Plasma turbulence near the lower-hybrid resonance

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We study non-linear processes in the frequency range close to the lower hybrid frequency ω_L. For sufficiently short-wavelength oscillations ($kv_{Ti} \gg \omega_{Hi}$) the main one of those is, for a wide temperature range, $T_e \gtrsim T_i$, induced scattering by electrons. We study the non-linear stage of the parametric instability in the frequency range $\omega \gtrsim \omega_L$. We show that if one is just above criticality plasma wave turbulence becomes strong. We obtain the equations describing the strongly non-linear regime. We study their properties.

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INTRODUCTION

The study of the possibility of plasma heating by a high-frequency electrical field and by powerful laser radiation pulses has recently excited a large amount of scientific and applied interest.[1–3] The choice of the optimum conditions for such a heating presupposes an understanding of the physics of the processes which take place as the result of the action of a powerful pumping wave on the plasma. Anomalously strong absorption of the energy of the external field is caused by the excitation and dissipation of a large number of plasma waves. Usually the characteristics of such a turbulent heating are determined by their non-linear interactions and at the present time only a start has made with its study.

Langmuir wave turbulence in an isotropic, isothermal plasma has been studied in most detail.[4–6] Recently we[7,8] have considered the excitation of plasma waves in a magneto-active plasma with frequencies much higher than the lower-hybrid frequency $\omega_L = \omega_{pi}\omega_H/(\omega_p^2+\omega_H^2)^{1/2}$ (ω_H, ω_p are here the electronic cyclotron and plasma frequencies). Thanks to induced scattering of plasma waves by ions the ionic component of the plasma is in this case very strongly heated; this heating can for sufficiently strong pumping be comparable to the heating of the electrons. We showed in[7,8] that the conditions for such a heating become more favorable at the limit of applicability of the theory—when the pumping frequency ω_0 approaches the lower-hybrid frequency ω_L. From this point of view it is very important to study non-linear effects in the immediate vicinity of the lower-hybrid resonance. A detailed study of this region is also of great independent interest because of the anomalously small thresholds for the excitation of the turbulence and the relative ease of introducing energy into the plasma.[9,10] This fact was just the reason why experimenters have paid so much attention to it.

The present paper is devoted to a theoretical study of the turbulence of hf plasma waves with frequencies $\omega \gtrsim \omega_L$. We derive in § 1 the equations which describe the evolution of hf waves when the non-linear interaction with the low-frequency motions in the plasma are taken into account. We show, in particular, the Hamiltonian nature of the interaction of the hf waves with the ion-cyclotron oscillations of arbitrary order and we determine the Hamiltonians for such an interaction. We also elucidate the role played by the various non-linear processes. For sufficiently short-wavelength hf waves ($kv_{Ti} \gg \omega_{Hi}$) the main non-linear effect near the lower-hybrid resonance turns out to be induced scattering by electrons. This circumstance is also retained in a non-isothermal plasma ($T_e \gg T_i$).

We consider in § 2 the non-linear stage of the parametric instability near the lower-hybrid resonance ($\omega_0 \sim \omega_L$). Before we study it we consider briefly the linear theory. We show that when we are sufficiently close to and above the threshold for the instability of the oscillations due to scattering by electrons there is a transfer into the range of angles $\theta = \measuredangle\, \mathbf{k}, \mathbf{H}_0$ close to $\pi/2$ at the same time leading to a condensation in the short-wavelength region of the spectrum $N_\mathbf{k} \sim N(\theta)\delta(k-k_0)$. When we are further above threshold, when the oscillations reach the range of angles

$$x=\cos\theta=(m/M)^{1/2}k_0 r_H \quad (r_H=v_{Te}/\omega_H),$$

the efficiency of the transfer in angle decreases steeply. The strong effect on the scattering processes simultaneously starts to show thermal corrections to the dispersion law for the hf waves. The direction of the spectral transfer then changes and after that it proceeds both in angle θ and in absolute magnitude of the wavevector. The weakening of the efficiency of the transfer leads to an accumulation of wave energy in the long-wavelength region of the spectrum ($k < k_0$) analogous to the condensation of Langmuir plasmons in the small-k region. The limits for the applicability of weak turbulence theory are then rapidly violated.

We obtain in § 3 the equations which describe the behavior of the strongly non-linear hf wave spectra. They turn out to be very different from the equations obtained by Zakharov[11] for the description of the Langmuir wave collapse in an isotropic plasma. We study several properties of these equations. The results of the present paper were published as a preprint.[12]

§1. NON-LINEAR PROCESSES NEAR THE LOWER-HYBRID RESONANCE

We consider short-wavelength ($kc \gg \omega_p$) potential plasma oscillations which propagate almost at right angles to the magnetic field so that $x = \cos\theta \ll 1$:

$$\omega_k=\omega_L(1+z^2)^{1/2}, \quad z=\sqrt{M/m}\cos\theta. \tag{1}$$

This formula does not take into account the effect of the thermal motion of the plasma on the dispersion law; this can be very important for $z<1$. We shall take this into account in what follows for actual cases. When the wavelength increases the oscillations become non-potential and as $k\to 0$ we shall have $\omega_k\to 0$. On the other hand, in the large-wavenumber region they are subject to strong damping due to collisionless dissipation mechanisms.

The evolution of the hf waves considered here due to non-linear interactions is adequately described by the equations from two-fluid hydrodynamics. If the nonlinearity is sufficiently small, so that the change in amplitude is small during a period of the oscillations it is convenient to change from the density and velocity variables to the normal wave amplitude

$$a_k=\frac{\omega_p}{\omega_H}\frac{\omega_k^2}{\omega_{pi}^2}\left(\frac{2n_0m(\omega_p^2+\omega_H^2)}{k^2\omega_k}\right)^{1/2}\frac{\delta n_k^i}{n_0} \tag{2}$$

