Electron bremsstrahlung from a nucleus in the field of a plane electromagnetic wave

A. V. Borisov and V. Ch. Zhukovskiĭ

Moscow State University
(Submitted July 16, 1975)
Zh. Eksp. Teor. Fiz. **70**, 477–483 (February 1976)

The cross section electron bremsstrahlung from a nucleus in the field of an arbitrary intensity electromagnetic wave is calculated in the equivalent-photon approximation. The cases of low and high wave intensities and the limiting transition to a constant field are considered.

PACS numbers: 25.30.Cg

1. INTRODUCTION

The development of laser technology has aroused interest in the study of various quantum processes in the field of an electromagnetic wave. Single-photon processes (emission, pair production, and annihilation) in the field of a wave have been thoroughly studied (see[1,2], and also[3]). With increasing field intensity and particle energy, an important role is assumed by electrodynamic processes of higher order in the fine-structure constant α. Effects such as the Moller scattering of an electron by an electron and Compton scattering in the field of a wave[4,5] have been recently investigated. Bremsstrahlung by a screened Coulomb center in the field of a wave was considered in the nonrelativistic approximation in[6]. In the relativistic case, in view of the complexity of the calculations, it is convenient to use the method of equivalent photons, [7–9] as was done in the calculation of the bremsstrahlung and photoproduction of pairs from a nucleus situated in a constant field. [10,11]

In this paper we use the procedure of[10,11] to calculate the cross section of the bremsstrahlung of a relativistic electron from a nucleus in the field of a plane electromagnetic wave.

2. BREMSSTRAHLUNG CROSS SECTION

We consider the collision of an electron with a spinless nucleus in the field of a plane circularly polarized wave with a vector potential[1)]

$$A=a_1\cos kx+\lambda a_2\sin kx,$$
$$a_1^2=a_2^2=a^2,\quad a_1a_2=0,\quad a_{1,2}k=0, \tag{1}$$

where $k=(\omega,\mathbf{k})$ is the wave vector, $a_{1,2}$ are the amplitudes, and $\lambda=\pm1$ determines the right-hand (left-hand) polarization of the wave.

The state of the electron in the field of the wave is described by the quasimomentum $p^\mu=(p_0,\mathbf{p})$, [1,2] which satisfies the condition $p^2=m_*^2\equiv m^2(1+\xi^2)$, where m is the electron mass and

$$\xi=e\sqrt{-a^2}/m \tag{2}$$

is a parameter of the wave intensity. The influence of the external field of the wave on the motion of the heavy nucleus will be neglected; the momentum of the nucleus will be designated by $Q(Q^2=M^2\gg m^2)$.

We choose a special reference frame, in which the electron prior to the collision is at rest, that is, $\mathbf{p}=0$, and the nucleus is relativistic ($|\mathbf{Q}|\gg M$) and moves along the wave propagation direction ($\mathbf{k}\parallel\mathbf{Q}$); in this system the scalar potential of the wave field is $A_0=0$. In invariant form, these conditions can be written in the following manner:

$$F^{\mu\nu}Q_\mu p_\nu=0, \tag{3}$$

where $F^{\mu\nu}$ is the field tensor.

In the indicated reference frame, the bremsstrahlung process can be regarded as the interaction of an electron with equivalent photons q of the nucleus, propagating along the direction $\mathbf{Q}\parallel\mathbf{k}$. The cross section of the process in the equivalent-photon approximation is written in the form

$$\sigma=\int\sigma^{\rm ph}dN, \tag{4}$$

where $\sigma^{\rm ph}$ is the cross section of the corresponding photoprocess in the field of the wave, with participation of real photons ($q^2=0$), dN is the spectrum of the equivalent photons in variant form:

$$dN=\frac{2}{\pi}Z^2e^2\ln\frac{\mu}{\varkappa_1}\frac{d\varkappa_1}{\varkappa_1}, \tag{5}$$

where Ze is the charge of the nucleus, and $\varkappa_1=2(pq)/m^2$, $\mu=pQ/mM$. We note that we have neglected the influence of the external field on the spectrum of the equivalent photons of the heavy nucleus ($M/m\gg1$).

