the same as for a linear layer. We must in that case in Eqs. (4.5), (4.7) replace ξ by ξ/n.

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Theory of nonuniform magnetic states in ferromagnets in the vicinity of second-order phase transitions

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A perturbation theory is constructed in a small parameter determined by the degree of closeness to a second-order phase-transition point. This makes it possible to take the demagnetizing field into account and to find all the quantities characterizing the magnetic-moment distribution, with any prescribed degree of accuracy in powers of the small parameter. It is shown that strong correlation effects lead to a nonuniform distribution of the magnetization over the thickness of a ferromagnetic plate. The character of the phase transition to the nonuniform state in finite samples in the vicinity of second-order phase transitions (i.e., near the Curie temperature and the phase-transition point with respect to the magnetic field) is investigated in detail. It is proved that, in ferromagnets of arbitrary shape, the energy of the demagnetizing field does not change the character of the phase transition.

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INTRODUCTION

It is well known that, in samples of finite size, a nonuniform distribution of the magnetization arises simultaneously with the appearance of the spontaneous magnetic moment. Far from the transition point the magnetic-moment distribution consists of an alternation of uniformly magnetized phase domains, separated by narrow intermediate layers inside which the magnetization vector rotates through 180°.[1] In this case the energy of an intermediate layer can be regarded as the surface energy, and the equilibrium configuration determined from the minimum of the energy of the intermediate layers and the magnetic dipole energy. The magnetiza-

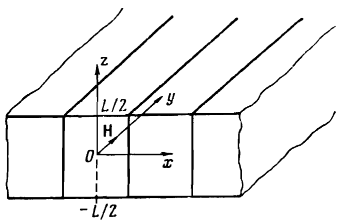

FIG. 1.

tion at the center of a domain is equal to the spontaneous magnetization for an infinite sample.

Shirobokov[2] obtained the periodic distribution of the magnetic moment without allowance for the demagnetizing field inside the sample. In the framework of this model the demagnetizing field and the period of the domain structure can be determined from the jump in the magnetization at the surface of the sample.[3–5] Such a treatment is valid when the static susceptibility χ of the ferromagnet is small ($\chi \ll 1$). In this case the back influence of the demagnetizing field on the magnitude of the magnetic moment can be neglected.

Near phase-transition points the susceptibility of a ferromagnet grows, with the result that the demagnetizing field has a substantial influence on the distribution of magnetization over the entire thickness of the plate. The length characterizing the nonuniformity of the magnetization across the plate becomes of the order of its thickness. In this case, allowance for the nonuniformity, over both the length and the thickness of the plate, is of fundamental importance for the determination of the type of phase transition and also the dependence on the temperature and magnetic field of the principal quantities characterizing the nonuniform state. In the vicinity of the Curie temperature such a nonuniform magnetic state was observed by Drabkin and coworkers in experiments on the scattering of polarized neutrons in nickel.[6]

The domain structure in ferroelectric crystals has been treated most systematically in[7] by one of the authors, in which the electric-field energy was calculated with allowance for the nonuniform distribution of polarization over the thickness of the plate, the shift in the Curie point was determined, and it was shown that the domain structure arises on account of the instability of static fluctuations with period corresponding to the width of a domain.

In the present paper we construct a rigorous quantitative theory of the nonuniform magnetic state in a ferromagnetic plate in the vicinity of second-order phase transitions. We consider a phase transition in a ferromagnet with anisotropy of the easy-axis type in a magnetic field H perpendicular to the anisotropy axis and parallel to the surface of the plate, far from the Curie point ($T \ll T_C$). In an infinite crystal with such geometry a phase transition with respect to the magnetic field occurs. At the phase-transition point H_C the magnetic moment lies along the field, and H_C is equal to the anisotropy field H_A. In addition, the phase transition with respect to the temperature in the absence of an external magnetic field is considered. It is shown that, in a sample of arbitrary shape, allowance for the dipole energy does not alter the character of the phase transition but only leads to a shift of the transition point and to the appearance of nonuniform magnetic ordering below the transition point. The magnitudes of the spontaneous magnetization and of the period of the nonuniformity in the ferromagnetic plate are obtained as functions of the degree of closeness to the transition point.

The discontinuities of the specific heat at the Currie point T_C and of the susceptibility at the phase-transition point H_C with respect to the magnetic field are determined. It is shown that they do not depend on the thickness of the plate and are smaller than in the infinite crystal by a factor of $\frac{9}{4}$. As the temperature (magnetic field) is lowered, the specific heat (susceptibility) of the ferromagnet increases. The rate of increase is proportional to the thickness L of the plate. This leads to the result that the total discontinuity in the specific heat (susceptibility) for $L \to \infty$ is equal to the discontinuity in the infinite crystal.

