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Surface quantum spin waves in a degenerate electron liquid in metals in a weak magnetic field

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We have discovered a possibility of propagation of surface waves in metals with a cylindrical Fermi surface; this is due to the electron Fermi-liquid interaction. We evaluate in the framework of the theory of a degenerate Fermi liquid the frequency spectra and damping coefficients and find the conditions for the existence of such waves.

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1. INTRODUCTION

It is well known[1] that the surface states of electrons which have a "glancing" motion along the surface of a metal determine the electromagnetic properties of metals in the uhf band (10^{10} to 10^{11} Hz) in relatively weak magnetic fields (1 to 10 gauss). The centers of the Larmor orbits of the "glancing" electrons are distributed outside the metal at distances which are almost equal to the Larmor radius (Fig. 1) and in momentum space the motion of such electrons corresponds to a closed orbit which bounds the hashed segment (Fig. 2). Transitions between the levels of the surface states lead to resonance absorption of radiation which manifests itself in experimentally observed oscillations of the surface impedance of the metal.

The theoretical possibility of the existence of surface waves in metals with a cylindrical Fermi surface near frequencies of the resonance transition between surface levels was discovered in[2]. In the same paper the impossibility was pointed out of the propagation of surface waves in metals with a spherical Fermi surface. On the other hand, up to now the problem has not been raised about the occurrence of effects which are connected with the electron surface states and which are at the same time caused by interelectron correlations. We shall present in the present paper the results of a theory of spin waves in an electron liquid, which takes into account the effect of electron surface states. The behavior of the surface spin waves found below enables us to confirm that the electron-electron interaction is one of the general causes for the existence of different kinds of surface waves and of the resonance properties of a metal near the frequencies of the electron transitions between surface levels. This enables us to disconnect the existence of absorption resonance and of surface waves from the necessity that there exist some particular shapes of the Fermi surface; this corresponds to a qualitative difference between the new kind of excitations which are considered by us and those predicted in[2].

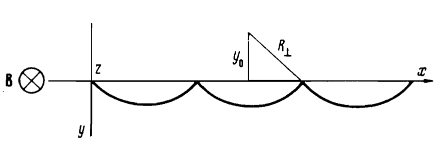

FIG. 1.

2. QUANTUM KINETIC EQUATION

To solve the problem in which we are interested we must first of all carry out a generalization of the quantum kinetic equation of a degenerate electron liquid (see, e.g.,[3]), taking into account the role of the electron surface levels. Bearing in mind that an electron state is characterized by a set of quantum numbers ν and assuming that the density matrix of the ground state of the electrons is diagonal we can write down the non-equilibrium density matrix in the following form:

$$\rho_{\nu'\nu}=\rho^0(\nu)\delta_{\nu'\nu}+\delta\rho_{\nu'\nu}e^{-i\omega t}. \tag{2.1}$$

If we are interested in linear problems we can write down for the non-equilibrium correction to the density matrix the following approximate equation (cf.[4]):

$$\{\hbar\omega-\varepsilon(\nu')+\varepsilon(\nu)\}\delta\rho_{\nu'\nu}+\{\rho^0(\nu')-\rho^0(\nu)\}\cdot \left\{-\frac{1}{c}\sum_{\mathbf{q}}\delta\mathbf{j}_{\nu'\nu}\delta\mathbf{A}_{\mathbf{q},\omega}+\sum_{\nu_2\nu_1}F_{\nu'\nu}^{\nu_1\nu_2}\delta\rho_{\nu_2\nu_1}\right\}=J_{\nu'\nu}^{st}. \tag{2.2}$$

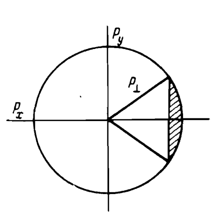

FIG. 2.

This equation is the quantum analog of the quasi-classical kinetic equation from the theory of a degenerate electron liquid (see, e.g., [5]). Here $\varepsilon(\nu)$ is the quasi-particle energy in the equilibrium state, $(\delta A_{q,\omega}$ is the amplitude of the potential of the non-equilibrium electromagnetic field,

$$\delta j_{\nu'\nu}(-q)=\delta_{\sigma'\sigma}e v_{\alpha'\alpha}(-q)-\mu_0 ci[q\sigma]_{\sigma'\sigma}I_{\alpha'\alpha}(-q),$$

$$v_{\alpha'\alpha}(-q)=\frac{1}{2m}\left\langle\alpha'\left|e^{iqr}\left(\hat{p}-\frac{e}{c}A_0\right)+\left(\hat{p}-\frac{e}{c}A_0\right)e^{iqr}\right|\alpha\right\rangle,$$

$$I_{\alpha'\alpha}(-q)=\langle\alpha'|e^{iqr}|\alpha\rangle.$$

We have here taken into account the fact that the complete set of quantum numbers includes orbital (α) and spin (σ) numbers ($\nu\equiv\alpha,\sigma$), while $e=-|e|$ is the charge of the electron, μ_0 the electron magnetic moment, σ the Pauli matrices vector, A_0 the vector potential of the static magnetic field, and m the electron effective mass corresponding to a quadratic momentum dependence of the energy. We note that the generalization to a more complex dispersion law for the electron energy does, as usual, not pose any difficulties.

The last term on the left-hand side of Eq. (2.2) describes the Fermi-liquid interaction between the electrons. Neglecting the spin-orbit interaction we have

$$F^{\nu_1\nu_2}_{\nu'\nu}=\varphi^{\alpha_1\alpha_2}_{\alpha'\alpha}\delta_{\sigma'\sigma}\delta_{\sigma_1\sigma_2}+\psi^{\alpha_1\alpha_2}_{\alpha'\alpha}(\sigma_{\sigma'\sigma}\sigma_{\sigma_1\sigma_2}). \tag{2.3}$$

Aiming to show up effects which are connected with spin waves, we restrict ourselves to the simplest model for the Fermi-liquid interaction (cf. [3]) when

$$\varphi^{\alpha_1\alpha_2}_{\alpha'\alpha}=0,$$

$$\psi^{\alpha_1\alpha_2}_{\alpha'\alpha}=\psi\sum_q I_{\alpha'\alpha}(-q)I_{\alpha_1\alpha_2}(q). \tag{2.4}$$

It is expedient for what follows to make a definite choice for the set of orbital quantum numbers α. We shall take them to be: the component p_z of the momentum along the direction of the constant magnetic field, the x-component p_x of the momentum (or the y-component $y_0=-cp_x/eB$ of the coordinate of the center of the electron Larmor orbit), and the energy quantum number n. The choice of such a representation is convenient, in particular, when the metal occupies the half-space $y>0$. We note here that the appearance of surface states of the electron is connected with the satisfying of the boundary condition that the electron wavefunction must vanish for $y=0$. In agreement with[6] we have

$$|\alpha\rangle=\frac{1}{2\pi\hbar}\exp\{i(p_z z+p_x x)\hbar^{-1}\}\Psi\left(\frac{y-y_0}{\lambda};a(p_x,n)\right). \tag{2.5}$$

Here

$$a(p_x,n)=\begin{cases}\dfrac{p_x^2}{2m\hbar\Omega}\left[1+\left(\dfrac{3\pi}{2}\right)^{2/3}\left(\dfrac{2n\hbar\Omega m}{p_x^2}\right)^{2/3}\right] & \text{— for "glancing" electrons}\\ n & \text{— for volume electrons}\end{cases} \tag{2.6}$$

Here $\lambda=(c\hbar/|e|B)^{1/2}$, $\Psi(\xi,a)$ is a parabolic cylinder function, and $\Omega=|e|B/mc$.

