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Quasilinear relaxation of a beam of fast ions in the tokamak

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Quasilinear relaxation of a beam of fast ions produced in a tokamak upon injection of a beam of fast neutral atoms is investigated theoretically. Relaxation of this type is assumed to be due to interaction between the ions and Alfvén waves. A beam moving along the magnetic field is considered. The shear of the magnetic force lines is neglected. It is shown that under these assumptions the quasilinear relaxation is much more rapid than the Coulomb relaxation due to pair collisions. It is concluded that the concept of a "two-component tokamak," which is based on the assumption of Coulomb relaxation of the fast ions, calls in general for revision.

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1. INTRODUCTION

According to present-day concepts, one of the main methods of obtaining a plasma with thermonuclear parameters in a tokamak is to inject a beam of fast neutral atoms (see, e.g., the reviews of Artsimovich[1] and Furth[2]). It is therefore important to study the dynamics of fast ions produced by ionizing these atoms, and in particular to study the relaxation of their thermodynamic non-equilibrium (non-Maxwellian) velocity distribution.

The velocity relaxation of fast ions in a tokamak has been the subject of many theoretical papers (reference to which can be found in[1,2]). Common to most hitherto performed theoretical investigations of the velocity relaxation of fast ions in a tokamak is the assumption that the only cause of this relaxation are the Coulomb collisions. One of the essential consequences of these investigations is the concept of the possibility of producing a so-called "two-component" thermonuclear tokamak reactor, i.e., a tokamak reactor whose plasma contains ions of two groups, "slow" (i.e., ions of the fundamental plasma component, obtained for example by Joule heating) and fast (injected) ions. The basis for this concept is the fact that the time of the Coulomb relaxation of the fast ions turns out to be just as long as the characteristic operating time of the two-component tokamak reactor.

It is clear from this that the concept of the two-component tokamak reactor may turn out to be untenable if it turns out that the distribution function of the injected ions is unstable and that the velocity relaxation of the ions, due to the reaction of the growing noise, turns out to be faster than the Coulomb relaxation. It is therefore important to carry out a thorough analysis of the instability of the ion beams and the associated processes of non-Coulomb relaxation. The latter call for further development of turbulence theory, particularly the quasi-linear theory.

Until recently, the linear approximation of plasma-oscillation theory has revealed no instabilities presenting any danger whatever to the problem of the two-component tokamak reaction (see, e.g., the article of Cordey and Houghton[3] and the references cited therein). The problem of the instability of ion beams in a tokamak was considered recently more fully, with allowance for the toroidal character of the magnetic field and the nonpotential character of the perturbations.[4] It was shown there that fast ions injected in a tokamak can lead to a buildup Alfvén waves. The purpose of the present paper is to investigate the quasi-linear relaxation of fast ions interacting with the Alfvén waves excited by them. It will be shown below that the quasi-linear relaxation due to the Alfvén instability is much faster than the Coulomb relaxation. At the characteristic parameters of tokamak reactors, the ratio of the relaxation times may amount to three orders of magnitude. The possible consequences of the observed anomalously fast relaxation of the ion beam for the problem of the two-component tokamak are discussed in the conclusion of the paper.

2. QUASI-LINEAR APPROXIMATION EQUATIONS

We represent the fast-ion velocity distribution function $F(\mathbf{v})$ in the form of two parts, averaged over the oscillations, $\bar{F}$, and perturbed, $\tilde{F}$, $F=\bar{F}+\tilde{F}$. We assume that the beam-particle velocity averaged over the oscillations is directed along the equilibrium magnetic field $\mathbf{B}_0$, $\mathbf{v}_0=v_{\parallel}\mathbf{e}_0$, where $\mathbf{e}_0\equiv\mathbf{B}_0/B_0$ (this corresponds to the case of longitudinal injection). Under this assumption, $\bar{F}$ is a function of two variables: the time t and the longitudinal velocity $v_{\parallel}$, $\bar{F}=F(v_{\parallel},t)$.