The spin-wave spectrum in a ferromagnetic plate in the vicinity of the phase transition in the magnetic field at H_C is considered. It is shown that the system becomes unstable with respect to a uniform distribution of magnetization as the transition point is approached from the side of fields $H > H_C$. The domain-structure periods obtained from the dynamic and static calculations coincide.

1. NONUNIFORM MAGNETIC STATE IN A UNIAXIAL FERROMAGNET IN A TRANSVERSE MAGNETIC FIELD

We shall consider a uniaxial ferromagnetic crystal in the form of a plane-parallel plate of thickness L. Let the anisotropy axis be directed perpendicular to the surface of the plate, along the z axis, and the external magnetic field **H** lie in the plane of the plate, along the y axis (Fig. 1). Then the Hamiltonian of the ferromagnet has the following form

$$\mathscr{H} = \int dv \left[\frac{1}{2}\alpha(\nabla\mathbf{M})^2 - \frac{1}{2}\beta M_z^2 - M_y H + \frac{H_D^2}{8\pi}\right], \quad (1.1)$$

where **M** is the density of the magnetic moment, α is the nonuniform-exchange constant, $\beta > 0$ is the anisotropy constant, and $\mathbf{H}_D$ is the demagnetizing field.

The equilibrium state of the ferromagnet is determined by the equations

$$[\mathbf{M}, \times \mathscr{H}_{\text{eff}}] = 0, \quad (1.2)$$

$$\text{rot}\,\mathbf{H}_D = 0, \ \text{div}\,(\mathbf{H}_D + 4\pi\mathbf{M}) = 0 \quad (1.3)$$

and the boundary conditions

$$H_D^x|_{z=\pm L/2} = H_D^{(e)x}|_{z=\pm L/2}, \quad H_D^z + 4\pi M_z|_{z=\pm L/2} = H_D^{(e)z}|_{z=\pm L/2},$$
$$\left.\frac{\partial M}{\partial z}\right|_{z=\pm L/2} = 0, \quad (1.4)$$

where $\mathscr{H}_{\text{eff}} = \alpha\nabla^2\mathbf{M} + M_z\mathbf{e}_z + H\mathbf{e}_y + \mathbf{H}_D$ is the effective magnetic field and $\mathbf{H}_D^{(e)}$ is the demagnetizing field in the vacuum. We do not consider the possibility of the existence of surface anisotropy, which can lead to nonuniform distributions of the magnetization[8,9] and to a change of the boundary condition

$$\frac{\partial \mathbf{M}}{\partial z}\bigg|_{z=\pm L/2}=0.$$

We shall confine ourselves to considering a strip-like domain structure. Since the distribution of magnetization along the y axis should be uniform, it follows from Eq. (1.2) that

$$h_x=-\alpha\nabla^2 m_x+\frac{m_x}{m_y}\left(\frac{H}{M_0}+\alpha\nabla^2 m_y\right),\quad h_y=0,$$
$$h_z=-\alpha\nabla^2 m_z-\beta m_z+\frac{m_z}{m_y}\left(\frac{H}{M_0}+\alpha\nabla^2 m_y\right),\qquad (1.5)$$

where $\mathbf{h}=\mathbf{H}_D/M_0$, $\mathbf{m}=\mathbf{M}/M_0$ and $M_0=|\mathbf{M}|$.

It is well known that, in an infinite crystal in a field perpendicular to the crystallographic axis and equal in magnitude to the anisotropy field ($H=H_A\equiv\beta M_0$), the magnetization vector lies in the direction of the magnetic field and a second-order phase transition occurs in the ferromagnet. In this section we shall investigate the character of the phase transition in samples of finite size, and the magnetization distribution in fields less than the field at the phase transition.

In the vicinity of the phase-transition point ($\xi=\beta-H/M_0\ll 4\pi,\beta$), the demagnetizing field (1.5) can be represented in the form

$$h_x=\beta m_x;\ h_z=-\alpha\nabla^2 m_z-\xi m_z+{}^1/_2\beta m_z^3;\qquad (1.6)$$
$$m_x,\ m_z\ll 1.$$

We note that, as can be seen from (1.1), in an anisotropic ferromagnet the strongest instability arises with respect to the appearance of m_z. Therefore, in the expressions (1.6) for the demagnetizing fields, we have taken into account nonlinear terms in m_z only, and have confined ourselves to the linear approximation in m_x.