The division of the electrons in volume and surface ones is performed according to the following features. For the volume electrons the center of the Larmor orbit lies in the bulk of the metal which means

$$y_0\gg R_\perp=\frac{c}{|e|B}(2m\varepsilon_F-p_z^2)^{1/2}. \tag{2.7}$$

Here $R_\perp$ is the classical radius of the Larmor orbit, and ε_F is the Fermi energy. In terms of the parameters of the wavefunction (2.5) condition (2.7) takes the form

$$y_0^2\gg 2\lambda^2 a(p_x,n).$$

For "glancing" electrons the condition is the opposite one:

$$\left|1-\frac{y_0^2}{2\lambda^2 a(p_x,n)}\right|\ll 1,$$

or, in other words,

$$(R_\perp^2-y_0^2)/R_\perp^2\ll 1. \tag{2.8}$$

Condition (2.8) means that for "glancing" electrons, for which $y_0<0$ and $y_0<|y_0|$, differs little from the Larmor radius $R_\perp$. We shall assume everywhere the energy $\hbar\Omega$ of a Larmor quantum to be small compared with the Fermi energy. For the surface electrons we can, according to[6], write down the following energy level spectrum:

$$\varepsilon_\alpha\equiv\varepsilon(p_z,p_x,n)=\frac{p_x^2}{2m}+\frac{p_z^2}{2m}+\left(\frac{3\pi}{2}\right)^{2/3}\left(\frac{p_x^2}{2m}\right)^{1/3}(n\hbar\Omega)^{2/3}. \tag{2.9}$$

For the volume states we have

$$\varepsilon(p_z,p_x,n)=p_z^2/2m+n\hbar\Omega. \tag{2.10}$$

To write down the kinetic equation we shall assume that the condition for quasi-classical behavior is satisfied:

$$|p_z'-p_z|\equiv\hbar|k_z|\ll|p_z|,\quad n'-n\ll n. \tag{2.11}$$

We shall in what follows be interested in perturbations for which $\hbar k_x=p_x'-p_x=0$, which corresponds to the equality $y_0'=y_0$. In connection with this we can write the non-equilibrium correction to the density matrix in the form:

$$\delta\rho_{\nu'\nu}=\delta\rho(p_z,p_x,n',n,k_z,\sigma',\sigma).$$

It is convenient to use the spin trace of the density matrix—the distribution function

$$\delta f(p_z,p_x,n',n,k_z)=\sum_\sigma\delta\rho(p_z,p_x,n',n,k_z,\sigma,\sigma)$$

and the vectorial spin distribution function which is the convolution of the density matrix and the Pauli matrix vector

$$\delta\sigma(p_z,p_x,n',n,k_z)=\sum_{\sigma'\sigma}\sigma_{\sigma'\sigma}\delta\rho(p_z,p_x,n',n,\sigma',\sigma).$$

We can use Eqs. (2.2) to write down kinetic equations for the distribution functions. The influence of the Fermi-liquid interaction manifests itself in the equation for the spin distribution function ($\delta\sigma^\pm=\delta\sigma^x\pm i\delta\sigma^y$). Such an equation can for the surface states be written in the following form:

$$\left[\omega-k_z v_z-\Omega(n'-n)\pm\Omega_0+\frac{i}{\tau}+\frac{i}{T}\right]\delta\sigma^\pm(p_z,p_x,n',n,k_z)$$
$$+\frac{\partial f_0}{\partial\varepsilon}\left[k_z v_z+(n'-n)\Omega\mp\Omega_0-\frac{i}{\tau}-\frac{i}{T}\right]\sum_{q>0}I_{n'n}(-q,p_x) \tag{2.12}$$
$$\times\left\{-\mu_0\delta B^\pm(0,q,k_z)+\psi\sum_{\substack{p_{1x}p_{1z}\\ rs}}I_{rs}(q,p_x)\delta\sigma^\pm(p_{1z},p_{1x},r,s,k_z)\right\}=\frac{i}{\tau}\frac{\partial f_0}{\partial\varepsilon}.$$

Here

$$f_0=2\rho_0(\varepsilon)=\begin{cases}2, & \varepsilon\leq\varepsilon_F,\\ 0, & \varepsilon>\varepsilon_F\end{cases}$$

$\delta B^{\pm}(0,q,k_z)$ is the Fourier component of the non-equilibrium magnetic field, τ is the momentum relaxation time, and T is the spin relaxation time. For electron spin resonance $T\gg\tau$. In the classical theory[5] it is just such a time T which determines the line width of the electron paramagnetic resonance, $\Omega_0=2\gamma B/\hbar$, γ is the effective electron magnetic moment. The summation in (2.12) is both over the volume and over the surface states, while

$$I_{n'n}(q,\ p_x)=\langle n',\ p_x|e^{iqy}|n,\ p_x\rangle. \tag{2.13}$$

The corresponding equation for surface electrons reads as follows:

$$\left[\omega-k_zv_z-\omega_{n'n}(p_x)\pm\Omega_0+\frac{i}{\tau}+\frac{i}{T}\right]\delta\sigma^{\pm}(p_z,p_x,n',n,k_z)$$
$$+\frac{\partial f_0}{\partial\varepsilon}\left[k_zv_z+\omega_{n'n}(p_x)\mp\Omega_0-\frac{i}{\tau}-\frac{i}{T}\right]\sum_{q>0}I_{n'n}(-q,p_x) \tag{2.14}$$
$$\times\left\{-\mu_0\delta B^{\pm}(0,q,k_z)+\psi\sum_{\substack{p_{1z}p_{1x}\\ rs}}I_{rs}(q,p_{1x})\delta\sigma^{\pm}(p_{1z},p_{1x},r,s,k_z)\right\}=\frac{i}{\tau}\frac{\partial f_0}{\partial\varepsilon}.$$

The characteristic frequency of the electron surface transitions has the form

$$\omega_{n'n}(p_x)=\frac{1}{\hbar}\left(\frac{3\pi}{2}\right)^{2/3}\left(\frac{p_x^2}{2m}\right)^{1/3}(\hbar\Omega)^{2/3}(n'^{2/3}-n^{2/3})=\left(\frac{p_x}{p_F}\right)^{2/3}\omega_{n'n}. \tag{2.15}$$

We have used here for the limiting value of the frequency the notation

$$\omega_{n'n}=\left(\frac{3\pi}{2}\right)^{2/3}\hbar^{-1}\varepsilon_F^{1/3}(\hbar\Omega)^{2/3}(n'^{2/3}-n^{2/3}). \tag{2.16}$$

The summation in (2.14) is over the volume and over the surface states.

3. EIGENFREQUENCIES OF THE SPIN DENSITY OSCILLATIONS

We turn first to obtaining the consequences of the kinetic Eqs. (2.12) and (2.14) for the spin distribution function under conditions such that we can completely neglect the effect of the variable magnetic field. For the quantity

$$\delta s^{\pm}(\mathbf{k})=\sum_{p_zp_xn'n}I_{n'n}(k_y,p_x)\delta\sigma^{\pm}(p_z,p_x,n',n,k_z), \tag{3.1}$$

which is the Fourier component of the spatial electron spin density we can then write down the following equation:

$$\delta s^{\pm}(0,k_y,k_z)+\sum_{k_y'>0}Q(k_z,k_y,k_y')\delta s^{\pm}(0,k_y',k_z)=0, \tag{3.2}$$

where

$$Q(k_z,k_y,k_y')=\sum_{p_zp_xn'n}\frac{\partial f_0}{\partial\varepsilon}I_{n'n}(k_y,p_x)I_{n'n}(-k_y',p_x)$$
$$\times\left[\omega-k_zv_z-\tilde{\omega}_{n'n}(p_x)\pm\Omega_0+\frac{i}{\tau}+\frac{i}{T}\right]^{-1}$$
$$\times\left\{\psi\left[k_zv_z+\tilde{\omega}_{n'n}(p_x)\mp\Omega_0-\frac{i}{T}\right]-\frac{i}{\tau}\left[\sum\frac{\partial f_0}{\partial\varepsilon}\right]^{-1}\right\}. \tag{3.3}$$

Here $\tilde{\omega}_{n'n}(p_x)$ equals $(n'-n)\Omega$ for volume states and it equals expression (2.15) for surface electrons. From a consideration of the electron gas model[2] it follows that surface waves cannot propagate in metals with a Fermi surface that is nearly spherical. To understand better the new possibilities which are opened up in principle by the interaction between the electrons, we focus our attention on a metal with a spherical Fermi surface. For such a model we obtain all the basic results, which can then be generalized to the case of an arbitrary Fermi surface.