Formula (4) is valid if the following conditions are satisfied (see, e.g., [9]):

$$-q^2\ll m^2,\quad -q^2/m^2\ll\varkappa_1,\quad \mu\gg1,\quad \mu/\varkappa_1\gg m/M. \tag{6}$$

The cross section of the photoprocess $\sigma^{\rm ph}$ in (4) is connected with the probability $w^{\rm ph}$ by the relation

$$\sigma^{\rm ph}=\frac{2q_0p_0}{m^2\varkappa_1}w^{\rm ph}, \tag{7}$$

where $w^{\rm ph}$ can be represented in the form of a sum of three terms:

$$w^{\rm ph}=w_1+w_{-1}+w_{02}, \tag{8}$$

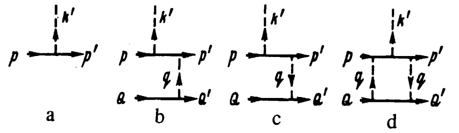

FIG. 1

with the following meanings: w_1 is the probability of the Compton scattering of the photon q by the electron in the field of the wave, w_{-1} is the probability of the emission of the photon k', accompanied by induced emission of the photon q (induced two-photon emission in the terminology of[12], in which the probabilities $w_{\pm 1}$ were obtained for an electron in a magnetic field). The term w_{02} describes the correction to the one-photon emission in the field of the wave, due to the absorption and emission of the photon q by the electron.

The probabilities w_i can be calculated from the probability obtained in[13], for the emission of the electron in the field of two monochromatic waves, by expanding in the intensity parameter of one of the waves.

As a result we obtain for the probability of the Compton effect in the field of the wave

$$w_1=\frac{\alpha m^2}{4p_0}\frac{4\pi e^2}{m^2 q_0}\sum_{s>-\varkappa_1/\varkappa}\int_0^{u_s}\frac{du}{(1+u)^3}\left[\frac{u^2}{2}S_1^2+\left(1+u+\frac{u^2}{2}\right)(S_2^2+S_3^2)\right],$$

$$S_1=(1+\xi^2)^{-1/2}\left(\frac{2\gamma}{y_g}J_{s+g}-\frac{2\sqrt{y_s-1}}{x_*}J_s\right),$$

$$S_2=\frac{\gamma}{y_g}\frac{y_s+y_g-2}{(y_s-1)^{1/2}}J_{s+g}-\frac{y_s-2}{x_*}J_s,$$

$$S_3=gJ_s+\frac{2\gamma}{x_*}(y_s-1)^{1/2}J_s'-\frac{2\gamma^2}{y_g}J_{s+g}', \tag{9}$$

where

$$u_s=(\varkappa_1+s\varkappa)/(1+\xi^2),\quad y_s=x_*+sy_*,\quad y_g=x_*-gy_*,$$

$$x_*=\varkappa_1/u(1+\xi^2),\quad y_*=\varkappa/u(1+\xi^2),\quad \varkappa=2kp/m^2,\quad \gamma=\xi/\sqrt{1+\xi^2},$$

$g=\lambda\lambda_1$, and λ_1 is the polarization of the photon q. In expressions (9), the Bessel functions J_s with integer index s depend on the argument

$$z=\frac{2\gamma}{y_*}(y_s-1)^{1/2}, \tag{10}$$

and the variable

$$u=kk'/kp' \tag{11}$$

(k' is the momentum of the emitted photon and p' is the quasimomentum of the electron in the final state) characterizes the spectral distribution of the probability. The probability w_{-1} of the induced two-photon emission is obtained from w_1 via the substitutions $\varkappa_1 \to -\varkappa_1$ and $g \to -g$; the explicit form of the probability w_{02} will not be presented here.

We substitute (8) and (7) in (4) and integrate over the spectrum of the equivalent photons $\varkappa_1$ from zero to ∞. We obtain the cross section in the form

$$\sigma=\int_0^\infty d\varkappa_1\frac{dN}{d\varkappa_1}\sum_s\int_0^{u_s}du\frac{d\sigma_s^\Phi}{du}. \tag{12}$$