It follows from (1.3)–(1.5) that, for equilibrium distributions of the magnetic moment, the energy (1.1) of the ferromagnet has the following form:

$$\mathscr{H}=-{}^1/_8\beta M_0^2\int dv\, m_z^4.\qquad (1.7)$$

The dipole magnetic energy, including the energy of the magnetic field in the vacuum, is completely taken into account in the expression (1.7). Since the energy of the ferromagnet is a continuous function of m_z, it follows from (1.7) that the transition to a state with $m_z\neq 0$ occurs without a jump in the magnetization m_z, i.e., such a transition is a second-order phase transition. We note that the form of the expression (1.7) does not depend on the shape and dimensions of the sample. Therefore, we can conclude that the phase transition is of the same type for any geometry of the sample.

We proceed to elucidate the character of the magnetization distribution in a ferromagnetic plate. From (1.3) and (1.6) we obtain an equation for the quantity m_z:

$$\left(\mu_\perp\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\left(\alpha\nabla^2 m_z+\xi m_z-\frac{\beta}{2}m_z^3\right)-4\pi\frac{\partial^2 m_z}{\partial z^2}=0,\qquad (1.8)$$

where $\mu_\perp=\frac{1}{4}\pi/\beta$. It is easy to show that, when the inequality $\xi\ll 4\pi,\ \beta$, which corresponds to sufficiently thick plates, is fulfilled, the nonuniformity of the magnetization in the volume of the ferromagnet is considerably less along the z axis than along the x axis, and $h_z\ll 4\pi m_z$. In this case the contribution of solutions of the surface type can be neglected, since the characteristic penetration depth of the surface solutions is proportional to $\alpha^{1/2}$ and is considerably less than the thickness L of the plate. Therefore, Eq. (1.8) can be brought to the following form:

$$\mu_\perp\frac{\partial^2}{\partial x^2}\left(\alpha\frac{\partial^2 m_z}{\partial x^2}+\xi m_z-\frac{\beta}{2}m_z^3\right)-4\pi\frac{\partial^2 m_z}{\partial z^2}=0.\qquad (1.9)$$

Here the boundary conditions have the form

$$4\pi m_z|_{z=\pm L/2}=h_z^{(e)}|_{z=\pm L/2},\quad h_x|_{z=\pm L/2}=h_x^{(e)}|_{z=\pm L/2}.\qquad (1.10)$$

Since, as was shown above, a second-order phase transition occurs in the ferromagnet, when the phase-transition point is approached from the side of the phase with $m_z\neq 0$ the magnetic-moment distribution tends to the form which is obtained from Eq. (1.9) without the cubic nonlinear term:

$$m_z\sim A\cos qz\cos kx.\qquad (1.11)$$

It is clear that, for a small departure from H_C, the spatial distribution of the magnetization should not differ significantly from the distribution (1.11). Therefore, we shall seek the solution of Eq. (1.9) in the form

$$m_z=\sum_{n=0}^{\infty}\lambda^{2n+1}A_{2n+1}(z)\cos(2n+1)kx,\qquad (1.12)$$

where λ is a formal small parameter characterizing the degree of closeness to the transition point, and $A_{2n+1}(z)$ is a function of z.

Confining ourselves to the approximation with terms up to and including λ^3, we obtain the following system of equations for the quantities $A_i(z)$:

$$4\pi A_1''(z)+\mu_\perp k^2(\xi-\alpha k^2)A_1(z)-{}^3/_8\mu_\perp k^2\beta\lambda^2 A_1^3(z)=0.$$
$$4\pi A_3''(z)-9\mu_\perp k^2(9\alpha k^2-\xi)A_3(z)-{}^9/_8\mu_\perp k^2\beta A_1^3(z)=0.\qquad (1.13)$$

The system of equations (1.13) admits two classes of solutions: symmetric and antisymmetric with respect to the center of the plate. It can be shown (and this follows from physical considerations) that a symmetric distribution is the more favorable, i.e., $A(z)=A(-z)$. The solution of the first equation of the system (1.13) is an elliptic sine with a maximum at $z=0$:

$$A_1(z)=A_1(0)\ \mathrm{sn}\,[\mathrm{K}(k_0)+kz\lambda A_1(0)\,(3\mu_\perp\beta/4\pi k_0^2)^{1/2},\ k_0],$$
$$k_0=\lambda A_1(0)\left(\frac{3\beta}{16(\xi-\alpha k^2)-3\beta\lambda^2 A_1^2(0)}\right)^{1/2}.\qquad (1.14)$$

where $\mathrm{K}(k_0)$ is a complete elliptic integral and k_0 is the modulus of the elliptic function.