We write the kernel Q in the form of three terms:

$$Q=Q_{\text{vol}}+Q_{\text{nr}}+Q_r, \tag{3.4}$$

due, respectively, to the volume electrons, the nonresonance surface electrons, and the surface electrons with a maximum value of the transition frequency $\omega_{n'n}$ which is close to the value of the frequency ω. Since we are interested in the case of rather weak fields when $\omega\gg\Omega$, Ω_0, we can completely neglect the effect of the magnetic field on the volume electrons. In that case

$$Q_{\text{vol}}(k_z,k_y,k_y')=\sum_{p_{\parallel}n}\frac{\partial f_0}{\partial\varepsilon}\frac{\pi^2\hbar^3v}{p_F^2}\left[\frac{B_0(k_zv_z+(n'-n)\Omega)+i\tau^{-1}}{\omega-(n'-n)\Omega-k_zv_z+i\tau^{-1}+iT^{-1}}\right]$$
$$\times J_{|n'-n|}\left(\frac{k_yv_\perp}{\Omega}\right)J_{|n'-n|}\left(\frac{k_yv_\perp}{\Omega}\right)=-\delta_{k_yk_y'}\left\{-B_0+\left[B_0\frac{\omega}{2kv}+\frac{i}{\omega\tau}\right]\ln\frac{\omega+kv}{\omega-kv}\right\}, \tag{3.5}$$

where $v=|\mathbf{v}|$ is the velocity on the Fermi surface, $B_0=m^2v/\hbar^3\pi^2$; $J_s(x)$ is a Bessel function, and $\mathbf{k}=(0,k_y,k_z)$. The contribution of the resonance electrons is given by the expression

$$Q_r(k_z,k_y,k_y')=-\frac{2}{(2\pi\hbar)^2}\int dp_x\,dp_zI_{n'n}(-k_y',p_x)$$
$$\times I_{n'n}(k_y,p_x)\delta(\varepsilon_F-\varepsilon(p_z,p_x,n))$$
$$\times\frac{B_0[k_zv_z+\omega_{n'n}(p_x)\mp\Omega_0]+i\tau^{-1}}{\omega\pm\Omega_0-\omega_{n'n}(p_x)-k_zv_z+i\tau^{-1}+iT^{-1}}\frac{\pi^2\hbar^3}{mp_F}, \tag{3.6}$$

$\varepsilon(p_z,p_x,n)$ is defined by (2.9). The possibility of separating the resonance term is connected with the fact that we are interested in the solution of Eq. (3.2) in the case

$$|\omega\pm\Omega_0-\omega_{n'n}|\ll\omega. \tag{3.7}$$

By virtue of the fact that $\Omega_0\ll\omega_{n'n}$ we shall in what follows neglect the Ω_0-dependence, drop the $\pm$ sign of δs, and rewrite condition (3.7) in the form

$$|\omega-\omega_{n'n}|\ll\omega. \tag{3.8}$$

We emphasize that the frequency ω becomes resonant with the maximum value of the frequency $\omega_{n'n}(p_x)$ given by Eq. (2.16). Condition (3.8) determines the numbers n' and n in Eq. (3.6).

It is clear from Eq. (2.15) that when we write $p_x=p_F\sin\theta$ for a spherical Fermi surface we can change in Eq. (3.6) to integration over the angle θ:

$$Q_r(k_z,k_y,k_y')=-\int_0^{\pi}d\theta\,I_{n'n}(-k_y',p_F\sin\theta)I_{n'n}(k_y,p_F\sin\theta)\frac{\hbar}{2mv}$$
$$\times\frac{B_0[k_zv\cos\theta+\omega_{n'n}\sin^{2/3}\theta]+i\tau^{-1}}{\omega-\omega_{n'n}\sin^{2/3}\theta-k_zv\cos\theta+i\tau^{-1}+iT^{-1}}. \tag{3.9}$$

Because of condition (3.8) the main contribution to the the integral (3.9) comes from the range of angles close to $\theta=\pi/2$, and therefore putting everywhere except in the resonance denominator $\theta=\pi/2$ we get

$$Q_r(k_z,k_y,k_y')=-I_{n'n}(-k_y',p_F)I_{n'n}(k_y,p_F)\frac{\hbar}{2mv}$$

$$\times\left[B_0+\frac{i}{\omega_{n'n}\tau}\right]2\sqrt{3}(\Delta+i\gamma)^{-1/2}\operatorname{arctg}\left[\frac{\pi}{2\sqrt{3}}(\Delta+i\gamma)^{-1/2}\right]. \quad (3.10)$$

Here

$$\Delta=\frac{\omega-\omega_{n'n}}{\omega_{n'n}}-\frac{3}{4}\left(\frac{k_zv}{\omega_{n'n}}\right)^2,$$

$$\gamma=(\omega_{n'n}\tau)^{-1}+(\omega_{n'n}T)^{-1}.$$

It follows from (3.10) that in the case where the argument of the arctangent is large compared to unity (and this is possible for small k_z) there appears a resonance factor $|(\Delta+i\gamma)^{-1/2}|\gg 1$ in Eq. (3.10) when $|\Delta|\ll 1$. We shall in what follows be interested in the case when the following conditions are satisfied:

$$k_zv\ll\omega_{n'n},\quad |\Delta|\ll 1. \quad (3.11)$$

We consider the contribution from the non-resonance electrons to the kernel of Eq. (3.3)

$$Q_{\rm nr}(k_z,k_y,k_y')=-\sum_{rs}{}'\,2\int\frac{dp_x\,dp_z}{(2\pi\hbar)^2}I_{rs}(-k_y',p_x)I_{rs}(k_y,p_x)$$

$$\times\delta(\varepsilon_F-\varepsilon(p_z,p_x,r))\frac{B_0[k_zv_z+\omega_{rs}(p_x)]+i\tau^{-1}}{\omega-\omega_{rs}(p_x)-k_zv_z+i\tau^{-1}+iT^{-1}}\frac{\pi^2\hbar^3}{mp_F}$$

$$=-\sum_{rs}{}'\int_0^{\pi}d\theta\,I_{rs}(-k_y',p_F\sin\theta)I_{rs}(k_y,p_F\sin\theta)\cdot$$

$$\frac{\hbar}{2p_F}\frac{B_0[\omega_{rs}\sin^{2/3}\theta+k_zv\cos\theta]+i\tau^{-1}}{\omega-\omega_{rs}\sin^{2/3}\theta-k_zv\cos\theta+i\tau^{-1}+iT^{-1}}. \quad (3.12)$$