(here δn^i is the variation in the ion density in the hf oscillations). The total energy of the oscillations is then

$$W=\int W_k\,d\mathbf{k}=\int\omega_k|a_k|^2\,d\mathbf{k},$$

while the hydrodynamical equations reduce to the form

$$\left(\frac{\partial}{\partial t}+i\omega_k\right)a_k=-\frac{i}{2}\frac{\omega_H^2}{(\omega_p^2+\omega_H^2)}\int\frac{k_1}{k}(\mathbf{k}\mathbf{u}_{k_1})\frac{\delta\tilde{n}_\varkappa}{n_0}a_{k_1}\delta_{\mathbf{k}-\mathbf{k}_1-\varkappa}\,d\mathbf{k}_1\,d\varkappa; \tag{3}$$

$$\mathbf{u}_k=-\frac{\omega_p^2}{k^2\omega_H^2}\left\{\omega_k\left(\mathbf{k}-\frac{\omega_H^2}{\omega_k^2}k_z\mathbf{h}\right)+i\omega_H[\mathbf{h}\times\mathbf{k}]\right\},\quad \mathbf{h}=\frac{\mathbf{H}_0}{H_0},$$

where $\delta\tilde{n}$ is the variation in the ion density (and also in the electron density as in what follows $kr_d=kv_{Te}/\omega_p\ll 1$) in the slow motions for which we can not apply a hydrodynamical description.

Equation (3) thus describes only the interaction with the low-frequency plasma motions, i.e., decay processes involving lf oscillations and the induced scattering of hf waves by particles. Only the decay processes involving long-wavelength non-potential oscillations drop out of consideration in this case. Their probability contains, however, a factor small in the parameter $v_T/c\ll 1$ and is considerably smaller than the probability for the non-linear processes considered below.

To close Eq. (3) it is necessary to take into account the effect of the hf waves on the evolution of lf perturbations. Taking this effect into account requires, in general, a kinetic description.

We consider to begin with decay processes involving lf plasma eigenoscillations—ion-cyclotron (IC) and ion-acoustic (IA) oscillations. In that case we can obtain (for details see[7,13])

$$\frac{\delta\tilde{n}_{\varkappa\Omega}}{n_0}=G_{\varkappa\Omega}\frac{\omega_H^2}{2n_0T_e(\omega_p^2+\omega_H^2)}\int\frac{k_2}{k_1}(\mathbf{k}_1\mathbf{u}_2)^*a_1a_2^*\delta_{\varkappa-\mathbf{k}_1+\mathbf{k}_2}\delta_{\Omega-\omega_1+\omega_2}\,d\mathbf{k}_{1,2}\,d\omega_{1,2}. \tag{4}$$

The Green function

$$G_{\varkappa\Omega}=\frac{\varepsilon_e}{\varepsilon}-1\approx\left\{1-\frac{\varkappa_z^2T_e}{M\Omega^2}-2\frac{T_e}{T_i}\sum_{\lambda=1}\frac{\lambda^2\omega_{Hi}^2\exp(-\varkappa_\perp^2v_{Ti}^2/\omega_{Hi}^2)I_\lambda}{\Omega^2-\lambda^2\omega_{Hi}^2}\right\}^{-1} \tag{5}$$

(here ε is the longitudinal permittivity, ε_e the electronic contribution to it, while $I_\lambda=I_\lambda(\varkappa_\perp^2v_{Ti}^2/\omega_{Hi}^2)$ is a Bessel function of an imaginary argument) has poles $\Omega=\Omega_\varkappa^\lambda$ ($\lambda=0,1,2,\dots$) corresponding to a resonance interaction of the beats produced by the hf waves with the ion-cyclotron and acoustic oscillations of the plasma. We can put Eqs. (3) to (5) in a simple form if we introduce the normal amplitudes of the lf waves $b_\varkappa^\lambda$ (the index $\lambda=0$ will refer to ion sound, and the indexes $\lambda=1,2,3$ will number the ion-cyclotron modes):

$$\frac{\delta\tilde{n}_\varkappa^\lambda}{n_0}=\mu_\varkappa^\lambda(b_\varkappa^\lambda+b_{-\varkappa}^{\lambda*}),\quad \mu_\varkappa^\lambda=(\mathrm{Res}\,G^\lambda/n_0T_e)^{1/2}. \tag{6}$$

In these formulae Res G^λ is the residue of the Green function in the point $\Omega=\Omega_\varkappa^\lambda$, while $\delta\tilde{n}^\lambda$ is the density variation in the λ-th oscillation.

One can easily verify directly that the equations of motion for the a_k and $b_\varkappa$ can be written in Hamiltonian form:

$$\dot{a}_k=-i\frac{\delta H}{\delta a_k^*},\quad \dot{b}_\varkappa^\lambda=-i\frac{\delta H}{\delta b_\varkappa^{\lambda*}};$$

$$H=\int\omega_k|a_k|^2\,d\mathbf{k}+\sum_{\lambda=0}^{\infty}\int\Omega_\varkappa^\lambda|b_\varkappa^\lambda|^2\,d\varkappa \tag{7}$$

$$+\sum_{\lambda=0}^{\infty}\int(V_{\varkappa k_1k_2}^\lambda b_\varkappa^\lambda a_{k_1}a_{k_2}^*+\text{c.c.})\,\delta_{\varkappa+\mathbf{k}_1-\mathbf{k}_2}\,d\varkappa\,d\mathbf{k}_1\,d\mathbf{k}_2.$$

The interaction matrix elements

$$V_{\varkappa k_1k_2}^\lambda=(V_{\varkappa k_2k_1}^\lambda)^*=\frac{\mu_\varkappa^\lambda\omega_H^2}{2(\omega_p^2+\omega_H^2)}\frac{k_1}{k_2}(\mathbf{k}_2\mathbf{u}_{k_1}) \tag{8}$$

contain complete information about three-wave decay processes involving the lf oscillations which we have considered.

The interaction with IA is very simple to construct. In this case the normalized coefficient is

$$\mu_\varkappa^0\approx(\Omega^0/2n_0T_e)^{1/2}. \tag{9}$$

In the small-wavenumber region

$$\Omega_s=\varkappa c_s=\varkappa\sqrt{T_e/M}\ll\omega_{Hi},\quad \Omega_\varkappa^0=\Omega_s|\cos\theta|,$$

sound of shorter wavelength must be considered to be unmagnetized, $\Omega^0=\Omega_s$.