Since Eqs. (1.13) are approximate, to the chosen degree of accuracy (1.14) can be written in the form

$$A_1(z)=A_1(0)\left[\left(1+\frac{3}{256}\frac{\beta\lambda^2A_1^2(0)}{\xi-\alpha k^2}\right)\cos q_1z-\frac{3}{256}\frac{\beta\lambda^2A_1^2(0)}{\xi-\alpha k^2}\cos 3q_1z\right], \tag{1.15}$$

where

$$q_1^2=\frac{\mu_\perp k^2}{4\pi}\left[\xi-\alpha k^2-\frac{9}{32}\beta\lambda^2A_1^2(0)\right]. \tag{1.16}$$

Substituting (1.15) into the second equation of the system (1.13), we obtain the solution for $A_3(z)$:

$$A_3(z)=B\,\mathrm{ch}\,q_3z-\frac{27}{256}\frac{\beta A_1^3(0)}{10\alpha k^2-\xi}\cos q_1z-\frac{\beta A_1^3(0)}{256\alpha k^2}\cos 3q_1z,$$
$$q_3=3q_1\left(\frac{9\alpha k^2-\xi}{\xi-\alpha k^2}\right)^{1/2}. \tag{1.17}$$

Knowing the distribution $m_z(x, z)$, we can determine $m_x(x, z)$ and $h_x(x, z)$ from the Maxwell equations and Eqs. (1.6):

$$h_x=\beta m_x=\frac{4\pi q_1}{\mu_\perp k}\lambda A_1(0)\left[\left(1+\frac{3}{256}\frac{\beta\lambda^2A_1^2(0)}{\xi-\alpha k^2}\right)\sin q_1z-\frac{9}{256}\frac{\beta\lambda^2A_1^2(0)}{\xi-\alpha k^2}\sin 3q_1z\right]\sin kx$$
$$-\frac{4\pi q_1}{\mu_\perp k}\lambda^3\left[B\sqrt{\frac{9\alpha k^2-\xi}{\xi-\alpha k^2}}\,\mathrm{sh}\,q_3z+\frac{9}{256}\frac{\beta A_1^3(0)}{10\alpha k^2-\xi}\sin q_1z+\frac{\beta A_1^3(0)}{256\alpha k^2}\sin 3q_1z\right]\sin 3kx. \tag{1.18}$$

For a given magnetization distribution the magnetic field in the vacuum for $z\geqslant L/2$ has the form

$$h_x^{(e)}=Ce^{-kz}\sin kx+De^{-3kz}\sin 3kx,$$
$$h_z^{(e)}=Ce^{-kz}\cos kx+De^{-3kz}\cos 3kx. \tag{1.19}$$

Substituting the expressions (1.15)–(1.19) into the boundary conditions (1.10), we obtain

$$\frac{q_1}{\mu_\perp k}\left[\sin\frac{q_1L}{2}-\frac{9}{256}\frac{\beta\lambda^2A_1^2(0)}{\xi-\alpha k^2}\sin\frac{3q_1L}{2}\right]=\cos\frac{q_1L}{2}-\frac{3}{256}\frac{\beta\lambda^2A_1^2(0)}{\xi-\alpha k^2}\cos\frac{3q_1L}{2}, \tag{1.20}$$
$$B=\frac{\beta A_1^3(0)}{128\alpha k^2\,\mathrm{ch}(q_3L/2)}\left[1+\frac{q_3}{3\mu_\perp k}\mathrm{th}\frac{q_3L}{2}\right]^{-1}\frac{\xi-\alpha k^2}{10\alpha k^2-\xi}\cos\frac{q_1L}{2}.$$

It can be seen from (1.16) that $q_1\ll k$. Then from (1.20) we obtain

$$q_1=\frac{\pi}{L}\left(1-\frac{2}{\mu_\perp kL}+o\left(\frac{1}{k^2L^2}\right)\right), \tag{1.21}$$

i.e., the magnetization m_z near the surface of the plate is considerably smaller than the magnetization at the center of a domain:

$$m_z\left(z=\frac{L}{2}\right)\approx\frac{\pi}{\mu_\perp kL}m_z(z=0).$$

We note that the decrease of m_z as the surface of the plate is approached leads to a substantial decrease of the magnetic field in the vacuum and, correspondingly, the dipole energy.

Equations (1.16) and (1.21) make it possible to determine the dependence of the magnetization at the domain center on the period of the nonuniformity:

$$\lambda^2A_1^2(0)\approx\frac{32}{9\beta}\left(\xi-\alpha k^2-\frac{4\pi^3}{\mu_\perp k^2L^2}\right). \tag{1.22}$$

To specify the distribution of magnetization in the ferromagnet there now remains a single arbitrary parameter—the inverse period k of the nonuniformity, which can be found from the minimum of the energy of the crystal:

$$\frac{\partial\mathscr{H}(k,\xi)}{\partial k}=0,\qquad\frac{\partial^2\mathscr{H}(k,\xi)}{\partial k^2}>0. \tag{1.23}$$

By means of (1.15)–(1.17) and (1.22), we can represent the energy (1.7) of the ferromagnet as a function of k in the form

$$\mathscr{H}=-\frac{3^2\beta M_0^2V}{2^9}\left[\lambda^4A_1^4(0)\left(1+\frac{2}{\mu_\perp kL}\right)-\frac{5\beta\lambda^6A_1^6(0)}{3\cdot 2^7}\left(\frac{1}{3\alpha k^2}+\frac{3}{\xi-\alpha k^2}+\frac{27}{10\alpha k^2-\xi}\right)\right]. \tag{1.24}$$

We note that to obtain the expression (1.24) it is necessary to calculate the magnetization to order λ^5. In view of their unwieldiness, we have not given the $\sim\lambda^5$ corrections to the magnetization.

