are relatively small. So the point is that in perpendicular pumping of spin-wave instability there are excited PSW with polar angle $\theta_k \approx 45°$, having a shorter lifetime and consequently a shorter range in comparison with the PSW excited in parallel pumping. In addition, the spectral density of the distribution of the spin waves, which determines the magnitude of the two-magnon scattering by the surface inhomogeneities, is minimal precisely for waves with $\theta_k = 45°$.[1] In view of these circumstances, the size effects described above were much weakened in this case. For example, whereas in parallel pumping of spin waves in a sample with $2r = 0.18$ mm there was no hard excitation at all, in the case of perpendicular pumping only the lower limit of the hard excitation of the PSW was shifted: for the sample with $2r = 0.52$ mm it was equal to 1300 Oe, as against 1500 Oe for the sample with $2r = 0.18$ mm, corresponding to excitation of spin waves with $k \approx 2 \times 10^5$ cm^{-1}. For waves with large k the scattering by the inhomogeneities, which increases in proportion to k, suppresses the hard excitation of the spin waves in the case of perpendicular pumping.

1)When a YIG sphere is magnetized along the easy axis, an instability appears in the lowest homogeneous mode of the low-frequency self-modulation of the magnetization, which has a zero gap in an unlimited according to the theory.[9] Allowance for the inhomogeneities causes the value of the gap to differ from zero.[11]

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On weakly nonlinear magnetoelastic oscillations in ferromagnets

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Weakly nonlinear magnetoelastic oscillations in ferromagnets, propagating in the direction of a uniform external magnetic field parallel to the magnetic anisotropy axis, are considered. A nonlinear parabolic equation that describes quasistationary disturbances of this type is found. It is shown that the magnetoelastic coupling has a qualitative effect on the modulation of a spin wave (modified by the magnetoelastic interaction) and also results in modulation of transverse sound (modified by the magnetoelastic interaction). It is further shown that the nonlinear excitation of a low-frequency modulated longitudinal sound wave by a high-frequency magnetization disturbance can take place under certain conditions.

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Nonlinear stationary magnetization disturbances in ferromagnets, and in particular, nonlinear periodic waves and solitons (solitary waves), were investigated in[1,2] but the time variation of the stationary profile was not discussed. On the other hand, considerable success has recently been achieved in investigating weakly nonlinear wave processes in media exhibiting spatial dispersion.[3,4] It was found that in a number of cases the evolution of the profile of a weakly nonlinear disturbance can be described by a nonlinear parabolic equation. This, in particular, is the case for magnetization disturbances in ferromagnets and antiferromagnets, which makes it possible to use a well developed mathematical apparatus in studying the latter.

In this paper we consider weakly nonlinear magnetoelastic oscillations propagating in a ferromagnet in the direction of a uniform external magnetic field parallel to the anisotropy axis. It is shown that a quasistationary disturbance of this type is also described by a nonlinear parabolic equation. On analyzing the coefficients of this equation we find that a relatively weak magnetoelastic coupling has a qualitative effect on the modulation of a spin wave (modified by the magnetoelastic interaction). This effect manifests itself both in changes in the characteristic parameters of the modulated wave, which are especially marked near the magnetoelastic resonance predicted by the linear theory, and in the appearance of a new region in which this wave can exist. We later consider the acoustic branch as modified by the magnetoelastic interaction and show that the nonlinearity associated with the magnetostrictive effects can lead to modulation of a transverse sound wave.

Finally, it is shown that when the group velocity of the high-frequency spin wave is close to the phase velocity of longitudinal sound, which (in the case under consideration) does not interact with the spin wave in the linear approximation, nonlinear excitation of a low-frequency modulated longitudinal sound wave by a high-frequency magnetization soliton can take place. This effect is analogous to the excitation of electroacoustic waves discussed in[4] (also see[5]).