The prime on the summation sign in (3.12) indicates that $r\neq n'$, $s\neq n$. The fact that under well-defined conditions the contribution from the sum of the non-resonance terms can be small compared to the contribution from the resonance term can be most simply demonstrated in the limit $k_z=0$ and $\omega\tau\gg 1$. We evaluate separately the real and imaginary parts of $Q_{\rm nr}(0,k_y,k_y')$. We have

$$\operatorname{Im}Q_{\rm nr}(0,k_y,k_y')=\sum_{rs}{}'\frac{3\pi\hbar}{2p_F}B_0\int d(\sin^{2/3}\theta)\operatorname{tg}\theta I_{rs}(-k_y',p_F\sin\theta)$$

$$\times I_{rs}(k_y,p_F\sin\theta)\delta\left(\frac{\omega}{\omega_{rs}}-\sin^{2/3}\theta\right) \quad (3.13)$$

$$=\frac{3\pi\hbar}{2p_F}\sum_{rs}{}'\left(\frac{\omega^3_{n'n}}{\omega_{rs}^3-\omega^3_{n'n}}\right)^{1/2}B_0I_{rs}\left(-k_y',p_F\left(\frac{\omega_{n'n}}{\omega_{rs}}\right)^{3/2}\right)I_{rs}\left(k_y,p_F\left(\frac{\omega_{n'n}}{\omega_{rs}}\right)^{3/2}\right)$$

In obtaining (3.13) we took condition (3.8) into account. The summation in (3.13) is over those r and s for which the condition

$$r^{2/3}-s^{2/3}>n'^{2/3}-n^{2/3}. \quad (3.14)$$

is satisfied. The real part of $Q_{\rm non\text{-}res}$ is given by the following expression:

$$\operatorname{Re}Q_{\rm nr}(0,k_y,k_y')$$

$$=-\sum{}'\frac{\pi\hbar\sqrt{3}}{2p_F}B_0I_{rs}(k_y,p_F)I_{rs}(-k_y',p_F)\left(\frac{r^{2/3}-s^{2/3}}{n'^{2/3}-n^{2/3}-r^{2/3}+s^{2/3}}\right)^{1/2}, \quad (3.15)$$

where the summation is over

$$r^{2/3}-s^{2/3}<n'^{2/3}-n^{2/3}.$$

Using (3.5), (3.10), (3.13), and (3.15) we can write the integral equation (3.2) in the form

$$\delta s(0,k_y,0)\left[1+B_0-B_0\frac{\omega}{k_yv}\ln\frac{\omega+k_yv}{\omega-k_yv}\right]-I_{n'n}(k_y,p_F)$$

$$\times\int_0^\infty\frac{dk_y'}{\pi}I_{n'n}(-k_y',p_F)\delta s(0,k_y',0)B_0\frac{\pi\hbar\sqrt{3}}{2mv}\left(\frac{\omega_{n'n}}{\omega-\omega_{n'n}}\right)^{1/2}$$

$$=\sum_{r^{2/3}-s^{2/3}<n'^{2/3}-n^{2/3}}\frac{\pi\hbar}{2p_F}B_0I_{rs}(k_y,p_F)$$

$$\times\int_0^\infty\frac{dk_y'}{\pi}I_{rs}(-k_y',p_F)\delta s(0,k_y',0)\left(\frac{r^{2/3}-s^{2/3}}{n'^{2/3}-n^{2/3}-r^{2/3}+s^{2/3}}\right)^{1/2}$$

$$-i\sum_{r^{2/3}-s^{2/3}>n'^{2/3}-n^{2/3}}\frac{3\pi\hbar}{2p_F}B_0I_{rs}\left(k_y,p_F\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}\right)^{3/2}\right)$$

$$\times\int_0^\infty\frac{dk_y'}{\pi}\delta s(0,k_y',0)I_{rs}\left(-k_y',p_F\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}\right)^{3/2}\right)$$

$$\times\left[\frac{(r^{2/3}-s^{2/3})^3}{(n'^{2/3}-n^{2/3})^3-(r^{2/3}-s^{2/3})^3}\right]^{1/2}. \quad (3.16)$$

The integration over k_y' is from zero to ∞, which corresponds to an even continuation of the non-equilibrium spin density into the region $y<0$. We note here that if we use the wavefunctions of the surface states[6] in the form which is valid in the region $y<|y_0|$:

$$\langle n,p_x|=\Psi\left(\frac{y-y_0}{\lambda},a(p_x,n)\right)$$

$$=\frac{1}{\lambda^{1/2}}\cos\left[\frac{2}{3}a(p_x,n)\left(1-\frac{(y+|y_0|)^2}{2a(p_x,n)\lambda}\right)^{3/2}-\frac{\pi}{4}\right], \quad (3.17)$$

where the notation is the same as in Eqs. (2.5), (2.6), and (2.8), and using an expansion of (3.17) in powers of $y/|y_0|$, we have

$$\Psi\left(\frac{y-|y_0|}{\lambda};a(p_x,n)\right)=\frac{1}{\lambda^{1/2}}\sin\frac{p_y(n,p_x)}{\hbar}y, \quad (3.18)$$

where

$$p_y(n,p_x)=(2m)^{1/2}\left(\frac{3\pi}{2}\right)^{1/3}\left(\frac{p_x^2}{2m}\right)^{1/6}(n\hbar\Omega)^{1/3}. \quad (3.19)$$

The wavefunction (3.18) satisfies the condition that it vanish at the metal boundary. Substituting the wavefunction (3.18) of the surface state into the matrix element $I_{n'n}(-k_y,p_F)$ we find that the latter does not vanish for k_y values that satisfy the condition

$$\hbar k_y=p_{n'}\pm p_n. \quad (3.20)$$

We have written here $p_y(n,p_F)=p_n$. One checks easily that for such k_y the condition $k_yv\gg\omega$ is satisfied which enables us to neglect the last term in the square brackets on the left-hand side of Eq. (3.16). We multiply the integral Eq. (3.16) by $I_{n'n}(-k_y,p_F)$ and integrate over k_y. We then get

$$\left[1+B_0-N_{n'n}B_0\frac{\pi\hbar\sqrt{3}}{mv}\frac{1}{\Delta^{1/2}}\right]\int_0^\infty\frac{dk_y'}{\pi}I_{n'n}(-k_y',p_F)\delta s(0,k_y',0)$$

$$=-\sum_{r^{2/3}-s^{2/3}>n'^{2/3}-n^{2/3}}\frac{3\pi\hbar i}{mv}B_0M_{rs}^{n'n}\left[\left(\frac{r^{2/3}-s^{2/3}}{n'^{2/3}-n^{2/3}}\right)^3-1\right]^{-1/2}$$

$$\times\int_0^\infty\frac{dk_y'}{\pi}I_{rs}\left(-k_y',\ p_F\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}\right)^{3/2}\right)\delta s(0,k_y',0)$$

$$+\sum_{r^{2/3}-s^{2/3}<n'^{2/3}-n^{2/3}}\frac{\pi\hbar\sqrt{3}}{mv}B_0N_{rs}^{n'n}\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}-1\right)^{-1/2}$$

$$\times\int_0^\infty\frac{dk_y'}{\pi}I_{rs}(-k_y',\ p_F)\delta s(0,k_y',0). \quad (3.21)$$

Here

$$N_{rs}^{n'n}=\int_0^\infty \frac{dk_y}{2\pi} I_{n'n}(-k_y, p_F) I_{rs}(k_y, p_F),$$

$$N_{n'n}\equiv N_{n'n}^{n'n},$$

$$M_{rs}^{n'n}=\int_0^\infty \frac{dk_y}{2\pi} I_{n'n}(-k_y, p_F) I_{rs}\left(k_y, p_F\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}\right)^{1/2}\right). \qquad (3.22)$$

