plained as follows. The particle density in the unperturbed field decreases sharply when $R_e > 8$ cm ($N(R_e = 10\text{ cm})/N(R_e = 8\text{ cm}) \lesssim 10^{-1}$). During the perturbation the drifting shells expand outwards to $R_e \sim 10$ cm and the particles go over from the region of absolute confinement, $\chi < \chi_c$, into the unstable state, $\chi' > \chi'_c$, which leads to an increase in the count of the second detector. The analytic expression of the functional dependence $n = n(\chi', \tau)$ (Fig. 4) can be represented in the form

$$\begin{aligned} &n=\exp(-\bar{D}\tau), \quad \bar{D}\approx 4\cdot 10^4 \exp(-a/\chi_\perp'),\\ &a=0.7, \quad \chi_\perp' \geqslant 0.1, \quad \alpha_e=\pi/2,\\ &a=0.22, \quad \chi_\perp' \geqslant 0.045, \quad \alpha_e \approx 34^\circ, \end{aligned} \tag{5}$$

where $\chi'_\perp$ is the peak value of the adiabaticity parameter. The quantity $\bar{D}$ is the effective coefficient of pitch-angle diffusion, and describes the rate of non-adiabatic losses due to "scattering" by the inhomogeneities of the magnetic field during the perturbation.

Thus, the adiabatic reversible variation in time of the magnetic field ($\tau_3 \mathring{H}/H \ll 1$) gives rise to a reversible evolution of the particles if by chance the condition $\chi' \lesssim \chi_c$ is fulfilled during the perturbation. The virtual coincidence of the critical value of $\chi_\perp$ for the static dipole field[2] with $\chi'_{\perp c}$ for the perturbed state is apparently explained by the fact that the inhomogeneity of the resultant field is determined primarily by the dipole term ($\nabla(H'_e - h) \approx \nabla H'_e$).

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Stationary Langmuir spectra in a non-uniform plasma

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We consider the effect of a weak inhomogeneity in the plasma on the structure of the Langmuir spectra, taking into account the induced scattering by ions. We determine, in the framework of a one-dimensional model, the general properties of the solution for the spectral density of the Langmuir oscillations with a source and a sink for plasmons. In the case when the role of the induced scattering is relatively small we obtain numerical solutions.

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The theory of a weakly turbulent plasma shows that the processes of induced scattering of Langmuir oscillations by ions lead to a plasmon energy spectral transfer to the small wavenumber region and in the case of a one-dimensional plasma to the formation of the so-called "Langmuir condensate" at $k = 0$.[1] By virtue of the plasmon-number conservation ($\omega_l^2 = \omega_p^2 + 3k^2 v_{Te}^2 = \text{const}$), their propagation in a non-uniform plasma is accompanied by drift in wavenumber space which prevents the formation of a condensate, so that, as we shall show below, there may exist stationary solutions for the spectral energy distribution of the Langmuir oscillations.

We consider the one-dimensional stationary problem with an external source which generates Langmuir waves in the direction of the density gradient in the plasma.[1)] Taking the direction of the z-axis in the direction opposite to that of the density gradient in the plasma and assuming, for the sake of argument, that the plasma is isothermal and that the Langmuir oscillations are generated in a narrow wavenumber interval close to $k_z = -k_0\ (r_D^{-1} \gg k_0 \gg (m_e/m_i)^{1/2} r_D^{-1})$ we get, neglecting the linear damping, thermal noise, and spontaneous scattering, for the one-dimensional spectral density W of the Langmuir oscillations[2)] (see, e.g., [3]):

$$\frac{1}{L}\frac{\partial W}{\partial x} = P\delta(x+x_0) + AW\int W(x')\psi(x+x')\operatorname{sign}(x'-x)\,dx'; \tag{1}$$

$$x = k_z/\Delta k, \quad x_0 = k_0/\Delta k, \quad \Delta k = {}^2/_3 (m_e/m_i)^{1/2} r_D^{-1},$$

$$L = \left| r_D \frac{\partial \ln \omega_p}{\partial z} \right|^{-1} \gg 1, \quad A = \frac{\pi}{18}\frac{m_e}{m_i},$$

$$\psi(y) = \frac{y}{\sqrt{2\pi}} a(y) \exp\left(-\frac{y^2}{2}\right),$$

$$a(y) = \left| 1 - \frac{y}{2} e^{-y^2/2} \int_{i\infty}^{y} e^{\tau^2/2}\, d\tau \right|^{-2},$$

$$r_D \int W(x(k_z))\, dk_z = U_l/nT,$$

where U_l is the energy density of the Langmuir oscillations, n the plasma density, $nT\omega_p P$ the volume power density of the Langmuir oscillations source.[3] As the function $a(y)$ is of order unity everywhere ($a(y)=1$ when $|y|\ll 1$, $a(y)=4$ when $|y|\gg 1$) we shall in what follows for the sake of simplicity put $a(y)=1$.