In describing weakly nonlinear magnetoelastic oscillations we shall start with the equations proposed in[6] for treating parametric phenomena in ferromagnets. For the case of disturbances propagating in the direction of a uniform applied magnetic field (along the z axis) and depending on the variables z and t, these equations take the form

$$\begin{gathered}\frac{\partial M^+}{\partial t}=-ig\left(B_0-4\pi M_z-\frac{2b_1}{M_0}\frac{\partial u_z}{\partial z}\right)M^+-igb_2\frac{\partial u^+}{\partial z}\\+ig\alpha\left(M_z\frac{\partial^2 M^+}{\partial z^2}-M^+\frac{\partial^2 M_z}{\partial z^2}\right),\quad M_z=(M_0^2-M_\perp^2)^{1/2},\\ \frac{\partial^2 u^+}{\partial t^2}=v_t^2\frac{\partial^2 u^+}{\partial z_2}+\frac{b^2}{\rho M_0^2}\frac{\partial}{\partial z}M_zM^+,\\ \frac{\partial^2 u_z}{\partial t^2}=v_l^2\frac{\partial^2 u_z}{\partial z^2}+\frac{b_1}{\rho M_0^2}\frac{\partial Mz^2}{\partial z},\end{gathered}\tag{1}$$

in which M^+ and u^+ are the circular components of the magnetization and the displacement, respectively, g is the electron gyromagnetic ratio, α is the inhomogeneous exchange constant, b_1 and b_2 are the magnetostriction constants, v_l and v_t are the velocities of longitudinal and transverse sound, ρ is the density of the ferromagnet, and $B_0=H+4\pi M_z$ is a constant magnetic induction that appears when the magnetostatic equations are integrated under the assumptions adopted concerning the character of the disturbances.[1] In writing Eqs. (1) we have neglected the anisotropy field as compared with the external field and have taken the conservation of $|\mathbf{M}|$ into account.

It is known that linearized spin waves propagating along the magnetic anisotropy axis interact only with transverse sound.[7] In the following we shall therefore be interested in weakly nonlinear disturbances whose fast phase corresponds to one of the branches of the linear dispersion equation for interacting spin and transverse sound waves. Bearing this in mind, we express M^+ and u^+ in the form

$$M^+=M_\perp\exp\{i(kz-\omega t)+i\varphi\},\quad u^+=u_\perp\exp\{i(kz-\omega t)+i\psi\}\tag{2}$$

and assume that $M_\perp$, φ, $u_\perp$, ψ, M_z, and u_z are slowly varying functions of the coordinates and time. It is easy to derive equations for these functions from Eqs. (1), but since they are very cumbersome we shall not give them here. We shall be interested in weakly nonlinear solutions of these equations that represent disturbances propagating along the z axis with some velocity V.

Now we pass to the wave system by means of the transformation $Z=z-Vt$, $T=t$, introduce the formally small parameter ε associated with the smallness of the amplitudes, define the reduced coordinate $\zeta=\varepsilon Z$ and time $\tau=\varepsilon^2 t$, and expand the slowly varying functions in power series in ε with slowly varying coefficients:

$$O(\zeta,\tau)=\sum_{n=0}^{\infty}\varepsilon^n O^{(n)}(\zeta,\tau).\tag{3}$$

On substituting expansion (3) into the equations for the slowly varying functions and equating the coefficients of different powers of ε to zero, we find that ω and k satisfy a linear dispersion equation for coupled spin and transverse sound waves[7]:

$$(\omega_\alpha-\omega_e(k))(\omega_\alpha^2-\omega_t^2(k))-gb_2^2k^2/\rho M_0=0,\tag{4}$$

where

$$\omega_e=g(B_0-4\pi M_0+\alpha M_0k^2),\quad \omega_t=kv_t,$$

V coincides with the group velocity $\partial\omega/\partial k$ of these waves, $u^{(1)}$ and $M^{(1)}$ are connected by the relation

$$\begin{gathered}u_\perp^{(1)}=s_\alpha M_\perp^{(1)},\\ s_\alpha=\frac{\omega_e-\omega_\alpha}{gb_2k\sin(\psi^{(0)}-\varphi^{(0)})},\quad \sin(\psi^{(0)}-\varphi^{(0)})=\pm1\end{gathered}\tag{5}$$

(the ambiguous sign is to be chosen so that $s>0$), and the longitudinal displacement is determined by the magnetization amplitude in accordance with the equation

$$\frac{\partial u_z^{(1)}}{\partial\zeta}=-\frac{b_1}{\rho M_0^2(V^2-v_l^2)}(M_\perp^{(1)})^2.\tag{6}$$