In accordance with the general concepts of quasi-linear theory,[6] the function $\bar{F}$ should satisfy the one-dimensional equation

$$\frac{\partial\bar{F}}{\partial t}=\frac{\partial}{\partial v_{\parallel}}D\frac{\partial\bar{F}}{\partial v_{\parallel}}+S, \tag{1}$$

where S is a certain source and $D=D(v_{\parallel},t)$ is the diffusion coefficient, which is a certain functional of the oscillation energy $W(\mathbf{k},\omega_k)$ ($\mathbf{k}$ is the wave vector of the oscillations and ω_k is their frequency) so that $D(v_{\parallel},t)=D[W]$.

The oscillation energy depends on the amplitude $\mathbf{E}$ of the perturbed electric field and on the dielectric tensor $\varepsilon_{\alpha\beta}$ of the plasma and is determined by the relation

$$W_{\mathbf{k}}=\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon_{\alpha\beta}^{(0)})\frac{E_\alpha^* E_\beta}{16\pi}, \tag{2}$$

where the zero superscript denotes the Hermitian part. The time dependence of the oscillation energy is determined in the quasi-linear approximation by the equation

$$\partial W_{\mathbf{k}}/\partial t=2\gamma_{\mathbf{k}}W_{\mathbf{k}}, \tag{3}$$

where $\gamma_{\mathbf{k}}$ is the growth rate of the oscillations and is a certain functional of $\bar{F}$, $\gamma_{\mathbf{k}}=\gamma_{\mathbf{k}}[\bar{F}]$.

The concrete forms of D and $\gamma_{\mathbf{k}}$ depend on the type of the excited oscillations and on the character of the resonant interaction of the particles with the oscillations. The oscillation type of interest to us constitutes Alfvén waves with a dispersion law

$$\omega_k^2=k_{\parallel}^2c_A^2, \tag{4}$$

where c_A is the Alfvén velocity ($c_A^2=B_0^2/4\pi Mn_0$, n_0 is the plasma density and M is the ion mass), $k_{\parallel}$ is the wave-vector component along the magnetic field, and $k_{\parallel}=\mathbf{kB}_0/B_0$. In the case of the tokamak (see, e.g.,[8]) we have

$$k_{\parallel}=(m-nq)/qR, \tag{5}$$

where R is the radius of the curvature of the magnetic axis of the torus, $q=aB_s/RB_\theta$ is the so-called margin coefficient of the tokamak, B_s and B_θ are the toroidal and poloidal magnetic fields,[9] a is the distance from the magnetic axis (the minor running radius of the torus), and m and n are integers characterizing the dependence of the perturbations on the cyclic coordinates θ (the minor azimuth of the torus) and φ (the major azimuth of the torus), so that $\mathbf{E}\sim\exp[i(m\theta-n\varphi)]$. It is assumed that the wave vector $\mathbf{k}$ is almost perpendicular to the equilibrium magnetic field $\mathbf{B}_0$, i.e., $k_{\parallel}\ll k_{\perp}$, where $k_{\perp}=[k_a^2+(m/a)^2]^{1/2}$, k_a is the radial component of the wave vector (the radial dependence of the perturbations is assumed in the form $\exp(ik_a a)$).

It is known,[7] that the electric field of Alfvén waves is oriented transversely to the equilibrium magnetic field $E_{\parallel}=\mathbf{E}\cdot\mathbf{B}_0/B_0=0$ (the approximation of infinitely large longitudinal conductivity) and coincides in direction with the transverse component of the wave vector (neglect of the Hall effect), so that $\mathbf{E}=\mathbf{k}(\mathbf{k}\cdot\mathbf{E})/k^2$. We assume that $\mathbf{k}_{\perp}\parallel\nabla a$, i.e., that the wave vector is almost radial, $k_a\gg m/a$. (Perturbations with $k_a\gg m/a$ are less sensitive to the shear of the magnetic-field force lines,[4] which we neglect.) Under these assumptions there remain in the right-hand part of (2) only the terms with $(\alpha,\beta)=1$ (the direction 1 is assumed to coincide with the direction of the minor radius). Recognizing also that at frequencies ω that are small in comparison with the ion cyclotron frequency $\omega_{Bi}\equiv e_iB_0/Mc$, where e_i is the ion charge (the low-frequency oscillations include also the Alfvén waves), and $\varepsilon_{11}^{(0)}=c^2/c_A^2$,[7] we reduce (2) to the form