As far as the interaction of hf waves with IC is concerned, simple expressions for its matrix elements can be obtained only in the limiting cases of magnetized ($\varkappa v_{Ti}\ll\omega_{Hi}$) or unmagnetized ions when the frequencies $\Omega_\varkappa^\lambda$ are close to the corresponding harmonic of the ion gyro-frequency $|\Omega^\lambda-\lambda\omega_{Hi}|\ll\omega_{Hi}$. Then

$$\mu_\varkappa^\lambda\approx\left(\frac{|\Omega_\varkappa^\lambda-\lambda\omega_{Hi}|}{n_0T_e}\right)^{1/2}. \tag{10}$$

A more detailed description of decay processes involving IC requires the application of numerical methods.[1)]

One should note that the IC and IA considered above exist only in the range of angles $\varkappa_z v_{Te} \gg \Omega_\varkappa^\lambda \gg \varkappa_z v_{Ti}$ and do not account for all lf potential oscillations. It is possible also in the opposite limiting case $\Omega_\varkappa^\lambda \gg \varkappa_z v_{Te}$ that weakly damped ion-cyclotron waves with frequencies[14] $\Omega \approx 2\omega_{Hi}$, $3\omega_{Hi}, \ldots$ can propagate; they correspond to the dispersion equation

$$1+\frac{\omega_p^2}{\omega_H^2}-2\frac{\omega_{pi}^2}{\varkappa^2 v_{Ti}^2}\sum_{\lambda=1}^{\infty}\frac{\lambda^2\omega_{Hi}^2\exp(-\varkappa_\perp^2 v_{Ti}^2/\omega_{Hi}^2)I_\lambda}{\Omega^2-\lambda^2\omega_{Hi}^2}=0.$$

However the matrix element for the interaction between them and the hf oscillations turns out to be small compared to (8).

We now estimate the time for the non-linear transfer due to the processes considered above.

Let the hf waves form a packet with characteristic wavevector $\mathbf{k}_0$. The non-linear transfer due to the decay

$$\omega_{\mathbf{k}_0}=\omega_{\mathbf{k}}+\Omega_\varkappa^\lambda,\quad \mathbf{k}_0=\mathbf{k}+\boldsymbol{\varkappa}\quad (\lambda=0,\ 1,\ 2,\ \ldots) \tag{11}$$

then occurs with a growth rate

$$\gamma_{nl}^\lambda(\mathbf{k})\sim\frac{|V^\lambda|^2 W_0}{\omega_{\mathbf{k}}\Omega_\varkappa^\lambda}\sim\frac{\omega_p^4\omega_H^2}{(\omega_p^2+\omega_H^2)^2\omega_L}\frac{W_0}{n_0T_e}\frac{|\Omega^\lambda-\lambda\omega_{Hi}|}{\Omega_\varkappa^\lambda}\frac{[\mathbf{k}\mathbf{k}_0]_z^2}{k^2k_0^2}. \tag{12}$$

The whole of its wavevector-dependence is determined by the last two factors. If we talk about the excitation of IC, the first of these factors reaches a maximum $\sim 1/\lambda$ in the region $\varkappa v_{Ti} \sim \omega_{Hi}$. We can thus at once state that the processes involving the first ion-cyclotron modes with wavelengths of the order of the Larmor radius of an ion, $r_{Hi} = v_{Ti}/\omega_{Hi}$, and ion sound are the fastest. The second factor determines the way the non-linear transfer velocity depends on the characteristic size of the hf turbulence. It is clear from (11) and (12) that the effective interaction of hf waves and IC is possible only in the case $k_0 v_{Ti} \sim \varkappa_{\max} v_{Ti} \sim \omega_{Hi}$. If, on the other hand, $k_0 \gg \varkappa_{\max}$, the characteristic growth rates $\gamma_{nl}^{1,2,\ldots}(\mathbf{k}_0)$ are small with a factor $(\varkappa/k_0)^2 \ll 1$.

It is important to note that the concentration of hf oscillations just in the short-wavelength region of the spectrum as follows from the results of [7,8] is very typical of plasma turbulence in the frequency range considered. The characteristic turbulence scale is then determined by the maximum possible wavevector $k_{\max}$ for which the considered hf waves still exist. In the case $\omega_p \gtrsim \omega_H$ this condition is $k_{\max} r_H < 1$. In that case, clearly $\varkappa_{\max} \sim k_{\max}$ is possible only in a strongly non-isothermal plasma, $T_e/T_i \gtrsim M/m$. If, however, $\omega_p \ll \omega_H$, the condition that the hf oscillations are not damped has the form

$$kv_{Te}<\min\{\omega_p(T_e/T_i)^{1/2},\ \omega_H\}.$$

In the opposite limit Landau damping occurs, either on unmagnetized ions or on electrons. We see that $k_{\max} \sim \varkappa_{\max}$ is possible then only in very strong magnetic fields, $\omega_{Hi} > \omega_{pi}$. Such situations will not be considered here by us. The conclusion that the interaction of IC with short-wavelength hf waves is weak gives us grounds for assuming that the main non-linear process for the latter will be the decay involving short-wavelength ($\varkappa v_{Ti} > \omega_{Hi}$) sound. This statement is, however, not always true. The fact is that unmagnetized IA exist only in a non-isothermal plasma and in the range of angles $z = (\varkappa_z/\varkappa)\sqrt{M/m} \gg 1$. If, however, the hf waves are concentrated in the region $z \ll 1$, it follows immediately from the conservation law (11) that the decay must proceed in a modulated form, i.e., in such a way that $\mathbf{k} \approx \mathbf{k}_0$. Its characteristic growth rate then acquires a small factor $\sim z^2 \ll 1$:

$$\gamma_{nl}^0\sim z^2\frac{\omega_p^4\omega_H^2}{(\omega_p^2+\omega_H^2)^2\omega_L}\frac{W_0}{n_0T_e}. \tag{13}$$