Finally, $M_\perp^{(1)}$ and $\varphi^{(0)}$ are determined by the closed set of equations

$$\begin{gathered}\frac{\partial M_\perp^{(1)}}{\partial\tau}=-\frac{\partial^2\omega}{\partial k^2}\frac{\partial}{\partial\zeta}\left((M_\perp^{(1)})^2\frac{\partial\varphi^{(0)}}{\partial\zeta}\right),\\ 2M_\perp^{(1)}\frac{\partial\varphi^{(0)}}{\partial\tau}=-\frac{\partial^2\omega}{\partial k^2}M_\perp^{(1)}\left(\frac{\partial\varphi^{(0)}}{\partial\zeta}\right)^2+\frac{\partial^2\omega}{\partial k^2}\frac{\partial^2M_\perp^{(1)}}{\partial\zeta^2}+\Delta(M_\perp^{(1)})^3,\end{gathered}\tag{7}$$

in which

$$\begin{gathered}\frac{\partial^2\omega}{\partial k^2}=-\frac{2(\omega-\omega_e)}{2\omega(\omega-\omega_e)+\omega^2-\omega_t^2}\left\{(V^2-v_t^2)-\frac{1}{2}\frac{\partial v_e}{\partial k}\frac{\omega^2-\omega_t^2}{\omega-\omega_e}\right.\\ \left.-\left(V-v_e-\frac{\omega-\omega_e}{k}\right)^2\frac{\omega^2-\omega_t^2}{(\omega-\omega_e)^2}\right\}\\ \Delta=\frac{g}{M_0}\frac{\omega^2-\omega_t^2}{\omega^2-\omega_t^2+2\omega(\omega-\omega_e)}\left\{\alpha k^2-4\pi+\frac{\omega-\omega_e}{gM_0}-\frac{4b_1^2}{\rho M_0^2(V^2-v_l^2)}\right\}\\ v_l=2g\alpha M_0k.\end{gathered}$$

We note that up to now we have considered only the case of nonresonant nonlinear generation of low-frequency longitudinal sound when the group velocity of the coupled magnetoelastic waves differs appreciably from the phase velocity v_1 of linear longitudinal sound ($V^2 - v_l^2 \sim \varepsilon^0$).

Equations (7) are well known in geometric optics, and the substitution $F = M_\perp^{(1)} e^{i\varphi^{(0)}}$ reduces them to the nonlinear parabolic equation[4]

$$2i\frac{\partial F}{\partial\tau}+\left(\frac{\partial^2\omega}{\partial k^2}\right)\frac{\partial^2 F}{\partial\zeta^2}+\Delta|F|^2F=0. \tag{7a}$$

The simplest solution of this equation is a nonlinear plane wave of constant amplitude whose frequency depends on the square of the amplitude. Because of Lighthill's condition,[8] this stationary solution is unstable against sufficiently long-wavelength perturbations for wave numbers satisfying the inequality

$$\nu=\frac{\Delta}{2}\left(\frac{\partial^2\omega}{\partial k^2}\right)^{-1}>0. \tag{8}$$

In the region in which a nonlinear wave of constant amplitude is unstable, there exist, generally speaking, stationary solutions of the type

$$\psi^{(0)}={}^1/_2C\Delta\tau,\quad M_\perp^{(1)}=M_\perp^{(1)}(\zeta) \tag{9}$$

in which the constant C and amplitude $M_\perp^{(1)}(\zeta)$ depend on the boundary conditions. In particular, in the case of a solitary wave,[1)] for which

$$\begin{gathered}M_\perp^{(1)}(\pm\infty)=0,\quad (\partial M_\perp^{(1)}/\partial\zeta)_{\zeta\to\pm\infty}=0,\quad M_\perp^{(1)}(\zeta_0)=M_{max},\\ (\partial M_\perp^{(1)}/\partial\zeta)_{\zeta=\zeta_0}=0\end{gathered} \tag{10}$$

(M_{max} is the maximum value of the transverse component of the magnetization, which is reached at some point ζ_0), we have

$$M_\perp^{(1)}(\zeta)=M_{max}\operatorname{sech}M_{max}\sqrt{\nu}(\zeta-\zeta_0). \tag{11}$$

Above we have presented a few well known results of the analysis of Eq. (7a). In the following we shall make these results more specific for application to the case of weakly nonlinear magnetoelastic oscillations under consideration. When the time comes for numerical estimates we shall use the following values:

$$\begin{gathered}M_0\approx10^3\ \mathrm{G},\quad H_0\approx10^4\,\mathrm{Oe},\quad \alpha\approx10^{-12}\ \mathrm{cm}^2,\quad v_t\approx3\cdot10^5\ \mathrm{cm/sec},\\ v_l\approx5\cdot10^5\ \mathrm{cm/sec},\quad |g|=2\cdot10^7\ \mathrm{Oe}^{-1}\cdot\mathrm{sec}^{-1},\quad b_1\approx b_2\approx10^7\ \mathrm{G}^2,\quad \rho\approx10\ \mathrm{g/cm}^3.\end{gathered}$$

It is known from the linear theroy that the effect of the magnetoelastic interaction on the dispersion becomes stronger on approaching the magnetoelastic resonance defined by the condition

$$\omega_e=-\omega_t=\omega_r<0. \tag{12}$$

The wave numbers corresponding to this condition are given by the formulas

$$k_r^{(1,2)}=\frac{v_t}{2|g|\alpha M_0}\left[1\mp\left(1-\frac{4|g|\alpha M_0|\omega_0|}{v_t^2}\right)^{1/2}\right]\qquad \omega_0=gH_0, \tag{13}$$

which yield $k_r^{(1)}\approx10^6\ \mathrm{cm}^{-1}$ and $k_r^{(2)}\approx10^7\ \mathrm{cm}^{-1}$.

The effect of the magnetoelastic coupling is negligible in the wave-number region in which $|\omega_e|$ and ω_t differ in order of magnitude. In the following we shall therefore always assume that $|\omega_e|\sim\omega_t$. We shall first consider the wave-number region

$$f=\mu\frac{|g|M_0\omega_t}{2(|\omega_e|-\omega_t)^2}\ll1,\quad \mu=\frac{b_2^2}{\rho v_t^2M_0^2}, \tag{14}$$

which we shall call the extraresonance region. Here the frequency separation characterizing the deviation of the dispersion of proper spin and elastic waves is proportional to the small parameter μ, and in the case of a spin wave it has the form[7]

$$\delta\omega=\omega+|\omega_e|\approx-\mu|g|M_0\omega_t^2/(\omega_e^2-\omega_t^2). \tag{15}$$

From this it follows that the coefficient Δ virtually coincides with the value

$$\Delta=-|g|M_0^{-1}(\alpha k^2-4\pi), \tag{16}$$

appropriate for a proper spin wave. As regards the derivative of the group velocity, when $||\omega_e|-\omega_t|\sim\omega_t$, it, too, is given by the expression

$$\partial^2\omega/\partial k^2=-2|g|\alpha M_0, \tag{17a}$$

corresponding to a proper spin wave, whereas closer to the magnetoelastic resonance we should have

$$\frac{\partial^2\omega}{\partial k^2}\approx-\frac{4f|\omega_e|}{\omega_e^2-\omega_t^2}(v_t+v_e)^2. \tag{17b}$$

Finally,

$$s\approx\mu\frac{\omega_t^2}{\omega_t^2-\omega_e^2}\frac{M_0}{kb_2}\sin^{-1}(\psi^{(0)}-\varphi^{(0)}), \tag{18}$$

Whence it follows that

$$\begin{gathered}\psi^{(0)}=\varphi^{(0)}+\pi/2,\quad \omega_t>|\omega_e|;\\ \psi^{(0)}=\varphi^{(0)}-\pi/2,\quad \omega_t<|\omega_e|.\end{gathered} \tag{19}$$

These results allow us to draw the following conclusions.

At sufficient distances from the resonance we have

$$\nu\approx\frac{1}{4M_0^2}\left(k^2-\frac{4\pi}{\alpha}\right), \tag{20}$$

from which it follows that the magnetoelastic coupling affects neither the width of the solitary wave nor the region in which such a wave can exist, that region being determined, as in the case of a proper spin wave, by the inequality

$$k^2>4\pi/\alpha. \tag{21}$$

This region usually lies to the right of the near magnetic resonance ($k=k_r^{(1)}$), and it does so lie for the numerical values that we have adopted. For wave numbers such that $|k^2-4\pi/\alpha|\sim4\pi/\alpha$ and under the assumption that $M_{\max}/M_0 \sim 10^{-1}$, the width of the soliton is $\Lambda \sim (\nu M_{\max}^2)^{-1/2} \sim 7\cdot 10^{-6}$ cm and decreases with increasing k, reaching the value $\Lambda \sim 10^{-6}$ cm at $k \sim k_r^{(2)}$. The part played by the magnetoelastic coupling is as follows: The solitary magnetization wave "accumulates" the disturbance of the elastic-displacement field. Then the perturbation of the transverse-displacement field is due to the linear coupling (5) and is therefore characterized by the same ζ dependence as the magnetization at the amplitude $u_{\perp\max} = sM_{\max}$. Numerical estimates show that $u_{\perp\max} \sim 10^{-12}$ cm for wave numbers such that $|k^2 - 4\pi/\alpha| \sim 4\pi/\alpha$ and decreases with increasing k, reaching a value $u_{\perp\max} \sim 10^{-13}$ cm at $k \sim k_r^{(2)}$. The perturbation of the longitudinal-displacement field is associated with the purely nonlinear magnetoelastic effect (6) and, as can easily be seen, leads to the ζ dependence

$$u_z^{(1)}(\zeta) = \frac{2b_t}{\rho(v_l^2 - V^2)}\left(k^2 - \frac{4\pi}{\alpha}\right)^{-1/2}\frac{M_{\max}}{M_0}\left\{1 - \operatorname{th}\frac{M_{\max}}{2M_0}\left(k^2 - \frac{4\pi}{\alpha}\right)^{1/2}|\zeta - \zeta_0|\right\}. \tag{22}$$

The amplitude of the longitudinal displacement is of the order of 10^{-12} cm and changes little within the range of wave numbers under consideration.

Close to the magnetic resonance, in the region where

$$|1 - |\omega_e|/\omega_t| \sim 10^{-2} - 10^{-3},$$

the magnetoelastic interaction leads to qualitative changes. In fact, in this region we have

$$\nu = \frac{|g|(\alpha k^2 - 4\pi)}{8fM_0 k_r v_t}\frac{\omega_e^2 - \omega_t^2}{(v_t + v_e)^2} \tag{23}$$

and the width of the soliton depends on the elastic characteristics of the medium. Further, it follows from Eq. (23) that in the vicinity of the near resonance, where $\alpha(k_r^{(1)})^2 - 4\pi < 0$, a soliton can appear for wave numbers $\omega_t > |\omega_e|$, whereas in the vicinity of the far resonance, where $\alpha(k_r^{(2)})^2 - 4\pi > 0$, a soliton can appear for wave numbers $\omega_t < |\omega_e|$. In other words, in both cases the region in which a soliton can arise lies to the right of the resonance value of k. We should note the appearance of a region in which there can exist a soliton corresponding to a spin wave modified by the magnetoelastic interaction for such values of k that $\Delta > 0$. This is due to the fact that $\partial^2\omega/\partial k^2$ for the modified spin wave is positive to the right of $k = k_r$. It is also important that the soliton broadens because

$$|\partial^2\omega/\partial k^2| \gg |\partial v_e/\partial k|$$

near the resonance. In particular, for the numerical values we are using we find $\Lambda^{(1)} \sim 10^{-4}$ cm and $\Lambda^{(2)} \sim 10^{-5}$ cm near the respective resonances $k_r^{(1)}$ and $k_r^{(2)}$. Finally, we note that on approaching the resonance, the intensity of the disturbance of the elastic-displacement field accompanying the magnetization disturbance increases strongly. Numerical estimates give

$$u_{\perp\max}^{(1)} \sim 10^{-10}\text{ cm},\quad u_{\perp\max}^{(2)} \sim 10^{-11}\text{ cm},\quad u_{z\max}^{(1)} \sim 10^{-9}\text{ cm},\quad u_{z\max}^{(2)} \sim 10^{-10}\text{ cm},$$

i.e., these quantities are one or two orders of magnitude larger than their values far from the resonance.

Now let us consider the modulation of the acoustic branch of the oscillations, whose frequency separation has the following form in the extraresonance region[2]:

$$\delta\omega = \omega + \omega_t \approx -\mu|g|M_0\omega_t/2(\omega_t - |\omega_e|). \tag{24}$$

It is convenient to use the function $F_u = u^{(1)}\exp(i\psi^{(0)})$, which satisfies a nonlinear parabolic equation with the coefficients $\partial^2\omega/\partial k^2$ and $\Delta_u = \Delta/s^2$. Since in the model under consideration the nonlinearity of the elasticity equations and the deviation from the linear dispersion law are due to magnetostriction, these coefficients owe their origin to the magnetoelastic interaction, and the leading terms in the expansion of these coefficients in the magnetoelastic interaction constant are given by the formulas

$$\Delta_u \approx -2|g|^2\rho\omega_t f^2\left[\alpha k^2 - 4\pi + \frac{\omega_t - |\omega_e|}{|g|M_0} - \frac{4b_t^2}{\rho M_0^2(V^2 - v_e^2)}\right], \tag{25}$$

$$\frac{\partial^2\omega}{\partial k^2} \approx -\frac{2v_t}{k}f\left[\frac{\omega_t}{\omega_t - |\omega_e|}\left(1 + \frac{v_e}{v_t}\right)^2 + 2\frac{\omega_t - |\omega_e|}{\omega_t} - 1 - \frac{3v_e}{2v_t}\right]. \tag{26}$$

From this we find that

$$\nu_u \approx f|g|^2\rho\omega_t(\omega_t - |\omega_e|)\frac{\alpha k^2 - 4\pi}{(v_t + v_e)^2}, \tag{27}$$

in the wave-number region $\omega_t \sim |\omega_e|$ of interest to us. Then a solitary wave corresponding to an acoustic wave modified by the magnetoelastic interaction will be described by Eq. (11) with $M_\perp$ replaced by $u_\perp$ and ν by ν_u. The amplitude of the magnetization disturbance will be related to $u_{\max}$ by the formula

$$M_{\max} = \frac{\sqrt{\mu\rho}|g|\omega_t M_0}{(|\omega_e| - \omega_t)\sin(\psi^{(0)} - \varphi^{(0)})}u_{\max}, \tag{28}$$

from which it follows that

$$\begin{aligned}&\psi^{(0)} = \varphi^{(0)} + \pi/2, \quad \omega_t < |\omega_e|;\\ &\psi^{(0)} = \varphi^{(0)} - \pi/2, \quad \omega_t > |\omega_e|.\end{aligned} \tag{29}$$

It follows from Eq. (27) that solitons can arise near both resonance values of the wave number. Moreover, in the vicinity of the near resonance the region in which a soliton can exist is determined by the condition $\omega_t < |\omega_e|$, and in the vicinity of the far resonance, by the condition $\omega_t > |\omega_e|$. In other words, in both cases the region in which a soliton can exist lies to the left of the resonance value of k. Using the numerical values adopted above together with the values $u_{\max}/a \sim 10^{-2}$ and $a \sim 10^{-7}$ cm (a is the lattice parameter), we obtain the respective values $\Lambda_u^{(1)} \sim 10^{-4}$ cm and $\Lambda_u^{(2)} \sim 10^{-5}$ cm for the width $\Lambda_u \sim (\nu_u u_{\max}^2)^{-1/2}$ of the soliton. Here the maximum deviation of the magnetization reaches $10^{-1}M_0$. Thus, we see that the widths of solitary waves corresponding to spin and elastic waves are of the same order near both magnetoelastic resonances. This is due to the fact that in the frequency region where $|\omega_e| \sim \omega_t$, these widths differ by the factor $\sqrt{f}$, which is of order unity close to the resonances.

In discussing modulated magnetoelastic waves in the intraresonance region we shall limit ourselves for simplicity to the exact resonance, i.e., we shall assume that the wave number is equal to one of the resonance values (13). Using the well known expression for the frequency separation at resonance,[7]

$$\delta\omega_r^{(\pm)}=\omega^{(\pm)}-\omega_r\approx\pm(^1/_2\mu|g|M_0|\omega_r|)^{1/2},\quad \omega_r=-k_r v_t, \tag{30}$$

and retaining only the leading terms, we obtain the following expressions for the coefficients in Eq. (7a):

$$\begin{gathered}\Delta^{(\pm)}\approx-\frac{|g|}{2M_0}\left[\alpha k^2-4\pi-\frac{\omega^{(\pm)}-\omega_r}{|g|M_0}-\frac{4b_1^2}{\rho M_0(V^2-v_t^2)}\right],\\ \frac{\partial^2\omega^{(\pm)}}{\partial k^2}\approx\frac{v_t^2}{4(\omega^{(\pm)}-\omega_r)}\left(1-\frac{4|g|\alpha M_0|\omega_0|}{v_t^2}\right).\end{gathered} \tag{31}$$

From this it follows that at $k=k_r^{(1)}$ we have

$$\nu_r^{(1)}\approx\frac{4\pi|g|}{M\,v_t^2}(\omega^{(\pm)}(k_r^{(1)})-\omega_r^{(1)}), \tag{32}$$

and a soliton can arise for the branch with positive deviation from resonance. In this case the width of the soliton will be $\Lambda^{(+)}(k_r^{(1)})\sim10^{-4}$ cm.

At $k=k_r^{(2)}$ we have

$$\nu_r^{(2)}\approx-\frac{|g|\alpha(k_r^{(2)})^2}{M_0(v_t+v_e)^2}[\omega^{\pm}(k_r^{(2)})-\omega_r^{(2)}], \tag{33}$$

and a soliton can arise for the branch with negative separation. In this case the width of the soliton is $\Lambda^{(-)}(k_r^{(2)})\sim10^{-5}$ cm.

