explain the anisotropy of the temperature dependences. Since the inequality $\omega\tau \gg 1$ is not satisfied in the investigated range of frequencies, some of the surface levels are not resolved in the impedance measurements and, therefore, the anisotropy data are only qualitative. At higher frequencies this range of temperatures cannot be investigated because the critical magnetic field is too low.[4]

The investigated singularity of the impedance appears in zero magnetic field due to magnetic surface levels and the width of the integrated absorption curve may be governed by two effects. If there are many unresolved levels and the width of a single level governed by the collision frequency is greater than the magnetic field interval in which the levels are located, the total width of the absorption peak is governed by the collision frequency and is independent of the observation frequency, as found by Sibbald *et al.*[3] In the second case the magnetic field interval within which the levels are located is greater than the width of an individual level. Then, the width of an absorption peak is governed by the resonance field of the lowest-energy level and depends on the frequency at which impedance is observed. It is clear from Fig. 6 that the second case applies to our investigation.

The width of the absorption peak was measured in the range of temperatures corresponding to rapid variation of dR/dH. It follows from our discussion that in this range there are several magnetic surface levels. We can then apply the treatment of the surface levels in normal metals. An analysis of the results on the basis of the dependences $\omega \propto H^{2/3}$ [2] is found to be in good agreement with the experimental data and extrapolation to zero frequency gives the average collision frequency of ~200 MHz for the investigated cylindrical sample. A similar absorption maximum in zero magnetic field is also observed in superconducting indium.

Thus, the singularity of the impedance of superconducting tin observed in the present study is due to normal excitations forming surface levels, whose number decreases with decreasing depth of penetration of the magnetic field.

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[1]M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. **39**, 212 (1960) [Sov. Phys. JETP **12**, 152 (1961)].
[2]R. E. Prange and Tsu-Wei Nee, Phys. Rev. **168**, 779 (1968).
[3]K. E. Sibbald, A. L. Mears, and J. F. Koch, Phys. Rev. Lett. **27**, 14 (1971).
[4]J. R. Maldonado and J. F. Koch, Phys. Rev. B **1**, 1031 (1970).
[5]A. B. Pippard, Proc. R. Soc. Ser. A **191**, 370 (1947).
[6]M. Spiewak, Phys. Rev. **113**, 1479 (1959).
[7]J. F. Koch and C. C. Kuo, Phys. Rev. **164**, 618 (1967).

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The thermoelectric field in superconductors

S. N. Artemenko and A. F. Volkov

Institute of Radio Engineering and Electronics, USSR Academy of Sciences
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The possibility of the appearance and measurement of a thermoelectric field E in a superconductor S is discussed theoretically. An equation is derived which describes the coordinate dependence of the field E in a superconductor with a nonzero energy gap. It is shown that the characteristic scale of the spatial variation of E is the distance l_b over which equilibrium is established between the branches of the energy spectrum and which, in pure superconductors, greatly exceeds the correlation length $\xi(T)$. It is shown that the field E arises in the presence of a temperature gradient near the boundary between S and a dielectric, a normal metal, or another superconductor, and falls off in S exponentially over the distance l_b.

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INTRODUCTION

The thermoelectric effect in homogeneous isotropic superconductors consists in the fact that in the presence in a superconductor of a temperature gradient T_x' there arises a thermocurrent $\beta T_x'$, which in an open sample is balanced by the superconducting current $e^* n_s v_s$.[1] Thus, the total current

$$j=\beta T_x'+e^* n_s v_s$$

is equal to zero, and so is the electric field.[1)]

Another picture arises if the superconductor is inhomogeneous in the direction of the temperature gradient, as when it borders on a dielectric D, a normal metal N, or another superconductor $\tilde{S}$ in which $\tilde{\beta} \neq \beta$. Then there arises near the boundary a quasiparticle-current divergence ($\mathrm{div}\,\mathbf{j}_n \neq 0$), since $\mathbf{j}_n = 0$ in D and N, and $\mathbf{j}_n \neq \tilde{\mathbf{j}}_n$ in the $S-\tilde{S}$ system. It has previously been shown on the basis of phenomenological equation[6,7] and equations obtained from a microscopic theory and valid for gapless superconductors with paramagnetic impurities[8] that in the presence of a nonzero divergence of $\mathbf{j}_n$ (or, which is the same, of $\mathbf{j}_s$) there arises in a superconductor the gauge invariant potential