Using Eq. (3.18) we get the following explicit expressions for (3.22):

$$N_{n'n}=\frac{R}{4\lambda^2}\equiv\frac{p_F}{4\hbar}, \qquad R=\frac{cp_F}{eB},$$

$$N_{rs}^{n'n}=\frac{1}{8}\frac{p_F}{\hbar}\left\{R^{-1}\left[\frac{p_r-p_s}{\hbar}\pm\frac{p_{n'}-p_n}{\hbar}\right]^{-1}\sin\left[\frac{p_r-p_s}{\hbar}\pm\frac{p_{n'}-p_n}{\hbar}\right]R\right.$$
$$-R^{-1}\left[\frac{p_r+p_s}{\hbar}\pm\frac{p_{n'}-p_n}{\hbar}\right]^{-1}\sin\left[\frac{p_r+p_s}{\hbar}\pm\frac{p_{n'}-p_n}{\hbar}\right]R$$
$$-R^{-1}\left[\frac{p_r-p_s}{\hbar}\pm\frac{p_{n'}+p_n}{\hbar}\right]^{-1}\sin\left[\frac{p_r-p_s}{\hbar}\pm\frac{p_{n'}+p_n}{\hbar}\right]R$$
$$\left.+R^{-1}\left[\frac{p_r+p_s}{\hbar}\pm\frac{p_{n'}+p_n}{\hbar}\right]^{-1}\sin\left[\frac{p_r+p_s}{\hbar}\pm\frac{p_{n'}+p_n}{\hbar}\right]R\right\},$$

$$M_{rs}^{n'n}=\frac{1}{8}\frac{p_F}{\hbar}\left\{\pm R^{-1}\left[\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}\right)^{1/2}\frac{p_r\mp p_s}{\hbar}\pm\frac{p_{n'}\mp p_n}{\hbar}\right]^{-1}\right.$$
$$\left.\times\sin\left[\left(\frac{n'^{2/3}-n^{2/3}}{r^{2/3}-s^{2/3}}\right)^{1/2}\frac{p_r\mp p_s}{\hbar}\pm\frac{p_{n'}\mp p_n}{\hbar}\right]R\right\}. \qquad (3.23)$$

In the left-hand side of the formula for $M_{rs}^{n'n}$ we imply in the braces a sum of the terms which differ by signs. We note that

$$p_n R\hbar^{-1}=\frac{cp_F}{\hbar eB}(2m)^{1/2}\left(\frac{3\pi}{2}\right)^{1/3}\left(\frac{p_F^2}{2m}\right)^{1/6}(n\hbar\Omega)^{1/3}=10^5 n^{1/3}B^{-2/3}\ [\mathrm{Oe}]. \qquad (3.24)$$

It follows from Eqs. (3.23) and (3.24) that $N_{rs}^{n'n}$ and $M_{rs}^{n'n}$ are always much smaller than $N_{n'n}$ except for those cases when the argument of the sine can vanish. This can occur for $N_{rs}^{n'n}$, if

$$r^{1/3}-s^{1/3}=n'^{1/3}-n^{1/3}, \qquad (3.25)$$

and for the argument of the sine in $M_{rs}^{n'n}$, if

$$\frac{r^{1/3}-s^{1/3}}{r^{1/3}+s^{1/3}}=\frac{n'^{1/3}-n^{1/3}}{n'^{1/3}+n^{1/3}}. \qquad (3.26)$$

However, even when conditions (3.25) or (3.26) are satisfied, the contribution from the real part of the sum of non-resonance terms will still be small, if

$$\Delta=\frac{\omega-\omega_{n'n}}{\omega_{n'n}}\ll\frac{n'^{1/3}+n^{1/3}}{r^{1/3}+s^{1/3}}-1, \quad r^{2/3}-s^{2/3}<n'^{2/3}-n^{2/3}, \qquad (3.27)$$

while the contribution from the imaginary part of the sum of non-resonance terms can be neglected under the conditions

$$\Delta=\frac{\omega-\omega_{n'n}}{\omega_{n'n}}\ll\left(\frac{r^{1/3}+s^{1/3}}{n'^{1/3}+n^{1/3}}\right)^6-1, \quad r^{2/3}-s^{2/3}>n'^{2/3}-n^{2/3}. \qquad (3.28)$$

Conditions (3.27) and (3.28) are satisfied for small numbers n', n less than a few times ten. Just for transitions with small numbers n' and n there are experimental data[1] about oscillations in weak magnetic fields.

If we take into account what we have just said, the dispersion equation for the surface spin oscillations near the frequency of the transition with numbers n' and n less than a few times ten takes the form

$$1+B_0=B_0\frac{\pi\sqrt{3}}{4\sqrt{\Delta}}. \qquad (3.29)$$

Hence we get the limiting value of the frequency of the surface oscillations for $k_z=0$:

$$\omega=\omega_{n'n}\left(1+\left(\frac{B_0}{1+B_0}\right)^2\frac{3\pi^2}{16}\right). \qquad (3.30)$$

The difference between the frequency of the surface spin oscillations and the value $\omega_{n'n}$ is caused by the Fermi-liquid interaction between the electrons. It is just the appearance of such a shift from the resonance frequency of the transition which enables us to speak about the possibility of the propagation of undamped spin oscillations in metals with a spherical Fermi surface.

Such a statement differs in principle from conclusions obtained when using a gas model of a metal with a spherical Fermi surface, in which the propagation of surface waves is impossible because of the strong collisionless Landau damping. We can therefore state that the electron-electron interaction is one of the general causes for the existence of different kinds of surface waves in the frequency range for transitions between surface levels and enables us to discard the idea that the presence of such waves is exclusively connected with cylindrical sections of the Fermi surface, as is done in the electron gas model. It is clear from (3.29) that the solution of the dispersion equation exists when the condition

$$B_0/(1+B_0)>0. \qquad (3.31)$$

is satisfied. In fact, condition (3.31) means $B_0>0$ for normal metals.

4. ALLOWANCE FOR DISSIPATIVE EFFECTS, THE FINITE WAVELENGTH, AND THE SHAPE OF THE FERMI SURFACE

In this section we consider the consequences from the integral equation (3.2) in which we drop the sum of the non-resonance terms which we can do, as we showed above, for values of n' and n less than a few times ten

$$\left(1+B_0+\frac{1}{\omega_{n'n}\tau}\right)\delta s(0,k_y,k_z)=\int_0^\infty\frac{dk_y'}{\pi}\delta s(0,k_y',k_z)$$
$$\times I_{n'n}(-k_y',p_F)I_{n'n}(k_y,p_F)\frac{\hbar}{2mv}\left[B_0+\frac{i}{\omega_{n'n}\tau}\right]$$
$$\times 2\sqrt{3}(\Delta+i\gamma)^{-1/2}\operatorname{arctg}\frac{\pi}{2\sqrt{3}}(\Delta+i\gamma)^{-1/2}, \qquad (4.1)$$

where the notation is similar to the one used when writing down Eq. (3.10). Assuming first of all that

$$|\Delta|\ll 1, \quad \omega_{n'n}\tau\gg 1,$$

we get as the condition that (4.1) can be solved the following dispersion equation:

$$1+B_0=\left[B_0+\frac{i}{\omega_{n'n}\tau}\right]\frac{\pi\sqrt{3}}{4}(\Delta+i\gamma)^{-1/2}. \qquad (4.2)$$

Introducing the notation $\mathrm{Re}\Delta=\tilde{\Delta}$ and $\mathrm{Im}\Delta+\gamma=\tilde{\gamma}$ we have the following equations:

$$(\tilde{\Delta}+\sqrt{\tilde{\Delta}^2+\tilde{\gamma}^2})^{1/2}=\sqrt{2}\frac{B_0}{1+B_0}\frac{\pi\sqrt{3}}{4}$$
$$(-\tilde{\Delta}+\sqrt{\tilde{\Delta}^2+\tilde{\gamma}^2})^{1/2}=\sqrt{2}[\omega_{n'n}\tau(1+B_0)]^{-1}\frac{\pi\sqrt{3}}{4}. \qquad (4.3)$$

It follows from (4.3) that solutions exist only when $\tilde{\Delta}>0$ and $B_0>0$. Solving the set (4.3) we find

$$\mathrm{Re}\,\Delta=[B_0^2-(\omega_{n'n}\tau)^{-2}]\frac{3\pi^2}{16(1+B_0)^2},$$
$$\mathrm{Im}\,\Delta=-\frac{1}{\omega_{n'n}\tau}-\frac{1}{\omega_{n'n}T}+\frac{1}{\omega_{n'n}\tau}\frac{3\pi^2}{16}\frac{2B_0}{1+B_0}. \qquad (4.4)$$

It follows from (4.4) that the eigenfrequencies of the spin waves lie above the frequency of the surface transition.