We see at once that in this situation the processes of the induced scattering of hf waves by particles turn out to be faster. Turning to describing them we shall assume that the characteristic frequency of the lf motions $\Omega \gg \omega_{Hi}$ and that the characteristic scale of the lf density modulation is much smaller than the ion Larmor radius ($\varkappa v_{Ti} \gg \omega_{Hi}$). Under those conditions the action of the magnetic field on the ions can be neglected and one can obtain for the Green function

$$G_{\varkappa\Omega}^0=-\frac{T_eL_eL_i}{T_eL_i+T_iL_e};$$
$$L_e=\int\frac{\varkappa_zv_zf_e^0\,dv_z}{\varkappa_zv_z-\Omega},\quad L_i=\int\frac{\boldsymbol{\varkappa}\mathbf{v}f_i^0\,d\mathbf{v}}{\boldsymbol{\varkappa}\mathbf{v}-\Omega}. \tag{14}$$

In a non-isothermal plasma in the range $\varkappa_z/\varkappa \gg \sqrt{m/M}$ it has a pole corresponding to the interaction with ion sound which has already been studied by us: $G \approx \Omega_s^2/(\Omega^2 - \Omega_s^2)$. In the range of angles $z \lesssim$ IC are strongly Landau-damped, but even in that case the Green function has close to the surface $\Omega/\varkappa_z v_{Te} \approx 1$ a pronounced maximum $\mathrm{Im}\, G \sim 1$ corresponding to scattering by electrons. Close to the surface $\Omega/\varkappa_z v_{Ti} \sim 1$, however, the Green function $G \ll 1$. Changing from the dynamical variables $a_{\mathbf{k}}$ to phase averages

$$\langle a_{\mathbf{k}}a_{\mathbf{k}'}^*\rangle=N_{\mathbf{k}}\delta(\mathbf{k}-\mathbf{k}'),$$

we get from (3) and (4) the following equation which describes the induced scattering of hf waves by electrons:

$$\dot N_{\mathbf{k}}\approx\int T_{\mathbf{k}\mathbf{k}'}N_{\mathbf{k}}N_{\mathbf{k}'}\,d\mathbf{k}';$$
$$T_{\mathbf{k}\mathbf{k}'}=\frac{\omega_H^2\omega_p^4}{(\omega_p^2+\omega_H^2)^2}\frac{[\mathbf{k}\mathbf{k}']_z^2}{2n_0T_ek^2k'^2}\,\mathrm{Im}\,G\left(\frac{\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}}{|k_z-k_z'|}\right). \tag{15}$$

This equation remains valid also in an isothermal plasma ($T_e \sim T_i$). In that case it is valid also for angles $k_z/k \gg \sqrt{m/M}$ where it describes already the scattering by ions (for details see[7]). The region $k_z/k \sim \sqrt{m/M}$ is a transition region; electrons and ions give here contributions to the scattering of hf waves which are of the same order of magnitude.[2)]

One can easily understand that for hf oscillations which propagate at angles $z < 1$ the scattering by electrons proceeds in such a way that the angle $a = \measuredangle\, \mathbf{k}, \mathbf{k}_0 \sim \pm\pi/2$. A characteristic growth rate for this process is

$$\gamma_{nl}^e\sim\frac{\omega_p^4\omega_H^2}{\omega_L(\omega_p^2+\omega_H^2)^2}\frac{W_0}{n_0T_e}. \tag{16}$$

It is larger by a factor z^{-2} than the decay growth rate γ_{nl}^0 (in a non-isothermal plasma) or the growth rate for scattering by protons (if $T_e \sim T_i$) which, as we saw, proceeds such that

$$k_z - k_z' \gg \sqrt{m/M}|\mathbf{k}-\mathbf{k}'|.$$

The scattering of short-wavelength ($kv_{Ti} \gg \omega_{Hi}$) hf oscillations by electrons close to the lower-hybrid resonance is thus in a wide temperature range, $T_e \gtrsim T_i$, the main non-linear process. We note also that the frequency range corresponding to the angles $z \lesssim 1$ is, as can be seen from (1), already very wide: $\omega_{\mathbf{k}} - \omega_L \lesssim \omega_L$.

§2. PARAMETRIC EXCITATION AND WEAK TURBULENCE OF hf WAVES

We consider the generation of hf waves under the action of a variable electrical field $\mathbf{E}_0(t) = \mathbf{E}_0 \cos\omega_0 t$ with frequency $\omega_0 \gtrsim \omega_L$. For not too large amplitudes of the external field when the growth of the waves γ_p is less than the characteristic frequencies Ω of the slow motions, one speaks of first and second order decay processes. They develop, respectively, near the surfaces

$$\omega_0 = \omega_{\mathbf{k}} + \Omega_{-\mathbf{k}}, \tag{17}$$

$$2\omega_0 = \omega_{\mathbf{k}} + \omega_{-\mathbf{k}}. \tag{17a}$$

These processes are also often called the decay and aperiodic instabilities.[2] If the low-frequency mode in (17) is strongly damped, we must understand by the decay instability the conversion of the external field into a plasma wave through the scattering by ions or electrons, depending on the angle θ_p at which the oscillations are produced. The magnitude of this angle is determined with great accuracy by the equation $\omega_0 = \omega(\cos\theta_p) = \omega(z_p)$. We easily get for the growth rate of the decay instability the following general relation:

$$\gamma_d = \frac{\omega_H^2 \omega_p^4 f(\theta, \chi)}{\omega_0^3 (\omega_p^2 + \omega_H^2)} \frac{E_0^2}{16\pi n_0 T_e} \operatorname{Im} G_{k,\omega_k - \omega_0},$$

$$f = \left[\cos\theta\cos\chi - \frac{\omega_0^2}{\omega_H^2}\sin\chi\cos\varphi\right]^2 + \left(\sin\chi\sin\varphi\frac{\omega_0}{\omega_H}\right)^2. \tag{18}$$

Here $\chi = \measuredangle\, \mathbf{E}_0, \mathbf{H}_0$ and the azimuthal angle φ of the wave-vector is reckoned from the plane through $\mathbf{E}_0$ and $\mathbf{H}_0$. The threshold value of E_0 is determined from the equation $\gamma_p = \gamma_{\mathbf{k}}$, where $\gamma_{\mathbf{k}}$ is the damping of the hf waves. In the range of angles $z \gg 1$

$$\gamma_{\mathbf{k}} = \nu_{ei}\left(1 + \frac{\omega_p^2}{\omega_p^2 + \omega_H^2}\right);$$

when $z \ll 1$

$$\gamma_{\mathbf{k}} = \nu_{ei}\left(z^2 + \frac{\omega_p^2}{\omega_p^2 + \omega_H^2}\right).$$