$$\Phi = {}^1/_2\dot{\chi} + e\varphi, \tag{1}$$

where χ is the phase of the order parameter and φ is the electric potential. This means that an electric field arises in the steady-state case in the superconductor, since

$$m\dot{\mathbf{v}}_s = \nabla\Phi + e\mathbf{E} = 0. \tag{2}$$

In the case of a gapless superconductor the equation for Φ has the form[8] 2)

$$12\frac{\sigma}{e\xi^2(T)}\Phi = -\frac{\partial j_n}{\partial x}. \tag{3}$$

Using the expression for the current

$$j = \sigma E + \beta T_x' + e' n_s v_s, \tag{4}$$

we see that the correlation length $\xi(T)$ serves as the characteristic variation length for Φ and E. Below we derive an equation analogous to (3) and valid for an ordinary superconductor with a gap. We shall show that it has the same form as Eq. (3), but that the length $\xi(T)$ should be replaced by a much greater length l_b characterizing the establishment of equilibrium between the branches of the quasiparticle spectrum ($p > p_0$ and $p < p_0$, where p_0 is the Fermi momentum). We shall find the thermoelectric field E in different cases3) and analyze the possibility of its experimental detection. In doing this below we shall assume that the temperature is close to the critical, i.e., that $\Delta \ll T$.

EQUATION FOR THE POTENTIAL

Let us consider a superconductor with a gap Δ, in which a temperature gradient T_x' exists. Let us derive for Φ an equation of the type of Eq. (3). If we do not take into account the inelastic quasiparticle-phonon collisions and the gap anisotropy, then we can verify that in the presence of a j_n divergence the potential Φ will increase without restriction. With that end in view, it is convenient to use the equations for the equal-time Green functions[11] and find the phase in the linear approximation, treating the longitudinal electric field $\mathbf{E} = -\Delta\varphi$ as a perturbation. As a result, we arrive at the Eq. (6) of the paper[10]. In order to obtain a finite value for Φ, it is necessary to take into account the inelastic collisions with the phonons characterized by a small (in comparison with Δ and T) frequency $\nu_{en} \approx T^3/\Theta_D^2$. On account of the smallness of ν_{en}, we should expect (and this is confirmed by the result) long characteristic lengths. Therefore, we can use for the derivation of the required equation the quasiclassical equations, the microscopic derivation of which is given by Aronov and Gurevich in[12]. Let us write the kinetic equation for the excitation distribution function in the form

$$\frac{\partial n}{\partial t} + \frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial n}{\partial \mathbf{x}} - \frac{\partial \varepsilon}{\partial \mathbf{x}}\frac{\partial n}{\partial \mathbf{p}} = I_{\text{imp}}(n) + I_{\text{ph}}(n), \tag{5}$$

where

$$\varepsilon = [(\xi+\Phi)^2+\Delta^2]^{1/2} + \mathbf{p}\mathbf{v}_s, \quad \xi = (p^2 - p_0^2)/2m,$$

I_{imp} and I_{ph} are the quasiparticle-impurity and quasiparticle-phonon collision integrals. This equation is valid if

$$v/\nu \equiv l \gg v/\Delta \equiv \xi(T)$$

and $l_b \gg \xi(T)$, where $v = p/m$, ν is the excitation-impurity collision rate, l is the mean free path with respect to scattering by impurities, and l_b is the characteristic field-attenuation length, which will be found below.

The determination of the solution to (5) becomes simplified if we assume that the relaxation of the odd—in the momentum p_x—part of n is due to scattering by the impurities, i.e., if we assume that $\nu \gg \nu_{en}$. We shall seek the solution to Eq. (5) in the form

$$n = n_0(\varepsilon) + n_1,$$

where n_0 is the Fermi distribution function. Let us expand the small correction n_1 in terms of the Legendre polynomials:

$$n_1 = a_0 + \mu a_1 + \ldots, \tag{6}$$

where $\mu = p_x/p$, while the coefficients a_0, a_1, ... depend on $\xi = v(p - p_0)$. Under the adopted assumption that the scattering is only weakly inelastic, we can restrict ourselves to the first terms, written out above, of the expansion of n_1, since the remaining terms will be small to the extent of the smallness of ν_{en}/ν. Let us substitute n into (5) and take into account the fact that $T = T(x)$. Then we obtain

$$\mu v \frac{\xi}{\varepsilon}\left[\frac{\partial n_1}{\partial x} - \frac{\partial n_0}{\partial \varepsilon}\varepsilon\frac{T_x'}{T}\right] = -\nu\frac{|\xi|}{\varepsilon}(n - \bar{n}) + I_{\text{ph}}(n_1). \tag{7}$$