We can now write the frequency spectrum and damping rate of the surface spin waves in the form

$$\omega=\omega_{n'n}\left\{1+\frac{3\pi^2}{16(1+B_0)^2}[B_0^2-(\omega_{n'n}\tau)^{-2}]-i(\omega_{n'n}T)^{-1}\right.$$
$$\left.-i(\omega_{n'n}\tau)^{-1}\left[1-\frac{3\pi^2}{16}\frac{2B_0}{1+B_0}\right]+\frac{3}{4}\left(\frac{k_zv}{\omega_{n'n}}\right)^2\right\}. \quad (4.5)$$

Hence it follows that the line width of the surface oscillations is determined by the momentum relaxation time in contrast to the electron spin resonance line the width of which is determined by the spin relaxation time.[5] Neglecting small corrections we can write the surface spin wave spectrum in the form

$$\omega=\omega_{n'n}\left\{1+\frac{3\pi^2}{16}\left(\frac{B_0}{1+B_0}\right)^2-i(\omega_{n'n}\tau)^{-1}+\frac{3}{4}\left(\frac{k_zv}{\omega_{n'n}}\right)^2\right\}. \quad (4.6)$$

Formula (4.6) extends the limiting Eq. (3.30) to the case of a finite wavelength and a finite value of $\omega_{n'n}\tau$. As in the preceding section we see that the spin wave spectrum can not lie below the surface transition frequency.

In order better to understand the way the surface oscillations spectrum depends on the shape of the Fermi surface we consider the dispersion Eq. (3.2) for a metal with a Fermi surface in the form of an ellipsoid of rotation. We neglect the non-resonance terms and dissipation and also assume that $k_z=0$. We put the origin in momentum space in the center of the ellipsoid in such a way that the axis of rotation makes an angle α with the p_z-axis.

We can find from the Bohr quantization rule the following expression for the frequency of the surface transition, which depends on the angle α:

$$\hbar\omega_{n'n}(p_z,\alpha)=\left(\frac{3\pi}{2}\right)^{2/3}\left(\cos^2\alpha+\frac{m_1}{m_3}\sin^2\alpha\right)^{1/3}(2m_1)^{-1/3}$$
$$\times[p_x^*(p_z,\alpha)]^{2/3}\left(\hbar\frac{eB}{m_1c}\right)^{2/3}(n'^{2/3}-n^{2/3}). \quad (4.7)$$

Here m_1 is the transverse effective mass of an electron on the Fermi surface, and m_3 the longitudinal one. When $m_1=m_3$ Eq. (4.7) changes to (2.15), as should be the case, because

$$p_x^{*2}(p_z,\alpha)=\left(\frac{\cos^2\alpha}{2m_1}+\frac{\sin^2\alpha}{2m_3}\right)^{-1}\left[\varepsilon_F-\frac{p_z^2}{2}\left(\frac{\sin^2\alpha}{m_1}+\frac{\cos^2\alpha}{m_3}\right)\right.$$
$$\left.+\frac{p_z^2\cos^2\alpha\sin^2\alpha(m_3-m_1)^2}{2m_1m_3(m_3\cos^2\alpha+m_1\sin^2\alpha)}\right]. \quad (4.8)$$

It follows from (4.7) that the surface transition frequency as function of p_z reaches a maximum which is equal to

$$\hbar\omega_{n'n}(0,\alpha)=\hbar\omega_{n'n}^{\max}(\alpha)=\left(\frac{3\pi}{2}\right)^{2/3}\left(\cos^2\alpha+\frac{m_1}{m_3}\sin^2\alpha\right)^{1/3}$$
$$\times\varepsilon_F^{1/3}\left(\hbar\frac{eB}{m_1c}\right)^{2/3}(n'^{2/3}-n^{2/3}). \quad (4.9)$$

when $p_z=0$. When $m_3>m_1$ ("prolate" ellipsoid) the largest maximum is reached for $\alpha=0$:

$$\hbar\omega_{n'n}^{\text{maximorum}}=\left(\frac{3\pi}{2}\right)^{2/3}\varepsilon_F^{1/3}\left(\hbar\frac{eB}{m_1c}\right)^{2/3}(n'^{2/3}-n^{2/3})=\hbar\omega_{n'n}^{\max}(0). \quad (4.10)$$

If $m_1>m_3$ ("oblate" ellipsoid) the largest maximum lies at $\alpha=\pi/2$:

$$\hbar\omega_{n'n}^{\text{maximorum}}=\left(\frac{3\pi}{2}\right)^{2/3}\varepsilon_F^{1/3}\left(\hbar\frac{eB}{m_3c}\right)^{2/3}\left(\frac{m_1}{m_3}\right)^{2/3}(n'^{2/3}-n^{2/3})=\hbar\omega_{n'n}^{\max}\left(\frac{\pi}{2}\right). \quad (4.11)$$

It follows from Eqs. (4.9) to (4.11) that the largest value of the surface transition frequency is always reached on a central section, but not always on an extremum with respect to α. In the case $m_1>m_3$ the central section corresponding to $\alpha=0$ is larger than the central section for $\alpha=\pi/2$.

By virtue of the fact that the maximum of the transition frequency is reached on a central section we can write the dispersion equation for the surface oscillations

$$1=\frac{B_0}{1+B_0}\bar{N}_{n'n}\int\frac{dp_z}{v_x(p_z,\alpha)}\frac{\omega_{n'n}(p_z,\alpha)}{\omega-\omega_{n'n}(p_z,\alpha)} \quad (4.12)$$

which α follows from (3.12), by using the expansion

$$\omega_{n'n}(p_z,\alpha)=\omega_{n'n}(0,\alpha)+{}^1/_2\omega''_{n'n}(0,\alpha)p_z^2 \quad (4.13)$$

in the form

$$1=\frac{B_0}{1+B_0}\bar{N}_{n'n}\frac{2^{3/2}}{v_x(0,\alpha)}\frac{\omega_{n'n}(0,\alpha)}{\omega-\omega_{n'n}(0,\alpha)}\sqrt{\frac{\omega_{n'n}(0,\alpha)-\omega}{\omega''_{n'n}(0,\alpha)}}$$
$$\times\operatorname{arctg}\left[p_z^*(\alpha)\sqrt{\frac{\omega''_{n'n}(0,\alpha)}{2[\omega_{n'n}(0,\alpha)-\omega]}}\right]. \quad (4.14)$$

Here

$$\bar{N}_{n'n}=\frac{\hbar}{4\pi}\int_{-\infty}^{\infty}dq\,|\langle n'|e^{iqy}|n\rangle|^2\left[\frac{1}{4\pi}\int\frac{ds}{|\mathbf{v}|}\right]^{-1},$$

$|\mathbf{v}|$ is the velocity on the Fermi surface and ds an element of that surface,

$$B_0=\psi\frac{2}{(2\pi\hbar)^2}\int\frac{ds}{|\mathbf{v}|},$$

the x-component of the electron velocity is given by the following expression:

$$v_x(0,\alpha)=\left.\frac{\partial\varepsilon}{\partial p_x}\right|_{p_z=0}=(2\varepsilon_F)^{1/2}\left(\frac{\cos^2\alpha}{m_1}+\frac{\sin^2\alpha}{m_3}\right)^{1/2}, \quad (4.15)$$