If the plasma is isothermal, $T_e \sim T_i$, and the lf component of the decay is strongly damped sound, $[\operatorname{Im} G]_{\max} \sim 1$. The threshold value of the field can be seen from (18) to decrease when ω_0 approaches ω_L and reaches a minimum when $0 < z_p < \sqrt{2}$, depending on the orientation of $\mathbf{E}_0$ and $\mathbf{H}_0$:

$$\left(\frac{W_0}{n_0 T_e}\right)_{\min} \sim \frac{\omega_L \nu_{ei}}{\omega_p^2 + \omega_H^2} \quad (\chi = 0), \quad \left(\frac{W_0}{n_0 T_e}\right)_{\min} \sim \frac{\omega_L \nu_{ei}}{\omega_p^2} \quad \left(\chi = \frac{\pi}{2}\right). \tag{19}$$

When T_e/T_i increases the minimum threshold is realized in the range of somewhat larger frequencies, $z_p \gg 1$, where decay is possible involving sound eigenoscillations, $[\operatorname{Im} G]_{\max} \sim (M/m)^{1/2}$. Its magnitude is then even further diminished:

$$\left(\frac{W_0}{n_0 T_e}\right)_{\min} \geqslant \frac{\nu_{ei}\omega_L}{\omega_p^2}\left(\frac{m}{M}\right)^{1/2} \quad (\chi \sim 1). \tag{20}$$

In the case of sound excitation Eq. (18) for the growth rate of the parametric instability is true only in the near-threshold region $(\gamma_p - \gamma_s) < \gamma_s$, where $\gamma_s = (\pi m/8M)^{1/2}\Omega_s$ is the ion sound damping. When one is further above threshold one finds easily that on the decay surface (17) $a_{\mathbf{k}} \propto \exp(\gamma_p t/2)$ and

$$\gamma_p = \frac{\omega_p^2 \omega_H f^{1/2}}{\omega_0(\omega_p^2 + \omega_H^2)^{1/2}}\left(\frac{kc_s}{\omega_0}\right)^{1/2}\left(\frac{E_0^2}{16\pi n_0 T_e}\right)^{1/2}.$$

We now consider the parametric excitation of IC. In that case $[\operatorname{Im} G]_{\max} \sim (\Omega^\lambda - \lambda\omega_{Hi})/\gamma_\lambda$ and it is clear from (18) that the smallest excitation threshold occurs for oscillations with a long wavelength $k^{-1} \sim r_{Hi}$:

$$\left(\frac{W_0}{n_0 T_e}\right)_{\min} \sim \frac{\gamma\gamma_\lambda}{(\Omega^\lambda - \lambda\omega_{Hi})}\frac{\omega_L^2}{\omega_p^2}$$

We consider briefly the aperiodic instability. It is independent of the characteristics of the lf motions and close to the decay surface (17a) it is described by Eq. (18) in which we must replace Im G by $T_e/(T_e + T_i)$. What we have said is valid up to amplitudes for which $\gamma_p \ll \{\varkappa v_{Ti}; \varkappa v_{Te}\}$. The ratio of the thresholds for the decay and the aperiodic instabilities is thus

$$[E_0^2]_d/[E_0^2]_a = T_e/(T_e + T_i)\,[\operatorname{Im} G]_{\max}.$$

In the case when the decay instability is a conversion through scattering by particles this ratio is ~ 1; if, however, for the lf motions $\gamma_\lambda \ll \Omega^\lambda$, the minimum threshold occurs, usually, for the decay instability.

We go over to a study of the non-linear stage of the parametric instability of short-wavelength ($kv_{Ti} \gg \omega_{Hi}$) hf oscillations. If the frequency $\omega_0 - \omega_L \lesssim \omega_L$, according to what we considered above they will be generated at angles $z \lesssim 1$ and afterwards will be transferred into the low-frequency region, mainly through scattering by electrons.

It will become clear in what follows that the region of excesses for which the plasma turbulence is weak is very small. Notwithstanding this a study of the general and very rough properties of weakly turbulent spectra is important for an understanding of strong turbulence. Pursuing henceforth just this goal in the present section we shall, to simplify the exposition, assume that the turbulence spectra are axially symmetric and put $\omega_H \sim \omega_p$. Excitation, non-linear transfer, and the damping of waves are then described by the kinetic equation

$$\left(\frac{\partial}{\partial t}+\gamma_k-\gamma_p\right)n_k=\frac{\omega_H^2}{4n_0T_e}\sqrt{\frac{m}{M}}\int n(k,z)n(k',z')\,\mathrm{Im}\,G\left(\frac{\omega_k-\omega_{k'}}{|kx-k'x'|}\right)dk'\,dz',$$
$$n(k,z)=2\pi k^2N_k. \qquad (21)$$

In the region $z\gg kr_H$ the transfer proceeds, as one can see easily, in such a way that in a single scattering process $|z-z'|\ll z$. This gives a basis for changing in (21) with respect to the variable z to the differential approximation, putting

$$\mathrm{Im}\,G\approx-\pi\frac{(kz-k'z')^2}{|zz'|}\frac{v_{Te}^2}{\omega_H^2}\frac{\partial}{\partial|z'|}\delta(|z|-|z'|)$$

(details of such a transition can be found in[7]). Equation (21) then becomes

$$\left(\frac{\partial}{\partial t}+\gamma_k-\gamma_p\right)n_k=\pi\sqrt{\frac{m}{M}}\frac{v_{Te}^2n(k,z)}{4n_0T_e}\int\left(k^2z\frac{\partial}{\partial z}\frac{n}{z}+\frac{k'^2}{z}\frac{\partial}{\partial z}nz\right)dk'. \qquad (22)$$

In its form it is similar to the equations of[7,8] which have been studied both analytically and using a computer. It is clear from the results of these papers that oscillations generated close to $z=z_p$ afterwards are transferred, due to scattering, to the region of lower frequencies and at the same time are stored rapidly in the large-k region forming there stable solutions in the shape of jets

$$n=n(x)\delta(k-k_0). \qquad (23)$$

The location of a jet k_0 is such that collisionless damping which increases fast in the large-k region is still small compared with collisional damping. In typical situations $k_0r_H\approx\frac{1}{5}$ to $\frac{1}{7}$ and depends weakly on the plasma parameters.