To find the bremsstrahlung spectrum $d\sigma/du$ we interchange in (12) the order of integration with respect to $\varkappa_1$ and u. As a result we get

$$\sigma=4\frac{Z^2e^6}{m_*^2}\left\{\sum_{s=-\infty}^{\infty}\int_0^\infty\frac{du}{u(1+u)^3}\int_{1+\xi^2-sy}^{\infty}\frac{dx}{x^2}\ln\left|\frac{\mu}{ux}\right|A_1^{(s)}\right.$$
$$\left.+\sum_{s=1}^{\infty}\int_0^{s\varkappa/(1+\xi^2)}\frac{du}{u(1+u)^3}\int_0^\infty\frac{dx}{x^2}\ln\left|\frac{\mu}{ux}\right|A_{02}^{(s)}\right\}=\sigma_1+\sigma_{-1}+\sigma_{02}, \tag{13}$$

where

$$A_1^{(s)}=\frac{u^2}{2}\langle S_1^2\rangle+\left(1+u+\frac{u^2}{2}\right)(\langle S_2^2\rangle+\langle S_3^2\rangle),$$

and in place of the variables x_* and y_*, defined in (9), we have introduced $x=(1+\xi^2)x_*=\varkappa_1/u$ and $y=(1+\xi^2)y_*=\varkappa/u$. Here the functions $\langle S_i^2\rangle$ are obtained from S_i^2 (9) by averaging over the polarizations λ_1 of the virtual photon q; the quantity A_{02} corresponds to the probability w_{02} (its explicit form will not be given here). We note also that in the derivation of (13) we took into account the relations $w_{-1}(\varkappa_1)=w_1(-\varkappa_1)$, $w_{02}(\varkappa_1)=w_{02}(-\varkappa_1)$.

The value of each of the terms σ_i in this formula can be explained with allowance for the interpretation given above for the corresponding terms in formula (8) for the probability of the photoprocesses: the terms σ_1 and σ_{-1} describe the contribution made to the cross section by the matrix elements of the bremsstrahlung upon emission or absorption of a photon q by a nucleus (diagrams b and c in Fig. 1); the term σ_{02} is due to the interference of the matrix element of the one-photon radiation in the field of the wave without interaction with the nucleus (diagram a of the same figure) and to bremsstrahlung with exchange of two photons q with the nucleus (diagram d), one of which is absorbed and the other emitted by the nucleus (each of the diagrams b, c, and d stands for an aggregate of diagrams that differ in the obvious permutation of the vertices).

3. ASYMPTOTIC FORMULAS FOR THE CROSS SECTION AND DISCUSSION OF RESULTS

The integration over the spectra of the equivalent photons $\varkappa_1$ in the expression (13) obtained above for the cross section is difficult for arbitrary values of the wave intensity parameter ξ. We shall therefore consider the asymptotic behavior of the cross section in two limiting cases, small and large ξ.

1) $\xi \ll 1$ (weak wave). In this case the argument (10) of the Bessel functions that enter in the expression for $A_i^{(n)}$ is small, and the main contribution is made by the terms with $s=0$, and ± 1. Expanding in terms of the parameter ξ, we obtain for the quantities $A_i^{(s)}$ the expressions

$$A_1^{(1)}=u^2\xi^2\left[\left(\frac{1}{x+y}-\frac{x+y-1}{xy}\right)^2+\left(\frac{x+y-1}{xy}\right)^2\right]$$
$$+\left(1+u+\frac{u^2}{2}\right)\xi^2\left\{\frac{1}{2}\left[\left(\frac{x+y-2}{xy}\right)^2+\left(\frac{2}{x+y}-\frac{x+y-2}{xy}\right)^2\right]\right.$$
$$\left.+\frac{1}{x^2}+\frac{1}{y^2}\right\}(x+y-1); \tag{14}$$

the expression for $A_1^{(-1)}$ (corresponding to $s=-1$) is

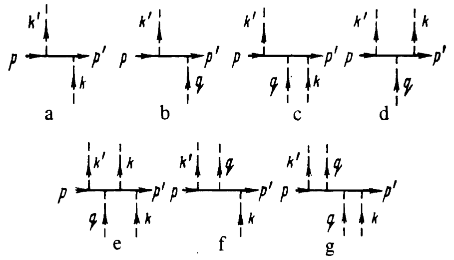

FIG. 2.

obtained from (14) by the substitution $y \to -y$.