$$W_{\mathbf{k}}=\frac{c^2}{c_A^2}\frac{|E_1|^2}{8\pi}. \tag{6}$$

The general expression for the growth rate of the Alfvén waves is[7]

$$\gamma_{\mathbf{k}}=-\omega_{\mathbf{k}}\operatorname{Im}\varepsilon_{11}/2\varepsilon_{11}^{(0)}. \tag{7}$$

This result is obtained by using the dispersion equation for the Alfvén waves[7]

$$\varepsilon_{11}-c^2k_{\parallel}^2/\omega^2=0, \tag{8}$$

in which we substitute

$$\omega=\omega_{\mathbf{k}}+i\gamma_{\mathbf{k}},\quad \varepsilon_{11}=\varepsilon_{11}^{(0)}+i\operatorname{Im}\varepsilon_{11}.$$

An expression for $\operatorname{Im}\varepsilon_{11}$ can be obtained if one knows the conduction tensor component σ_{11}, henceforth designated $\sigma_{11}^{(1)}$, by using the relation

$$\operatorname{Im}\varepsilon_{11}=4\pi\sigma_{11}^{(1)}/\omega. \tag{9}$$

By definition we have

$$j_1=\sigma_{11}E_1, \tag{10}$$

where j_1 is the radial component of the electric field.

To find σ_{11} it is therefore necessary to calculate the current j_1. Since $\sigma_{11}^{(1)}$ corresponds to interaction with oscillations of the resonant particles, it is required to calculate the part of the current connected only with these particles. In accordance with the results of[4,5], this part of the current is determined by the relation

$$j_1 = e_i \int v_1 \tilde{F}\, dv_\parallel, \tag{11}$$

where v_1 is the radial projection of the particle drift velocity $\mathbf{v}$. The drift velocity $\mathbf{v}$ is due to the curvature of the magnetic-field force lines and in the case of a longitudinal beam it is given by

$$\mathbf{v} = v_\parallel^2 [\mathbf{e}_0 \times (\mathbf{e}_0 \nabla)\mathbf{e}_0]/\omega_{Bi}. \tag{12}$$

To calculate the perturbed distribution function $\tilde{F}$ we use, as in[4,5], the drift kinetic equation. According to Rudakov and Sagdeev[10] the drift kinetic equation for the function $f(\varepsilon, \mu, t)$ ($\varepsilon = v^2/2$) is the energy of the particle of unit mass and $\mu = v_\perp^2/2B$ is the suitably normalized magnetic moment of the particle) is written in the form

$$\frac{\partial}{\partial t}\left(\frac{B}{v_\parallel} f\right) + \operatorname{div}\left[\left(\mathbf{B} + \frac{B}{v_\parallel}\mathbf{v}_E\right) f\right] + \frac{\partial}{\partial \varepsilon}\left\{\frac{fB}{v_\parallel}\left[\mu \frac{\partial B}{\partial t} + \frac{\mathbf{v}_E \nabla B}{B}(2\varepsilon - \mu B)\right]\right\} = S, \tag{13}$$

where $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}'$ is the total magnetic field ($\mathbf{B}'$ is the perturbation), and $\mathbf{v}_E = c\mathbf{E} \times \mathbf{e}_0/B$ is the electric drift velocity. For the longitudinal beam of interest to us, the linearized part of this equation reduces to

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_\parallel \nabla\right)\tilde{F} = -(\mathbf{v}_E \nabla \ln B_0) v_\parallel \frac{\partial \bar{F}}{\partial v_\parallel}. \tag{14}$$

We have neglected here the coordinate dependence of the function $\bar{F}$, $\nabla \bar{F} = 0$.