The physical reason for the energy accumulation in the short-wavelength region of the spectrum is, as was shown for the analogous situation in[7,8], the increase of phase volume of the oscillations with increasing k.

If we use (23), Eq. (12) becomes one-dimensional. For excesses $\gamma_p/\gamma\gg1$ its stationary solution is concentrated mainly outside the generation region and is independent of the finer details of the growth rate γ_p:

$$n(k,z)=\frac{2}{\pi}\sqrt{\frac{M}{m}}\frac{\gamma n_0T_e}{(k_0v_{Te})^2}(z-z_0),\quad z_0=z_p-\eta,$$
$$\eta=\int_0^{z_p}\frac{\gamma_p}{\gamma}dz\sim\left(\frac{\gamma_p}{\gamma}\right)_{max}k_0r_H. \qquad (24)$$

Knowing the turbulence spectrum one can easily determine the energy flux in the plasma:

$$Q=2\int_0^{z_p}\omega_k\gamma_p(x)n(x)dx\sim\frac{\omega_p^4}{\omega_H^2\omega_L}\frac{W_0^2}{n_0T_e}. \qquad (25)$$

(One must state that the formula given here remains valid also when $\omega_H\lesssim\omega_p$.) This energy is almost completely transferred to the electrons as a result of scattering and taking electron-ion collisions into account.

We now note that the solution (24) obtained here, like Eq. (12) itself, is applicable only in the range of angles $z>k_0r_H$ which is filled already for relatively small excesses above the threshold:

$$(\gamma_p/\gamma)_{max}\lesssim(k_0r_H)^{-1}.$$

For smaller z the transfer along the spectrum occurs in a non-differential way. It is then noteworthy that at the same time as taking into account the fact that the transfer is non-differential it is necessary to retain the thermal corrections to the dispersion law (1):

$$\omega_k=\omega_L\left(1+\frac{z^2}{2}+\frac{y^2}{2}\right),\quad y^2=k^2R^2=k^2\left(\frac{3}{4}+3\frac{T_i}{T_e}\right)r_H^2. \qquad (26)$$

How does allowance for the thermal dispersion affect the nature of the transfer into the range of angles which are non-differential in z? It is clear that oscillations which are concentrated for $z_p>y>z$, in the variable y again will be scattered in a differential way and are now stored in the region of even larger z until we have $y\sim z$. For $z\ll k_0r_H$ the oscillations will thus be concentrated mainly close to $y=z$, so that we can put in (21)

$$n(k,z)=n(z)\delta(k-|z|R^{-1})$$

and change again to a one-dimensional equation

$$\gamma=\frac{\omega_H^2}{4n_0T_e}\left(\frac{m}{M}\right)^{1/2}\int n(z')\,\mathrm{Im}\,G\left[\frac{z^2-z'^2}{\xi(z^2+z'^2)}\right]dz',\quad \xi=\left(\frac{3}{4}+3\frac{T_i}{T_e}\right)^{-1/2}. \qquad (27)$$

We further note that the kernel Im G is an odd function of its argument and is mainly concentrated in the range [− 1, 1] of its values. This makes it possible to put for smooth solutions of (27)

$$\mathrm{Im}\,G\approx\delta(z'-\xi z)-\delta(z'-z/\xi),\quad \xi\gtrsim1.$$

The equation obtained has an exact solution

$$n(z)=\left(\frac{M}{m}\right)^{1/2}\frac{4n_0T_e}{\omega_H^2\ln\xi^2}\ln\frac{z}{z_1}. \qquad (28)$$

The location of the end of the jet z_1 and with it also the characteristics of the turbulence must, as in (24), be expressed in terms of the excess η. To do this we evaluate the energy flux along the spectrum $P(z)$ which we define as

$$\left(\frac{\partial}{\partial t}+\gamma\right)W(z)=\frac{\partial}{\partial z}P.$$

In the differential region ($z>k_0r_H$)

$$P_d\sim\frac{n_0T_e\omega_L\gamma^2}{(k_0v_{Te})^2}(z-z_0).$$

In the opposite limit

$$P_N\sim\frac{n_0T_e\omega_L\gamma^2}{\omega_H^2\ln\xi^2}\left(z\ln\frac{z}{z_1}-z+z_1\right).$$

Equating these fluxes in the transition region $z\sim k_0r_H$ we determine $z_1\ll k_0r_H$:

$$z_1\approx k_0r_H\exp\left[-\eta^2/k_0^3r_H^3\right],\quad (\eta>z_p\sim1).$$

We study now the distribution of the energy over the spectrum. If we let z approach $k_0 r_H$ from the side of small and large z we get, respectively,

$$n_- \sim \left(\frac{M}{m}\right)^{1/2} \frac{\gamma n_0 T_e}{\omega_H^2} \frac{\eta^2}{(k_0 r_H)^3}, \quad n_+ \sim \left(\frac{M}{m}\right)^{1/2} \frac{\gamma n_0 T_e}{(k_0 v_{Te})^2} \eta. \tag{29}$$

The energy of the oscillations accumulates therefore in the region of small k, z forming a long-wavelength core in the turbulence spectrum with a relative height $n_-/n_+ \sim \eta/k_0 r_H \gg 1$ which increases steeply with increasing excess above the instability threshold. The reason for such a condensation of energy in the small z, k region consists in the steep weakening of the efficiency of the transfer along the spectrum.

We elucidate the limits of the applicability of the results given here. The kinetic Eq. (15) is valid as long as the non-linear growth rate γ_{nl} is less than the characteristic frequency of the lf motions. In our case $\Omega \sim \varkappa_z v_{Te} \sim \omega_L (k r_H)^2$. This means that the condition for the applicability of (15) is the same as the random phase criterion in the weak turbulence theory (see[11,17]). Expressed in terms of the parameters of the problem this condition gives

$$\frac{\gamma_p}{\gamma} \lessapprox \left(\frac{m}{M}\right)^{1/2} \frac{\omega_H}{\nu_{ei}} (k_0 r_H)^2. \tag{30}$$

For larger excesses the parametric turbulence turns out to be strong (such excesses can easily be reached in experiments on the hf heating near the lower-hybrid resonance[18–20]).