We have used for I_{imp} the τ-approximation. Averaging Eq. (7) over the angles, we obtain the equation

$$\frac{v}{3}\frac{\xi}{\varepsilon}\frac{\partial a_1}{\partial x} = \bar{I}_{\text{ph}}(a_0), \tag{8}$$

where $\bar{I}_{\text{ph}}(a_0)$ is the linearized and angle-averaged excitation-phonon collision integral. Multiplying (7) by μ and integrating over μ, we find a_1:

$$a_1 = -\operatorname{sgn}\xi \cdot l\left[\frac{\partial a_0}{\partial x} - \frac{\partial n_0}{\partial \varepsilon}\varepsilon\frac{T_x'}{T}\right]. \tag{9}$$

With the aid of (8) and (9) we can find the equation for Φ and the expression for the current. Let us compute the change in the number of particles in the superconductor, after substituting the distribution function (6) into the expression for N[12]:

$$\delta N = \delta\int d\tau[u_p^2 n + v_p^2(1-n)]$$
$$= \frac{p_0 m}{\pi^2}\int d\xi\left\{\Phi\left[\frac{\Delta^2}{2\varepsilon^3}(2n_0 - 1) + \frac{\partial n_0}{\partial \varepsilon}\left(\frac{\xi}{\varepsilon}\right)^2\right] + \frac{\xi}{\varepsilon}a_0\right\}.$$

Near the critical temperature we can neglect the first term in the square brackets. Substituting δN into the Poisson equation, we obtain

$$\frac{\partial^2\Phi}{\partial x^2}=k_{TF}^2\left[\Phi\int d\xi\frac{\partial n_0}{\partial\xi}\left(\frac{\xi}{\varepsilon}\right)^2+\int d\xi\frac{\xi}{\varepsilon}a_0\right], \tag{10}$$

where $k_{TF}^{-1}=(6\pi e^2N/\varepsilon_F)^{-1/2}$ is the Thomas–Fermi screening length. Since the left-hand side in (10) is of the order of $l_b^{-2}\Phi$, where l_b satisfies the inequality

$$l_b^{-1}\ll\xi^{-1}(T)\ll k_{TF},$$

the left-hand side of the expression (10) can be neglected. Thus, the potential Φ can be found from the neutrality condition $\delta N=0$ and is equal, near T_c, to

$$\Phi=\int d\xi\frac{\xi}{\varepsilon}a_0, \tag{11}$$

i.e., the potential is expressible in terms of the even—in the momentum p_x—part of n_1, in the same way as in a normal metal. In a normal metal the integral in (11) is proportional to the difference between the number of electrons and the number of holes. In a superconductor this integral is proportional to the difference between the number of electron-like excitations (i.e., excitations belonging to the $\xi>0$ branch) and the number of hole-like excitations ($\xi<0$).

Let us now consider the expression for the current density j. Using (6), we obtain for j the expression

$$j=e\int d\tau\,\mu v n+eNv_s=j_n+j_s, \tag{12}$$

where

$$j_s=2en_sv_s=eN\left(1+\int_{-\infty}^{+\infty}d\xi\frac{\partial n_0}{\partial\varepsilon}\right)v_s,$$

$$j_n=\frac{eN}{p_0}\int_{-\infty}^{\infty}d\xi\,a_1. \tag{13}$$

Let us substitute into (13) the expression (9) for a_1. With allowance for (11) and the fact that the characteristic variation scale for ξ near T_c is $\xi\sim T$, we obtain

$$j_n=-\frac{\sigma}{e}\frac{\partial\Phi}{\partial x}+\beta\frac{\partial T}{\partial x}, \tag{14}$$

where

$$\sigma=e^2N\tau/m,\quad \beta/\sigma=\pi^2T/3e^2\varepsilon_F.$$

Let us derive the equation for Φ. Let us multiply Eq. (8) by ξ/ε and integrate over ξ. Then we have in the principal approximation in Δ/T with allowance for (13) the equation

$$\frac{ev^2\tau}{3\sigma}\frac{\partial j_n}{\partial x}=\int d\xi\frac{\xi}{\varepsilon}\bar{I}_{ph}(a_0)=\left\langle\frac{\xi}{\varepsilon}\bar{I}_{ph}\right\rangle. \tag{15}$$

Let us compute the right-hand side. Let us substitute the expression for the collision integral. Then

$$\left\langle I_{ph}\frac{\xi}{\varepsilon}\right\rangle=\frac{\pi\zeta_{ph}}{2\Theta_D^2}\int d\xi\,d\xi'\,d\omega\frac{\xi}{\varepsilon}\omega^2\left\{F_-(\xi,\xi')\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)+F_+(\xi,\xi')\left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\right\}$$

$$=-\frac{\pi\zeta_{ph}}{2\Theta_D^2}\int d\xi\,d\xi'\,d\omega\,\omega^2\frac{\Delta^2}{\varepsilon\varepsilon'}(\xi+\xi')(F_--F_+);$$

here

$$F_-(\xi,\xi')=\delta\{[n'(1-n)(1+N_\omega)-n(1-n')N_\omega]\delta(\varepsilon'-\varepsilon-\omega)+[n'(1-n)N_\omega-n(1-n')(1+N_\omega)]\delta(\varepsilon-\varepsilon'-\omega)\},$$

$$F_+(\xi,\xi')=\delta\{[N_\omega(1-n)(1-n')-nn'(1+N_\omega)]\delta(\varepsilon+\varepsilon'-\omega)\},$$

$n'=n(\xi')$, N_ω is the equilibrium phonon distribution function, the δ in front of the curly brackets denotes that the expressions in the curly brackets should be linearized with respect to n_1. The function $F_\pm(\xi,\xi')$ is even (odd) with respect to the interchange $\xi\rightleftarrows\xi'$. We used this fact in the derivation of the last equality. The matrix element of the interaction with the phonons has been written in the form

$$g^2=2\pi^2\zeta_{ph}/p_0m,\quad \zeta_{ph}\sim1.$$

Substituting the linearized expression for $F_\pm$, we find after simple transformations that

$$\left\langle\bar{I}_{ph}\frac{\xi}{\varepsilon}\right\rangle=-\int d\xi\,a_0(\xi)\frac{\xi}{\varepsilon}\nu_b(\varepsilon)\approx-\nu_b\int d\xi\,a_0(\xi)\frac{\xi}{\varepsilon}, \tag{16}$$

where

$$\nu_b(\varepsilon)=\frac{\pi^2\zeta_{ph}}{\Theta_D^2}\frac{\Delta^2}{\varepsilon}\int_\Delta\frac{d\varepsilon'}{\varepsilon'(\varepsilon'^2-\Delta^2)^{1/2}}[\theta(\varepsilon-\varepsilon')(1-n_0'+N_{\varepsilon-\varepsilon'})(\varepsilon-\varepsilon')^3+(\varepsilon+\varepsilon')^3(n'+N_{\varepsilon+\varepsilon'})-\theta(\varepsilon'-\varepsilon)(\varepsilon'-\varepsilon)^3(n'+N_{\varepsilon'-\varepsilon})].$$

Near T_c the dominant contribution to $\nu_b(\varepsilon)$ is made by the first and second terms. Computing them, we find

$$\nu_b(\varepsilon)=2\pi^2\zeta_{ph}(\varepsilon^2\Delta/\Theta_D^2)\,\mathrm{cth}\,(\varepsilon/2T). \tag{17}$$

Near T_c the characteristic variation scale for ξ in a_0 is $\xi\sim T$; therefore, in the formula (16) $\nu_b=\nu_b(T^*)$, where $\nu_b(T^*)$ is determined by the expression (17), while $T^*\sim T$.

The frequency ν_b was introduced by Clarke and Tinkham[13] and computed (by another method) by Tinkham.[14] As can be seen from the relation (16), it characterizes the rate of relaxation of the odd—in ξ—part of the distribution function, i.e., the rate of establishment of equilibrium between the populations of the branches of the quasiparticle spectrum.

Substituting into (15) the expressions (11) and (16), we obtain the sought equation

$$\frac{\sigma}{el_b^2}\Phi=-\frac{\partial j_n}{\partial x},\quad l_b^2=\frac{1}{3}\frac{v^2}{\nu\nu_b}. \tag{18}$$

From (14) and (18) follows the equation for Φ:

$$\frac{d^2\Phi}{dx^2}-l_b^{-2}\Phi=\frac{\beta}{\sigma}e\frac{d^2T}{dx^2}. \tag{19}$$

Thus, for a superconductor with a gap the characteristic variation scale for the longitudinal electric field $\mathbf{E}=-\nabla\Phi/e$ is the length l_b, which exceeds the mean free path. This length is also much greater than the correlation length $\xi(T)$.