The relative part played by the induced scattering in the spectral drift of the plasmons when we take non-uniformity into account is characterized by the parameter $\alpha = APL^2$.

When $\alpha\ll 1$, when the scattering processes play a small role, we have the following solution of Eq. (1) (to first order in α)[4]:

$$W=\begin{cases} 0 & \text{if} \quad x<-x_0 \\ PL\left(1+\dfrac{\alpha}{\sqrt{2\pi}}\displaystyle\int_{-x_0}^{x}\exp\left[-\frac{(x+x')^2}{2}\right]dx'\right) & \text{if} \quad x>-x_0 \end{cases} \tag{2}$$

i.e., the induced scattering by ions expresses itself only in a certain increase in the level of the spectral density for[5] $0<x<x_0$:

$$W=PL(1+\alpha); \quad x\gg 1, \quad x_0-x\gg 1. \tag{3}$$

To see the form of the solution for $\alpha\gtrsim 1$, we determine some of its general properties.

1. Integrating (1) (using the condition $W(-\infty)=0$), we get

$$W=0 \quad \text{if} \quad x<-x_0, \qquad W=PL \quad \text{if} \quad x-x_0\gg 1,$$

$$\lim_{\varepsilon\to 0} W(-x_0+\varepsilon)=PL \qquad (\varepsilon>0).$$

2. When $|x|\gg 1$ and $x_0-|x|\gg 1$ we can, assuming that the characteristic scale of the change in W is much larger than unity, get from (1) the following relations between $W^+\equiv W|_{x>0}$ and $W^-\equiv W|_{x<0}$:

$$W^+(x)-W^-(-x)=ALW^+(x)W^-(-x),$$

$$\ln\frac{W^+(x)}{PL}=ALW^-(-x). \tag{4}$$

One sees easily that the only solutions of the set (4) are

$$W^+=W_0^+=\text{const}, \qquad W^-=W_0^-=\text{const}.$$

When $\alpha\gg 1$

$$W_0^+=PL(1-1/\alpha)e, \tag{5}$$

$$W_0^-=PL/\alpha=1/AL. \tag{6}$$

3. One shows easily that against the constant-level background $W_0^\pm$ there can exist solutions that have the character of oscillations, $W^\pm=W_0^\pm+\tilde W^\pm$. Assuming the amplitude of the oscillations to be small and linearizing Eq. (1) with respect to them we get

$$\frac{\partial \tilde W^\pm}{\partial x}=\pm ALW_0^\pm\int \tilde W^\mp(x')\frac{(x+x')}{\sqrt{2\pi}}\exp\left[-\frac{(x+x')^2}{2}\right]dx'$$

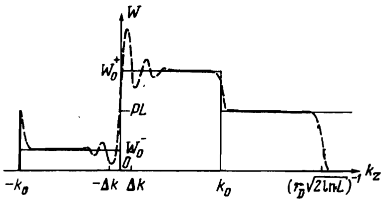

FIG. 1.

Writing $\tilde W^\pm$ in the form

$$\tilde W^\pm=\tfrac{1}{2}(W_1^\pm e^{\pm i\varkappa x}+\text{c.c.}),$$

we have for the complex amplitudes

$$W_1^+=ALW_0^+e^{-\varkappa^2/2}W_1^-,$$

$$W_1^-=-ALW_0^-e^{-\varkappa^2/2}W_1^+.$$

Hence

$$\varkappa^2=\ln(W_0^+W_0^-(AL)^2)+i\pi m, \tag{7}$$

$$(m=\pm 1,\ \pm 3,\ \pm 5,\ldots),$$

$$W_1^+/W_1^-=\pm i(W_0^+/W_0^-)^{1/2}. \tag{8}$$

For $\alpha\gg 1$ we get for the oscillatory solution with the smallest value of $|\operatorname{Im}\varkappa|$ ($m=\pm 1$)

$$\operatorname{Re}\varkappa=(1+\ln\alpha)^{1/2}, \qquad |\operatorname{Im}\varkappa|=\pi/4(1+\ln\alpha)^{1/2}.$$

Although the solution which has the form of oscillations was obtained in the linear approximation in their amplitudes and for the case where $|x|\gg 1$, the possibility for the existence of such solutions gives us grounds for assuming that for $\alpha\gtrsim 1$ the change from W_0^- to W_0^+ in the region where $|x|\lesssim 1$ must in all likelihood be accompanied by the appearance of non-linear oscillations in W which are damped with increasing $|x|$.

