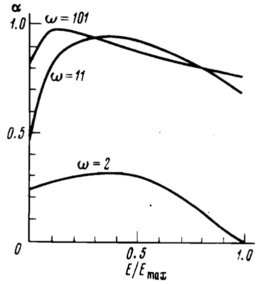

FIG. 4. The angular distribution of the outgoing photon for the same processes to which the curves in Fig. 3 correspond. Along the ordinate axis is plotted the quantity α, which is determined from the expression $d\sigma/dEd^2\Omega_{k_2} \sim 1-\alpha(\mathbf{e}_1\mathbf{e}_2)^2$ (E is the electron energy).

have poles at electron energies corresponding to the photoelectric effect from these less excited levels. These poles will naturally disappear if we take the level widths into account.

Figure 2 shows the plots of the dependence of the cross section for light scattering with electron ionization from the 2s and 2p states on the energy of the outgoing electron. As can be seen from this figure, the cross section has a pole only when the ionization is from the 2p level.

Figures 3 and 4 show the cross sections for electron emission and the angular distributions of the outgoing photon for three values of the incident-photon frequency; initially, the electron is in the ground state. At constant ω_2 the cross section monotonically decreases with increasing ω_1. If, on the other hand, we fix the outgoing-electron energy E, then the monotonic decrease of the cross section may not occur, since E may approach a pole singularity of the cross section as the incident-photon frequency increases.

The asymptotic form, (13), of the amplitude is also applicable in the case when $n_2 \to \infty$. From it we easily obtain that $d\sigma/dE \to$ const as $E \to 0$.

In conclusion, I take the opportunity to thank V. S. Polikanov and L. A. Sliv for a discussion of the present paper and for a number of useful comments.

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Passage of low-energy particles through a nonstationary potential barrier and the quasi-energy spectrum

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The passage of a particle through a narrow potential barrier with a periodically varying depth is investigated. A set of wave functions with a definite quasi-energy are constructed and the concept of scattering eigenphases and eigenamplitudes is used. It is shown that the quasi-energy spectrum is a continuous spectrum with an infinite degree of degeneracy. Examples are presented of cases when there exist a discrete (nondecaying) quasi-energy state superimposed on the continuous-spectrum background. The effect of total reflection of particles from a nonstationary potential barrier is discovered and found to be of a resonance nature.

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1. INTRODUCTION. FORMULATION OF THE PROBLEM

The interest in the problem of the passage of a quantum particle through a potential barrier with a periodically varying depth is explained by its connection with the theory of the interaction of laser radiation with matter—in particular, with the theory of many-photon ionization (see, for example, the monograph by Baz', Zel'dovich, and Perelmov[1] and the references cited therein). So far, different versions of the quasi-classical approximation were usually used here, i.e., the barrier was assumed to be sufficiently wide. Besides the papers cited in[1], let us also mention Varshalovich and D'yankonov's paper[2], in which the modulation of a beam of high-energy electrons by a wide potential barrier periodically dependent on the time was investigated.

In the present paper we investigate the opposite case (i.e., passage through a narrow barrier, within which the quasi-classical approximation is inapplicable), which, apparently, has not been investigated before in the literature. The particle energy E is assumed to be low, so that the change in it which arises from the interaction with the barrier (and which is a multiple of the oscillation frequency ω of the latter) is not negligible (in contrast to the situation considered in[2]). For a particle of low energy ($kd \ll 1$, $k=\sqrt{2E}$, d is the width of the barrier) we can neglect the finite dimensions of the barrier and replace it by a potential of zero radius. Assuming that the Hamiltonian $H(t)$ depneds harmonically on time, we obtain ($\hbar = m = 1$)

$$H(t)=-{}^1/_2\partial^2/\partial x^2-(\alpha+\lambda\cos\omega t)\delta(x); \qquad (1.1)$$

here the potential is concentrated at the point $x=0$ and λ characterizes the magnitude of the harmonic perturbation. It is often more convenient to use another formulation of the problem: the wave function $\Psi(x, t)$ everywhere satisfies the time-dependent Schrödinger equation for a free particle

$$\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2}-i\frac{\partial}{\partial t}\right)\Psi(x,t)=0, \qquad (1.2)$$

except at the point $x=0$, where we impose the boundary condition

$$\frac{\partial\Psi}{\partial x}\Big|_{x=-0}^{x=+0}=-2(\alpha+\lambda\cos\omega t)\Psi(0,t). \qquad (1.3)$$