For the case of tokamak with round cross section, when[9]

$$B_0 = B_s[1 + (a/R)\cos\theta],$$

we obtain from (14), by the method of integrating along the trajectories,[4]

$$\tilde{F} = -\frac{cE_t}{2B_s R} v_\parallel \frac{\partial \bar{F}}{\partial v_\parallel}\left(\frac{e^{i\theta}}{\omega - v_\parallel/qR} - \frac{e^{-i\theta}}{\omega + v_\parallel/qR}\right). \tag{15}$$

We have neglected in (15) terms of smallness $v_\parallel/c_A$. (Account is taken of the fact that in the presently considered variants of fast-ion injection into a tokamak the velocity of these ions is assumed to be small in comparison with the Alfvén velocity, albeit higher than the thermal velocity v_{Ti} of the ions of the principal plasma component, i.e., $v_{Ti} < v_\parallel < c_A$). In addition, just as in[4], we assume $|k_\parallel| \ll 1/qR$.

Using the relations given above, we obtain in analogy with[4]

$$\gamma_k = \frac{\pi}{8R^2 n_0}\int v_\parallel^3 \frac{\partial \bar{F}}{\partial v_\parallel}\delta\left(\omega_k - \frac{v_\parallel}{qR}\right) dv_\parallel. \tag{16}$$

By the same token we have determined in explicit form the functional dependence of the growth rate of the oscillation energy on the average distribution function $\bar{F}$, see Eq. (3).

Thus, to obtain a closed system of quasi-linear equations it remains only to find the expression for the diffusion coefficient D, which enters in Eq. (1). To this end we average the drift kinetic equation (13) over the perturbations and reduce it to the form

$$\frac{\partial \bar{F}}{\partial t} + \frac{\partial}{\partial v_\parallel}\langle(\mathbf{v}_E \nabla \ln B_0)\tilde{F}\rangle = 0, \tag{17}$$

where the angle brackets represent averaging. Substituting here $\tilde{F}$ from (15) we arrive at Eq. (1), in which

$$D = \frac{\pi v_\parallel^2}{4R^2 M n_0}\int W(k_\parallel, t)\,\delta\left(\omega_k - \frac{v_\parallel}{qR}\right) dk_\parallel. \tag{18}$$

Equations (1), (3), (16), and (18) comprise the system of the sought quasi-linear-approximation equation describing the interaction of the longitudinal ion beam with the Alfvén waves in the tokamak. This system of quasi-linear equations has no analog in the case of a plasma situated in a homogeneous magnetic field, i.e., in a field with $R = \infty$, so Eqs. (16) and (18) with $R = \infty$ lead to $\gamma = 0$ and $D = 0$.

3. ANALYSIS OF QUASI-LINEAR EQUATIONS

We use the quasi-linear equations obtained above for a numerical analysis of the dynamics of the fast ions produced when a beam of neutral atoms injected into a tokamak becomes ionized. Assume that at $t \leq 0$ there are no fast ions, so that $\bar{F}(v_\parallel, t) = 0$ if $t \leq 0$. Let a stationary fast-ion source $S(v_\parallel)$ with a velocity distribution that has a maximum at $v_\parallel = v_\parallel^0$ be turned on at the instant $t = 0$.

Assuming $v_\parallel^0$ to be the unit of the velocity scale and $\tau_0 \equiv 2\pi R_0 q\lambda/v_\parallel^0$ the unit of the time scale (λ is a certain dimensionless constant introduced to facilitate the calculations), and the plasma density n_0 to be the unit of the density scale, and changing over to dimensionless variables in the system of equations (1), (3), (18), and (16), we obtain

$$\frac{\partial F}{\partial \tau} = \frac{\partial}{\partial \mathscr{E}} D_{\mathscr{E}} \frac{\partial F}{\partial \mathscr{E}} + J(\mathscr{E}), \quad D_{\mathscr{E}} = A\mathscr{E}^2 w(\mathscr{E}, \tau),$$
$$w = \exp\left(\int_0^\tau \Gamma\, d\tau\right), \quad \Gamma = B\mathscr{E}^{5/2}\frac{\partial F}{\partial \mathscr{E}}. \tag{19}$$

