"Current" state of electrons in the field of a sound wave and amplification of sound

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The effect of the gap produced in the spectrum of one-dimensional electrons by an intense sinusoidal sound wave on the electron dynamics and on the transfer of energy to the wave in sound amplification is considered. A gap of sufficient magnitude produces a large acoustoelectric current. In this case, all the applied electric energy is transferred to the wave.

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The interaction of an intense sound wave with a one-dimensional system, consisting of electrons located in an external electric field and impurities is considered. This model can be used for the description of the propagation and amplification of sound in a semimetal or a semiconductor, located in a strong magnetic field, when only the lowest Landau level is occupied.[1)]

The "superlattice" created by the sound, and the band structure of electrons, were first considered by Keldysh.[3] For one-dimensional electrons, the effect of the gap on the spectrum is most significant when the sound wavelength λ and the Fermi momentum of the electrons p_0 are connected by the relation

$$\lambda \approx \pi\hbar/p_0, \tag{1}$$

i.e., at low temperatures the gap is located near the Fermi surface.

We shall assume that the effect of electrons on the sound velocity is insignificant, which corresponds to a small number of carriers in the semimetal or semiconductor. The opposite case, when the sound velocity decreases almost to zero because of the contribution of the electrons, was considered by Fröhlich.[4]

A time-independent, periodic potential and the gap in the spectrum, the value of which we denote by 2Δ, are realized in a set of coordinates moving with the wave. If the gap is sufficiently great;

$$\Delta > \hbar\omega \approx 2\hbar p_0 s \tag{2}$$

(s, ω are the velocity and frequency of the sound), it exists also in the laboratory system of coordinates. The electrons occupying the states below the gap practically cannot rise to the upper levels: the probability of a transition across the gap due to collisions with impurities at $\hbar\omega \ll \Delta$ is proportional to $(\Delta/\varepsilon_0)^{\Delta/\hbar\omega}$ and is very small (the ratio of Δ to the Fermi energy ε_0 will be assumed to be small). These electrons create a large acoustoelectric current.[5] In an external electric field, because of the impenetrability of the gap, the electrons cannot accumulate energy and the entire power jE is transferred to the wave[6] (E is the value of the electric field and j in the current, equal in order of magnitude to esn_0, where n_0 is the number of electrons in 1 cm^3). The condition (2) recalls the condition of stability of the current in a superconductor.[7] By analogy, we can call the described state of the electrons in the field of the wave the "current" state.

We consider here also the case of a lower intensity of the wave,

$$\hbar(\nu\omega)^{1/2} \ll \Delta \ll \hbar\omega, \tag{3}$$

$\nu = \tau^{-1}$ is the frequency of collisions with impurities.[2)] Here two processes are in competition: the transfer of energy to the wave (amplification) and heating of the electrons with their hopping across the gap. If the applied electric field is not too large, i.e., electric breakdown of the gap does not occur (Zener breakdown[11]), the discontinuity in the distribution function of the electrons at the gap leads to a new mechanism of amplification, thanks to which, as under conditions (2), a large part of the electric energy is transferred to the wave. For effective sound amplification, it is necessary that τ be sufficiently large, and that the applied electric field increase with time (see Sec. 2).

In the concluding part of the work (in Sec. 3), some generalizations of the considered model are discussed briefly, as well as the stability of the solutions, and numerical estimates are made.

1. FORMULATION OF THE PROBLEM. THE KINETIC EQUATION FOR ELECTRONS

Let the electrons interact with the wave through a deformation potential with a constant Λ. The Hamiltonian of the considered system has the form

$$H = \int dx \left\{ \frac{d}{2} [(U_t')^2 + s^2 (U_x')^2] + n_0 \psi^+ \left[\Lambda \frac{\partial U}{\partial x} + \frac{p^2}{2m} + \chi V_{\text{imp}} + eEx \right] \psi \right\}; \tag{4}$$

here d is the density of the crystal, $U(x, t)$ describes the wave, m is the effective mass of the electron, ψ^+ and ψ^- are the creation and annihilation operators, $\chi \times V_{\text{imp}}$ is the potential of the impurities, E is the uniform electric field. In this model, the total energy of the particles and the sound wave is conserved. For $E \neq 0$, the carriers receive energy from the electric field and transfer part of it (different under different conditions) to the sound field. The interaction of the particles with each other is not taken into account in explicit form in (4). In the approximation of the self-consistent field, it leads only to a change in the coefficient of interaction with the wave.