At the phase-transition point $H=H_C$ ($\xi=\xi_C$),

$$\left.\frac{\partial\mathscr{H}(k,\xi)}{\partial k}\right|_{H=H_C}=0,\qquad\left.\frac{\partial^2\mathscr{H}(k,\xi)}{\partial k^2}\right|_{H=H_C}=0. \tag{1.25}$$

From the system (1.25) we find the field H_C at the phase transition and the period D_C of the nonuniformity:

$$H_C=H_A-\frac{4\pi M_0}{L}\left(\frac{\pi\alpha}{\mu_\perp}\right)^{1/2}=H_A-\frac{2\pi M_0}{\mu_\perp}\frac{D_C^2}{L^2}, \tag{1.26}$$
$$D_C=\frac{2\pi}{k_C}=(4\pi\mu_\perp\alpha L^2)^{1/4},\qquad\xi_C=2\alpha k_C^2=\frac{4\pi^{3/2}\alpha^{1/2}}{\mu_\perp^{1/2}L}. \tag{1.27}$$

As can be seen from (1.26), after the phase transition, H_C is displaced toward lower fields with decrease of the thickness of the plate, and the period of the nonuniformity has the well known dependence $D\sim L^{1/2}$. We note that (1.26) coincides with the formula for H_C obtained in the paper[10] of Goldstein and Muller, in which the boundaries of stability of the uniform phase in samples of finite size were determined.

It follows from the requirement $\xi\ll 4\pi$, β that our treatment is valid for samples with thickness $L\gg(\pi\alpha/\mu_\perp)^{1/2}$.

Using the expressions (1.22)–(1.24) it is easy to obtain the field dependences of the magnetization m_z, the period D, and the energy of the ferromagnet in the vicinity of the phase-transition field H_C:

$$m_z(x=0;\ z=0)=\frac{4}{3}\left(\frac{2\Delta\xi}{\beta}\right)^{1/2}\left(1-\frac{7^2}{2^63^3}\frac{\Delta\xi}{\xi_C}\right), \tag{1.28}$$
$$D=D_C+\frac{L}{16\pi^2}\Delta\xi, \tag{1.29}$$
$$\mathscr{H}=-\frac{2M_0V}{9}(\Delta\xi)^2\left(1+\frac{161}{6^4}\frac{\Delta\xi}{\xi_C}\right), \tag{1.30}$$

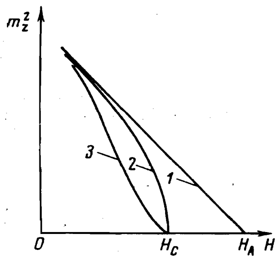

FIG. 2. Dependence of the magnetization of the ferromagent on the magnetic field: 1) $m_z^2(H)$ is the square of the magnetization in the uniform case; 2) $m_z^2(x=0;\ z=0)$ is the square of the magnetization at the domain center; 3) $m_z^2(x=0;\ z=L/2)$ is the square of the magnetization at the surface of the plate.

where $\Delta\xi=\xi-\xi_C=(H_C-H)/M_0$. It can be seen from formulas (1.28)–(1.30) that the small expansion parameter λ^2 is $\Delta\xi/\xi_C$, i.e., the results obtained are valid in the region of magnetic fields in which $\Delta\xi \ll \xi_C$.

We note that the quantity m_z^2 has a linear dependence on the magnetic field near the transition point, but the magnitude of $\partial m_z^2(z=0)/\partial H$ at $H=H_C$ is greater than in the uniform case by a factor of $\frac{16}{9}$. With increase of $\Delta\xi$ (decrease of the magnetic field) the magnetization becomes linear in the thickness of the sample, and the rate of increase of the magnetization decreases at the center of the domain and increases at the edges of the plate (see Fig. 2). The period of the nonuniformity grows with decrease of the magnetic field.