Here $\mathscr{E} = (v_\parallel/v_\parallel^0)^2$ is the dimensionless energy of the ions, $F(\mathscr{E}) = \varepsilon_0 \bar{F}(v_\parallel)/v_\parallel n_0$ is a dimensionless distribution function, $\varepsilon_0 = v_\parallel^{02}/2$, $\tau = t/\tau_0$, $\Gamma = 2\gamma\tau_0$ is the growth rate of the noise energy in units of τ_0, $w = W/W_0$ is the dimensionless density of the oscillation energy, W_0 is a certain initial level of the oscillation energy density, and $J(\mathscr{E}) = \varepsilon_0 S\tau_0/v_\parallel n_0$ is a function characterizing the source. The constants A and B in (19) stand for

$$A = (2\pi)^{3/2} W_0 \lambda / R_0 \rho_0^{1/2} B_0 \varepsilon_0^{1/2}, \quad B = \pi^2 q\lambda. \tag{20}$$

We note also that, when working with the function $W(\mathscr{E})$, we have, in fact, in mind a function of the wave

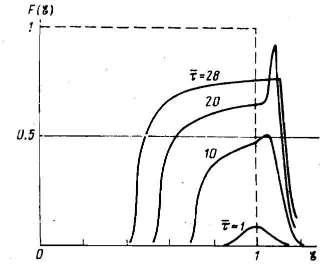

FIG. 1. A time variation of the fast-particle distribution function.

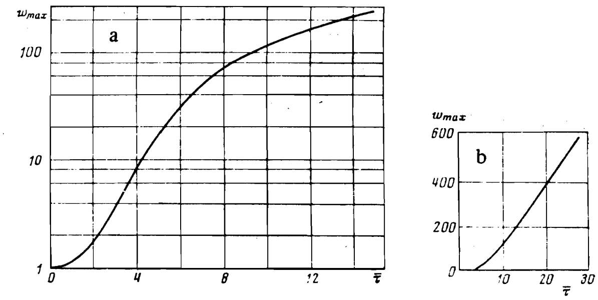

FIG. 3. Time dependence of the maximum value of the spectral density of the oscillation energy. a) Initial stage of the process (semi-logarithmic scale, b) overall picture of the process (linear scale).

number $k_\parallel$, which is connected with $\mathscr{E}$ by the resonance condition $\omega_{\mathbf{k}} = k_\parallel c_A = v_\parallel/qR$, so that the argument $\mathscr{E}$ in w actually stands for

$$\mathscr{E} = (qRc_A)^2 k_\parallel^2/2\varepsilon_0. \tag{21}$$

The system (19) was solved numerically under the assumption that the energy distribution of the injected particles is of the form

$$J(\mathscr{E}) = a \exp[-(\mathscr{E}-1)^2/b^2]. \tag{22}$$

The values of the constants were assumed to be $a = 10^{-3}$, $b = 0.1$, $A = 10^{-4}$, $B = 50$. The choice of these values is explained by the following considerations. The constant a, which characterizes the source power, should be such that the fast-ion density remain less than the plasma density during the entire quasi-linear relaxation process investigated by us. (This corresponds to the situation in real experiments. Furthermore, in the opposite case the assumption that the contribution of the fast ions to the dielectric constant is small would not hold.) The constant b, which characterizes the energy spread of the source, should be small, in agreement with the relatively small spread of real injectors. On the other hand, in the case of a very small energy spread of the fast ions, the system of quasi-linear equations presented above cannot be used, since according to a paper by one of us,[4] a strong instability of hydrodynamic type is possible in this case. The very small value of the constant A is due to the fact that this constant characterizes the ratio of the initial noise energy level to the energy that the fast particles would have if their density were of the order of the plasma density (accurate to $v_\parallel^0/c_A$). As to the constant B, it follows from its definition that it is uniquely connected with the margin coefficient q. Since the constant B contains also the parameter λ introduced above, our calculations, which pertain to a fixed B, can be used for arbitrary q if the parameter λ is suitably chosen. (This indeed constitutes the meaning of the introduced λ.)

