the estimates obtained that for $\tau = 10^{-9}$ sec generation by beats requires intensities four–five orders of magnitude higher than the intensities required for generation by absorption, i.e., 10^{11}–10^{12} W/cm^2.

It is interesting to compare the characteristic intensities of resonance and nonresonance pumps. This can be done for broad-band pumping, using the result of the paper[4] for J^* in the case of resonance pumping by beats:

$$\frac{J^*\,(\text{resonance beats})}{J^*\,(\text{nonresonance beats})} \approx \frac{\Gamma_0}{\Omega_0}. \tag{7.5}$$

The analogous ratio for absorption is, as follows from (3.23), simply the ratio of the coefficients of two-phonon and one-phonon resonance absorption:

$$\frac{J^*\,(\text{resonance absorption})}{J^*\,(\text{nonresonance absorption})} \approx \left(\frac{\Gamma_0}{\Omega_0}\right)^2, \tag{7.6}$$

i.e., the resonance and nonresonance pump efficiencies differ less in generation by beats than in generation by absorption.

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Polaritons in inhomogeneous crystals

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An exact solution of the problem of the motion of a polariton in an inhomogeneous crystal is found with allowance for spatial dispersion, if the energy of the bottom of the exciton band (optical phonons) linearly depends on the coordinates. Outside the region of the turning point for excitons, the polaritons represent either excitons or electromagnetic waves. In the region of the turning point, where the effects of the mixing of exciton and electromagnetic waves are large, transformation of some waves into others takes place, where the efficiency of such a conversion process depends on the degree of inhomogeneity. The properties of electromagnetic waves upon propagation in the directions of increasing and decreasing energy of the bottom of the band are found to be different, a reflected wave being present in the first case and absent in the second. The difference is also manifest in the dependence of the fraction of electromagnetic wave energy transformed into exciton energy on the direction of motion. The non-equivalence of opposite directions becomes very pronounced in the case of a gradual inhomogeneity; for one direction of propagation the electromagnetic wave is completely reflected, and for the other direction it is completely transformed into excitons (optical phonons). The latter process may be utilized for the generation of a coherent exciton beam. The physical nature of the phenomenon is explained and criteria are discussed for the applicability of the results to inhomogeneities of another type.

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The problem of determining the spectrum of polaritons in an inhomogeneous crystal has a number of characteristic features distinguishing it from the corresponding problem in a homogeneous medium. From a macroscopic point of view, the distinctive feature consists in the fact that in the presence of spatial dispersion it is impossible to characterize an inhomogeneous medium by a dielectric constant which depends on the wave vector. However, if the effects of spatial dispersion are unimportant, one can introduce a dielectric constant that depends on the frequency and coordinates, but has in the polariton part of the spectrum singularities as a function of the coordinates. From a microscopic point of view, polaritons represent mixed states of the electromagnetic field and excitons or phonons. In an inhomogeneous crystal the role of the effects of mixing changes upon movement of the polariton from one region of space to another. As we shall see, this may strongly influence the motion of the quasiparticles and may also lead to their mutual interconversion.

The exact solution of the problem of the motion of polaritons in a medium with a specific (depending linearly on the coordinates) inhomogeneity is found in the present article. The question of the applicability of these results to inhomogeneities of another type is also discussed.

The investigation is carried out for the case of polaritons which are formed when electormagnetic waves and excitons are mixed (i.e., for light-excitons[1]). However, the results can be applied to an investigation of the interaction of electromagnetic waves with phonons and magnons in inhomogeneous crystals.

1. FORMULATION OF THE PROBLEM AND GENERAL SOLUTION

In order to determine the spectrum of mixed electromagnetic and exciton waves in an inhomogeneous crystal, it is necessary to find a consistent solution of the time-dependent Schrödinger equation and of Maxwell's equations. An electromagnetic wave of frequency ω excites an exciton, an oscillating dipole moment appears which in turn influences the propagation of the electromagnetic wave. We shall assume that the frequency ω is close to the excitation frequency of some exciton band, and the influence of the other bands on the propagation of electromagnetic waves can be neglected. Let the inhomogeneity of the system consist in the fact that the variation of the energy of the bottom of the exciton band is a linear function of the single variable z. The z dependence of the exciton effective mass can be neglected if "smooth" states are investigated (a slowly varying field over the period of the lattice). We shall investigate polariton waves propagating along the inhomogeneity, i.e., along the z axis.