The susceptibility $\hat{\chi}$ of the system is defined by the equation $\chi_{ik}=-\partial^2\mathscr{H}/\partial H_i\partial H_k$, from which, with the aid of (1.30), we can obtain the discontinuities and field dependence of the longitudinal susceptibility:

$$\chi_{yy}=\Delta\chi_{yy}\left(1+\frac{161}{2^4 3^3}\frac{\Delta\xi}{\xi_C}\right),\quad \Delta\chi_{yy}=\frac{4}{9\beta}. \qquad (1.31)$$

From this it can be seen that the jump in the susceptibility at the transition point does not depend on the thickness of the plate and is smaller by a factor of $\frac{9}{4}$ than in the phase transition in the infinite crystal.

The susceptibility χ_{yy} grows with decrease of the magnetic field, the rate of growth being $\sim L$. In the limit $L\to\infty$ the quantity $\partial\chi_{yy}/\partial H\to\infty$, which is equivalent in practice to the usual jump in the susceptibility. The dependence $\chi_{yy}(H)$ is shown schematically in Fig. 3.

2. SPIN-WAVE SPECTRUM IN A FERROMAGNETIC PLATE IN A TRANSVERSE MAGNETIC FIELD

To determine the spin-wave spectrum in a ferromagnetic plate in an external magnetic field perpendicular to the anisotropy axis, we shall use the Maxwell equations (1.3) and the equations of motion of the magnetic moment

$$\dot{\mathbf{M}}=g[\mathbf{M},\mathscr{H}_{\text{eff}}] \qquad (2.1)$$

together with the boundary conditions (1.4). Here g is the gyromagnetic ratio.

In the vicinity of the phase transition ($|\xi|\ll 4\pi,\ \beta$) on the side of high magnetic fields ($H>H_C$), we write Eq. (2.1) in the form

$$\omega_0 h_x=-i\omega m_z+\omega_0\beta m_x,$$
$$\omega_0 h_z=i\omega m_x-\omega_0(\alpha\nabla^2 m_z+\xi m_z), \qquad (2.2)$$

where $\omega_0=gM_0$ and ω is the spin-wave frequency.

Substituting (2.2) into the Maxwell equations, we obtain

$$\omega^2\frac{\partial^2 m_z}{\partial x^2}+\mu_\perp\beta\omega_0^2\frac{\partial^2}{\partial x^2}\left(\alpha\frac{\partial^2}{\partial x^2}+\xi\right)m_z-4\pi\beta\omega_0^2\frac{\partial^2 m_z}{\partial z^2}=0. \qquad (2.3)$$

The solution of Eqs. (2.3) and (1.3) with the boundary conditions (1.10) will be sought in the form

$$m_z=(a\cos qz+b\sin qz)e^{ikx},$$
$$m_x=\frac{ie^{ikx}}{\mu_\perp\beta\omega_0^2}\left[\omega(a\cos qz+b\sin qz)+4\pi\omega_0\frac{q}{k}(-a\sin qz+b\cos qz)\right]. \qquad (2.4)$$

Substituting the solution (2.4) into Eq. (2.3), we find the spectrum of the spin waves in the ferromagnet:

$$\omega^2=\beta\omega_0^2[4\pi q^2/k^2+\mu_\perp(\alpha k^2-\xi)]. \qquad (2.5)$$

From the boundary conditions (1.10) there follows a second equation relating the spin-wave frequency to the wave vectors k and q:

$$\omega^2+\mu_\perp\beta^2\omega_0^2(\operatorname{tg}{}^1/_2qL-\operatorname{ctg}{}^1/_2qL)-\mu_\perp^2\beta^2\omega_0^2k/q=0. \qquad (2.6)$$

Since $q\ll k$, we represent Eq. (2.6) in the form

$$\operatorname{tg}{}^1/_2qL=\mu_\perp k/q,\quad \operatorname{tg}{}^1/_2qL=-q/\mu_\perp k, \qquad (2.7)$$

whence it follows that

$$q=\frac{\pi n}{L}\left(1-\frac{2}{\mu_\perp kL}\right), \qquad (2.8)$$

where $n=1,\ 2,\ldots$

Equations (2.5) and (2.8) determine the dispersion equation of the spin waves in a ferromagnetic plate:

$$\omega_n^2=\beta\omega_0^2[4\pi^3n^2/k^2L^2+\mu_\perp(\alpha k^2-\xi)], \qquad (2.9)$$

whence it can be seen that the functions $\omega_n(k)$ have a minimum at $k=k_n\approx k_C\sqrt{n}$ (Fig. 4). The values of ω_n at the minimum are equal to

$$\omega_n^2=\beta\mu_\perp\xi_C\omega_0^2(n-1+\Delta\xi/\xi_C), \qquad (2.10)$$

where $\Delta\xi=(H-H_C)/M_0$. For the branch $n=1$ the frequency $\omega\to 0$ as $\Delta\xi\to 0$, corresponding to instability of

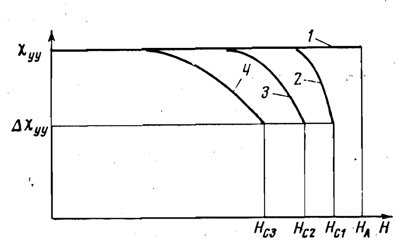

FIG. 3. Dependence of the susceptibility of the ferromagnet on the magnetic field: 1) susceptibility of a uniform ferromagnet; 2, 3, 4) susceptibility of a ferromagnet with a nonuniform distribution of the magnetic moment; $H_C=H_{Ci}(L_i)$; $L_1>L_2>L_3$; $H_C(L\to\infty)=H_A$.