For $A_1^{(0)}$ ($s=0$) we have

$$A_1^{(0)} = \frac{u^2}{2}\left\{\frac{4(x-1)}{x^2} - 4\xi^2\left[\frac{1}{x^2} + \frac{2(x-1)^2}{x^2y^2} + \frac{2(x-1)}{x(x^2-y^2)}\right]\right\}$$
$$+\left(1+u+\frac{u^2}{2}\right)\left\{1+\left(\frac{x-2}{x}\right)^2 - 2\xi^2\left[\frac{(x-1)(x-2)^2}{x^2y^2} + \frac{2(x-2)}{x^2}\right.\right.$$
$$\left.\left. + \frac{(x-2)^2}{x(x^2-y^2)} + \frac{x-1}{y^2} - \frac{x}{x^2-y^2}\right]\right\}. \qquad (15)$$

We note that $A_{-1}^{(s)}(x) = A_1^{(s)}(-x)$ (this relation was used in the derivation of formula (13) for the cross section).

Finally, we present an expression for the interference term A_{02}:

$$A_{02} = 2u^2\xi^2\left[\frac{2(y-1)}{y(x^2-y^2)} - 2\left(\frac{y-1}{xy}\right)^2 - \frac{1}{y^2}\right]$$
$$+2\xi^2\left(1+u+\frac{u^2}{2}\right)\left[\frac{(y-2)^2}{y(x^2-y^2)} - (y-1)\left(\frac{y-2}{xy}\right)^2 - \frac{2(y-2)}{y^2}\right.$$
$$\left. + \frac{y}{x^2-y^2} - \frac{y-1}{x^2}\right]. \qquad (16)$$

Figure 2 shows diagrams corresponding to expansion, in terms of ξ, of the probability w^{ph} of the photoprocess (see formula (8)) accurate to terms $\sim \xi^2$. The functions $A_1^{(1)}$, $A_1^{(-1)}$, and $A_{-1}^{(-1)}$ correspond here to the diagrams c, d, and f, respectively, the functions $A_1^{(0)}$ correspond to interference of the diagrams b and e, and the functions A_{02} correspond to interference of diagrams a and g. We see that the photons k and q enter in the expansion in symmetrical fashion. This explains why A_{02} (16) is obtained from $A_1^{(0)}$ (15) to means of the substitutions $x \to y$ and $y \to x$ in the terms $\sim \xi^2$.

For the cross section, with allowance for the terms $\sim \xi^2$, we have the expression

$$\sigma = 4\frac{Z^2e^6}{m^2}\left\{\int_0^\infty \frac{du}{u(1+u)^3}\left[\int_{1-y}^\infty \frac{dx}{x^2}\ln\left|\frac{\mu}{ux}\right| A_1^{(1)}\right.\right.$$
$$\left. + \int_{1+y}^\infty \frac{dx}{x^2}\ln\left|\frac{\mu}{ux}\right| A_1^{(-1)} + \xi^2\left(\frac{\partial}{\partial\xi^2}\int_{1+\xi^2}^\infty \frac{dx}{x^2}\ln\left|\frac{\mu}{ux}\right| A_1^{(0)}\right)_{\xi^2=0}\right]$$
$$\left. + \int_0^\varkappa \frac{du}{u(1+u)^3}\int_0^\infty \frac{dx}{x^2}\ln\left|\frac{\mu}{ux}\right| A_{02}\right\}. \qquad (17)$$

where $A_i^{(s)}$ are defined in (14)–(16).

When integrating over the spectrum of x in (17), a distinction must be made between two cases: a) $y = \varkappa/u < 1$ ($u > \varkappa$); b) $y > 1$. Let us consider them separately.

a) $y < 1$. In this region of the spectrum, the interference term $d\sigma_{02}/du$, as is seen from (17), makes no contribution. Integration with respect to x in the logarithmic approximation with allowance for the relation $A_1^{(-1)}(y) = A_1^{(1)}(-y)$ yields the following expression for the spectrum:

$$d\sigma = 4\frac{Z^2e^6}{m^2}\frac{du}{u(1+u)^2}\left\{\left(\frac{4}{3}+\frac{u^2}{1+u}\right) + \xi^2\left[\left(\frac{4}{3} - \frac{8}{3}\frac{3-2y^2}{y^2(1-y^2)}\right.\right.\right.$$
$$\left. + \frac{8}{y^4}\ln|1-y^2| + \frac{8}{y^3}\ln\left|\frac{1+y}{1-y}\right|\right) + \left(1+\frac{u^2}{2(1+u)}\right)\left(-\frac{8}{3} - \frac{11}{3y^2}\right.$$
$$\left.\left.\left. + \frac{10}{3y^2(1-y^2)} + \frac{1}{3y^2}\left(\frac{1+y^2}{1-y^2}\right)^2 + \frac{2}{y^2}\ln|1-y|^2\right)\right]\right\}\ln\frac{\mu}{u} = d\sigma_0 + d\sigma'. \qquad (18)$$