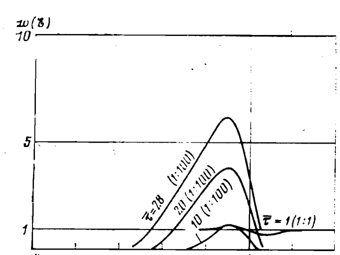

FIG. 2. Time evolution of the oscillation spectrum.

The result of the numerical calculation is shown in Figs. 1–3, in which $\bar{\tau} = 100\,\tau$.

The initial state corresponding to the instant of time $t = 0$ is characterized by a zero value of the function $F(\mathscr{E})$ in Fig. 1 and by a unity value of the function $W(\mathscr{E})$ in Fig. 2. At $\bar{\tau} = 1$, the system already contains a certain number of fast particles (see Fig. 1), which interact resonantly with the oscillations that lead to the growth of the oscillation energy in the region $\mathscr{E} < 1$, where $\partial F/\partial\mathscr{E} > 0$, and to its decrease at $\mathscr{E} > 1$, corresponding to $\partial F/\partial\mathscr{E} < 0$ (Fig. 2). The time interval $\bar{\tau} < 1$, however, does not exceed the reciprocal growth rate characteristic of this time interval, so that $w(\mathscr{E}, \bar{\tau} = 1)$ differs little from $w(\mathscr{E}, \bar{\tau} = 0)$—see Fig. 2.

At $\bar{\tau} = 10$, the density of the continuously injected fast ions turns out to be larger by one order of magnitude than at $\bar{\tau} = 1$—see Fig. 1. (The fast-ion density at any particular instant of time is characterized by the area under the corresponding curve of Fig. 1. The area under the dashed curve in Fig. 1 corresponds to the density of the cold plasma.) By that instant of time, owing to the large duration of the process of the particle interaction with the oscillations, and owing to the increase of the growth rate of the oscillations (due to the increase of the particle density), the oscillation energy density turns out to be greatly increased (see Figs. 2 and 3). The reaction of the oscillations on the dynamics of the injected particles leads to an appreciable distortion of their energy spectrum (Fig. 1). The most important circumstance connected with this distortion is the slowing down of the fast ions. The same effect is typical also of the usual quasi-linear relaxation—in the absence of a source.

The curves of Fig. 1, which correspond to $\bar{\tau} = 20$ and 28, also point to a tendency to formation of a plateau on the distribution function, $\partial F/\partial\mathscr{E} \to 0$.[6] The formation of the plateau, however, is made difficult because particles with $\mathscr{E} \approx 1$ are continuously injected and produce a flux of particles over the spectrum $\mathscr{E} \leqslant 1$.

One more important circumstance typical of the one-dimensional quasi-linear relaxation process is the ever increasing slope of the front of the distribution function in the low-velocity region,[6] owing to the smallness of the diffusion coefficient at small $\mathscr{E}$ (i.e., in the region of $\mathscr{E}$ where the number of resonant particles is small and the oscillation level is low). As seen from Fig. 1, this tendency is observed also in the continuous-injection case investigated by us.