If $\lambda\to 0$, then the problem goes over into the steady-state problem, and for $\alpha>0$ the potential has one bound state with energy $E=-\alpha^2/2$. In the $\lambda=0$ case the coefficients of transmission T and reflection R are smooth functions of the energy:

$$T=\frac{k^2}{k^2+\alpha^2},\quad R=\frac{\alpha^2}{k^2+\alpha^2};\quad k=\sqrt{2E}. \qquad (1.4)$$

However, in the nonstationary problem, as will be shown below (Sec. 4), there arises interesting resonance effects.

The second important aspect of the problem consists in the fact that, using it as an example, we can investigate some general properties of the quasi-energy spectrum. The concept of quasi-energy was introduced by Zel'dovich[1,3] to describe quantum systems with a periodic dependence of the Hamiltonian on time. The quasi-energy ε and the corresponding solution to the time-dependent Schrödinger equation, $\Psi_\varepsilon(t)$, are defined by the condition

$$\Psi_\varepsilon(t+\tau)=e^{-i\varepsilon\tau}\Psi_\varepsilon(t),\quad \tau=2\pi/\omega,\quad 0\leqslant\varepsilon\leqslant\omega. \qquad (1.5)$$

It is to be expected that the use of the quasi-energy concept will be fruitful in, for example, the theory of many-photon ionization[4,5] in precisely those cases where the quasi-classical and perturbation-theory methods are inapplicable. However, the results that have practically been attained here thus far are not so great, and one of the reasons for this clearly consists in the fact that very little is known about the nature of the quasi-energy spectrum for a real problem. In fact, the quasi-energy problem has been solved for the oscillator with a variable frequency[1] and for an electron in the field of an electromagnetic wave,[4] but of greatest interest are the cases when the corresponding steady-state problem has a continuous, as well as a discrete, spectrum. The latter is precisely the case for the Hamiltonian (1.1) (for $\alpha>0$), as well as for the majority of real physical problems. Therefore, in the present paper we construct quasi-energy solutions (Secs. 2 and 3), and discuss their connection with the scattering problem (Sec. 4).

2. THE QUASI-ENERGY SOLUTIONS

It is natural to seek the wave functions with a definite quasi-energy ε (see (1.2)) in the form of a sum of the terms

$$c_n\exp[ik_n|x|-i(\varepsilon+n\omega)t],\quad k_n=[2(\varepsilon+n\omega)]^{1/2}, \qquad (2.1)$$

where n is an integer ($-\infty<n<+\infty$). The expressions (2.1) satisfy the condition (1.5), and are symmetric—in the coordinate x—solutions to the time-independent Schrödinger equation (1.2) (it is clear that the zero-radius potential can in no way influence the antisymmetric solutions, which vanish at the point $x=0$). For the coefficients c_n we obtain from the boundary condition (1.3) trinomial recurrence relations which are characteristic of problems with a harmonic time dependence and which, generally speaking, do not break off as $n\to\pm\infty$. For $n<0$ we should choose the solution in (2.1) that decreases as $|x|\to\infty$, i.e., for which $ik_n=-\varkappa_n$, where $\varkappa_n=[2|\varepsilon+n\omega|]^{1/2}>0$. For $n\geqslant 0$, k_n is real and we can take both the positive and the negative values of the square root in (2.1), which facilitates the construction of the quasi-energy solutions (the recurrence relations can then be terminated).