THE THERMOELECTRIC FIELD AND THE POSSIBILITY OF ITS EXPERIMENTAL DETECTION

Let us consider the boundary of a superconductor with a dielectric. Then for a uniform temperature gradient (constancy of the heat flux $q_x' \approx \varkappa T_{xx}'' = 0$) the solution to (19) that satisfies the boundary condition $j_n(0) = 0$ is[10]

$$E(x) = -\frac{1}{e}\Phi_x' = E_N e^{-x/l_b}, \quad E_N = -\frac{\beta}{\sigma}T_x', \tag{20}$$

i.e., there arises at the boundary of the superconductor a thermoelectric field which is equal in magnitude to the field in a normal metal and which falls off exponentially over the distance l_b into the superconductor. If $T \to T_c$, then l_b, as follows from its definition and from the expression (17), increases without restriction. Thus, a continuous transition is realized, as in the Meissner effect, to the normal metal; as $T \to T_c$ the thermoelectric field occupies an ever-increasing volume of the superconductor.

With the aid of Eq. (19) we can investigate the boundary between a superconductor and a normal metal. So as to be able to use the quasiclassical method, let us assume that Δ varies over a characteristic distance x_0 greater than $\xi(T)$ (although, qualitatively, the result remains valid also for $x_0 \sim \xi(T)$), i.e., let us assume that

$$\Delta(x) = \begin{cases} 0, & x \leqslant 0, \quad N\text{-region} \\ \Delta_\infty, & x \gg x_0, \quad S\text{-region} \end{cases},$$

where $\xi(T) \ll x_0 \ll l_b(\infty)$. Such a situation is realized, for example, in a film of thickness w ($v\nu^{-1} \ll w \ll \lambda$, where λ is the Meissner skin thickness) located in a longitudinal magnetic field that varies along the coordinate. The coefficient l_b^{-2} varies in proportion to $\Delta(x)$. Integrating Eq. (19) over a layer of thickness $x_0 \ll x \ll l_b(\infty)$, we obtain that the potential Φ and the field E are continuous at the S–N boundary. Therefore, the solution to (19) in the S region is again the function (20), this result being valid irrespective of whether the field E_N in the region N is produced by the temperature gradient or by the flowing current. The computation of E_N in the latter case for gapless superconductors has been carried out on the basis of Eq. (3) in the papers[6,15] 4).

The penetration of the thermoelectric field from the N region to a depth of l_b in the S region will lead to the increase of the thermoelectromotive force, ε_N, of the S–N system by a value equal to $\varepsilon_N(l_b/L_N)$, where L_N is the dimension of the N region. As we approach T_c, the contribution of the S region will increase. Such an effect can, apparently, be experimentally observed by measuring the variation of the thermoelectromotive force, ε_T, of a superconductor in the intermediate state. Then ε_T should increase in the same manner as the resistance, discovered by Pippard and his collaborators,[18] of such a system increased.

The thermoelectric field can arise not only in an inhomogeneous superconductor, but also in a homogeneous superconductor with a nonuniform temperature gradient.

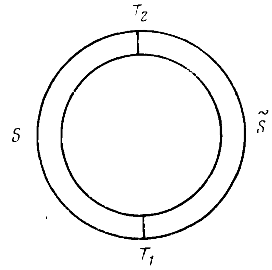

FIG. 1. Contour composed of superconductors S and $\tilde{S}$ and used for the observation of the thermoelectric effect. $T_{1,2}$ are the temperatures of the contacts.

Let, for example, the temperature in S vary according to the law

$$T = T_0 + \Delta T|x|/L.$$

Then from Eq. (19) we obtain the potential distribution

$$\Phi = -\frac{\beta e l_b}{\sigma}\frac{\Delta T}{L}\exp\left\{-\frac{|x|}{l_b}\right\}$$

and field distribution

$$E = \frac{\beta}{\sigma}\frac{\Delta T}{L}\operatorname{sgn} x \cdot \exp\left\{-\frac{|x|}{l_b}\right\}.$$