In the cases indicated above the fundamental system can be obtained in the same way, for example, as[2], where the propagation of electromagnetic waves in an impurity crystal was investigated. The solution of the Schrödinger equation and Maxwell's equations reduces to finding the solution of the following system of equations:

$$(\hbar\omega-\mathscr{E}_0-D'z)\Psi(z)+\frac{\hbar^2}{2m^*}\frac{d^2\Psi}{dz^2}=-\frac{pE(z)}{v_0^{1/2}}, \tag{1}$$

$$\frac{d^2E}{dz^2}+\frac{\omega^2}{c^2}\varepsilon_0E(z)=-\frac{4\pi p^*\omega^2}{v_0^{1/2}c^2}\Psi(z), \tag{2}$$

where $E(z)$ is the electric field strength of the electromagneticwave and $\Psi(z)$ is the probability amplitude for the excitation of excitons, p is the dipole moment, referred to an elementary cell, of the transition to the excited state, ε_0 denotes the contribution to the dielectric constant from the other states of the crystal which are far removed from the exciton band under consideration; we assume that ε_0 does not depend on ω and z; $\mathscr{E}_0$ denotes the excitation energy of the bottom of the exciton band for $z=0$, D' is a coefficient characterizing the inhomogeneity, and v_0 denotes the volume of an elementary cell.

Equations (1) and (2) are valid for Frenkel excitons as well as for excitons in the Wannier–Mott model.

Let us solve the system of Eqs. (1) and (2) by Laplace's method

$$\Psi(z)=\int_C\Psi(k)e^{ikz}dk,\qquad E(z)=\int_C E(k)e^{ikz}dk. \tag{3}$$

The condition

$$\Psi(k_2)e^{ik_2z}-\Psi(k_1)e^{ik_1z}=0 \tag{4}$$

must be satisfied at the ends k_1 and k_2 of the contour C. The specific choice of the contour depends on the process under investigation. This choice will be made below.

Substitution of relations (3) into Eqs. (1) and (2) leads to the following solution:

$$\Psi(z)=a\int_C f(k)\,dk, \tag{5}$$

where a is a constant,

$$f(k)=\exp\left\{ik\tilde{z}+i\beta\frac{k^3}{3}\right\}\left(\frac{k+k_0}{k-k_0}\right)^{i\gamma}, \tag{6}$$

$$\tilde{z}=z-z_0,\quad z_0=(\hbar\omega-\mathscr{E}_0)/D',\quad \beta=\hbar^2/2m^*D', \tag{7}$$

$$k_0=\omega\varepsilon_0^{1/2}/c,\qquad \gamma=2\pi\omega^2|p|^2/c^2v_0k_0D'. \tag{8}$$

At the point $\tilde{z}=0(z=z_0)$ the energy $\hbar\omega$ is equal to the excitation energy of the bottom of the exciton band.

We shall call this point the turning point although this designation is arbitrary. The true turning point for excitons would be the point $\tilde{z}=0$ in the absence of the interaction of excitons with electromagnetic waves.

Condition (4) can be satisfied if the ends of the contour C depart at infinity into a region where $f(k)\to 0$. Investigation of the integrand expression (6) indicates that the contour may depart at infinity into the following sectors of the complex k plane:

for $\beta>0$

$$0<\arg k<\pi/3,\quad 2\pi/3<\arg k<\pi,\quad 4\pi/3<\arg k<5\pi/3;$$

for $\beta<0$

$$\pi/3<\arg k<2\pi/3,\quad \pi<\arg k<4\pi/3,\quad 5\pi/3<\arg k<2\pi.$$

The function $f(k)$ has two branch points, $k=\pm k_0$. A cut exists between the points $-k_0$ and k_0, and crossing this cut leads the contour onto another sheet of the Riemann surface.

At large distances from the turning points ($\tilde{z}\to\pm\infty$) the regions near the saddle points $k_{1,2}=\pm(\tilde{z}/\beta)^{1/2}$ and the regions near the branch points $k=\pm k_0$ give the major

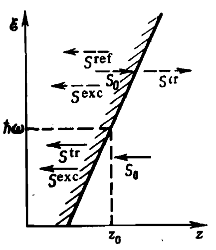

FIG. 1. Dependence of the energy of the bottom of the exciton band on the coordinate. The arrows indicate the directions of the energy fluxes of the electromagnetic waves and excitons; the solid arrows are for case A and the dotted arrows—for case B.

contribution to the integral (5). In the first case the contributions to the integral correspond to exciton waves, and in the second case they correspond to electromagnetic waves—waves in an inhomogeneous medium, but still not bound among themselves. The latter property is associated with the fact that at large distances from the turning point the frequency ω differs markedly from the excitation frequency of the bottom of the exciton band. Then the effects of mixing are negligible and the waves become independent. However, the relationships between the amplitudes of the exciton and electromagnetic waves are different as $\tilde{z} \to -\infty$ and $\tilde{z} \to \infty$.

Thus, the individual polariton waves are not independent in the presence of an ihhomogeneity: a conversion of one kind of wave into the other kind takes place in the vicinity of the turning point. In order to investigate the energy relationships of these conversions, it is sufficient to investigate the asymptotic behavior of the solution (5) as $\tilde{z} \to \pm\infty$. By choosing different contours C, we investigate the most important processes in practice.

2. INTERCONVERSION OF POLARITON WAVES AT AN INHOMOGENEITY

Let the slope of the inhomogeneity be such that $D' > 0$.

A. An electromagnetic wave is incident from the right on the turning point, the effective mass of the exciton is positive, $\beta > 0$ (Fig. 1, the solid arrows). The integration contour C is shown in Fig. 2. For $\tilde{z} > 0$ the integrand rapidly decreases with removal of a point on the contour upwards from the real axis. A region of order $k \sim 1/\tilde{z}$ gives a contribution to the integral. Let us make the substitution $i(k+k_0)\tilde{z} = t$ in the integrand, and let us expand the integrand function in a series in powers of the small parameter $t/k_0\tilde{z}$. As a result we obtain the following asymptotic expression for $\Psi(z)$ as $\tilde{z} \to +\infty$:

$$\Psi(z) \approx \frac{2\pi a}{\tilde{z}\Gamma(-i\gamma)}(2k_0\tilde{z})^{-i\gamma} \times \exp\left\{-ik_0\tilde{z} - i\beta\frac{k_0^3}{3} - \frac{\gamma\pi}{2}\right\}\left[1 - \left(\beta k_0^2 + \frac{\gamma}{2k_0}\right)\frac{1+i\gamma}{\tilde{z}}\right], \tag{9}$$

where $\Gamma(-i\gamma)$ is the gamma function. Relation (9) describes an electromagnetic wave which is incoming at the turning point.

Now let us consider the region $\tilde{z} < 0$. Let us deform the contour C in such a way that it approaches the point $-k_0$ from below, since for $\tilde{z} < 0$ the integrand of expression (5) decreases (Fig. 2) in the lower half-plane of the complex k plane. In order to evaluate the integral in this case, it is also necessary to take into account the contribution from the saddle point $k_1 = -(|\tilde{z}|/\beta)^{1/2}$, the contribution being counted twice, on two sheets of the Riemann surface. Thus, it is necessary to consider the region near $k = -k_0$ and $k = k_1$ in order to evaluate the integral for $\tilde{z} \to -\infty$. This corresponds to the fact that two polariton waves may exist in the present region: exciton and electromagnetic.