4. By integrating Eq. (1) we can obtain the relation

$$2\int W(x)\frac{e^{-2x^2}}{\sqrt{2\pi}}dx=W_0^+ +\frac{1}{AL}\ln\frac{W_0^-}{PL},$$

from which it follows (see (4) to (6)) that the integral of W over the region $|x|\lesssim 1$ is, by order of magnitude, not larger than PL.

Taking the general properties of the solution, obtained above, into account we may expect that the spectrum of the Langmuir oscillations has for $\alpha\gtrsim 1$ the form shown in Fig. 1.

We solved Eq. (2) for $\alpha\leq 1$ numerically by the method of successive approximations ($x_0=10$). Fig. 2 shows the results of the numerical calculation for $\alpha=0.1$; $\alpha=0.5$; and $\alpha=1.0$. The peak in the spectral density at $x\sim 1$ corresponds to the appearance of oscillations in W when changing from W_0^- to W_0^+. However, since the damping rate of the oscillatory solution (see (7)) is ap-

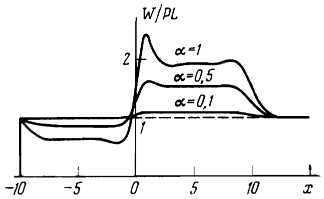

FIG. 2.

proximately equal to $\sqrt{(\pi/2)}$ at $\alpha = 1$, it is practically completely damped even after half a period.

The analysis of Eq. (1) and the results of a numerical calculation thus show that the presence of a weak non-uniformity makes it possible that there exist stationary Langmuir spectra of the kind considered (with a plasmon source and sink). We note that in a uniform plasma there are no finite stationary spectra (formation of the "condensate"). It is not excluded that stationary solutions are also possible in a non-uniform plasma when there are no external sources. A qualitative analysis of Eq. (1) for that case shows that the position k_1 of the spectrum, its width δk and its energy U_l are (for $r_D k_1 < (m_e/m_i)^{1/2}$) interconnected through the relation:

$$\left(\frac{m_e}{m_i}\right)^{1/2}\frac{U_l}{nT}Lr_D^2k_1\delta k\sim 1.$$

According to the criterion for the modulational instability[4] $U_l/nT > r_D^2\,\delta k^2$ it follows from this that when

$$\frac{U_l}{nT} < \left(\frac{m_e}{m_i}\right)^{1/3}(k_1 r_D L)^{-2/3}$$

the strong turbulence effects will not manifest themselves, i.e., such a spectrum will be stable. However, this problem, like the problem of the stability of the solutions (with sources) found above, needs an additional study.

[1]Lundin[2] has considered the evolution of the spectrum of the Langmuir oscillations (without sources) close to the minima of the density of a weakly non-uniform plasma.

[2]In order that we could use near $k = 0$ the equations of geometric optics for W, it is necessary that the width of the region in which the geometric optics approximation ($\delta k \sim (r_D^2 L_0)^{1/3}$) is inapplicable be smaller than the width of the kernel of the integrand in (1) (Δk). Hence we have for the characteristic size of the non-uniformity: $L_0 > (m_i/m_e)^{3/2} r_D$.

[3]In deriving (1) we neglected a term with the derivative $\partial W/\partial z$. This neglect turns out to be valid when

$$(k_0 r_D)^2\left(\left|\frac{\partial \ln P}{\partial z}\right| + \left|\frac{\partial \ln k_0}{\partial z}\right|\right) \ll 1.$$

[4]Taking the linear Landau damping into account shows that when $kr_D \approx (2\ln L)^{-1/2}$ (we assume that $k_0 r_D < (2\ln L)^{-1/2}$) the solution decreases fast (plasmon "sink").

[5]Such an increase in the level of W corresponds in fact to taking into account the term $2N/k$ in Eq. (2.5) in Lundin's paper.[2] If the plasmons moved close to the plasma density minimum along a closed trajectory (without source or sink) the solution with $\partial W/\partial k = 0$ for $|k| > \Delta k$ could not be a stationary one as an increase in the level of W would at each reflection lead to an increase of W with time; this agrees with the results of the above-mentioned paper.

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