Attention is also called in Fig. 1 to the narrow maximum, due to the earlier stage of the relaxation process, which appears on the distribution function. This maximum is formed in the region $\mathscr{E} > 1$, shifts gradually to the right, becomes narrower, and finally disappears. This phenomenon can be understood by comparing Figs. 1 and 2. According to Fig. 2, during the initial stage of the process there is a certain decrease of the oscillation energy with $\mathscr{E} > 1$. This is due to the fact that the derivative $\partial F/\partial \mathscr{E}$ is negative at $\mathscr{E} > 1$, see Fig. 1. Owing to the low noise level at $\mathscr{E} > 1$ the noise-induced diffusion of particles with $\mathscr{E} > 1$ is also very small, whereas in this region, just as in the region $\mathscr{E} < 1$, the particles are continuously injected. As a result, particles are accumulated at $\mathscr{E} > 1$, and this explains the appearance of the peak on the $F(\mathscr{E})$ curves of Fig. 1 at $\mathscr{E} > 1$. The peak produced in this manner has a positive derivative $\partial F/\partial \mathscr{E}$ in a certain interval of $\mathscr{E} > 1$ and is therefore unstable. As a result of the development of the instability and of the reaction of the growing noise with $\mathscr{E} > 1$, the peak begins to "become stationary" on the side of the positive derivative $\partial F/\partial \mathscr{E}$, advancing as it were the oscillation spectrum into the region $\mathscr{E} > 1$, after which the peak decreases to the level typical of the region $\mathscr{E} < 1$ and therefore plays no role in the succeeding stages of the relaxation process.

Since the number of fast ions is constant, as a result of their continuous injection, we are faced with the problem of determining the characteristic value of the growth rate of the oscillations. In this connection, Fig. 3 shows the time dependence of the maximum value of the spectral density of the oscillation energy. Figure 3a illustrates more graphically the initial stage, while Fig. 3b shows the general picture of the oscillation growth. It is seen from Fig. 3a that at $\bar{\tau} \leqslant 5$ the growth of the oscillation amplitudes is almost exponential with a rate approximately equal to $(2.6\,\bar{\tau})^{-1}$ (in the appropriate units). This result can be interpreted as a sort of mutual compensation of the increase of the growth rate, due to the continuous increase of the number of injected particles, and the quasi-linear decrease of the growth rate due to the reaction of the oscillations on the distribution function of these particles. With time, however, as the oscillation level increases, the quasi-linear effect becomes predominant, so that the growth rate of the noise slows down. According to Fig. 3b, at $\bar{\tau} > 12$ the oscillation energy density increases only linearly, and not exponentially as at $\bar{\tau} \lesssim 5$.

4. DISCUSSION OF RESULTS

From the analysis in Sec. 3 it follows that during the course of the quasi-linear relaxation the front of the distribution function, shifting to the left over Fig. 1, reaches a value $\mathscr{E} = \frac{1}{2}$ after a time

$$t_{1/2} \approx 4\cdot 10^2 R/v_\parallel^0. \quad (23)$$

This characteristic quasi-linear-relaxation time under the typical thermonuclear conditions $v_\parallel^0 = 5\times 10^8$ cm/sec and $R \approx 5\times 10^2$ cm amounts to

$$t_{1/2} \approx 4\cdot 10^{-4} \text{ sec.} \quad (24)$$

In the absence of instability, the fast-ion relaxation, as is well known, is determined by the Coulomb collisions with the particles of the principal "cold" component of the plasma. At a plasma density $n \approx 5\times 10^{13}$, the characteristic Coulomb relaxation time of an ion with velocity $v^0 = 5\times 10^8$ cm/sec is of the order of [11]

$$t_{\text{Coul}} \approx 0.3 \text{ sec.} \quad (25)$$

It follows therefore from our calculations that the quasi-linear relaxation is fast by three orders of magnitude that the Coulomb relaxation:

$$t_{1/2}/t_{\text{Coul}} = 10^{-3}. \quad (26)$$

This means that the concept of the "two-component tokamak," [2] which is based on the notion that the fast-on relaxation is of the Coulomb type, must in general be reviewed. A more definite judgement of the consistency of this concept calls for a more complex investigation of the instabilities of the ion beams injected into the tokamak, and of the possibilities of suppressing these instabilities.