The contribution to the integral coming from the immediate vicinity of the point k_0 is calculated in the same way as for $\tilde{z} > 0$, but the contribution from the region $k \approx k_1$ is calculated by the method of steepest descents. Here it is necessary to take the condition $|k_1| \gg k_0$ as $|\tilde{z}| \to \infty$ into consideration. After integration we obtain

$$\Psi(z) \approx \Psi^{el}(z) + \Psi^{exc}(z), \quad \tilde{z} \to -\infty; \tag{10}$$

$$\Psi^{el}(z) = \frac{2\pi a}{\tilde{z}\Gamma(-i\gamma)}(-2k_0\tilde{z})^{-i\gamma}\exp\left\{-ik_0\tilde{z} - i\beta\frac{k_0^3}{3} - \frac{3\gamma\pi}{2}\right\}\left[1 + O\left(\frac{1}{\tilde{z}}\right)\right], \tag{11}$$

$$\Psi^{exc}(z) \approx \frac{a\sqrt{\pi}}{(-\tilde{z}\beta)^{1/4}}\exp\left\{-i\frac{2}{3}\tilde{z}\left(\frac{\tilde{z}}{\beta}\right)^{1/2} - \frac{i\pi}{4}\right\}(1 - e^{-2\pi\gamma})\left(1 - \frac{2i\gamma k_0\beta^{1/2}}{|\tilde{z}|^{1/2}}\right), \tag{12}$$

where $\Psi^{el}(z)$ is the probability amplitude for the excitation of molecules, corresponding to an electromagnetic wave, and $\Psi^{exc}(z)$ is the analogous probability amplitude corresponding to an exciton wave.

From relations (9)–(12) follow criteria on $\tilde{z}$, for which one can confine attention to the maximal terms

$$|\tilde{z}|^{1/2} \gg 2\gamma k_0 \beta^{1/2}, \tag{13}$$

$$\tilde{z} \gg |1 + i\gamma|(\beta k_0^2 + \gamma/2k_0). \tag{14}$$

One can neglect retardation in the investigation of exciton waves upon fulfillment of the criterion (13). The dispersion law of the electromagnetic waves is the same as in a homogeneous medium upon fulfillment of the criterion (14). These criteria are more stringent, the weaker the inhomogeneity.

Let us find the electric field strength from Eq. (1). Substituting the value of Ψ from formulas (9) and (10) into Eq. (1), we obtain the following result as $\tilde{z} \to \infty$

$$E(z) \approx \frac{D'\sqrt{v_0}}{p}\frac{2\pi a}{\Gamma(-i\gamma)}(2k_0\tilde{z})^{-i\gamma}\exp\left\{-ik_0\tilde{z} - i\beta\frac{k_0^3}{3} - \frac{\gamma\pi}{2}\right\}, \tag{15}$$

and for $\tilde{z} \to -\infty$ we find

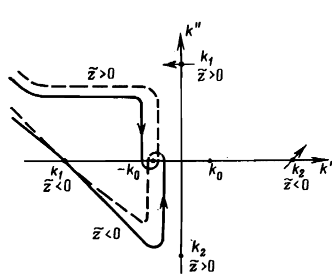

FIG. 2. Integration contours of the integral (5) associated with investigation of case A.

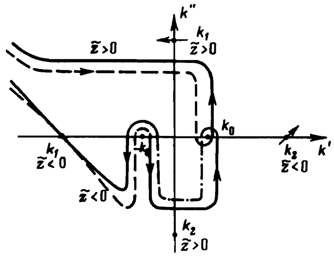

FIG. 3. Contours of integration of the integral (5) for investigation of case B. The solid, dashed, and dot-dashed lines pass on different sheets of the Riemann surface.

$$E(z) \approx \frac{D'\sqrt{v_0}}{p}\frac{2\pi a}{\Gamma(-i\gamma)}(-2k_0\tilde{z})^{-i\gamma}\exp\left\{-ik_0\tilde{z}-i\beta\frac{k_0^3}{3}-\frac{3\gamma\pi}{2}\right\}. \tag{16}$$

The electric field, accompanying the exciton wave, rapidly decreases with increasing distance from the turning point ($E^{\text{exc}} \sim |\tilde{z}|^{-5/4}$) and one can neglect it.

Let us examine the energy relationships. In a medium with spatial dispersion, the energy flux consists of two terms: a flux of electromagnetic energy and a flux of "mechanical" energy, carried by the particles of the medium.[3] The latter is related to the motion of excitons. For regions of space far away from the turnign point, the exciton and electromagnetic waves are independent, and their energy fluxes can be calculated by the usual method.

The flux of incident electromagnetic energy is given by

$$S_0=c\sqrt{\varepsilon_0}|E|^2/2\pi=2\pi D'\omega|a|^2(1-e^{-2\pi\gamma}). \tag{17}$$

From formula (16) we obtain the result

$$S_{\text{tr}}^{\text{el}}=S_0e^{-2\pi\gamma} \tag{18}$$

for the flux of electromagnetic energy in the transmitted wave. Let us calculate the energy flux of the exciton wave according to the formula

$$S^{\text{exc}}=(\hbar\omega/2m^*)(\Psi^* p_{\text{exc}}\Psi-\Psi p_{\text{exc}}\Psi^*), \tag{19}$$

where p_{exc} is the momentum operator of the excitons.

Substituting the value Ψ^{exc} from formula (12) into formula (19), we find

$$S^{\text{exc}}=S_0(1-e^{-2\pi\gamma}). \tag{20}$$

One can easily show that the exciton wave's flux of electromagnetic energy and, on the other hand, the electromagnetic wave's flux of mechanical energy fall off rapidly with increasing $|\tilde{z}|$, and these fluxes can be neglected.

Thus, after passage through the turning point the electromagnetic wave with flux S_0 which is incident on the inhomogeneity is decomposed into two waves—the electromagnetic wave (18) and the exciton wave (20). Let us analyze the obtained results in terms of their dependence on the parameter γ. For a physical analysis of the results it is convenient to represent expression (8) for γ in the following form:

$$2\pi\gamma=2\pi^2\mathscr{E}_p/D'\lambda, \tag{21}$$

where $\mathscr{E}_p=4\pi|p|^2/v_0\mathscr{E}_0$ denotes the energy of the longitudinally-transverse splitting for excitons, $D'\lambda$ is the change over a distance equal to the wavelength of the electromagnetic wave in the energy of the bottom of the band due to the inhomogeneity.

From formulas (18) and (20) it follows that if $2\pi\gamma \ll 1$, then

$$S_{\text{tr}}^{\text{el}}=(1-2\pi\gamma)S_0, \quad S^{\text{exc}}=2\pi\gamma S_0. \tag{22}$$

In this case the energy losses by the electromagnetic waves on excitation of excitons are small. In the other limiting case $2\pi\gamma \gtrsim 1$

$$S_{\text{tr}}^{\text{el}}\approx 0, \quad S^{\text{exc}}\approx S_0. \tag{23}$$

Thus, a complete conversion of the electromagnetic wave into an exciton wave takes place in the presence of a gradual inhomogeneity. It follows from relationships (20) and (21) that complete conversion will occur if the change in energy of the bottom of the band over a wavelength is smaller than $\varepsilon_p/10$.

B. The electromagnetic wave is incident on the turning point from the left, $\beta>0$ (Fig. 1, the dotted arrows). The integration contour for this case is shown in Fig. 3. As $\tilde{z}\to\infty$ the contour encompasses the point k_0, and only one wave exists, moving from the turning point to the right. As $\tilde{z}\to-\infty$ it is necessary to deform the contour such that it will pass below the point k_0. As a result the contour will pass through the saddle point k_1 and will twice, on different sheets of the Riemann surface, encompass the point $-k_0$. The integral over this part of the contour describes a wave, moving from the turning point to the left, i.e., a reflected wave.

Omitting the detailed calculations, let us write down the result for the energy fluxes of the reflected $S_{\text{ref}}^{\text{el}}$, transmitted $S_{\text{tr}}^{\text{el}}$, and exciton S^{exc} waves as a function of the incident flux S_0.

As $\tilde{z}\to-\infty$

$$S_{\text{ref}}^{\text{el}}=-(1-e^{-2\pi\gamma})^2S_0, \quad S^{\text{exc}}=-e^{-2\pi\gamma}(1-e^{-2\pi\gamma})S_0, \tag{24}$$

but for $\tilde{z}\to\infty$

$$S_{\text{tr}}^{\text{el}}=e^{-2\pi\gamma}S_0. \tag{25}$$

From relations (24) and (25) it follows that upon incidence on an inhomogeneity in one case the electromagnetic waves are not reflected at all, and in the other case they are partially reflected from the turning point. For $2\pi\gamma \ll 1$ the intensity of the reflected wave is small, and the intensities of the exciton and transmitted waves are determined by formulas (22). On the other hand, for $2\pi\gamma \gtrsim 1$ we have

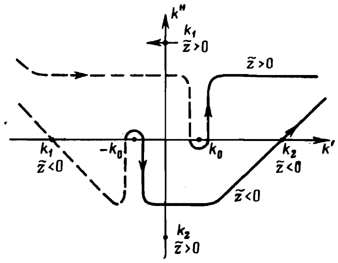

FIG. 4. Integration contours of the integral for investigation of case C.

$$S^{\mathrm{el}}_{\mathrm{ref}} \approx -S_0, \quad S^{\mathrm{exc}} \approx S^{\mathrm{el}}_{\mathrm{tr}} \approx 0. \tag{26}$$

in contrast to expressions (23). In this case the electromagnetic waves are completely reflected from the turning point.

Thus, the inhomogeneity behaves as a membrane which transmits electromagnetic energy completely, converting it into the energy of excitons when the motion is in the direction of decreasing energy of the bottom of the exciton band, and completely reflecting it when the motion of the electromagnetic wave is in the direction of increasing energy of the bottom of the exciton band.

C. An exciton wave is incident on the turning point from the left, $\beta > 0$. In the neighborhood of the turning point the exciton wave will be partially converted into electromagnetic waves. Such a problem may arise in connection with investigation of the problem of excitonic luminescence. The integration contour for this case is shown in Fig. 4. In the neighborhood of the turning point the energy flux S^{exc} of the excitons is converted into the energy flux $S^{\mathrm{exc}}_{\mathrm{ref}}$ of the reflected excitons and the energy fluxes $S^{\mathrm{el}}_{\mathrm{ref}}$ and $S^{\mathrm{el}}_{\mathrm{tr}}$ of the electromagnetic waves which are moving in different directions. Computation of the energy fluxes leads to the following results:

as $\tilde z \to -\infty$

$$S^{\mathrm{exc}}_{\mathrm{ref}} = -e^{-4\pi\gamma} S_0^{\mathrm{exc}}, \quad S^{\mathrm{el}}_{\mathrm{ref}} = -e^{-2\pi\gamma}(1-e^{-2\pi\gamma}) S_0^{\mathrm{exc}}; \tag{27}$$

as $\tilde z \to \infty$

$$S^{\mathrm{el}}_{\mathrm{tr}} = (1-e^{-2\pi\gamma}) S_0^{\mathrm{exc}}. \tag{28}$$

For $2\pi\gamma \ll 1$ (a "steep" inhomogeneity) the excitons are completely reflected; for $2\pi\gamma \gtrsim 1$ (a gradual inhomogeneity) the excitons are completely de-excited, i.e., transformed into electromagnetic waves propagating in the direction of the exciton's motion.

D. The case of a negative effective mass, $D' > 0$, $\beta < 0$. For $m^* < 0$ the exciton states are localized to the right of the turning point z_0 (Fig. 1). In the neighborhood of the turning point, excitons with $k \approx k_1 < 0$ are excited by the electromagnetic waves, but they transport energy in the positive direction of the z axis (since the effective mass of the excitons is negative).

It turns out that the results obtained in cases A, B, and C (formulas (18), (20), (24), and (25)) remain valid, but the expressions describing exciton fluxes in the region $z < z_0$ should be carried over into the region $z > z_0$ and their signs should be reversed. The formulas for the flux of electromagnetic energy remain unchanged.

3. DISCUSSION

1. *Criterion for the applicability of the obtained results to other problems.* Let us assume that the coordinate dependence of the energy of the bottom of the exciton band is determined by the function $D(z)$. The turning point z_0 is found from the relation

$$\hbar\omega = \mathscr{E}_0 + D(z_0). \tag{29}$$

Let us expand $D(z)$ in a series in powers of $\tilde z$. One can neglect the quadratic terms if the condition

$$D''(z_0)\tilde z \ll D'(z_0). \tag{30}$$

is satisfied. If the criteria (13) and (14) for the applicability of the asymptotic expansion are valid in the region of fulfillment of condition (30), then in the region of interaction between the waves ($z \sim z_0$) one can regard the inhomogeneity as linear, and all of the formulas derived in the preceding section are valid. It is only necessary to assume that the quantity $D'(z_0)$ depends on ω according to formula (29). Thus, the criterion for the applicability of the obtained results to a potential of different shape has the form