Indeed, let us first consider the solutions into which only terms the type (2.1) with $n>0$ enter. It is easy to verify that the solutions

$$\Psi_{\varepsilon n}(x,t)=\{b_{n-1}\sin(k_{n-1}|x|)e^{i\omega t}+\sin(k_n|x|+\delta_{\varepsilon n})+b_{n+1}\sin(k_{n+1}|x|)e^{-i\omega t}\}e^{-i(\varepsilon+n\omega)t} \qquad (2.2)$$

satisfy the boundary condition (1.3) if

$$b_{n-1}k_{n-1}=b_{n+1}k_{n+1}=-\lambda\sin\delta_{\varepsilon n},\quad k_n\operatorname{ctg}\delta_{\varepsilon n}=-\alpha,$$

i.e., if $\delta_{\varepsilon n}$ coincides with the phase of the wave function of the continuous spectrum for the energy $k_n^2/2$ in the case of a constant potential ($\lambda=0$).

Let us turn to the quasi-energy solution that includes terms with $n<0$ and that can be represented in the form

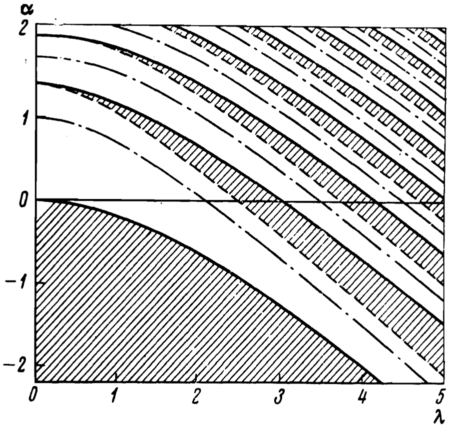

FIG. 1. Curves of constant values of the quasi-energy corresponding to total particle reflection from a nonstationary barrier. Dashed curves: $\varepsilon = 0$; dot-dash curves: $\varepsilon = 0.5$; solid curves: $\varepsilon = 1.0$. The regions of the parameters α and λ where the total-reflection effect does not exist (the "forbidden bands") are hatched.

$$\Psi_{\varepsilon 0}(x,t) = b_1 \sin(k_1|x|)\, e^{-i(\varepsilon+\omega)t} + \{b_{0s}\sin(k_0|x|) + b_{0c}\cos(k_0 x)\}\, e^{-i\varepsilon t} + \sum_{n=-1}^{-\infty} c_n \exp[-\varkappa_n|x| - i(\varepsilon+n\omega)t]. \quad (2.3)$$

Let us write out the recurrence relations for the coefficients:

$$b_1 k_1 = -\lambda b_{0c}, \quad b_{0s} k_0 = -\alpha b_{0c} - {}^1/_2\lambda(b_1 + c_{-1}); \quad (2.4)$$

$$c_{-1}\varkappa_{-1} = \alpha c_{-1} + {}^1/_2\lambda(b_{0c} + c_{-2}),$$

$$c_n \varkappa_n = \alpha c_n + {}^1/_2\lambda(c_{n-1} + c_{n+1}), \quad n \leq -2. \quad (2.5)$$

It is necessary to impose on the coefficients c_n the requirement that they decrease sufficiently rapidly as $n \to -\infty$. As is shown in the Appendix, this leads to a situation in which one solution, $\Psi_{\varepsilon 0}$, exists for any ε. To this solution corresponds a definite value of the ratio $b_{0s}/b_{0c} = \cot\delta_{\varepsilon 0}$ (the phase $\delta_{\varepsilon 0}$ is thus different from its value for $\lambda = 0$). This result becomes especially clear if we take into account the fact that the trinomial recurrence relations are similar to second-order differential equations, and, like the latter, have two linearly independent solutions. One of these solutions (the regular solution) can be chosen such that it decreases as $n \to -\infty$. Then the other is an increasing solution, and the series in (2.3) for it diverges.

Of special interest for what follows is the case when $b_{0c} = b_1 = 0$ and $\Psi_{\varepsilon 0}$ assumes the form

$$\Psi_{\varepsilon 0} = b_{0s}\sin(k_0|x|)\, e^{-i\varepsilon t} + \sum_{n=-1}^{-\infty} c_n \exp[-\varkappa_n|x| - i(\varepsilon+n\omega)t]. \quad (2.6)$$

For given α and λ, a solution of the form (2.6) exists only for certain values of $\varepsilon = \tilde{\varepsilon}$; there occurs at the corresponding energy total particle reflection from the barrier (Sec. 4). If, say, we fix α and vary λ, then the dependence $\tilde{\varepsilon}(\lambda)$ has a typical band character with allowed and forbidden bands. In Fig. 1 we show in the plane of the variables λ and α "iso-quasi-energy" curves, $\tilde{\varepsilon}(\lambda, \alpha)$, obtained as a result of a numerical calculation for the recurrence relations (2.5); the forbidden bands are hatched.