The term $d\sigma_0$ which does not depend on ξ yields here the well-known spectrum of bremsstrahlung from a nucleus in the free case (see[8], p. 455), if we put $u = (E - E')/E'$, $\mu = E/m$, where E and E' are the initial and final energies of the electron in the rest system of the nucleus in the absence of an external field ($F_{\mu\nu} = 0$). The terms $d\sigma' \sim \xi^2$ describe the influence exerted on the emission spectrum by the external field of the wave. We note that as $y \to 1$ the cross section (18) acquires a resonant character: it diverges like $(1-y)^{-2}$. This resonance is due to the fact that the electron in the intermediate state (see Fig. 2) can lie on the mass shell (at $x = 0$, i.e., $(qp) = q^2 = 0$), and then the bremsstrahlung process reduces to two independent processes: elastic scattering of the electron by the nucleus with exchange of a virtual photon q, and Compton scattering of a photon k on an electron, with the radiation at $y = 1$ directed strictly forward relative to the motion of the initial electron. The same region ($y = 1$) yields the maximum frequency of the scattered photons in the Compton effect on moving electrons: $\omega'_{\max} = E\varkappa/(1+\varkappa)$. This divergence can be eliminated by introducing radiative corrections to the Green's function of the electron with allowance for the screening of the Coulomb field of the nucleus (see[6]). A resonant interaction of this type was considered also in[14]. Thus, the result (18) is valid in a region far from resonance, that is, at $(y-1)^2 \gg \xi^2$.

b) $y > 1$ ($u < \varkappa$). In this case the integral of $A_1^{(1)}$ in (17) diverges at the point $x = 0$, but this divergence is offset by the interference term A_{02}. The result turns out to be finite and coincides with (18). Thus, both at $u < \varkappa$ and at $u > \varkappa$ the cross section is described by one and the same formula (18).

2) $\xi \gg 1$ (strong wave). The parameter ξ can be made large by decreasing the frequency ω of the wave at a fixed intensity amplitude $F(\xi = eF/m\omega)$. This case reduces therefore to the process of bremsstrahlung in a constant crossed field, which was considered in[10]. The limiting transition to a crossed field in the expressions for the probabilities of the photoprocesses $w_{\pm1,02}$ can be effected in the same manner as in[2] for single-photon processes in the field of a wave.

Using the limiting expressions obtained in this manner, we obtain from (4), (7), and (8), for bremsstrahlung in the field of a strong ($\xi \to \infty$) wave, a cross section that coincides with the cross section calculated in[10] for the radiation in a constant field. In particular, for the hard end of the spectrum we have the following expression for the correction to the cross section, due to the constant external field:

$$d\sigma' = \frac{8Z^2e^6}{m^2}\frac{du}{u(1+u)^2}\left(\frac{\chi}{u}\right)^2\left(\frac{86}{15}+\frac{5u^2}{1+u}\right)\ln\frac{\mu}{u},$$
$$\chi = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}p^\nu)^2}, \quad u \gg \chi. \tag{19}$$

We note that this result follows also from formula (18), which was obtained for $\xi \ll 1$ in the limit $u \gg \varkappa$, if we put $\xi\varkappa/2 = \chi$. This is explained by the fact that the time $\tau \sim E/m^2u$ of formation of the hard end of the spectrum (see[9]) is small in comparison with the period $1/\omega$ of the external wave.

The authors are deeply grateful to A. A. Sokolov for constant interest in the work.

1)We use the metric (+ – – –) and the system of units $\hbar = c = 1$, $\alpha = e^2 = 1/137$.

1A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 46, 776, 1768 (1964) [Sov. Phys.-JETP 19, 529, 1191 (1964)].