In completing our discussion of the intraresonance region, we note that

$$s_r^{(\pm)}\approx\pm(2\rho M_0k_rv_t)^{-1/2}\sin^{-1}(\psi^{(0)}-\varphi^{(0)}). \tag{34}$$

Numerical estimates give the following values for the amplitudes of the transverse elastic displacement:

$$u_{\perp\,max}(k_r^{(1)})\sim10^{-9}\text{ cm},\quad u_{\perp max}(k_r^{(2)})\sim10^{-10}\text{ cm}.$$

Now let us consider the excitation of low-frequency longitudinal sound vibrations by high-frequency weakly nonlinear magnetization disturbances under such conditions that the phase velocity of longitudinal sound differs little from the group velocity of the high-frequency disturbance, i.e., such that

$$(V+v_l)/v_l=\pm\varepsilon^2,\quad |\varepsilon|\ll1. \tag{35}$$

We shall assume that condition (35) is satisfied far from the magnetoelastic resonances of the linear theory, where the magnetoelastic interaction between spin waves and transverse sound waves can be neglected. For the case under discussion, this is equivalent to assuming that $b_2=0$. Since $V=2g\alpha M_0k$ for the spin wave, it follows from condition (35) that the wave number of the high-frequency exciting disturbance will be $k\approx v_l/2|g|\alpha M_0\sim10^7$ cm^{-1}. Thus, it is a matter of a short-wavelength high-frequency magnetization disturbance, for which, however, a continuous description can still be used.

As the initial set of equations we can use Eqs. (1) with $b_2=0$. Using a coordinate system moving with the group velocity of the high-frequency disturbance and assuming a weak spatial dependence of the unknown quantities ($\zeta=\varepsilon Z$), we can put these equations for the steady-state case ($\partial/\partial\tau\equiv0$) in the form[3)]:

$$\begin{gathered}\varepsilon V\frac{\partial M_\perp}{\partial\zeta}=\varepsilon^2g\alpha M_zM_\perp\frac{\partial^2\varphi}{\partial\zeta^2}+2g\alpha\varepsilon M_z\frac{\partial M_\perp}{\partial\zeta}\left(k+\varepsilon\frac{\partial\varphi}{\partial\zeta}\right),\\ \left(\varepsilon V\frac{\partial\varphi}{\partial\zeta}+\omega\right)M_\perp=gH_0M_\perp-\frac{2gb_1}{M_0}M_\perp w+\varepsilon^2g\alpha M_\perp\frac{\partial^2M_z}{\partial\zeta^2}\\ -g\alpha\varepsilon^2M_z\frac{\partial^2M_\perp}{\partial\zeta^2}+g\alpha M_zM_\perp\left\{k^2+2\varepsilon k\frac{\partial\varphi}{\partial\zeta}+\varepsilon^2\left(\frac{\partial\varphi}{\partial\zeta}\right)^2\right\},\\ \pm2\varepsilon^2v_l^2\frac{\partial w}{\partial\zeta}=-\frac{b_1}{\rho M_0^2}\frac{\partial M_\perp^2}{\partial\zeta},\end{gathered} \tag{36}$$

where $w=\partial u_z/\partial z$.

From the third of Eqs. (36) we find that the expansions of w and $M_\perp$ in powers of ε begin with the second-order terms, and indeed

$$\frac{\partial w^{(2)}}{\partial\zeta}=\frac{b_1}{2v_l^2\rho M_0^2}\frac{\partial M_\perp^{(2)}}{\partial\zeta}. \tag{37}$$

Expanding the slowly-varying phase φ and the frequency ω in powers of ε, we obtain the correction to the frequency from the second of Eqs. (36),

$$\omega^{(2)}=-\frac{2gb_1}{M_0}w^{(2)}+g\alpha M_0\left(\frac{\partial\varphi^{(0)}}{\partial\zeta}\right)^2-\frac{g\alpha M_0}{M_\perp^{(2)}}\frac{\partial^2M_\perp^{(2)}}{\partial\zeta^2}, \tag{38}$$

while the first of Eqs. (36) gives

$$\frac{\partial}{\partial\zeta}\left(M_\perp^{(2)}\frac{\partial\varphi^{(0)}}{\partial\zeta}\right)=0. \tag{39}$$