$\omega_{n'n}(0,\alpha)$ is given by Eq. (4.9):

$$\omega''_{n'n}(0,\alpha)=-\frac{1}{\hbar}\left(\frac{3\pi}{2}\right)^{2/3}\left(\cos^2\alpha+\frac{m_1}{m_3}\sin^2\alpha\right)^{1/2}\varepsilon_F^{-2/3}$$
$$\times\left(\hbar\frac{eB}{m_1c}\right)^{2/3}(n'^{2/3}-n^{2/3})\left\{\frac{\sin^2\alpha}{m_1}+\frac{\cos^2\alpha}{m_3}-\frac{\cos^2\alpha\sin^2\alpha(m_3-m_1)^2}{m_1m_3(m_3\cos^2\alpha+m_1\sin^2\alpha)}\right\}, \quad (4.16)$$

$$p_z^*(\alpha)=(2\varepsilon_F)^{1/2}\left(\frac{\sin^2\alpha}{m_1}+\frac{\cos^2\alpha}{m_3}\right)^{-1/2}. \quad (4.17)$$

In order that undamped solutions of Eq. (4.14) exist in the general case it is necessary that two inequalities be simultaneously satisfied:

$$\frac{\omega_{n'n}(0,\alpha)-\omega}{\omega''_{n'n}(0,\alpha)}>0,\quad \frac{\omega_{n'n}(0,\alpha)-\omega}{\omega_{n'n}(0,\alpha)}B_0<0.$$

As for an ellipsoid the transition frequency reaches a maximum on a central section, the resonance frequency is larger than such a maximum value and, correspondingly, the constant B_0 must be positive.

In the particular case of a Fermi surface which is nearly cylindrical, when the transition frequency is independent of p_z, we get from (4.12) an expression for the surface spin resonance frequency

$$\omega=\omega_{n'n}(0,\alpha)\left[1+\frac{2B_0}{1+B_0}\frac{\bar{N}_{n'n}p_z^*(\alpha)}{v_x(0,\alpha)}\right]. \quad (4.18)$$

To determine the quantities which occur in Eq. (4.18) using Eqs. (4.15) and (4.17) we must put $m_3 \gg m_1$.

In the opposite case of weakly oblate Fermi surfaces, when the argument of the arctangent in Eq. (4.9) turns out to be large compared to unity, the frequency shift depends not linearly, but quadratically on B_0 (cf. [5]):

$$\omega=\omega_{n'n}(0,\alpha)\left\{1-\frac{2\omega_{n'n}(0,\alpha)}{\omega''_{n'n}(0,\alpha)}\left[\frac{\pi B_0}{1+B_0}\bar{N}_{n'n}\right]^2\right\}. \qquad (4.19)$$

Equations (4.14), (4.18), and (4.19) solve the problem of the limiting value of the frequency of the surface spin oscillations spectrum in the model of a metal with an ellipsoidal Fermi surface which is inclined at an angle α to the magnetic field.

5. COUPLED SURFACE SPIN-ELECTROMAGNETIC WAVES

In the foregoing we neglected the non-equilibrium magnetic field. In the present section we aim to explain what is the result of taking into account the non-equilibrium magnetic field when we write down the dispersion equation for surface waves coupled to spin-electromagnetic oscillations. In a metal occupying the half-space $y > 0$ we shall look for a surface H-wave, propagating along the magnetic field with a wavevector k_z. In this sense the statement of the problem is analogous to the one given in[2].

The expression for the Fourier component of the x-component of the current density, obtained by using quantum kinetic equations, in which the interaction is approximated by a single constant B_0 is the same as the one obtained earlier in the gas model[2] when the electrons are reflected specularly from the boundary. We can therefore immediately write down a set of equations for the quantities $\delta s(0, k_y, k_z)$ and the Fourier component of the non-equilibrium magnetic field $\delta B(0, k_y, k_z)$. In fact, the set consists of Eq. (3.2), in the right-hand side of which we must take into account the non-equilibrium magnetic field, and the Maxwell equation. The quantity $Q(k_z, k_y, q)$ occurring in (3.2) is determined by Eqs. (3.3), (3.5), and (3.10). We have

$$\delta s(0,k_y,k_z)\,[1+B_0]-B_0\frac{\pi\sqrt{3}\hbar}{2mv\Delta^{1/2}}\int_0^\infty\frac{dq}{\pi}I_{n'n}(k_y,p_F)I_{n'n}(-q,p_F)\,\delta s(0,q,k_z)$$

$$=-\frac{m\sqrt{3}}{2\pi\hbar^2}\frac{1}{\Delta^{1/2}}I_{n'n}(k_y,p_F)\int_0^\infty\frac{dq}{\pi}I_{n'n}(-q,p_F)\,\mu_0\delta B(0,q,k_z)+\mu_0\frac{m^2v}{\pi^2\hbar^3}\delta B(0,k_y,k_z),$$

$$2\delta B'(0)+(k_z^2+k_y^2)\,\delta B(0,k_y,k_z)=\frac{3\pi i\omega_{Le}^2\omega}{4c^2v(k_z^2+k_y^2)^{1/2}}\delta B(0,k_y,k_z)$$

$$-\frac{3\sqrt{3}\hbar\omega_{Le}^2}{4c^2p_F}I_{n'n}(-k_y,p_F)\frac{1}{\Delta^{1/2}}\int_0^\infty dq\,I_{n'n}(-q,p_F)\,\delta B(0,q,k_z)+4\pi k_z^2\mu_0\delta s(0,k_y,k_z). \qquad (5.1)$$

Here

$$\delta B'(0)=\frac{\partial}{\partial y}\delta B_y|_{y=0},$$

μ_0 is the electron magnetic moment. Solving the set (5.1) for $\delta B(0, k_y, k_z)/\delta B'(0)$, we get

$$\frac{\delta B(0,k_y,k_z)}{\delta B'(0)}=2\left[k_z^2+k_y^2-\frac{3\pi i\omega_{Le}^2\omega}{4c^2v(k_z^2+k_y^2)^{1/2}}-k_z^2\mu_0^2\frac{4m^2v}{\pi\hbar^3}\right]^{-1}\times\left[-1+I_{n'n}(k_y,p_F)\frac{\alpha_{n'n}(k_z)\Xi}{1+\beta_{n'n}(k_z)\Xi}\right]. \qquad (5.2)$$

We have introduced here the following notation:

$$\Xi=\frac{3\sqrt{3}\hbar\omega_{Le}^2\pi}{8c^2p_F\Delta^{1/2}}+\frac{2k_z^2\mu_0^2m\sqrt{3}\hbar^{-2}}{1+B_0}\frac{1}{\Delta^{1/2}}\frac{1}{1+B_0-B_0N_{n'n}\pi\hbar\sqrt{3}(mv)^{-1}(\Delta)^{-1/2}}$$

$$\alpha_{n'n}(k_z)=\frac{2}{\pi}\int_0^\infty dk_yI_{n'n}(k_y,p_F)\left[k_z^2+k_y^2-\frac{3\pi i\omega_{Le}^2\omega}{4c^2v(k_z^2+k_y^2)^{1/2}}-k_z^2\mu_0^2\frac{4m^2v}{\pi\hbar^3}\right]^{-1}$$

$$\beta_{n'n}(k_z)=\frac{2}{\pi}\int_0^\infty dk_yI_{n'n}(k_yp_F)I_{n'n}(-k_y,p_F)\times\left[k_z^2+k_y^2-\frac{3\pi i\omega_{Le}^2\omega}{4c^2v(k_z^2+k_y^2)^{1/2}}-k_z^2\mu_0^2\frac{4m^2v}{\pi\hbar^3}\right]^{-1}.$$