Thus, the thermoelectric field changes sign and the thermoelectromotive force is equal to zero ($\Phi(-\infty) = \Phi(+\infty)$). A thermoelectric field of similar form arises at the boundary between two superconductors with different β coefficients, and leads to an observable effect. In fact, let us consider a superconducting ring consisting of the superconductors S and $\tilde{S}$ (see Fig. 1). Let us integrate the expression, (12), for the total current along the contour, assuming that the thickness of the ring is greater than the depth of penetration:

$$\oint \frac{j}{n_s}\,dl = -\left(\frac{\sigma}{n_s} - \frac{\tilde{\sigma}}{n_{\tilde{s}}}\right)\frac{\Phi_1}{e} + \left(\frac{\beta}{n_s} - \frac{\tilde{\beta}}{n_{\tilde{s}}}\right)(T_2 - T_1) + \frac{(2e)^2}{cm}(nW_0 - W) = 0. \tag{21}$$

Here we have taken into account the fact that $\Phi_1 = -\Phi_2$; W_0 is the flux quantum and W is the magnetic flux in the contour. Let us determine the potential Φ_1 from the condition for the continuity of the current j_n at the S–$\tilde{S}$ interface:

$$-\sigma l_b^{-1}\Phi_1 + \beta\frac{T_2 - T_1}{L}e = \tilde{\sigma}\tilde{l}_b^{-1}\Phi_1 - \tilde{\beta}\frac{T_2 - T_1}{L}e,$$

where L is the length of the superconductor S and also of $\tilde{S}$. Finding from this the potential Φ_1, and substituting it into (21), we obtain the expression for the flux W:

$$W = nW_0 + \frac{cm}{(2e)^2}\left[\left(\frac{\beta}{n_s} - \frac{\tilde{\beta}}{n_{\tilde{s}}}\right) - \left(\frac{\sigma}{n_s} - \frac{\tilde{\sigma}}{n_{\tilde{s}}}\right)\frac{\beta + \tilde{\beta}}{\sigma l_b + \tilde{\sigma}\tilde{l}_b}\frac{l_b \tilde{l}_b}{L}\right](T_2 - T_1).$$

The second term in the square brackets is due to the presence of a thermoelectric field near the contacts. It is, in order of magnitude, smaller than the first principal term by a factor of L/l_b.

Notice that the Peltier effect does not occur at the

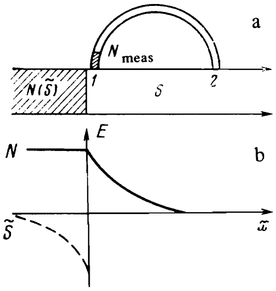

FIG. 2. N–S or S–$\tilde{S}$ system (Fig. a) allowing the measurements of the thermoelectric field E (Fig. b) in the superconductor S. The dashed curve in Fig. 2b represents the field E in the superconductor $\tilde{S}$.

S–$\tilde{S}$ contact. Indeed, the heat flux $\mathbf{q}$ for $T\to T_c$ is determined by the same expression that determines the flux in a normal metal[12]:

$$\mathbf{q}=\Pi\mathbf{j}_n-\varkappa\nabla T. \tag{22}$$

Let the temperature gradient be equal to zero; then $\mathbf{j}_n=0$. Therefore, no heat is released (absorbed) when a superconducting current passes through the contact.

The Peltier effect occurs at an S–N contact. Furthermore, it is worth noting that in a superconductor, near an S–N contact, as well as an S–$\tilde{S}$ contact when $\nabla T\neq 0$, heat will be absorbed or released, depending on the direction of the current j_n, over the distance l_b over which the superconducting current is converted into a quasiparticle current. In this case heat is released on one, and absorbed on the other side of the S–$\tilde{S}$ contact when $\nabla T\neq 0$.

A more direct proof of the existence of the thermoelectromotive force in a superconductor can be obtained, using for the measurement the system schematically represented in Fig. 2a. Such a system was used for the superconductor-resistance measurement in[19]. The temperature gradient T_x' at the S–N or S–$\tilde{S}$ contact will lead to the appearance of a field E that falls off with increasing distance from the contact into the S region (Fig. 2b) and, consequently, to a potential difference between the points 1 and 2:

$$\mathscr{E}_{12}=\frac{\beta}{\sigma}l_bT_x' \text{ for } S\text{–}N,\quad \mathscr{E}_{12}=\frac{l_b\tilde{l}_b(\tilde{\beta}\sigma-\beta\tilde{\sigma})}{(\tilde{\sigma}l_b+\sigma\tilde{l}_b)\tilde{\sigma}}T_x' \text{ for } S\text{–}\tilde{S}.$$