Starting out from (4), we can obtain the kinetic equation for the electrons and the equation for the sound wave. Writing down the equation for the electrons, we can assume the amplitude of the wave to be constant:

$$U(x,\ t)=A\sin(k_0x-\omega t). \tag{5}$$

It is convenient to transform to a set of coordinates moving with the sound wave. The equation for the density matrix of the electrons has the form

$$\begin{gathered} i\hbar\dot{\hat{\rho}}=[\hat{H}_0,\hat{\rho}]+eE[\bar{x},\hat{\rho}]+\chi[V_{\rm imp},\hat{\rho}],\\ \hat{H}=\frac{p^2}{2m}-s\hat{p}+2\Delta\cos(k_0x),\quad \Delta=\frac{1}{2}\Lambda k_0A. \end{gathered} \tag{6}$$

In Eq. (6), we must carry out averaging over the location of the impurities. In the lowest approximation in χ^2 and E, the result (in the interaction representation) has the form

$$i\hbar\dot{\tilde{\rho}}=eE[\tilde{x},\tilde{\rho}]-\frac{i\chi^2}{\hbar}\int\limits_{-\infty}^{t}d\tau\,\overline{[\tilde{V}(t),[\tilde{V}(\tau),\tilde{\rho}(\tau)]]}. \tag{7}$$

The bar denotes averaging. Upon neglect of small oscillations of the impurities together with the wave, the potential $\tilde{V}(t)$ in (7) depends on time in the following fashion:

$$\tilde{V}_{\alpha\beta}(t)=[V_{\rm imp}(x+st)]_{\alpha\beta}\exp\{i(H_\alpha^0-H_\beta^0)t\}.$$

The characteristic numbers of H_0 are the quasimomentum p, $-\hbar k_0/2\le p\le\hbar k_0/2$ and the band number σ. We need only the diagonal in p matrix elements (a closed expression is obtained for them from Eq. (7)).

Generally speaking, the diagonal ρ^σ and nondiagonal matrix elements $\rho^{\sigma\sigma'}$ (the latter are responsible for the transfer of energy to the wave) are not separated in Eq. (7). However, for

$$\hbar\nu\ll\Delta,\quad eE\lambda\ll\Delta^2/\varepsilon_0 \tag{8}$$

we have $\rho^{\sigma\sigma'}\ll\rho^\sigma$. We are particularly interested in this case (the second condition means that there is no electric breakdown of the gap). Expanding (7), we obtain the following closed equation for ρ^σ:

$$\dot{\rho}_p^\sigma=eE\frac{\partial\rho^\sigma}{\partial p}-\nu_0v_0\sum_{\pi\sigma'}\int\limits_{-\hbar k_0/2}^{\hbar k_0/2}dp'\,A_\pi^{\sigma\sigma'}(p,p')\,\delta[\omega_{pp'}^{\sigma\sigma'}+s(p-p'-\pi\hbar k_0)](\rho_p^\sigma-\rho_{p'}^{\sigma'}),$$

where

$$\begin{gathered} \nu_0=\frac{(2\pi)^2}{\hbar^2}\frac{|\mathscr{V}(0)|^2N_{\rm imp}}{v_0},\quad v_0=\frac{\hbar k_0}{2m},\\ A_\pi^{\sigma\sigma'}(p,p')=\sum_{\alpha\beta}|(C_{p\sigma}^\alpha)^*C_{p'\sigma'}^{\alpha-\pi}C_{p\sigma}^\beta(C_{p'\sigma'}^{\beta-\pi})^*]; \end{gathered} \tag{9}$$

$C_{p\sigma}^\alpha$ are the coefficients of the expansion of the eigenfunctions in plane waves:

$$\langle q|p,\sigma\rangle=\sum_\varkappa C_{p\sigma}^\varkappa\delta(p-q-\varkappa\hbar k_0),\quad \varkappa=0,\pm1\ldots, \tag{10}$$

$\mathscr{V}(0)$ is the Fourier component of the impurity potential (which is assumed to be short-range), $N_{\rm imp}$ is the number of impurities. The nondiagonal elements $\rho_p^{\sigma\sigma}$ are expressed in terms of ρ_p^σ;

$$\begin{gathered} \rho_p^{\sigma\sigma'}=-\frac{1}{\hbar\omega_p^{\sigma\sigma'}}[eE\hat{x}_p^{\sigma\sigma'}(\rho_p^{\sigma'}-\rho_p^\sigma)+\hat{I}_{\sigma\sigma'}\{\rho_p^\sigma\}];\\ \hbar\omega_p^{\sigma\sigma'}=(H_0)_p^\sigma-(H_0)_p^{\sigma'}, \end{gathered} \tag{11}$$

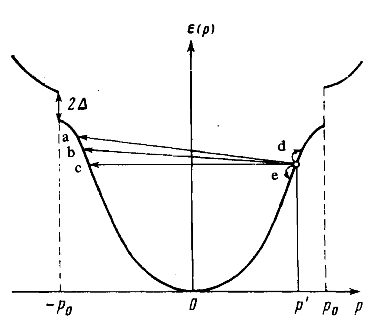
I denotes the collision integral.

The relations (11) enable us to obtain the estimates (8). Calculation of $\rho_p^{\sigma\sigma'}$ in detail is not required, since we can compute the transfer of energy to the wave from the energy balance (see Sec. 2). For the three-dimensional case, an equation similar to (9) was obtained in Ref. 9. Equation (9) has a complicated form. It can be simplified by expanding the collision integral in the small parameter (Δ/ε_0) and considering the cases $\Delta\gg\hbar\omega$ and $\hbar\nu\ll\Delta\ll\hbar\omega$ separately. In the lowest approximation in Δ/ε_0, the expressions for the energy levels of H_0 and the coefficients $C_{p\sigma}^\alpha$ have the form[12]

$$\begin{gathered} \varepsilon_{1,2}=\frac{1}{2m}[u^2+p_0^2]\\ \pm\frac{1}{2}\left[\left(\frac{2up_0}{m}\right)^2+4\Delta^2\right]^{1/2},\\ u=p-p_0,\quad \varepsilon(-p)=\varepsilon(p),\\ 0\le p\le p_0=\hbar k_0/2,\\ C_{p1}^0=\cos\varphi,\quad C_{p2}^0=\sin\varphi,\\ C_{p1}^1=-\sin\varphi,\quad C_{p2}^1=\cos\varphi,\\ \operatorname{tg}2\varphi=-m\Delta/p_0u. \end{gathered} \tag{12}$$

The quantities u and φ are introduced for compactness of expression. The indices 1 and 2 denote the lowest and next lowest bands (higher ones are not needed). In this approximation, five transitions are retained in the collision integral (see the figure).

In the case $\Delta\gg\hbar\omega$, further simplification is achieved by expansion of the collision integral in s, which converts it into a differential operator. In the lowest approximation in Δ/ε_0, scattering across the gap is absent (see the figure) and the distribution functions of the



Energy spectrum of electrons in the system of the wave. The arrows indicate transitions in scattering by impurities from the state p' to the state with energy: a) $\varepsilon(p')+2p's+\hbar\omega$, b) $\varepsilon(p')_-+2p's$, c) $\varepsilon(p')+2p's-\hbar\omega$, d) $\varepsilon(p')+\hbar\omega$, e) $\varepsilon(p')-\hbar\omega$.

different bands are not connected. It is convenient to separate the symmetric and antisymmetric parts of ρ_p^σ:

$$\rho_\pm^\sigma(p)=\rho_p^\sigma\pm\rho_{-p}^\sigma.$$

Expansion in s requires rather lengthy calculations, the result of which are

$$\dot\rho_-=-\nu_0 v_0\left\{\frac{2A(p)}{v}\rho_-+\left[\frac{sB(p)}{v^2}-\frac{eE}{\nu_0 v_0}\right]\frac{\partial\rho_+}{\partial p}\right\}.$$

$$\dot\rho_+=\nu_0 v_0\left\{\frac{s^2}{2}\frac{\partial}{\partial p}\left[\frac{\Gamma(p)}{v^3}\frac{\partial\rho_+}{\partial p}\right]+\frac{\partial}{\partial p}\left[\left(\frac{sB(p)}{v^2}+\frac{eE}{\nu_0 v_0}\right)\rho_-\right]\right\},\quad(13)$$

$$A(p)=1+\frac{1}{2}\sin^2(2\varphi),\quad B(p)=2p-k_0\mp k_0\cos(2\varphi),$$

$$\Gamma(p)=(2p-k_0)^2(1+{}^1/_2\sin^2(2\varphi))\mp2k_0(2p-k_0)\cos(2\varphi)+k_0^2.\quad v=\partial\varepsilon/\partial p.$$

The signs $\mp$ correspond to the lower and upper bands. The boundary conditions follow from the periodicity in the quasimomentum $\rho(p+2p_0)=\rho(p)$ and have the form

$$\rho_-(p_0)=0,\quad\rho_+'(p_0)=0.\quad(14)$$

It is seen from the first equation (13) that ρ_- has two characteristic times. The quasiequilibrium is established in the time $\sim\nu_0^{-1}$, after which $\dot\rho_-\approx0$ and ρ_- depends on the time only through ρ_+. Eliminating ρ_-, we obtain an equation of the diffusion type:

$$\dot\rho_+=\nu_0 v_0\frac{(\hbar\omega)^2}{2}\frac{\partial}{\partial\rho}\left\{\frac{{}^3/_2\sin^2(2\varphi)+(eE/\hbar\omega\nu_0 v_0)^2v^4}{v^3(1+{}^1/_2\sin^2 2\varphi)}\frac{\partial\rho_+}{\partial p}\right\}=\frac{\partial}{\partial p}\left[R(p)\frac{\partial\rho_+}{\partial p}\right].\quad(15)$$

The equation has the form of the equation of continuity and is satisfied by the condition of the vanishing of the flow at the boundaries:

$$R(p)|_{p=0,\,p=p_0}=0.\quad(16)$$

We proceed to the case $\Delta\ll\hbar\omega$. The change in the spectrum and additional transformations in the collision integral (see the figure) are unimportant here. Equation (9) takes the simple form:

$$\dot\rho=eE\frac{\partial\rho}{\partial p}+\frac{\nu_0 p_0}{|p+2ms|}[\rho(p)-\rho(-p-2ms)].\quad(17)$$

The existence of the gap appears only in the boundary conditions:

$$\rho(p_0+0)=\rho(-p_0-0),\quad p(p_0-0)=\rho(-p_0+0).\quad(18)$$

(In Eqs. (17) and (18) we have transformed from the quasimomentum to the ordinary momentum.) It was shown previously that the results are valid if there is no electric breakdown of the gap. At the same time, amplification of the sound is possible only in a rather strong electric field, when electrons gather an energy not less than $\hbar\omega$ over the path length. Taken together, these requirements give

$$vms\leqslant eE\leqslant\frac{\pi^2}{4}\frac{\Lambda^2}{\varepsilon_0\lambda},\quad(19)$$

which is possible if

$$\hbar^2(\nu\omega)\leqslant{}^1/_2\pi\Delta^2.\quad(20)$$

In motion without collisions, the electrons are reflected from the boundaries of the gap, but collisions with impurities push the electrons across the gap (into the regions $|\varepsilon-\varepsilon_0|\leqslant\hbar\omega$). If

$$s_E=eE/m\nu\gg s,\quad(21)$$

then Eq. (17) can be expanded in s. The same result is obtained from (13) and (15) if $|p_0-p|\gg p_0\Delta/\varepsilon_0$:

$$\dot\rho_+^\sigma=\frac{(eE)^2}{\nu_0 p_0}\frac{\partial}{\partial p}\left[p\frac{\partial\rho_+^\sigma}{\partial p}\right],\quad\rho_-^\sigma=-\left(ms-\frac{eE}{\nu}\right)\frac{\partial\rho_+^\sigma}{\partial p}\quad(22)$$

($\sigma=1$ or 2). The boundary conditions for $p=p_0$ differ from (16). We obtain them by comparing the flow from (22) to the flow across the gap obtained from (17) by direct integration over the region $|\varepsilon-\varepsilon_0|\leqslant\hbar\omega$:

$$\left.\frac{\partial\rho_+^{(1)}}{\partial p}\right|_{p_0-0}=\frac{\nu_0^2ms}{(eE)^2}[\rho_+^{(1)}(p_0)-\rho_+^{(2)}(p_0)].\quad(23)$$

The relations (22) and (23) determine the problem completely.

2. TRANSFER OF ENERGY AND SOUND AMPLIFICATION

We shall use two systems of coordinates: one moving with the wave and the other the laboratory system. In the wave system, the total energy of the electrons $\mathscr{E}$ and the momentum $\mathscr{P}$ are expressed in terms of the distribution function. From (9), we have

$$\frac{d}{dt}\mathscr{E}=\frac{d}{dt}\langle\varepsilon\hat\rho\rangle=\langle\varepsilon\hat I\hat\rho\rangle+eE\langle\hat v\hat\rho\rangle,\quad(24)$$

where $\hat v=[x,\hat H]$ is the velocity operator, the brackets $\langle\,\rangle$ denote summation over states;

$$\frac{d}{dt}\mathscr{P}=\langle\hat p\hat\rho\rangle=\langle[\hat p,H]\rho\rangle+\langle\hat p\hat I\hat\rho\rangle+eE\langle1\cdot\rho\rangle.\quad(25)$$

The momentum obtained from the electric field is partially scattered by impurities and partially transferred to the sound wave. The energy $\mathscr{E}'$ in the laboratory system is connected with $\mathscr{E}$ and $\mathscr{P}$ by the Galilean transformation

$$\frac{d}{dt}\mathscr{E}'=\frac{d}{dt}(\mathscr{E}+s\mathscr{P})=s\langle[p,H]\rho\rangle+eE\langle(s+v)\rho\rangle\quad(26)$$

(it must be taken into account that $\langle(\varepsilon+sp)\hat I\rho\rangle=0$, since the scattering from impurities is elastic in the laboratory system).

The physical picture of the energy transfer is very simple. The modulation of the electron density

$$\delta n(x)\sim n_0\frac{\Delta}{\varepsilon_0}[\cos(k_0x)+\xi\sin(k_0x)]=n^{(1)}(x)+n^{(2)}(x).$$

corresponds to a wave of the form (5) for $\Delta\ll\varepsilon_0$. The shift (which is proportional to ξ) of the electron density relative to the "crystal lattice," produced by the wave is created by the action of the electric field and the flow of the impurities. Using (10) we can easily establish the fact that ρ^{σ} enters only into $n^{(1)}$, and $\rho^{\sigma\sigma'}$, generally speaking, enters in both $n^{(1)}$ and $n^{(2)}$. The density $n^{(1)}$ leads only to a small (because of the small number of electrons) change in the sound velocity, and $n^{(2)}$ is responsible for the energy transfer to the wave.

For calculation of the transferred power $\dot{W}$, knowledge of $\rho^{\sigma\sigma'}$ is unnecessary, since we can use the energy balance (26). For the case $\hbar^2 (\nu\omega) \ll \Delta^2 \ll \hbar^2\omega^2$, using (22), we obtain

$$W=-\int_0^\infty dp\frac{p^2}{2m}\frac{d}{dp}\left[\frac{(eE)^2}{\nu(p)}\frac{d\rho_+}{dp}\right]$$
$$+eE\int_0^\infty dp\left[\frac{p}{m}\left(ms-\frac{eE}{\nu(p)}\right)\frac{d\rho_+}{dp}\right]+s\frac{d}{dt}\int_0^\infty dp\,p\left(\frac{eE}{\nu(p)}-ms\right)\frac{d\rho_+}{dp}. \quad (27)$$

Integrating by parts, and taking the discontinuity of $\rho_+(p)$ at the point p_0 into account, we obtain the simple formula

$$W\approx eEsn_0[\rho_+(p_0-0)-\rho_+(p_0+0)] \quad (\nu(p)\approx\nu_0,\ \nu\rho\gg\dot{\rho}). \quad (28)$$

which explicitly demonstrates the role of the gap in the considered mechanism of sound amplification. The gap makes difficult the energy absorption by the electrons, because of which the energy transfer to the wave increases. The value of the gap is not important if it is not broken down by the electric field. The change of $\dot{W}$ with time is determined by Eqs. (22) and (23). They can not be solved analytically even for E that is time-independent (in this case, the Laplace transform of $\rho(t)$ is expressed in terms of Bessel functions, but the inversion cannot be carried out explicitly). Nevertheless, without solving the equations, we can make it clear when effective sound amplification is possible. Equations (27) and (23) permit us to connect W and n_2—the number of particles passing through the gap:

$$W=\frac{2p_0eE}{\nu_0}\dot{n}_2.$$

$\dot{W}$ is expressed in terms of (Δ^2) (see (4), (6)):

$$W=\frac{4s^2d}{\Lambda^2}(\dot{\Delta^2})=N\frac{1}{c\Lambda}(\dot{\Delta^2}),$$

where N is the number of atoms in 1 cm^3 and $c\sim 1$ is a numerical coefficient. The value of E is chosen to be as large as possible, close to breakdown (see (19)). As a result, we obtain the relation

$$\frac{(\dot{\Delta^2})}{\Delta^2}=\frac{2c}{\pi}\left(\frac{n_0}{N}\frac{\Lambda}{\varepsilon_0}\right)(\omega\tau)\frac{p_0}{ms}\frac{\dot{n}_2}{n_0}.$$

If

$$\omega\tau\gg\frac{\pi}{2c}\frac{ms}{p_0}\left(\frac{N}{n_0}\frac{\varepsilon_0}{\Lambda}\right) \quad (29)$$

or

$$\tau\gg\frac{\pi}{4c}\frac{N}{n_0}\frac{\hbar}{\Lambda}, \quad (30)$$

the growth of the gap takes place more rapidly than the transfer of particles to the upper band.[3)] The applied electric field should increase approximately as $\exp(2\gamma_0 t)$, where

$$\gamma_0=\frac{c}{2\pi}\omega\left(\frac{n}{N}\frac{\Lambda}{\varepsilon_0}\right). \quad (31)$$

Here $\Delta\sim\exp(\gamma_0, t)$, γ_0 is of the same order of magnitude as the sound growth increment in the linear stage of amplification (as $\Delta\to 0$). Upon reaching the threshold $\Delta=\hbar\omega$, leakage through the gap abruptly decreases with increase in the wave. For $\Delta\gg\hbar\omega$ the change in the energy of the electrons and the wave is determined by Eqs. (15) and (26). It has not been possible to solve Eq. (15) analytically. It is essential that the only stationary solution of (14) and (15) is

$$\rho=\text{const}=n_1/n_0\leqslant 1 \quad (32)$$

(n_1 is the number of particles in the lower band).[4)] The quantity $\rho(t)$ approaches (32) in a time of the order of $\tau\,(p_0/ms_E)^2$. Within this time, a current $j_1=esn_1$ is established in the laboratory system and a power j_1E transferred to the wave (the particles in the upper band continue to be heated independently). The assertion that $\rho_1(p)<1$ throughout the entire depth of the band is not absurd. The particles are "locked" on the high energy side and a distribution that is far from equilibrium is established in the strong electric field. The existence (neglecting the rare "multiphonon" transitions) of a steady current independent of the electric field gives grounds for calling the state of the electrons a "current" state by analogy with superconductivity. The steady current exists even for $E=0$; however, its calculation has meaning only in the more general model, for example, with account of collisions of particles from different bands.

3. ADDITIONAL REMARKS, STABILITY OF THE SOLUTION AND NUMERICAL ESTIMATES

A real semimetal or semiconductor frequently has a multi-valley spectrum. Intervalley scattering from impurities, not taken into account in the assumed model, is usually weaker than the intravalley scattering. If the particles of the other valleys are little heated by the field (for example, this is the case for heavy holes), the intervalley transitions will increase the number of electrons below the gap, i.e., they will favor the amplification of the sound. The collisions of particles from different bands (intraband collisions in the one-dimensional case do not change the distribution function) play a similar role: if the holes are in states close to the ground state, the electrons can only lower the total energy as a result of collisions with them. The stability of the state of the system is determined by the value of the fluctuations of its parameters.