The foregoing results and their conclusions, however, must not weaken the interest in the very problem of injection of fast particles, since the energy of the injected particles, which goes over in the course of the relaxation into the energy of Alfven oscillations, can then be absorbed by the particles of the principal component of the plasma.[1)]

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Stimulated Raman scattering and the penetration of an electromagnetic wave into an inhomogeneous plasma

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We use the equations of non-linear electrodynamics to formulate relations that enable us to determine the noise level and the intensity of the pumping wave in an inhomogeneous medium under conditions of convective parametric instability. Using these relations we study the effect of stimulated Raman scattering on the penetration of an electromagnetic wave into a rarefied inhomogeneous plasma.

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INTRODUCTION

Parametric instabilities in an inhomogeneous plasma can be either drift (convective)[1–6] or absolute[7–15] instabilities. In the first case the departure of growing waves from the region where they interact resonantly with the pumping wave leads to the establishing of a stationary state. We can then determine not only the noise amplitude but also the way it varies with the intensity of the pumping wave in an inhomogeneous plasma. In the present paper we consider how a decay-type parametric instability which occurs in a rarefied plasma—stimulated Raman scattering (SRS)—affects the penetration of the pumping wave.

We use in the first section the phenomenological equations of non-linear electrodynamics to formulate the initial relations for a self-consistent determination for the noise level and the intensity of the pumping wave in an inhomogeneous medium. We determine in the second section, from the solution of the dispersion relation, the growth coefficients for SRS in a plasma. We obtain in the third section a non-linear differential equation to determine the pump wave intensity. We give in the fourth section the results of a numerical solution of that equation for a linear variation of the plasma density. In the conclusion we discuss the application of the results to a laser plasma.

We show in the paper that SRS has practically no effect on the propagation of the pumping wave when its intensity is small. When the intensity increases this effect becomes important. The distance over which the pumping wave can travel without practically changing its amplitude is proportional to the wavelength of the incident wave and to the plasma temperature, and inversely proportional to the intensity. At large distances the intensity of the pumping wave decreases with distance according to a hyperbolic law. We show that the nature of the wave penetration does not depend on the steepness of the increase in density when the plasma density varies according to a power law.

1. GENERAL RELATIONS

1. We start our considerations with the equation for the electrical field strength $\mathbf{E}$ in an arbitrary material medium[6]

$$\mathrm{rot\,rot}\,\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=0, \tag{1.1}$$

where the induction vector $\mathbf{D}$ is connected with the field $\mathbf{E}$ by a non-linear material equation which in the quadratic approximation has the form[17]

$$D_i(\mathbf{r},t)=\int d\mathbf{r}'\int_{-\infty}^{t}dt'\Big\{\varepsilon_{ij}(\mathbf{r},t;\mathbf{r}',t')E_j(\mathbf{r}',t') + \int d\mathbf{r}''\int_{-\infty}^{t'}dt''\,\varepsilon_{ijl}(\mathbf{r},t;\mathbf{r}',t';\mathbf{r}'',t'')E_j(\mathbf{r}',t')E_l(\mathbf{r}'',t'')\Big\}; \tag{1.2}$$

ε_{ij} and ε_{ijl} are, respectively, the linear and quadratic permittivity tensors of the medium.

We assume that the field in the medium is the sum of the field of a strong pumping wave $\mathbf{E}_0$ and of weaker fluctuation fields $\delta\mathbf{E}$:

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0(\mathbf{r},t)+\delta\mathbf{E}(\mathbf{r},t). \tag{1.3}$$

We substitute Eq. (1.3) into Eq. (1.2) and average over a statistical ensemble. As a result we get for the pumping wave the equation

$$(\mathrm{rot\,rot}\,\mathbf{E}_0)_i+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t}dt'\,d\mathbf{r}'\,\varepsilon_{ij}(\mathbf{r},t;\mathbf{r}',t')E_{0j}(\mathbf{r}',t')=-\frac{1}{c^2}\frac{\partial^2\tilde{D}_i}{\partial t^2}, \tag{1.4}$$

where the vector $\tilde{\mathbf{D}}$ determines the non-linear effect of the fluctuation fields on the pumping wave:

