331 (1975) [Sov. Phys. Solid State 17, 205 (1975)].
[8]R. H. Bude, Phys. Rev. 99, 1105 (1955).
[9]L. D. Didukh and I. V. Stasyuk, Fiz. Metal. Metalloved. 26, 435 (1968).
[10]A. A. Berdyshev, Fiz. Tverd. Tela 8, 1382 (1966) [Sov. Phys. Solid State 8, 1104 (1966)].
[11]A. A. Berdyshev and B. M. Letfulov, Fiz. Tverd. Tela 14, 2119 (1972) [Sov. Phys. Solid State 14, 1824 (1973)].
[12]V. N. Rybin, Fiz. Tekh. Poluprovodn. 7, 499 (1973) [Sov. Phys. Semicond. 7, 353 (1973)].

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Size effects in parametric excitation of spin waves in ferrites

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The nonlinear properties of yttrium iron garnet (YIG) ferrite spheres with dimensions comparable with the mean free path of parametrically excited spin waves are investigated at a pumping frequency of 9370 MHz, using a dielectric cavity made of polycrystalline rutile. The damping constant of the spin waves is found to be dependent on the wave vector and on the sample diameter. It is also observed that the imaginary part of the nonlinear susceptibility, the threshold for the appearance of self-modulation of the magnetization, and the hard excitation of spin waves all depend significantly on the efficiency of scattering of the waves by surface inhomogeneities.

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INTRODUCTION

Most experiments on parametric excitation of spin waves in ferrites were performed on samples whose dimensions greatly exceeded the free path l of the excited waves.[1] As a result, the boundaries of the ferrite did not exert a noticeable influence on the nonlinear properties; in particular, it has been established that the threshold of the parametric excitation of the short-wave spin waves and the associated field damping parameter ΔH_k do not depend on the state of the surface, although if we disregard the condition $l \ll r$, where r is the radius of the investigated ferrite sphere, then ΔH_k should be strongly influenced by the quality of the surface finish.[2]

This paper is devoted to an investigation of the nonlinear properties of a ferrite sphere made of yttrium iron garnet (YIG), the initial radius of which was 0.51 mm. The radius of the sphere was then decreased to 0.26 mm, and ultimately to 0.09 mm. In the latter case, the condition $l \approx r$ was satisfied. The quality of the surface finish was in all cases the same—the surface was polished with diamond paste with grain dimension smaller than $l \approx r$. The measurements were performed at a pump frequency 9370 MHz, in both the pulsed regime (pulse duration 200 μsec, pulse repetition frequency 50 Hz) and in the cw regime. To increase the sensitivity of the experimental setup, since we measured nonlinear properties of such small ferrite samples, smaller in volume by more than two orders of magnitude than the customarily employed ferrites, the cavity resonator[3] was replaced by an open dielectric resonator of rectangular form, made of polycrystalline rutile.[4] This circumstance led also to a considerable lowering of the threshold power of the excitation of the spin waves; for example, in the field H_c—the field of the minimal threshold of the spin-wave instability, at which spin waves are excited with a wave vector **k** close to zero—the threshold power was of the order of several hundred microwatts. The samples were oriented in such a way that the direction of the easy-magnetization axis [111] coincided with the direction of the constant magnetic field $\mathbf{H}_0$.

EXPERIMENTAL RESULTS AND DISCUSSION

1. The results of an investigation of the threshold of the parametric instability in parallel pumping are shown in Fig. 1. For short-wave spin waves in constant magnetic fields $H_0 < H_c = 1540$ Oe, up to a sample diameter 0.52 mm inclusive, within an experimental accuracy limit ±0.5 dB the instability threshold does not depend on the sample diameter. The threshold is appreciably increased, however, for the sample of 0.18 mm diam-

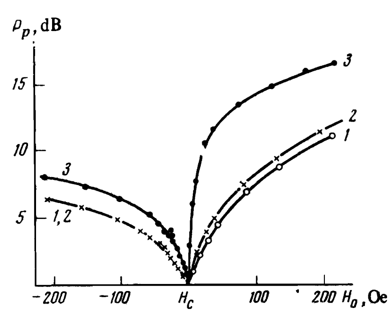

FIG. 1. Dependence of the threshold of parametric-instability excitation threshold on the constant magnetic field H_0 in the case of parallel pumping P_p. Curves 1, 2, and 3 correspond to YIG spheres 1.02, 0.52, and 0.18 mm in diameter. The absolute value of the threshold field at the point H_c amounts to 0.27 Oe for curves 1 and 2 and 0.34 Oe for curve 3; $\mathbf{H}_0 \parallel [111]$.

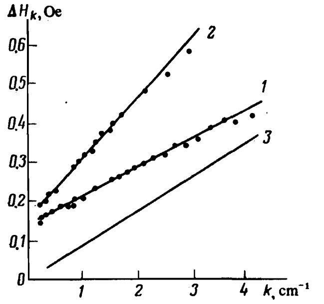

FIG. 2. Dependence of the spin-wave damping parameter ΔH_k on the wave vector k for YIG spheres of diameter 1.02 mm (curve 1) and 0.18 mm (curve 2). Line 3 was obtained by subtracting curve 1 from curve 2; $\mathbf{H}_0 \parallel [111]$.

eter. The value of ΔH_k calculated from the threshold of the parametric instability, as a function of the value of the wave vector, is shown in Fig. 2. The absolute error of ΔH_k does not exceed 15%, and the relative error, which influences, in particular, the slope of the curves in Fig. 2, is 2%. We have seen that the plots of $\Delta H_k = f(k)$ are evidently not the same for samples with diameters $2r = 1.02$ mm and $2r = 0.18$ mm, and they differ mainly in the slopes of curves 1 and 2 on Fig. 2. Whereas the slope of curve 1 is governed mainly by three-magnon relaxation processes,[5] contributing to the slope of curve 2, besides these processes, are additional relaxation processes connected with the scattering of the spin waves by the surface inhomogeneities. The increment of the spin-wave relaxation due to this scattering, henceforth designated ΔH_{ks}, is equal to the difference between curves 2 and 1 and is shown by line 3 in Fig. 2. We see that ΔH_{ks} depends linearly on k and is given approximately by

$$\Delta H_{ks} = Ak, \quad A = 9\cdot 10^{-7} \text{ Oe}\cdot\text{cm}.$$

If it is assumed that the main mechanism of spin-wave scattering by the surface inhomogeneities is two-magnon scattering, then this behavior of ΔH_{ks} is naturally explained by the fact that the probability of two-magnon scattering increases with increasing spectral density of the spin-wave distribution in k-space, which in turn increases together with k. It appears also that an important role is played by the fact that the free path of the spin wave increases somewhat with increasing k. For example, according to Fig. 2, for $k = 10^5$ cm^{-1} the mean free path is $l = 0.05$ mm, and for $k = 4\times 10^5$ cm^{-1} we have $l = 0.1$ mm. It must be noted, however, that the two-magnon scattering is not the only possible cause of the scattering of the spin waves by the inhomogeneities of the crystal—these inhomogeneities can also lead to an appreciable renormalization of the three-magnon scattering processes. A discussion of the influence of such a renormalization is premature, since this question has not been investigated theoretically so far. One can only propose, on the basis of the analogy with the usual three-magnon scattering, that in this case, too, ΔH_{ks} should increase with increasing k.

2. The right-hand side of the curves in Fig. 1 at $H_0 > H_c$ corresponds to the threshold of the excitation of magnetostatic waves, the number of which is $k < 10^4$ cm^{-1}. It is known that the excitation threshold of such waves depends on the state of the sample surface, since the ferrite can no longer be regarded as an infinite medium—the length of the magnetostatic wave becomes comparable with the dimensions of the sample.[6] For magnetostatic waves, in contrast to the exchange spin waves considered above, the influence of the surface inhomogeneities manifests itself most strongly for waves at small k, and it is maximal for homogeneous precession of the magnetization.

It is easy to see from Fig. 1 that the threshold of the excitation of the magnetostatic waves depends on the sample diameter; this is most clearly noticeable on going from a sample of diameter $2r = 0.52$ mm to a sample with $2r = 0.18$ mm. Particular notice should be taken of the fact that curve 3 in Fig. 1 is so steep that its behavior cannot be described by any of the previously proposed approximations of the dependence of the parameter of the damping of the magnetostatic waves on the value of $H_0 - H_c$ or on the direction of propagation of these waves relative to the constant magnetic field connected with $(H_0 - H_c)$ by a dispersion relation.[3,7]

3. Figure 3 shows the dependence of the imaginary part of the nonlinear susceptibility χ'' of a sample of 0.18 mm diameter on the constant magnetic field H_0 at different excesses of the pump power P over the threshold level P_p, measured at a given field H_0. The curves in Fig. 3 differ in shape from the analogous plots for samples whose dimensions greatly exceed the range of the spin wave. By way of example, the dashed line of Fig. 3 shows the dependence of the imaginary part of the nonlinear susceptibility of a YIG sphere of 2.11 mm diameter on the constant magnetic field, plotted from the results of[8]. It is seen that in the case of a sphere of 0.18 mm diameter there is a noticeable singularity in the behavior of χ'' near the three-magnon-coalescence field $H_{3m} = 1015$ Oe,[1] below which nonlinear damping due to the coalescence of two parametrically excited spin waves (PSW) into a single combined spin wave becomes possible. The influence of the nonlinear damping increases with decreasing ratio P/P_p; at $P/P_p = 1$ dB the value of χ'' in the field H_{3m} is approximately one-fifth that in fields close to H_c, at $P/P_p = 3$ dB the

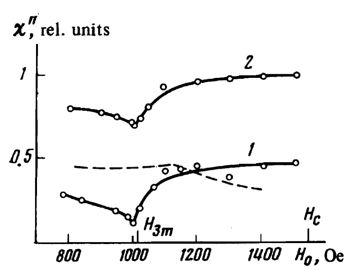

FIG. 3. Dependence of the imaginary part of the nonlinear susceptibility χ'' in parallel pumping on the constant magnetic field H_0 at excess of the pump over the threshold level $P/P_p = 1$ dB (curve 1) and $P/P_p = 3$ dB (curve 2). The sample is a YIG sphere of 0.18 mm diameter; $\mathbf{H}_0 \parallel [111]$. Dashed curve—experimental dependence for χ'' on H_0 for a YIG sphere of 2.11 mm diameter; $\mathbf{H}_0 \parallel [111]$, $P/P_p = 1$ dB.[8]

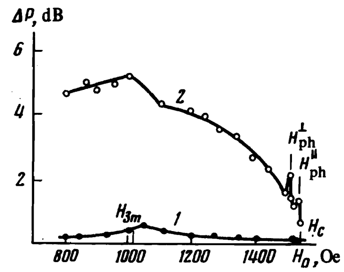

FIG. 4. Gap ΔP between the power at which the low-frequency self-modulation of the magnetization sets in and the threshold power of the parametric instability in the case of parallel pumping, as a function of the constant magnetic field H_0 for samples of 1.02 mm diameter (curve 1) and 0.18 mm diameter (curve 2); $H_0 \parallel [111]$.

difference is by a factor of 1.5, and at $P/P_p = 5$ dB the difference amounts to only several percent.

All these facts can be explained in the following manner: The phase mechanism that limits the amplitudes of the parametrically excited spin waves and predominate in large crystals[9] is now weakened by the two-magnon scattering, which decreases the phase correlation in the PSW system.[10] The relative contribution of the nonlinear damping increases as a result. However, at large ratios P/P_p the phase mechanism again predominates, since it is governed by processes in which the participating number of magnons is larger than in nonlinear damping (four- and three-magnon processes, respectively), by virtue of which the contribution of the phase mechanism to the limitation of the PSW amplitude increases more rapidly with increasing power than the contribution of the nonlinear damping.

4. In the case of samples whose dimensions are comparable with l, an appreciable change takes place in the character of the low-frequency self-modulations of the magnetization. This is clearly illustrated by Fig. 4, which shows the gap ΔP between the threshold of the self-modulation and the instability threshold as a function of the constant magnetic field H_0, for two different samples, one of which has $r \approx l$. For the larger sample, the gap reaches its maximum value,[1)] not exceeding 0.5 dB, near the field H_{3m}. For the smaller sample, the gap is much larger, and increases with decreasing H_0. In addition, the $P = f(H_0)$ curve has three singularities in the field $H_0 = H_{3m}$ and in the field $H^{\parallel}_{ph} = 1533$ Oe and $H^{\perp}_{ph} = 1510$ Oe corresponding to the intersection of the spin-wave spectrum with the longitudinal and transverse branches of the elastic oscillations. A comparison of curve 2 on Fig. 4 with the dependence of the parameter of the spin-wave damping by the inhomogeneities on the wave vector, as shown in Fig. 2, demonstrates convincingly that the gap of the self-modulations of the magnetization is proportional to the efficiency of the scattering of the spin waves by the surface inhomogeneities.

The singularities of curve 2 on Fig. 4 indicate that the magnitude of the gap depends also on the nonlinear damping of the PSW which results from the coalescence of two PSW into a new quasiparticle—magnon or phonon—even though this dependence is not strongly pronounced against the background of the action of the scattering by the inhomogeneities. The singularities in the behavior of ΔP in the fields H_{3m}, $H^{\parallel}_{ph}$, and $H^{\perp}_{ph}$ are probably connected with the singularity, which exists in these fields, of the density of states for which the coalescence processes are allowed, by virtue of which the influence of the nonlinear damping increases resonantly in this case.[9] The available experimental results enable us therefore to conclude that the nonlinear damping, in analogy with the two-magnon scattering of spin waves by inhomogeneities,[11] increases the gap between the threshold of the onset of self-modulation and the threshold of the spin-wave instability.

5. Size effects manifest themselves also in hard excitation of spin waves in ferrites.[9] In large ferrite spheres with $2r \gg l$, the lower limit of the constant magnetic field at which hard excitation is possible lies near the saturation field $H_s \approx 580$ Oe. With decreasing diameter, this limit shifts upwards, amounting to 800 Oe for a sphere of 0.52 mm diameter. In the investigation of the smallest sphere with $2r = 0.18$ mm, no hard excitation of PSW was observed at all in the entire range of constant magnetic fields. Thus, the scattering of spin waves by surface inhomogeneities influences significantly the hard excitation of spin waves in ferrites. The reason for this phenomenon becomes understandable if we consider the interaction of the PSW with the thermal spin waves with allowance for the two-magnon scattering by the inhomogeneities, an interaction that is characterized by the damping parameter ΔH_{ks}. Two-magnon scattering "intermixes" the spin states with equal frequency, and it becomes more difficult for the parametric waves to decrease the population of that section of the spin-wave spectrum which is responsible for the PSW relaxation. It can be shown that the jump in the imaginary part of the nonlinear susceptibility $\Delta\chi''$, due to the hard excitation of the PSW, by means of which it is observed experimentally, decreases like ΔH_{ks}^{-2}. Within the framework of such an explanation, the hard excitation, owing to the influence of the two-magnon scattering, does not vanish at all; only the value of $\Delta\chi''$ decreases. This decrease is larger, the larger the wave vector of the PSW. The fact that experiment reveals no hard excitation means that $\Delta\chi''$ is less than the apparatus sensitivity threshold, which in our case was not worse than $4\pi\chi'' \approx 10^{-3}$. For comparison we indicate that for a sphere of 1.02 mm diameter, in a field $H_0 = 800$ Oe, the jump of the susceptibility amounted to $4\pi\chi'' \approx 2\times 10^{-2}$, whereas for a sphere of 0.52 mm diameter, in the same field, the hysteresis observed was at the very limit of the sensitivity of the apparatus. However, for the last sample we have $4\pi\chi'' \sim 10^{-2}$ already at H_0 850 Oe. All the values of $4\pi\chi''$ cited above are approximate, since measurements of the absolute values of χ'' in a dielectric resonator are subject to a large measurement error, on the order of 50%. The error in the relative measurements shown in Fig. 3 did not exceed 10%.

6. All the experiments described above were performed with parallel pumping of spin-wave instability. However, the size effects manifest themselves also in the case of perpendicular pumping, although here they

are relatively small. So the point is that in perpendicular pumping of spin-wave instability there are excited PSW with polar angle $\theta_k \approx 45°$, having a shorter lifetime and consequently a shorter range in comparison with the PSW excited in parallel pumping. In addition, the spectral density of the distribution of the spin waves, which determines the magnitude of the two-magnon scattering by the surface inhomogeneities, is minimal precisely for waves with $\theta_k = 45°$.[1] In view of these circumstances, the size effects described above were much weakened in this case. For example, whereas in parallel pumping of spin waves in a sample with $2r = 0.18$ mm there was no hard excitation at all, in the case of perpendicular pumping only the lower limit of the hard excitation of the PSW was shifted: for the sample with $2r = 0.52$ mm it was equal to 1300 Oe, as against 1500 Oe for the sample with $2r = 0.18$ mm, corresponding to excitation of spin waves with $k \approx 2\times 10^5$ cm^{-1}. For waves with large k the scattering by the inhomogeneities, which increases in proportion to k, suppresses the hard excitation of the spin waves in the case of perpendicular pumping.

1)When a YIG sphere is magnetized along the easy axis, an instability appears in the lowest homogeneous mode of the low-frequency self-modulation of the magnetization, which has a zero gap in an unlimited according to the theory.[9] Allowance for the inhomogeneities causes the value of the gap to differ from zero.[11]

1Ya. A. Monosov, Nelineinyĭ ferromagnitnyĭ rezonans (Nonlinear Ferromagnetic Resonance), Nauka, 1971.
2R. W. Damon, Magnetism—A Treatise on Modern Theory and Materials, eds. G. T. Rado and H. Suhl, I, chap. 11, Acad. Press, New York, 1963.
3E. Schlomann, J. J. Green, and U. Milano, J. Appl. Phys. 31, 386 S (1960).
4G. A. Melkov, Radiotekh. Élektron. 17, 2027 (1972).
5A. G. Gurevich, Magnitnyĭ rezonans v ferritakh i antiferromagnetikakh (Magnetic Resonance in Ferrites and Antiferromagnets), Nauka, 1973.
6G. A. Petrakovskiĭ, Izv. Vyssh. Uchebn. Zaved. Fiz. 6, 125 (1962).
7A. P. Safant'evskiĭ, Yu. R. Shil'nikov, and O. G. Minkina, Fiz. Tverd. Tela 13, 1927 (1971) [Sov. Phys. Solid State 13, 1616 (1972)].
8G. A. Melkov, Zh. Eksp. Teor. Fiz. 61, 373 (1971) [Sov. Phys. JETP 34, 198 (1972)].
9V. E. Zakharov, V. S. L'vov, and S. S. Starovinets, Usp. Fiz. Nauk 114, 609 (1974) [Sov. Phys. Usp. 17, 896 (1975)].
10V. E. Zakharov and V. S. L'vov, Fiz. Tverd Tela 14, 2913 (1972) [Sov. Phys. Solid State 14, 2513 (1973)].
11V. S. L'vov, Preprint IYaF SO AN SSSR (Nuc. Phys. Inst., Siberian Div. Acad. Sci.) No. 8-73, Novosibirsk, 1973.

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On weakly nonlinear magnetoelastic oscillations in ferromagnets

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Weakly nonlinear magnetoelastic oscillations in ferromagnets, propagating in the direction of a uniform external magnetic field parallel to the magnetic anisotropy axis, are considered. A nonlinear parabolic equation that describes quasistationary disturbances of this type is found. It is shown that the magnetoelastic coupling has a qualitative effect on the modulation of a spin wave (modified by the magnetoelastic interaction) and also results in modulation of transverse sound (modified by the magnetoelastic interaction). It is further shown that the nonlinear excitation of a low-frequency modulated longitudinal sound wave by a high-frequency magnetization disturbance can take place under certain conditions.

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Nonlinear stationary magnetization disturbances in ferromagnets, and in particular, nonlinear periodic waves and solitons (solitary waves), were investigated in[1,2] but the time variation of the stationary profile was not discussed. On the other hand, considerable success has recently been achieved in investigating weakly nonlinear wave processes in media exhibiting spatial dispersion.[3,4] It was found that in a number of cases the evolution of the profile of a weakly nonlinear disturbance can be described by a nonlinear parabolic equation. This, in particular, is the case for magnetization disturbances in ferromagnets and antiferromagnets, which makes it possible to use a well developed mathematical apparatus in studying the latter.

In this paper we consider weakly nonlinear magnetoelastic oscillations propagating in a ferromagnet in the direction of a uniform external magnetic field parallel to the anisotropy axis. It is shown that a quasistationary disturbance of this type is also described by a nonlinear parabolic equation. On analyzing the coefficients of this equation we find that a relatively weak magnetoelastic coupling has a qualitative effect on the modulation of a spin wave (modified by the magnetoelastic in-