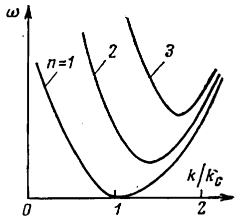

FIG. 4. Spin-wave spectrum in a ferromagnetic plate in a transverse magnetic field ($H = H_C$).

the ferromagnet with respect to the appearance of a nonuniform state.

Thus, the values obtained from the solution of the dynamical problem for the period of the critical mode and for the field at which the instability of the uniform state appears coincide exactly with the values found in the preceding section for, respectively, the period of the nonuniformity and the critical field at which the second-order phase transition occurs.

3. NONUNIFORM MAGNETIC STATE IN A FERROMAGNET NEAR THE CURIE POINT

We shall consider a uniaxial ferromagnetic crystal in the form of a plane-parallel plate, with the crystallographic axis directed perpendicular to the surface of the plate. We write the free energy of the ferromagnet in the form

$$F=\int dv\left\{\frac{1}{2}\alpha(\nabla\mathbf{M})^2+\frac{1}{2}\beta(M_x^2+M_y^2)+\frac{\delta\mathbf{M}^2}{4M_0^2}[\mathbf{M}^2-2\tilde{M}^2(T)]+\frac{\mathbf{H}_D^2}{8\pi}\right\}, \qquad (3.1)$$

where $\tilde{M}(T)$ is the equilibrium value of the magnetization at temperature T in the uniform case in the absence of a field, $M_0=\tilde{M}(T=0)$, $\delta \sim T_C/\mu M_0$ is the exchange constant and μ is the Bohr magneton. The other symbols correspond to those introduced earlier. The expression (3.1) is the expansion of the free energy of the ferromagnet in the order parameter M.[11]

We obtain the equilibrium distribution of the magnetization in the ferromagnet from the equation of state

$$\delta F/\delta\mathbf{M}=0 \qquad (3.2)$$

and the Maxwell equations (1.3). It follows from Eq. (3.2) that

$$\mathbf{H}_D=-\alpha\nabla^2\mathbf{M}+\beta(\mathbf{M}-M_z\mathbf{e}_z)+\delta\mathbf{M}M_0^{-2}(\mathbf{M}^2-\tilde{M}^2). \qquad (3.3)$$

When (3.3) and (1.3) are taken into account, the expression (3.1) takes the form

$$F=-\frac{\delta}{4M_0^2}\int dv\,\mathbf{M}^4. \qquad (3.4)$$

The free energy is a continuous function of the temperature, and, therefore, it follows from (3.4) that the transition from the paramagnetic phase ($\mathbf{M}=0$) to the ferromagnetic phase occurs without a jump in the magnetization, i.e., such a transition is a second-order phase transition.[1)] This can also be seen from the fact that the gradient energy and demagnetization energy are proportional to the square of the magnetization ($H_D \sim M$; cf. (1.3)). Consequently, these terms can alter the coefficients of M^2 (but not of M^4) in the expansion of the free energy. Such a renormalization of the coefficients can lead only to a shift in the transition point, but cannot alter the character of the transition.

We shall confine ourselves to considering a strip domain structure near the Curie point. In the approximation $\xi \ll 4\pi$, β, Eqs. (3.3) can be brought to the form

$$h_x=\beta m_x,\quad h_y=\beta m_y,$$
$$h_z=-\alpha\nabla^2 m_z-\tilde{\xi}m_z+\delta m_z^3, \qquad (3.5)$$

where

$$\mathbf{m}=\mathbf{M}/M_0,\quad \mathbf{h}=\mathbf{H}_D/M_0,\quad \tilde{\xi}=\delta\tilde{M}^2(T)/M_0^2,$$

and for a strip domain structure $h_y=m_y=0$. In[12] it was shown that for $\tilde{\xi}<\beta$ the magnetic moment changes in magnitude but not in direction, and therefore $m_y=0$.