One easily sees that the situation described here turns out to be very close to the one arising when Langmuir waves are parametrically excited in a plasma without a magnetic field.[4,5] The accumulation of the oscillations in the long-wavelength region of the spectrum leads in that case, as was shown in a number of papers,[11,21,22] to their collapse—a strongly non-linear energy dissipation mechanism in a plasma.

§3. STRONG TURBULENCE EQUATIONS

We state the problem of describing hf waves in the strongly non-linear regime. In the simplest case one can judge the evolution of the strongly non-linear spectra using the instability of a large amplitude wave. The basic set of Eqs. (3), (4) have an exact solution:

$$a = \frac{W_0^{1/2}}{\omega_{\mathbf{k}_0}^{1/2}} \exp\{-i\omega_{\mathbf{k}_0} t + i\mathbf{k}_0 \mathbf{r}\}, \quad \delta\tilde{n} = 0.$$

Linearizing it on the basis of this background we can easily get for perturbations proportional to $e^{-i\omega t + i\varkappa\cdot r}$ the dispersion equation ($z_0 \lesssim 1$)

$$1 + \frac{W_0}{n_0 T_e} \Gamma G_{\varkappa\omega} \varkappa^2 \sin^2\alpha \left[\frac{(\mathbf{k}_0+\varkappa)^{-2}}{-\omega+\omega_{\mathbf{k}_0+\varkappa}-\omega_{\mathbf{k}_0}} + \frac{(\mathbf{k}_0-\varkappa)^{-2}}{-\omega+\omega_{\mathbf{k}_0-\varkappa}-\omega_{\mathbf{k}_0}}\right] = 0. \tag{31}$$

Here $a = \measuredangle\, \mathbf{k}_0, \varkappa$; $\Gamma = M(\omega_L \omega_p^2)/m(\omega_p^2 + \omega_H^2)$, while the dispersion law for $\omega_\mathbf{k}$ in the case of a strong magnetic field ($\omega_H \gg \omega_p$) has the form (26) with $R^2 = 3 T_i r_d^2 / T_e$.

In our problem there does not enter a detailed analysis of Eq. (31). We consider the most important case of perturbations with $\varkappa \gg k_0$. In the static limit, $\varkappa_z v_{Te} \gg \omega$, the Green function $G_{\varkappa\omega} \approx -T_e/(T_e + T_i)$. Putting $a = \pi/2$ in (31) we get

$$\omega^2 = (\omega_\varkappa - \omega_L)\left(\omega_\varkappa - \omega_L - \frac{T_e}{T_e+T_i} \Gamma \frac{W_0}{2 n_0 T_e}\right).$$

The wave is unstable when

$$\omega_\varkappa - \omega_L < \frac{T_e}{T_e+T_i} \Gamma \frac{W_0}{2 n_0 T_e}.$$

The maximum growth rate

$$\gamma_{max} \sim \frac{T_e}{T_e+T_i} \Gamma \frac{W_0}{n_0 T_e} \tag{32}$$

is reached when

$$\omega_\varkappa - \omega_L \approx \frac{T_e}{T_e+T_i} \Gamma \frac{W_0}{4 n_0 T_e}.$$

We note now that the condition for the static approximation in the maximum of the growth rate can be fulfilled only when we have the condition $y \sim z$; in the opposite case the magnitude of the growth rate is significantly less than γ_{max}.

Therefore, the instability with

$$\begin{aligned} \varkappa^2 r_H^2 &\sim \frac{M}{m} \frac{W_0}{n_0 T_e} \quad (\omega_H \lessapprox \omega_p), \\ \varkappa^2 r_d^2 &\sim \frac{M}{m} \frac{\omega_p^2}{\omega_H^2} \frac{W_0}{n_0 T_e} \quad (\omega_H \gg \omega_p). \end{aligned} \tag{33}$$

proceeds fastest. We see that even for very small amplitudes,

$$\frac{W_0}{n_0 T_e} \sim \max\left\{\frac{m}{M}; \frac{m}{M} \frac{\omega_H^2}{\omega_p^2}\right\},$$

the oscillations in the region $k r_H \sim 1$, $k r_d \sim 1$ are excited; it is then important to take into account collisionless dissipation and the non-linear effects.

Knowing the characteristic times and scales of the development of the instability we can obtain equations describing the evolution of long-wavelength plasma oscillations. The description of strongly non-linear wave processes, including coherent effects, can in a natural way be performed in the r-representation. Splitting off the slow time-dependence in the hf electrostatic potential,

$$\varphi_{el} = \psi \exp(-i\omega_L t) + \text{c.c.},$$

we get for the potential ψ from (3) and (14) in the static approximation the equation

$$\nabla^2\left(i\psi_t + \omega_L \frac{R^2}{2} \Delta\psi\right) - \frac{\omega_L}{2} \frac{M}{m} \Delta_z \psi - \frac{e^2 \omega_p^2}{2 m \omega_L (T_e+T_i)(\omega_p^2+\omega_H^2)} \operatorname{div}([\nabla\psi \times \nabla\psi^*]_z [\mathbf{h} \times \nabla\psi]) = 0. \tag{34}$$

The condition for the realization of the static approximation in the r-representation means that the second and third terms in (34) are equal. The non-linear term contains only differentiation in the direction at right angles to the magnetic field. The equation obtained is thus essentially two-dimensional. Changing to the dimensionless variables

$$\psi\to 2\frac{\omega_L}{\omega_p}\frac{R}{e}[m(\omega_p^2+\omega_H^2)(T_e+T_i)]^{1/2}\psi,$$

$$t\to t/\omega_L,\quad r\to Rr,$$

we can write it in the form

$$\nabla_\perp^2(i\psi_t+\nabla_\perp^2\psi)-2\,\mathrm{div}([\nabla\psi\times\nabla\psi^*]_z[\mathbf{h}\times\nabla\psi])=0 \qquad (35)$$