3. THE EIGENPHASES AND THE QUASI-ENERGY SPECTRUM

The simple solutions $\Psi_{\varepsilon n}$ constructed above are orthogonal (in the coordinate x) for different ε, as follows from the general theory.[1,3] For one and the same ε and different n there is no orthogonality in the usual sense. Let us now proceed to the construction of a natural—for the present problem—set of orthogonal quasi-energy solutions.

Let us use the concept of eigenphases and eigenamplitudes introduced earlier in Demkov and Rudakov's paper[6] for the steady-state problem of scattering by nonspherical scatterers. The use of eigenphases and eigenamplitudes in the nonstationary case is also quite natural and fruitful; thus, the knowledge of them allows us to write down immediately the solution to the scattering problem (see[7], as well as Sec. 4 of the present paper). The wave function corresponding to an eigenphase is defined by the requirement that the coefficients for the ingoing and outgoing waves differ only by a phase factor. In our case this clearly leads to a solution of the type (cf.[6])

$$\Phi_\varepsilon(x,t) = \sum_{n=0}^{\infty} \frac{A_n}{k_n^{1/2}} \sin(k_n|x| + \eta_\varepsilon)\, e^{-i(\varepsilon+n\omega)t} + \sin\eta_\varepsilon \sum_{n=-1}^{-\infty} c_n \exp[-\varkappa_n|x| - i(\varepsilon+n\omega)t], \quad (3.1)$$

where η_ε is the eigenphase ($|\eta_\varepsilon| < \pi$), while the coefficients c_n and A_n satisfy the recurrence relations (2.5) (with b_{0c} replaced by $A_0/k_0^{1/2}$) and the relations

$$\cot\eta_\varepsilon A_n = -\left\{\frac{\alpha}{k_n}A_n + \frac{\lambda}{2}\left[\frac{A_{n+1}}{(k_n k_{n+1})^{1/2}} + \frac{A_{n-1}}{(k_n k_{n-1})^{1/2}}\right]\right\}, \quad n \geq 1,$$

$$\cot\eta_\varepsilon A_0 = -\left[\frac{\alpha}{k_0}A_0 + \frac{\lambda}{2}\frac{A_1}{(k_0 k_1)^{1/2}} + \frac{\lambda}{2k_0^{1/2}}c_{-1}\right]. \quad (3.2)$$

Thus, we arrive at a generalized eigenvalue problem, (3.2), for the quantity $\cot\eta_\varepsilon$, upon the solution of which we obtain a set of eigenvalues $\cot\eta_{\varepsilon\nu}$ and eigenfunctions $\Phi_{\varepsilon\nu}$ characterized by the vector whose components are the coefficients $A_{n\nu}$ ($n = 0, 1, \ldots$). For a given ε all the coefficients c_n do not, up to normalization factors, depend on the index ν, since, as was shown during the construction of the solution $\Psi_{\varepsilon 0}$, they are determined by the condition $c_n \to 0$ as $n \to -\infty$, the ratio A_0/c_{-1} being also determined then. The assignment of the latter quantity corresponds to the assignment of the logarithmic derivative at the boundary for a second-order differential equation defined over a semi-infinite interval (such an equation, as has already been pointed out, is analogous to trinomial recurrence relations). The set of functions $\Phi_{\varepsilon\nu}(x, t)$ thus constructed is complete in the sense that any symmetric solution of the time-dependent Schrödinger equation can be expanded in terms of the functions.

When we go over to the steady-state problem (i.e., for $\lambda \to 0$), the eigenphases $\eta_{\varepsilon\nu}$ tend to their unperturbed values. Using this fact, we can conveniently number the phases $\eta_{\varepsilon\nu}$ ($\nu = 0, 1, \ldots$) in such a way that $\eta_{\varepsilon n} \to \delta_{\varepsilon n}$ (as $\lambda \to 0$).

It is easy to verify that the orthogonality condition for the eigenamplitudes $A_{n\nu}$ is fulfilled, i.e., that

$$\sum_{n=0}^{\infty} A_{n\nu}A^{*}_{n\nu'}=\delta_{\nu\nu'}. \qquad (3.3)$$

The quasi-energy spectrum obtained by us is continuous both in the sense that the ε values continuously fill the segment $0 \leq \varepsilon < \omega$ and because the functions $\Phi_{\varepsilon n}$ are not quadratically integrable. Furthermore, the spectrum is degenerate to an infinite degree, since for each ε there are infinitely many solutions, $\Phi_{\varepsilon n}$ ($n = 0, 1, \ldots$), corresponding to different eigenphases.

The formulated properties of the spectrum are sufficiently general for realistic problems. There, however, arises the question whether the existence of a discrete (i.e., a quadratically integrable) quasi-energy state is possible. We shall now construct examples of such states.

Let us consider two identical and in-phase oscillating zero-radius potentials separated by a distance of $2a$:

$$H(t)=-\frac{1}{2}\frac{\partial^2}{\partial x^2}-(\alpha+\lambda\cos\omega t)[\delta(x+a)+\delta(x-a)]. \qquad (3.4)$$

Let us seek the solution analogous to $\Psi_{\varepsilon 0}$ in the form

$$\Psi_{\varepsilon 0}^{(\pm)}(x,t)=\sum_{n=-1}^{-\infty} c_n^{(\pm)}\left\{\begin{matrix}\mathrm{ch}\,\varkappa_n x\\ \mathrm{sh}\,\varkappa_n x\end{matrix}\right\}e^{-i(\varepsilon+n\omega)t}+b^{(\pm)}\left\{\begin{matrix}\cos k_0 x\\ \sin k_0 x\end{matrix}\right\}e^{-i\varepsilon t} \quad \text{for } |x|<a, \qquad (3.5a)$$

$$\Psi_{\varepsilon 0}^{(\pm)}(x,t)=\sum_{n=-1}^{-\infty} d_n^{(\pm)}\exp[-\varkappa_n|x|-i(\varepsilon+n\omega)t] \quad \text{for } x>a. \qquad (3.5b)$$

Here the upper signs and the functions pertain to the solutions that are symmetric in the coordinate x; the lower signs, to the antisymmetric solutions. The solutions (3.5) are quadratically integrable, but they exist only under the additional condition that

$$\left\{\begin{matrix}\cos k_0 a\\ \sin k_0 a\end{matrix}\right\}=0. \qquad (3.6)$$

Introducing the new coefficients

$$D_n^{(\pm)}=c_n^{(\pm)}\left\{\begin{matrix}\mathrm{ch}\,\varkappa_n a\\ \mathrm{sh}\,\varkappa_n a\end{matrix}\right\}=d_n^{(\pm)}\exp(-\varkappa_n a),$$

we obtain

$$\{-\alpha-\varkappa_n[1\pm\exp(-2\varkappa_n a)]^{-1}\}D_n^{(\pm)}=-{}^1/_2\lambda(D_{n+1}^{(\pm)}+D_{n-1}^{(\pm)}), \quad n\leq -2;$$
$$\{-\alpha-\varkappa_{-1}[1\pm\exp(-2\varkappa_n a)]^{-1}\}D_{-1}^{(\pm)}=-{}^1/_2\lambda D_{-2}^{(\pm)}; \qquad (3.7)$$
$$k_0 b^{(\pm)}\left\{\begin{matrix}\sin k_0 a\\ \cos k_0 a\end{matrix}\right\}=-\lambda D_{-1}^{(\pm)}.$$

The recurrence relations (3.7) differ little from (2.5) and lead to "iso-quasi-energy" curves similar to those shown in Fig. 1 (let us recall that the quantity a here depends on ε—see (3.6)). Now these curves correspond to a discrete quasi-energy state which is superimposed on a continuous-spectrum background and which we were able to construct, using the additional parameter a of the problem.

4. SCATTERING

In the problem of the scattering of a particle of energy E by a nonstationary barrier, the quasi-energy clearly is $\varepsilon = E - n_0\omega$, $n_0 = [E/\omega]$, where the symbol $[z]$ denotes the integral part of the number z. To the left of the barrier (i.e., in the region $x < 0$) we should assign an incident wave; the remaining waves are scattered waves:

$$\Psi(x,t)=e^{ikx-iEt}+F_R e^{-ikx-iEt}+\sum_{\substack{n\neq n_0\\ n>0}} F_{n_0\to n}\exp[-ik_n x-i(\varepsilon+n\omega)t]+o(1) \quad \text{for } x<0; \qquad (4.1a)$$

$$\Psi(x,t)=F_T e^{ikx-iEt}+\sum_{\substack{n\neq n_0\\ n>0}} F_{n_0\to n}\exp[ik_n x-i(\varepsilon+n\omega)t]+o(1) \quad \text{for } x>0. \qquad (4.1b)$$

Here F_R and F_T are the amplitudes of the elastically reflected and transmitted waves ($R = |F_R|^2$, $T = |F_T|^2$); the $F_{n_0\to n}$ are the amplitudes of the inelastic scattering, which, evidently, occurs symmetrically to the right and left and is accompanied by absorption by the particle (as well as by a kick to the barrier when this is energetically possible) of an integral number $n - n_0$ of quanta of energy ω. The condition for particle-number conservation has the form

$$|F_R|^2+|F_T|^2+2\sum_{n\neq n_0}\frac{k_n}{k_{n_0}}|F_{n_0\to n}|^2=1. \qquad (4.2)$$

The scattering amplitude can, with the aid of standard transformations,[6,7] be written in terms of the scattering phases:

$$F_{n_0\to n}=-\frac{1}{2}\sum_{\nu=0}^{\infty}\left(\frac{k_{n_0}}{k_n}\right)^{1/2}\exp(2i\eta_{\varepsilon\nu})A^{*}_{n_0\nu}A_{n\nu},$$
$$F_R=-\frac{1}{2}\left\{1+\sum_{\nu=0}^{\infty}\exp(2i\eta_{\varepsilon\nu})|A_{n_0\nu}|^2\right\}, \qquad (4.3)$$
$$F_T=-\frac{1}{2}\left\{-1+\sum_{\nu=0}^{\infty}\exp(2i\eta_{\varepsilon\nu})|A_{n_0\nu}|^2\right\}.$$

As always, the passage of the phase $\eta_{\varepsilon\nu}$ (as it increases) through a number that is a multiple of π leads to resonances in the scattering. In the problem under consideration one such case, which corresponds to a solution of the type (2.6), is extremely important. In fact, the latter corresponds to the vanishing of the phase $\eta_{\varepsilon 0}$, with $A_{n0} = \delta_{n0}$, i.e., particles with $E = \tilde{\varepsilon} < \omega$ are scattered only elastically. This can be directly verified, taking into account the fact that the solution to the Schrödinger equation

$$\Psi(x,t)=\Psi_{\tilde{\varepsilon}0}(x,t)/2b_0-{}^1/_2\sin k_0 x e^{-i\tilde{\varepsilon}t} \qquad (4.4)$$

has precisely the form (4.1) and corresponds to total particle reflection ($\Psi(x, t) = o(1)$ as $x \to +\infty$). Thus, at those quasi-energy values when the solution (2.6) exists (see Fig. 1) there occurs total reflection from the barrier. It is interesting to note that total reflection is unattainable for stationary barriers of finite width.

To illustrate the behavior of the coefficients of transmission T and reflection R near the resonance energy

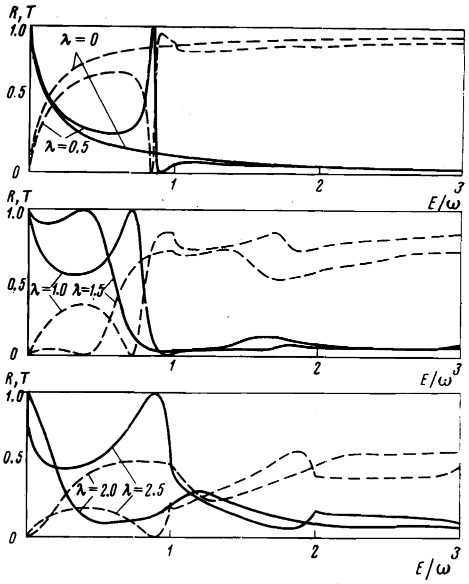

FIG. 2. The coefficients of reflection R (solid curves) and transmission T (dashed curves) of a particle for a nonstationary δ-potential for $\alpha = 0.5$ and different values of λ. As λ increases, the resonance characterizing the region of total reflection moves towards the lower-energy region and broadens. The quantity $1 - R - T$ characterizes the total probability for all the inelastic processes. When E is a multiple of ω, there exist threshold singularities (vertical tangents) connected with the opening of a new channel (the loss by the particle of several photons of energy ω).

and at different values of the parameters, we carried out a computer calculation,[1)] the results of which are shown in Fig. 2. As can be seen, at small λ the formulas (1.4) are valid for all E, with the exception of a very narrow region near the resonance. As λ increases, the width of the resonance increases and the resonance itself moves towards the low-energy region until it finally disappears, which corresponds to the forbidden (hatched) region in Fig. 1. As λ increases further, the resonance reappears at $E=\omega$ and again moves towards the low-energy region. It can clearly be seen from the plots that the total-reflection effect has a resonance character.

5. CONCLUSION

The effect, discovered in the present paper, of total reflection from a nonstationary barrier is of fundamental interest. The possibility of the use of the resonance character of the effect to, for example, produce monoenergetic electrons upon their passage through a thin film whose work function is modulated is not excluded. For sufficiently slow electrons the thickness of the film can be neglected ($kd \ll 1$), and the reflection will be total at the resonance energy. For a better selection, we can use multiple reflection. Furthermore, such a method of obtaining monoenergetic electrons would allow us to smoothly vary the value of the selected energy by varying the angle between the surface of the film and the electron beam.

As to the general conclusions about the quasi-energy spectrum, the intrinsic properties (the continuity and degeneracy with infinite multiplicity) of the spectrum make the practical application of the quasi-energy concept in the theory of many-photon processes difficult. The application of the eigenphase concept introduced here may be useful, since it provides a natural classification for degenerate states with a given quasi-energy. To the logarithmic singularities of the eigenphases correspond, generally speaking, quasi-stationary quasi-energy states, i.e., quasi-energy states possessing finite lifetimes (widths).

Attention should especially be drawn to the nontrivial possibility of the existence of discrete quasi-energy states in the continuous spectrum. It may be thought that they arise in our problem bceause of the total reflection of waves incident on each of the delta-potentials. Upon the proper choice of the distance between the potentials, the interference leads to the formation between the potentials of a standing, nondecaying wave. Such type of "blocking" arises in the problem under consideration simply and naturally, and has an interference character. Notice also that the effect is not unique to the one-dimensional case. Thus, the antisymmetric solution (3.5) is the radial function for the three-dimensional problem with the potential

$$U(r, t) = (\alpha + \lambda \cos \omega t)\, \delta(r - a)$$

(a thin spherical layer with an oscillating penetration factor).

APPENDIX

We are interested in the solution to the recurrence relations (2.5) that decreases as $n \to -\infty$. For this solution it is not difficult to find the asymptotic form

$$c_n = \left(\frac{\lambda^2 e}{8\omega}\right)^{|n|/2} \exp\left(\alpha \sqrt{\frac{2|n|}{\omega}}\right) |n|^{-\zeta} \left[1 + O\left(\frac{1}{n}\right)\right], \qquad \zeta = \frac{|n|}{2} + \frac{1}{\omega}\left(\varepsilon - \frac{\lambda^2}{4}\right) + \frac{1}{4}. \tag{A.1}$$

Let us, for the sake of brevity, consider the case when $b_{0\varepsilon} = 0$ ($\delta_{\varepsilon 0} = 0$; the general case can be investigated in similar fashion). Then the recurrence relations can be obtained as a condition for the minimum of the functional

$$I\{c\} = \sum_{n=-1}^{-\infty} c_n^2 \varkappa_n - \lambda \sum_{n=-1}^{-\infty} c_n c_{n-1} \tag{A.2}$$

under the additional condition that

$$\sum_{n=-1}^{-\infty} c_n^2 = 1.$$

Let us reformulate the problem: Assuming that ε and λ are given, we shall find α from the minimum of the functional $I\{c\}$. It is necessary to prove the existence of a sequence of the c_n that causes the functional $I\{c\}$ to have a minimum. For this purpose, it is sufficient to prove the compactness in l_2 of the set of sequences defined by the condition $I\{c\} < A$. On account of the inequalities

$$I\{c\} \geqslant \sum_{n=-1}^{-\infty} \varkappa_n c_n^2 - \lambda \sum_{n=-1}^{-\infty} c_n^2 , \quad (A.3)$$

a sufficient condition for this is the compactness of the set of sequences that are such that $\Sigma \varkappa_n c_n^2 < A$. The latter assertion can easily be verified with the aid of the criterion for compactness[8] if we take into account the monotonic increase of the coefficients $\varkappa_n$.

Similar arguments enable us to prove the existence of bound states in the case of two delta-wells.

Notice that the analog of the considered situation in the case of differential equations is the assertion that the spectrum of systems with a potential that increases without restriction is discrete.[9]

1)It is convenient to employ the following procedure in practical computations. At sufficiently large $|n|$, we can use the asymptotic form (A.1) for the recurrence relations. Then, with the aid of the recurrence relations, we successively find the coefficients c_n, moving from the two sides (positive and negative n) right up to $n = n_0$, where we carry out a "matching" with allowance for the incident wave. After this a reverse run over the recurrence relations gives all the amplitudes $F_{n_0 \to n}$.

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Investigation of the possibility of designing a recombination gasdynamic O_2 laser

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An experimental and theoretical study was made of the possibility of designing a recombination gasdynamic oxygen laser operating in the visible range, as suggested by A. S. Biryukov et al. {Zh. Eksp. Teor. Fiz. 67, 2064 (1974), [Sov. Phys.-JETP 40, 1025 (1975)]}. The negative results obtained in the search for laser action with the aid of a shock tube indicated that the gain of transitions in the first positive system of the Herzberg bands was less than 10^{-3} cm^{-1}. The reported calculations indicated that although the population inversion in these transitions could reach values of 10^{18} cm^{-3}, the gain should not exceed 2×10^{-6} cm^{-1}, which was in agreement with the results of an experimental check.

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1. A recently published paper[1] is concerned with one of the possibilities of solving the pressing problem of development of chemical gas lasers emitting in the visible range. The main idea behind these lasers is the utilization of various chemiluminescence reactions as a result of which molecules are formed in excited electronic states. However, in spite of the fact that there is a large number of such reactions with a considerable quantum efficiency (see, for example[2,3]), nobody has yet been able to build a chemical laser emitting in the visible range.

Fairly attractive chemical reactions which can be used in such a laser are those involving recombination reactions producing molecules with a small number of atoms. The simplest reactions of this type are those involving three-particle recombination of atoms producing diatomic molecules. A specific example is

$$O(^3P) + O(^3P) + M \to O_2(i) + M, \quad (1)$$

where $O_2(i)$ is an oxygen molecule in an i-th bound electronic state correlated with the ground state of ^3P of the oxygen atom. Six such states are known: $X^3\Sigma_g^-$, $a^1\Delta_g$, $b^1\Sigma_g^+$, $C^3\Delta_u$, $A^3\Sigma_u^+$, $c^1\Sigma_u^-$ (see Fig. 1 in Ref. 1); we shall denote these by $i = 1, \ldots, 6$, respectively. The states can be divided arbitrarily into two groups. The minima of the potential curves of the states $i = 4, 5, 6$, located higher than those in the other group ($i = 1, 2, 3$), are shifted toward greater internuclear distances.

It is suggested in[1] that the reaction (1) be used in