2N. B. Narozhnyĭ, A. I. Nikishov, and V. I. Ritus, Zh. Eksp. Teor. Fiz. 47, 930 (1964) [Sov. Phys. -JETP 20, 622 (1965)].

3A. A. Sokolov and I. M. Ternov, Relyativistskiĭ elektron (The Relativistic Electron), Nauka, 1974.

4V. P. Oleĭnik, Zh. Eksp. Teor. Fiz. 52, 1049 (1967) [Sov. Phys.-JETP 25, 697 (1967)].

5V. P. Oleĭnik, Zh. Eksp. Teor. Fiz. 53, 1997 (1967) [Sov. Phys.-JETP 26, 1132 (1968)].

6I. V. Lebedev, Opt. Spektrosk. 32, 120 (1972).

7V. N. Grivob, V. A. Kolkunov, L. B. Okun', and V. M. Shekhter, Zh. Eksp. Teor. Fiz. 41, 1839 (1961) [Sov. Phys.-JETP 14, 1308 (1962)].

8V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), pt. 1, Nauka, 1968 [Pergamon, 1971)].

9V. N. Baĭer, V. M. Katkov, and V. S. Fadin, Izluchenie relyativistskikh élektronov (Radiation of Relativistic Electrons), Atomizdat, 1973.

10V. Ch. Zhukovskiĭ, Zh. Eksp. Teor. Fiz. 66, 9 (1974) [Sov. Phys.-JETP 39, 4 (1974)].

11A. V. Borisov and V. Ch. Zhukovskiĭ, Yad. Fiz. 21, 579 (1975) [Sov. J. Nucl. Phys. 21, 301 (1975)].

12V. Ch. Zhukovskiĭ and N. S. Nikitina, Zh. Eksp. Teor. Fiz. 64, 1169 (1973) [Sov. Phys.-JETP 37, 595 (1973)].

13V. Ch. Zhukovskiĭ and I. Kheppman, Vestn. Mosk. Univ. Fiz. Astron. 6, 671 (1970).

14M. V. Fedorov, Zh. Eksp. Teor. Fiz. 68, 1209 (1975) [Sov. Phys.-JETP 41, 601 (1975)].

Translated by J. G. Adashko

Two-photon processes in a Coulomb field in the dipole approximation

A. I. Ignat'ev

Leningrad Institute of Nuclear Physics, USSR Academy of Sciences

(Submitted August 9, 1975)
Zh. Eksp. Teor. Fiz. 70, 484–492 (February 1976)

An expression for the amplitude of two-photon processes in a Coulomb field is derived. The expression is an analytic function of not only the photon energy, but also of the quantum numbers of the initial and final electron states. The formulas obtained are used to calculate the cross section for light scattering for both bound-bound and bound-free electron transitions.

PACS numbers: 32.10.Vc

1. INTRODUCTION

In recent years two-photon processes in a Coulomb field have been the subject of a number of papers. Apart from the fact that these processes describe many physical phenomena, they are also of great interest as a model for investigating more complex systems.

The probability for the two-photon decay of the metastable $2s$ level of the hydrogen atom was computed in[1] by the method of approximate numerical summation of series. In[2] an approximate (semiquantitative) formula was derived for coherent light scattering from the ground state of the hydrogen atom. The cross sections for coherent $(1s \to 1s)$[3] and Raman $(1s \to 2s)$[4] light scattering and for the two-photon ionization of the $2s$ level[5] have been computed with the aid of the Schwartz–Tiemann method.

Only comparatively recently were analytic expressions for the amplitude of two-photon transitions between certain low-lying excited states of the hydrogen atom obtained with the aid of one or another integral representation of the Green function for a charged particle in the Coulomb field.[6–8] In 1967 Gavrila[9] expressed the amplitude for coherent light scattering from the $1s$ state in terms of the hypergeometric functions. The same result was independently obtained together with expressions for the two-photon $1s \rightleftarrows 2s$ transition amplitudes by Vetchinkin and Khristenko[10] and Granovskiĭ.[11] Further, Zon, Manakov, and Rappoport[12] have shown that the bound–bound and bound–free transition amplitudes can be expressed in terms of linear combinations of the hypergeometric functions. A similar result was obtained in[13] by Gorshkov and Polikanov, who used the momentum representation of the