If we make the dimensions of the normal region, N_{meas}, in the measuring circuit sufficiently small, then we can, in principle, make the difference between the temperatures at the boundaries of N_{meas} and, hence, the thermoelectromotive force in N_{meas} vanish. There will then arise in the contour a current $I=\mathscr{E}_{12}/R$, which can be measured (here R is the resistance of the region N_{meas}). Indeed, let us integrate the field E around a contour passing through the measuring circuit and the superconductor S:

$$\oint E\,dl=\int_{N_{meas}}E\,dl+\int_1^2 E\,dl+\delta V=0.$$

The first term is the potential drop across the normal metal N_{meas}, the second term is the potential difference in S between the points 1 and 2, and the last term is the potential jump across the S–N_{meas} contact, due to the nonequilibrium state obtaining in S. Near the critical temperature $\delta V\sim(\Delta/T)^2\Phi$,[20] and this term can be neglected. In the superconductor forming the measuring circuit the field is zero, since $\Phi=0$ in it (we assume that the distance between the points 1 and 2 is much greater than l_b), and the contribution to the integral from the integration along the semicircle is equal to zero. Thus, the potential difference $\mathscr{E}_{12}$, between the points 1 and 2 will give rise in the metal N_{meas} to a current that will produce in N_{meas} a potential drop balancing $\mathscr{E}_{12}$.

Let us note one important circumstance. We have thus far spoken of the thermoelectric field $eE=-\Phi_x'$ as an electric field, and we have identified the potential Φ with the electrostatic potential φ. However, in the case when the superconductor S is inhomogeneous, or when the temperature dependence of the chemical potential μ_0 is important, under E should be understood the gradient of the electrochemical potential of the quasiparticles, $\mu_{ec}=\mu_0+e\varphi$, just as is done in the theory of thermoelectric phenomena in normal metals.[21] The gradient of μ_{ec} gives rise to the quasiparticle current

$$j_q=-\sigma\nabla\mu_{ec}$$

the expression for which is, near T_c, the same as in the case of a normal metal. In a measuring circuit consisting of a normal metal (N_{meas}) the current is produced precisely by the potential difference μ_{ec}. We neglected the gradient of μ_0, since it is quite small:

$$\nabla\mu_{ec}\sim(T/\varepsilon_F)^2\nabla T.$$

The appearance of an electric field in a superconductor as a result of the dependence of μ_0 on temperature is considered in[7]. However, this field, $e\tilde{E}=-\nabla\mu_0(T)$, is not a thermoelectric field, since in this case the gradient of the electrochemical potential is equal to zero (the second term in (2) was neglected by the authors of[7]):

$$\nabla(\mu_0+e\varphi)=0.$$

The field $e\tilde{E}=-\nabla\mu_0(T)$ does not lead to an electromotive force, and does not make any contribution to the above-discussed experiments. For example, in the scheme shown in Fig. 2a the integral

$$\int_{N_{meas}}\nabla\mu_0(T)\,dl$$

vanishes, since the temperature in N_{meas} is assumed to be a constant.

In our case it is precisely the gradient of the electrochemical potential of the quasiparticles that is different from zero, i.e.,

$$eE=-\nabla(\mu_0+e\varphi)\neq 0;$$

therefore, a current arises in the measuring circuit shown in Fig. 2a. On the other hand, the difference between the "chemical potentials" of the pairs (or the order-parameter phase difference) does not, naturally lead to the appearance of current in the measuring circuit. The force acting on the condensate in the superconductor S is equal to zero:

$$\nabla(\mu_0+e\varphi)+\nabla\Phi=0,$$

i.e., in the case under consideration the fields exert different forces on the condensate and the quasiparticles (it can be shown that the electrochemical potentials of the quasiparticles and the pairs do not coincide[6,13]).

CONCLUSION

Thus, in the presence of a temperature gradient near the boundary of a superconductor with a dielectric, a normal metal, or another superconductor there arises a thermoelectric field E. The field E also arises in the volume of the superconductor if the temperature gradient is not a constant with respect to the coordinate, i.e., if $T''_{xx}\neq 0$. The attenuation length of the field E in pure superconductors near T_c can attain macroscopic values

$$l_b=(T/\Delta)^{1/2}v/(\nu\nu_{en})^{1/2}\approx 10^{-1}\text{ cm}$$

when $\nu_{en}\approx T^3/\Theta_D^2\sim 10^9$ sec^{-1}, $\nu\sim 10^{10}$ sec^{-1}, $v\sim 10^8$ cm/sec, and $T/\Delta\sim 10$. Therefore, the thermoelectric field E in superconductors and the thermoelectromotive force connected with this field can be measured in an experiment.