(we shall drop the index ⊥ in what follows). Similarly to the equations describing the collapse of Langmuir waves[11,19] (35) can be written in Hamiltonian form

$$\nabla^2 i\psi_t+\delta\mathscr{H}/\delta\psi^*=0,$$

$$\mathscr{H}=\int(|\Delta\psi|^2+[\Delta\psi\times\Delta\psi^*]_z^2)d\mathbf{r}. \qquad (36)$$

It follows immediately from this way of writing the equations that the Hamiltonian $\mathscr{H}$ is conserved. It is also easy to prove that the conservation laws for the total number of quasi-particles $I=\int|\nabla\psi|^2d\mathbf{r}$, for the total momentum

$$\mathbf{P}_l=\frac{i}{2}\int(\nabla_k\psi^*\nabla_l\nabla_k\psi-\mathrm{c.c})\,d\mathbf{r}$$

and its moment, which are valid in the case of the Langmuir wave collapse[11,24] also are valid in the framework of (34), (35). Notwithstanding the above-mentioned similarity, the problem of the collapse and its criterion (in the case of the Langmuir wave collapse, as a rule, a sufficient condition for the production of a singularity is the fact that the Hamiltonian becomes negative) is very complicated and, possibly, not susceptible to an analytical approach. One can, however, use very general considerations to reach an important conclusion about the properties of the solutions of (35). To do this we consider its stationary states $\psi(\mathbf{r},t)=\psi(\mathbf{r})\exp(iq_0^2t)$. One shows easily that, as in[22], for such solutions we have always $\mathscr{H}=0$. An arbitrary initial condition can therefore as the result of evolution never reach a stationary state. On the other hand, it follows from the conservation of the Hamiltonian that when $\mathscr{H}<0$, $\psi(\mathbf{r},t)$ can not spread out without limits.

We note that the solution of (35) cannot be spherically symmetric as its non-linear part vanishes. We can, however look for ψ in the form

$$\psi=\Phi(r)e^{im\varphi}.$$

We then get for the function $\Phi(r)$ the one-dimensional equation

$$\hat{K}\left(i\Phi_t+\frac{1}{r}\hat{K}\Phi\right)+2m^2\Phi\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}|\Phi|^2=0,$$

$$\hat{K}=\frac{\partial}{\partial r}r\frac{\partial}{\partial r}-\frac{m^2}{r}. \qquad (37)$$

The high order of this equation makes it very difficult to study it analytically. However, one can reach some general conclusions about the behavior of the solution as a function of the number m. Indeed, we introduce a characteristic amplitude of Φ and a scale for its change δr. For sufficiently large $m(m\gg r/\delta r)$ we can neglect the differential part in the operator $\hat{K}$. One sees easily that there is no collapse in the framework of the equation which one then obtains. On the other hand, for a sufficiently high level of non-linearity $(m<r\Phi/\delta r)$ we have

$$\mathscr{H}=2\pi m^2\int\left[\frac{|\Phi|^2}{r}-\left(\frac{\partial}{\partial r}|\Phi|^2\right)^2\right]\frac{dr}{r}<0.$$

For the collapse the first modes of (37) are thus the most dangerous.

A numerical experiment, recently performed in Ref. 23, has shown that within the framework of Eq. (37) collapse occurs for $m=1,2$. Confirmation of the idea of a collapse is also provided by the recent experiments[20] in which the formation of regions where the field is localized and which are stretched out along H_0 as well as their self-compression was directly observed. The heating of the plasma was, in agreement with what has been said above, accompanied by the appearance of an appreciable number of accelerated particles.

The existence of a strongly non-linear energy dissipation mechanism at the lower-hybrid resonance must show up in a strong effect on the nature of the plasma heating. From that point of view it is important to have a comprehensive study of the lower-hybrid collapse; first of all, a study of it at large intensities,

$$\frac{W}{n_0T_e}\gg\frac{m}{M}\left(1+\frac{\omega_p^2}{\omega_H^2}\right),$$

when powerful dissipation mechanisms must be included.

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1) Like the description of the dispersion law of ion-cyclotron oscillations (see[14,15]).

2) The same conclusion was reached in a recent paper.[16]

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Electromagnetic waves with a discrete spectrum in metallic ferromagnets

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The spectrum, attenuation, and polarization of electromagnetic waves with a discrete spectrum in an isotropic metallic ferromagnet are investigated theoretically as functions of the external magnetic field **H**, the wave vector **k**, and the angle between **H** and **k**. Such waves can exist because, in the case of a definite ratio of the wavelength to the cyclotron radius of the conduction electrons, the absorption of the electromagnetic wave energy by the electrons as a result of Landau damping becomes small. The interaction between the electromagnetic wave in the ferromagnet with the magnetic subsystem alters the character of the wave propagation as compared with that in a normal metal. In particular, in a weak external magnetic field, the Landau damping of the wave at points far from the points of the discrete spectrum becomes less than unity. This fact is important for the excitation and experimental observation of electromagnetic waves with a discrete spectrum.

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INTRODUCTION

It has long been assumed that electromagnetic waves with a frequency much smaller than the plasma frequency cannot propagate in metals to a depth greater than the skin depth. Konstantinov and Perel'[1] were the first to show that this is not the case under certain circumstances. The existence of electromagnetic oscillations in a metal is due to the presence of a strong magnetic field ($l \gg R$), which localizes the electrons in a region with dimensions of the order of the cyclotron radius R; l is the free path length of the conduction electrons. Localization of the electrons makes the electromagnetic-wave damping connected with dissipative currents small. In addition to this damping, there are resonance mechanisms of absorption of energy of the electromagnetic wave by electrons in the metal. This includes cyclotron absorption and Landau damping. The resonance damping of the wave is determined by the electrons for which the phase relation

$$k_z\bar{v}_z+N\Omega=\omega,\ N=0,\ \pm1,\ \pm2,\dots \qquad (1)$$

is satisfied. Here ω is the frequency of the wave, $\Omega = eB/mc$ is the cyclotron frequency, e and m are the values of the charge and the effective mass of the conduction electron in the direction of **B**. The Landau damping turns out to be most important for short waves, whose length is much less than the cyclotron radius.[2] The electron interacts most effectively with the field of such a wave on those portions of its trajectory where it moves almost parallel to the planes of equal phase of the wave. In Fig. 1, this is in the vicinity of the points A and B. The value of the absorption of energy of the wave by the electron will change as a function of the