$$D'(z_0)/D''(z_0) \gg (1+\gamma^2)^{1/4}(\beta k_0^2 + \gamma/2k_0). \tag{31}$$

2. *Physical interpretation of the effect of asymmetry in the properties of polaritons with respect to the direction of propagation.* For a very gradual inhomogeneity, one can assume that in each region of space the polariton branches have the same shape as in a homogeneous crystal (Fig. 5). As the polariton moves from the right to the turning point, the bottom of the exciton band descends slowly in the region of space corresponding to its position. On Fig. 5 this is equivalent to movement of the point, characterizing the state of the polariton, upward along curve 1. For a very slow variation of the energy of the bottom of the band there is a gradual transition of the polariton state along branch 1 from the region $\omega < \mathscr{E}_0/\hbar$, where branch 1 describes electromagnetic waves, to the region $\omega > \mathscr{E}_0/\hbar$, where branch 1 describes excitons. Thus, in the present case complete conversion of electromagnetic energy into excitonic energy takes place in connection with the polariton motions. Incidence of the electromagnetic

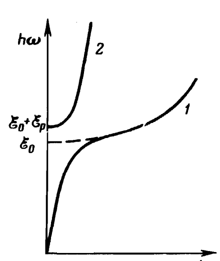

FIG. 5. The dispersion law for a polariton in a homogeneous crystal.

waves from the left onto the turning point is equivalent to downward motion along the dispersion curve (Fig. 5). In this case a gradual transition to branch 1 is impossible, and the electromagnetic waves are completely reflected from the turning point.

3. *Some remarks.* One can describe processes involving the scattering of excitons by lattice vibrations and defects by introducing an imaginary correction to the exciton's energy, $\mathscr{E}_0 \to \mathscr{E}_0 - i\Gamma/2$. In this connection all preceding formulas remain in force, but $\tilde{z}$ is a complex number and, according to formula (12), the wave function and the energy flux of the excitons are attenuated in space. In this case the transformation of the electromagnetic-wave energy into excitons is an irreversible process and should be observed as the absorption of electromagnetic waves.

The difference between the fluxes of the incident and transmitted electromagnetic waves does not depend on the direction of motion of the incident wave and is given by $(1 - e^{-2\pi\gamma}) S_0$. The presence of a reflected wave and the fraction of energy transformed into excitonic energy depend to a marked degree on the direction of motion of the incident wave. It should be noted that the asymmetry in the properties of polaritons with respect to the direction of propagation in an inhomogeneous crystal is not related to the effects of spatial dispersion, but is related to the singularity of the dielectric constant. In order to neglect spatial dispersion, one should set $m^* \to \infty$ in the initial formulas and one should set $\beta \to 0$ in the solution (5). In this case the integral (5) exists if $\mathscr{E}_0$ has an imaginary correction (otherwise it diverges for $\tilde{z} = 0$). After neglecting the effects of spatial dispersion, it is found that all of the formulas for electromagnetic energy fluxes remain unchanged (with the exception of case C, Sec. 2). Energy fluxes of the excitons are not present. Naturally, the law of energy conservation associated with the passage of the electromagnetic wave through the inhomogeneity is still not satisfied. This is explained by the fact that, with the introduction of an imaginary correction and by discarding the exciton fluxes, we arrive at an examination of a dissipative process. Allowing for spatial dispersion would explicitly allow us to trace where the transformed energy of the electromagnetic field goes (in the present case it goes into the excitation of excitons), and would allow us to investigate the inverse process (the emission of electromagnetic waves—case C, Sec. 2).

The investigated effects may be applied in order to generate a coherent beam of excitons. In the usual method for the excitation of excitons, involving the absorption and creation of a phonon, excitons moving in different directions are created. If the exciton's mean free path $l \sim \tilde{z}$ where $\tilde{z}$ satisfies condition (13), then in the presence of a gradual inhomogeneity light may be completely transformed into excitons which are moving in a single direction.

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