Considering only weakly nonlinear disturbances of the solitary-wave type, we obtain

$$w^{(2)}=\frac{b_1}{2\rho v_l^2M_0^2}(M_\perp^{(2)})^2,\quad \frac{\partial\varphi^{(0)}}{\partial\zeta}=0 \tag{40}$$

from Eqs. (37) and (39). Then by substituting Eqs. (40) into Eq. (38), we can express the correction to the frequency in terms of the maximum value of the transverse component of the magnetization,

$$\omega^{(2)}=-\frac{gb_1^2}{2\rho v_l^2M_0^3}M_{max}^2>0, \tag{41}$$

and can also determine the spatial dependences of $M^{(2)}$ and $u_z^{(2)}$ in the form

$$\begin{gathered}M_\perp^{(2)}(\zeta)=M_{max}\operatorname{sech}M_{max}\left(\frac{b_1^2}{2\rho v_l^2\alpha M_0^4}\right)^{1/2}(\zeta-\zeta_0),\\ u_z^{(2)}(\zeta)=\left(\frac{M_{max}^2\alpha}{2\rho v_l^2}\right)^{1/2}\left\{1-\operatorname{th}M_{max}\left(\frac{b_1^2}{2\rho v_l^2\alpha M_0^4}\right)^{1/2}|\zeta-\zeta_0|\right\}.\end{gathered} \tag{42}$$

This result is similar to the result given in[4] for the excitation of electroacoustic waves. It is important that the parameters of the modulated waves under consideration are determined by the magnetostrictive characteristics of the medium; numerical estimates give $\Lambda\sim10^{-3}$ cm.

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[1]) We shall not pause here to consider other possible stationary solutions of this type, e.g., nonlinear waves with periodically varying amplitude.

[2]) We shall consider only the branch of transverse acoustic vibrations that interacts strongly with the spin branch.

[3]) For simplicity we neglect the effect of demagnetization.

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Superconducting contacts with a nonequilibrium electron distribution function

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The effect of nonequilibrium electrons on the current-voltage characteristics of superconducting contacts is found. In the case of bulk superconductors, when the length of the contact $a \gg \xi\tau^{1/4}$ superconductivity is stimulated in the contact and the current through the contact increases considerably under small voltages. In film contacts nonequilibrium effects lead to suppression of the superconductivity. The current-voltage characteristic in this case has a portion with negative resistance and this leads to experimentally observable voltage discontinuities.

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Superconducting contacts (film bridges, point contacts, bulk superconductors, etc.) possess, in a number of cases, volt-ampere characteristics that differ from the hyperbolic dependence found for sufficiently short contacts.[1] For example, portions corresponding to voltage discontinuities at constant current can appear in the current-voltage characteristics of contacts.[2,3] A possible explanation of these effects is that the energy distribution function of the electrons in the contact is a nonequilibrium function.

At currents exceeding the critical value a normal component of current flows through the contact and gives rise to a change in the electron distribution function. As a result the superconducting order parameter and, correspondingly, the magnitude of the superconducting current through the contact change. The changes in the current-voltage characteristic of the contact which then arise depend substantially on the dimensions of the contact.

When a current flows through the contact, the order parameter and the gap in the electron spectrum are smaller in the region of the contact than outside the contact. Electrons whose energy is less than Δ_0, the value of the gap outside the region of the contact, cannot go beyond the boundaries of the contact. For these electrons the time τ_ε for establishment of thermal equilibrium is determined by the collisions with phonons and is very long at low temperatures. Therefore, the distribution function of these electrons is greatly changed even in a weak electric field.

If the dimensions of the contact are small, the change in the distribution function of these electrons does not lead to substantial changes in the current-voltage characteristic of the contact. However, if the size a of the contact exceeds the characteristic length $\eta = \xi\tau^{1/4}$ (here ξ is the "size" of a superconducting pair and $\tau = (T - T_c)/T_c$; T_c is the critical temperature), then this change in the electron distribution function leads to stimulation of superconductivity in the contact. As a result, even in a weak electric field, a large increase in the current through the contact arises.

Electrons whose energy is higher than the value of the gap outside the contact diffuse out of the contact. Their relaxation time is determined by the diffusion rate. The effect of these electrons on the volt-ampere characteristic of the contact is substantially different for bulk-superconductor contacts (the three-dimensional case) and for film bridges. In the three-dimensional case the change in the distribution function of such electrons is small and can be disregarded. In a film, however, diffusion of the electrons is made difficult because of the two-dimensional character of their motion, and the electron distribution function is proportional to the logarithm of the long energy-relaxation time.