From (1.3) and (3.5) we obtain an equation for m_z, determining the distribution of the magnetization in the sample:

$$\mu_\perp\frac{\partial^2}{\partial x^2}\left(\alpha\frac{\partial^2 m_z}{\partial x^2}+\tilde{\xi}m_z-\delta m_z^3\right)-4\pi\frac{\partial^2 m_z}{\partial z^2}=0. \qquad (3.6)$$

As can easily be seen, this equation coincides with Eq. (1.9) for the distribution of m_z near the phase-transition point with respect to the magnetic field, and, therefore, all the results can be obtained from the formulas of the first section by the replacement $\xi\to\tilde{\xi}$, $\beta\to 2\delta$. In particular, the dependence of the Curie temperature T_C on the sample thickness L has the following form:

$$T_C=T_0-\tilde{\xi}_C/\xi', \qquad (3.7)$$

where $\xi'=-\partial\tilde{\xi}/\partial T$ at $T=T_C$, and T_0 is the Curie point in the infinite crystal. The period of the nonuniform state is determined by formula (1.29), and the temperature dependence of the specific heat and the specific-heat discontinuity at the Curie point are respectively equal to

$$C=\Delta C\left(1+\frac{161}{2^4 3^3}\frac{\Delta\tilde{\xi}}{\tilde{\xi}_C}\right),\quad \Delta C=\frac{2\xi'^2M_0^2V}{9\delta}T_C. \qquad (3.8)$$

The specific heat of the ferromagnet behaves analogously to the longitudinal susceptibility χ_{yy} near the phase transition in a transverse magnetic field.

In conclusion we shall make certain remarks concerning the applicability of the theory of phase transitions in a ferromagnet of finite dimensions, developed above. The perturbation theory constructed is valid for

$$\Delta\xi\ll\xi_C\ll 4\pi. \qquad (3.9)$$

On the other hand, the whole treatment was based on the Landau theory of phase transitions, which is applicable for sufficiently large $\Delta\xi$ ($\Delta\xi\gg\Delta\xi_f$), when the fluctuations of the order parameter can be neglected.[13] For a ferromagnet in a transverse magnetic field for $T\ll T_C$, estimates of the fluctuation[2)] give

$$\Delta\xi_\Phi=\frac{\beta}{\pi^4\delta}\left(\frac{T}{T_C}\right)^2.$$

For example, for the garnet $Y_{2.6}Gd_{0.3}Pb_{0.1}Fe_{3.8}Ga_{0.1}O_{12}$ the parameters $\beta\sim 35$, $\delta\sim 3\times 10^4$, $\alpha\sim 1.5\times 10^{-10}$ cm^2.[5] Then $\Delta\xi_f\sim 10^{-5}(T/T_C)^2$ and $\xi_C\sim 10^{-4}/L$[cm] (1.27). In this case, with the aid of (3.9) we can find the range of thicknesses L (10^{-5}cm $\ll L \ll (T_C/T)^2$) for which our analysis can be used.

Because of the presence of fluctuations, the results obtained for the phase transition near the Curie point are valid only for ferromagnets with long-range interaction.

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1) In[3] it was concluded that a first-order phase transition that is almost second-order occurs in finite samples. This is connected with the fact that the nonuniformity of the magnetization over the thickness of the plate was not taken into account in that paper.

2) The zero-point oscillations give a negligibly small contribution $\Delta\xi_f$.

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Directed collisions between indicator ions containing short-lived nuclei and neighboring atoms in single crystals

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We consider the spectral distribution of γ quanta (or particles) emitted by short-lived excited (or compound) nuclei that move in a crystal and are produced in nuclear reactions induced by parallel beams of monoenergetic particles. By orienting the single crystal with respect to the beam it is possible to produce, with high probability, directed collisions between ions containing short-lived nuclei and neighboring atoms of the crystal. It is found that the velocity change due to scattering can alter the spectrum significantly if the lifetime τ of the nuclei is comparable with the time of flight to the neighboring atoms. Directed collisions permit therefore observation of short-lived compound or γ-excited nuclei with $\tau\sim 10^{-16}$–10^{-14} sec and measurement of their lifetime. Possible applications of the directed-collision technique to the investigation of the local structure of crystals and of vibration dynamics are discussed.

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A method based on controlled atomic collisions in single crystals has been proposed[1,2] to measure the lifetimes of ultra-short-lived nuclei and to analyze the structure of the crystal lattice. It is well known that the crystal lattice exerts an appreciable influence on the motion of fast charged particles, ions, and atoms, as well as on the character of many atomic and nuclear processes that occur in the lattice.[3] The ordered arrangement of the atomic nuclei and of the electrons produces a large anisotropy of the electronic and nuclear stopping losses, producing relatively free "channels" in certain directions of the crystal, and practically blocking the motion in other directions. These orientational singularities of the particle motion manifest them-