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Note added in proof (*February* 16, 1976). In a recently published paper (J. Low Temp. Phys. **20**, 207 (1975)), A. Schmid and G. Schön have, in particular, derived for the potential on the basis of the Eliashberg equations in the limit of dirty superconductors ($\nu\gg\Delta$) an equation of the type of the Eq. (6) of our paper, and have computed the attenuation depth of the electric field in the superconductor near its boundary with a normal metal. However, the conclusion drawn in that paper that, as the temperature approaches the critical temperature, the attenuation depth for the field varies from a value of the order of l_b to a value of the order of the correlation length $\xi(T)$ seems to us to be incorrect. Let us also note that near T_c we can replace the function a_0 in the formula (16) by its value in the zeroth order in Δ/T and obtain for ν_b the exact result: $\nu_b=7\pi^2\zeta(3)\,\zeta_{ph}\Delta T^2\Theta_D^{-2}$.

1) If, on the other hand, the superconductor is anisotropic or inhomogeneous in the transverse—to the temperature gradient—direction (for example, along the y axis), then the velocity $\mathbf{v}_s$ will depend on y. Consequently, curl $\mathbf{v}_s\sim\mathbf{H}\neq 0$, and there will arise in the superconductor a circulating current[1-3] that can be measured experimentally.[4,5]

2) We assume the modulus of the order parameter is equal to the equilibrium value, since it varies in second order in T'_x, which is of no interest to us. Notice that to the potential Φ in the theory of superfluid helium corresponds a correction to the chemical potential, h, for which an equation similar to (3) is valid.[9]

3) The thermoelectric field near the boundary of a superconductor with a dielectric was found earlier.[10]

4) A longitudinal electric field arises also in the vicinity of the center of a vortex moving under the action of a transport current.[16,17]

1. V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **14**, 177 (1944).
2. B. T. Geĭlikman and V. Z. Kresin, Kineticheskie i nestatsionarnye yavleniya v sverkhprovodnikakh (Kinetic and Nonstationary Phenomena in Superconductors), Nauka, 1972.
3. Yu. M. Gal'perin, V. L. Gurevich, and V. I. Kozub, Pis'ma Zh. Eksp. Teor. Fiz. 17, 687 (1973) [JETP Lett. 17, 476 (1973)].
4. N. V. Zavaritskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 205 (1974) [JETP Lett. **19**, 126 (1974)].
5. P. M. Selzer and W. M. Fairbank, Phys. Lett. **A48**, 279 (1974).
6. T. J. Rieger, D. J. Scalapino, and J. E. Mercereau, Phys. Rev. Lett. **27**, 1787 (1971).
7. S. Putterman and B. De Bruyn Ouboter, Phys. Rev. Lett. **24**, 50 (1970).
8. L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
9. I. M. Khalatnikov, Teoriya sverkhtekuchesti (The Theory of Superfluidity), Nauka, 1971.
10. S. N. Artemenko and A. F. Vokov, Pis'ma Zh. Eksp. Teor. Fiz. 21, 662 (1975) [JETP Lett. 21, 313 (1975)].
11. A. F. Volkov and Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **65**, 2038 (1973) [Sov. Phys. JETP **38**, 1018 (1974)].
12. A. G. Aronov and V. L. Gurevich, Fiz. Tverd. Tela **16**, 2656 (1974) [Sov. Phys. Solid State **16**, 1722 (1975)].
13. M. Tinkham and J. Clarke, Phys. Rev. Lett. **28**, 1366 (1972).
14. M. Tinkham, Phys. Rev. **B6**, 1747 (1972).
15. A. F. Volkov, Zh. Eksp. Teor. Fiz. **66**, 758 (1974) [Sov. Phys. JETP **39**, 366 (1974)].
16. M. Yu. Kupriyanov and K. K. Likharev, Pis'ma Zh. Eksp. Teor. Fiz. 15, 349 (1972) [JETP Lett. 15, 247 (1972)].
17. Chia-Ren Hu and R. S. Thompson, Phys. Rev. **B6**, 110 (1972).
18. A. B. Pippard, J. G. Shepherd, and D. A. Tindall, Proc. Roy. Soc., London, **A324**, 17 (1971).
19. M. L. Yu and J. E. Mercereau, Phys. Rev. Lett. **28**, 1117 (1972).
20. A. F. Volkov and A. V. Zaĭtsev, Zh. Eksp. Teor. Fiz. **69**, 2222 (1975).
21. L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz, 1959 (Eng. Transl., Pergamon, London, 1960).

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