In the case $\Delta>\hbar\omega$, a decrease in the total momentum of the electrons, due to filling of the states above the gap is energetically unfavorable (as in a superconductor, this is equivalent to the existence of a gap in the laboratory system and to stability of the "current" state). For $\hbar(\nu\omega)^{1/2} \ll \Delta \ll \hbar\omega$ (and in the stationary state for $\Delta \gg \hbar\omega$) the value of the gap has no effect on the dynamics of the electrons, or on the fluctuations of the gap and of the electrons, and the fields are independent. By virtue of (28), $\Delta(x) \sim eE/\Delta(x)$, and the spatial inhomogeneities of the gap are relatively equalized. The reaction of the electron system to the perturbations of the electric field changes from metallic to dielectric upon approach to the steady state; the electron fluctuations are small. The reason for the instability can be the electric breakdown in places of small amplitude of the wave, which in turn can be a consequence of the nonmonochromatic character of the initial wave packet.

The possibility of generation of acoustic noise is studied within the framework of the assumed model. Generation of all waves with $k = 2p/\hbar$, for which $\rho(p) > 0$, is possible. In the linear approximation, the increment $\gamma(p)$ is expressed in terms of γ_0 from (31):

$$\gamma(p) \sim \gamma_0 \rho_-(p) \sim \gamma_0 s/s_E \ll \gamma_0$$

(see (14), (19), and (23)).

We note the effect on the system of a smooth external potential $\Phi(x)$. In the laboratory system, the particles move in correspondence with the conservation law:

$$\varepsilon^a(p) + sp + \Phi(x) = \text{const},$$

and the motion of the particles in the energy layer of thickness $\delta/\Phi(x)$, adjacent to the gap, is limited in space ($\delta\Phi(x)$ is the characteristic variation of the potential). These particles are not accelerated by the field and do not make a contribution to the current.

In conclusion, we carry out numerical estimates, using the parameters of Bi. The electrons become "one-dimensional" in fields of the order of 10^4–10^5 V. If p_0 is identical in order of magnitude with the Fermi momentum for $H = 0$, for different orientations of the magnetic field, the condition $\hbar\omega \approx 2p_0 s$ means $\omega \sim 2$–$20 \times 10^{11}\ \text{sec}^{-1}$. The elastic parameters have the following values:

$$\Lambda \sim 5\ \text{eV} \approx 8 \cdot 10^{-12}\ \text{erg}\ [^{13}],\quad d = 10^7/\text{cm}^3,\quad s = 2 \cdot 10^5\ \text{cm/sec}.$$

For c from (29), we have ($N \sim 3 \times 10^{22}$)

$$c = N\Lambda/4ds^2 \approx 0.15.$$

The condition (30) takes the form $\tau \gg 10^{-10}$ sec. In very pure Bi, it is possible to obtain $\tau \sim 10^{-8}$.

We also estimate the value of the breakdown field for $\Delta \sim \hbar\omega \sim 10^{-16}$ erg. From (19), we get $E \sim 10^{-3} \approx 0.3$ V/cm. The sound field with $\Delta \sim 10^{-16}$ erg corresponds to an energy flux of several W/cm^2. Finally, in conversion to temperature, the size of the gap is of the order of 1 °K. It is desirable that the initial temperature of the sample not exceed this value.

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[1)]Reduction to the one-dimensional case is carried out in the same way as in other kinetic problems in a strong magnetic field. The problem of the conductibility is considered in the works of Adams and Holstein[1] and Abrikosov.[2]

[2)]The researches of Zil'berman[8] and Laĭkhtman and Pogorelskiĭ[9] were devoted to related questions, the interaction of the wave with three-dimensional electrons was considered under the assumption of their small departure from equilibrium. There is also the work of Gal'perin, Kagan and Kozub,[10] where the capture of "classical" electrons by the sound wave is considered.

[3)]In a semimetal with overlapping bands, a situation is possible in which $\varepsilon_0(H) = \text{const}$; then $n_0 \sim H$. For point-like impurities, $\tau \sim H$.

[4)]The expression for $\rho(p)$ in Ref. 5 is in error: only $\rho = \text{const}$ satisfies the condition of zero flow across the boundary.

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