The remaining notation is similar to that used in (3.21). Integrating Eq. (5.2) over k_y and equating it to the free space impedance $4\pi i\omega/c^2|k_z|$ we get a dispersion equation for coupled spin-electromagnetic surface oscillations:

$$\frac{1}{|k_z|}=\frac{1}{\pi}\int_0^\infty dk_y\frac{\delta B(0,k_y,k_z)}{\delta B'(0)}=-f(k_z)+\frac{\alpha_{n'n}^2(k_z)\Xi}{1+\beta_{n'n}(k_z)\Xi},$$

$$f(k_z)=\frac{2}{\pi}\int_0^\infty dk_y\left[k_z^2+k_y^2-\frac{3\pi i\omega_{Le}^2\omega}{4c^2v(k_z^2+k_y^2)^{1/2}}-k_z^2\mu_0^2\frac{4m^2v}{\pi\hbar^3}\right]^{-1}. \qquad (5.3)$$

The expression for the surface impedance of the metal obtained from (5.2) has in the long-wavelength limit the form

$$Z(k_z\to 0)=\frac{16\pi\omega\delta}{3^{3/2}c^2}e^{-i\pi/3}+\frac{4\pi i\omega}{c^2}\frac{\alpha_{n'n}^2(0)\Xi}{1+\beta_{n'n}(0)\Xi}. \qquad (5.4)$$

The first term describes here the impedance of the metal when there is no magnetic field and when the electrons are reflected specularly from the surface; $\delta=(3\pi\omega_{Le}^2\omega/4c^2v)^{-1/3}$ is the anomalous skin depth. The vanishing of the denominator of the second term in the right-hand side of (5.4) corresponds to the possibility of the excitation of surface waves and describes the resonance properties of the impedance.

We rewrite the dispersion Eq. (5.3) in a more convenient form:

$$\frac{3\sqrt{3}\hbar\omega_{Le}^2\pi}{8c^2p_F}\frac{1}{\Delta^{1/2}}+k_z^2\frac{2\mu_0m\sqrt{3}}{\hbar^2(1+B_0)^2}\left[\sqrt{\Delta}-\frac{B_0}{1+B_0}N_{n'n}\frac{\pi\hbar\sqrt{3}}{mv}\right]^{-1}=-\operatorname{Re}\frac{1+|k_z|f(k_z)}{\beta_{n'n}(k_z)-\alpha_{n'n}^2(k_z)|k_z|}. \qquad (5.5)$$

The solution of this equation can be written in the form

$$\sqrt{\Delta}=\frac{B_0D\Pi(k_z)-A-Gk_z^2\pm((B_0D\Pi(k_z)-A-Gk_z^2)^2+4\Pi(k_z)B_0DA)^{1/2}}{2\Pi(k_z)}, \qquad (5.6)$$

where we have used the notation

$$A=\frac{3\sqrt{3}\,\omega_{Le}^2\hbar\pi}{8c^2p_F},\quad G=\frac{2\mu_0^2m\sqrt{3}}{\hbar^2(1+B_0)^2},\quad D=\frac{\pi\sqrt{3}}{4(1+B_0)},$$

$$\Pi(k_z)=\operatorname{Re}\frac{1+|k_z|f(k_z)}{\beta_{n'n}(k_z)-\alpha_{n'n}^2(k_z)|k_z|}.$$

To obtain the dispersion Eq. (5.5) we have separated the resonance term in the current of the surface electrons and have neglected the displacement current in the Maxwell equations. This means that the dispersion Eq. (5.5) is valid in the region

$$\omega_{n'n}/c<k_z<\omega_{n'n}/v. \quad (5.7)$$

In this range

$$k_z^2G<\frac{2\mu_0^2m\sqrt{3}\,\omega_{n'n}^2}{\hbar^2 v}=10^{-7}\,\mathrm{cm}^{-1}\ll A=10^2\,\mathrm{cm}^{-1}.$$

Hence we get from (5.6) two branches of surface oscillations

$$\sqrt{\Delta}=-\frac{A}{\Pi(k_z)}\left(1+\frac{Gk_z^2}{\Pi(k_z)B_0D+A}\right), \quad (5.8)$$

$$\sqrt{\Delta}=B_0D\left(1-k_z^2G\frac{1}{\Pi(k_z)B_0D+A}\right). \quad (5.9)$$

The expression for $\Pi(k_z)$ contains the quantities $\alpha_{n'n}(k_z)$ and $\beta_{n'n}(k_z)$, in which we put $k_z=0$ because of (5.7), and which we evaluate by the stationary phase method, using the explicit form of the surface state wavefunctions (3.18) and the notation $k^{\pm}_{n'n}=(p_{n'}\pm p_n)/\hbar$:

$$\alpha_{n'n}(0)=\frac{2}{\pi}\int_0^\infty dk_y\frac{k_y}{k_y^3-i\delta^{-3}}I_{n'n}(k_y,p_F)$$

$$=\frac{1}{\lambda}\left[\frac{k^+_{n'n}}{(k^+_{n'n})^3-i\delta^{-3}}-\frac{k^-_{n'n}}{(k^-_{n'n})^3-i\delta^{-3}}\right],$$

$$\beta_{n'n}(0)=\frac{2}{\pi}\int_0^\infty dk_y\frac{k_y}{k_y^3-i\delta^{-3}}I_{n'n}(k_y,p_F)I_{n'n}(-k_y,p_F)$$

$$=\frac{R}{2\lambda^2}\left[\frac{k^+_{n'n}}{(k^+_{n'n})^3-i\delta^{-3}}-\frac{k^-_{n'n}}{(k^-_{n'n})^3-i\delta^{-3}}\right].$$

For a frequency $\omega=10^{11}$ Hz and $B=10$ Oe we get $R=cp_F/eB=1$ cm, $\lambda=10^{-4}$ cm, $\delta=10^{-5}$ cm, $k^{\pm}_{n'n}=(n'^{1/3}\pm n^{1/3})\cdot 10^5\,\mathrm{cm}^{-1}$, n', $n\leq 10$, $\alpha_{n'n}(0)\approx(1+i)\cdot 10^{-6}\,\mathrm{cm}\approx(1+i)\cdot 0.1\delta$,

$$|\beta_{n'n}(0)|\approx 0{,}1\delta\cdot 10^4\approx 10^{-2}\,\mathrm{cm},\qquad \frac{3\sqrt{3}\,\hbar\omega_{Le}^2\pi}{8c^2p_F}\,\mathrm{Re}\,\beta_{n'n}(0)=10^{-1}.$$

Then, the following condition is satisfied:

$$\mathrm{Re}[\beta_{n'n}(0)-\alpha^2_{n'n}(0)|k_z|]>0.$$

Hence, waves with the spectrum (5.8) cannot propagate in an electron liquid with a spherical Fermi surface, which corresponds to the result of[2]. The second branch (5.9) caused by the interaction between the electrons describes weakly damped oscillations with a spectrum

$$\omega=\omega_{n'n}\left\{1+B_0^2\frac{3\pi^2}{16(1+B_0)^2}\left[1-\frac{2\mu_0^2m\sqrt{3}\,k_z^2}{\hbar^2(1+B_0)^2}\left(\frac{3\sqrt{3}\,\hbar\omega_{Le}^2\pi}{8c^2p_F}+\frac{B_0}{1+B_0}\frac{\sqrt{3}\pi}{4\beta_{n'n}(0)}\right)^{-1}\right]+\frac{3}{4}\left(\frac{k_zv}{\omega_{n'n}}\right)^2\right\}-\frac{i}{\tau}. \quad (5.10)$$

The wave spectrum (5.10) differs from the spin wave spectrum (4.6) by the small second terms in the braces, which are proportional to k_z^2.

It is shown in[2] that an equation such as (5.8) has a solution for the case of cylindrical Fermi surfaces. However, for the spectrum of the waves which are then obtained it is characteristic that their frequency is smaller than the transition frequency $\omega_{n'n}$. This distinguishes such waves qualitatively from those studied in the present paper.

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