The function $\Delta(Q)$ has a minimum at the point $Q = \Delta_0/2 - \mu$. In analogy with the previous discussion, from (22) with the condition $\mu < Q$, $\Delta \ll Q$ we obtain an equation for Q_{opt}:

$$Q_{opt}\ln\frac{2(\mu + Q_{opt})}{\Delta_0} = \frac{\mu}{2}. \tag{27}$$

It is obvious that $\mu + Q_{opt} > \frac{1}{2}\Delta_0$, i.e., the value of Q_{opt} is greater than that which corresponds to the minimum in the curve $\Delta(Q)$. We shall show that the corresponding value Δ_{opt} is smaller than the value of Δ at $Q = 0$.

Since $\mu \ll \Delta_0$ and $\mu + Q_{opt} > \frac{1}{2}\Delta_0$, we have $\mu \ll Q_{opt}$. The system of equations (26), (27) determines the value Δ_{opt} corresponding to Q_{opt}. Taking into account the condition $\mu \ll Q_{opt}$, for Δ_{opt} we obtain the expression

$$\Delta_{opt} = \Delta_0 \exp\{-Q_{opt}/\mu\}.$$

It is easy to see that with the assumptions made above the value Δ_{opt} is smaller than the value of Δ at $Q=0$, which is given by formula (23).

Thus, the dependence of δF on Δ for $Q=Q_{opt}$ corresponds to the curve 2 in Fig. 3. It can be seen from a comparison of the curves 1 and 2 that the uniform state of a superconductor with inverted population and electron-electron repulsion is stable.

Because of the perfect paramagnetism[22] of such superconductors an external magnetic field penetrates, oscillating, into the sample, and this leads to oscillations of the gap, the amplitude of which is proportional to the magnitude of the field.

1(a) G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)]; (b) L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].

2V. F. Elesin, Yu. V. Kopaev, and R. Kh. Timerov, Zh. Eksp. Teor. Fiz. **65**, 2343 (1973) [Sov. Phys. JETP **38**, 1170 (1964)].

3L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela (Leningrad) **6**, 2791 (1964) [Sov. Phys. Solid State **6**, 2219 (1965)].

4V. V. Kapaev and Yu. V. Kopaev, Zh. Eksp. Teor. Fiz. **69**, 2171 (1975) [Sov. Phys. JETP. **42**, 1103 (1975)].

5P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).

6A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].

7T. M. Rice, Phys. Rev. B2, 3619 (1970).

8V. F. Elesin, Zh. Eksp. Teor. Fiz. **66**, 1755 (1974) [Sov. Phys. JETP **39**, 862 (1974)].

9L. R. Testardi, Phys. Rev. **B4**, 2189 (1971).

10W. H. Parker and W. D. Williams, Phys. Rev. Lett. **29**, 924 (1972).

11C. S. Owen and D. J. Scalapino, Phys. Rev. Lett. 28, 1559 (1972).

12G. A. Sai–Halasz, C. C. Chi, A. Denenstein, and D. N. Langenberg, Phys. Rev. Lett. **33**, 215 (1974).

13P. Hu, R. C. Dynes and V. Narayanamurti, Phys. Rev. **B10**, 2786 (1974).

14A. I. Golovashkin, K. V. Mitsen, and G. P. Motulevich, Zh. Eksp. Teor. Fiz. **68**, 1408 (1975) [Sov. Phys. JETP **41**, 701].

15J.-J. Chang and D. J. Scalapino, Phys. Rev. B10, 4047 (1974).

16V. G. Baru and A. A. Sukhanov, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 209 (1975) [JETP Lett. **21**, 93 (1975)].

17R. A. Vardanyan and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **65**, 2327 (1973) [Sov. Phys. JETP **38**, 1162 (1974)].

18L. V. Keldysh, Usp. Fiz. Nauk **100**, 514 (1970) [Sov. Phys. Uspekhi **13**, 292 (1970)].

19A. I. Rusinov, Do Chan Kat, and Yu. V. Kopaev, Zh. Eksp. Teor. Fiz. **65**, 1984 (1973) [Sov. Phys. JETP **38**, 991 (1974)].

20V. L. Ginzburg, Usp. Fiz. Nauk **101**, 185 (1970) [Sov. Phys. Usp. **13**, 335 (1970)].

21D. A. Kirzhnits and Yu. V. Kopaev, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 379 (1973) [JETP Lett. **17**, 270 (1973)].

22V. M. Galitskiĭ, V. F. Elesin, and Yu. V. Kopaev, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 50 (1973) [JETP Lett. **18**, 27 (1973)].

23A. G. Aronov and V. L. Gurevich, Zh. Eksp. Teor. Fiz. **65**, 1111 (1973) [Sov. Phys. JETP **38**, 550 (1974)].

24A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, M., 1962 (English translation published by Pergamon Press, Oxford, 1965).

Translated by P. J. Shepherd.