that the principal directions of the matrix s^j_k coincide with the "Kasner axes," and the Kasner exponents p_i are determined by the equations

$$p_i=1-\frac{2s_i}{s_1+s_2+s_3},$$

where s_1, s_2, s_3 are the eigenvalues of the matrix s^j_k.

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Properties of second vacuum pole P' in the theory of the pomeron as a Goldstone particle

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It is proposed that the pomeron is a Goldstone particle that appears upon spontaneous symmetry breaking of the system of vacuum poles and P and P'. The properties of the pomeron are barely affected by the interaction and are determined by its bare (unrenormalized) characteristics. The properties of P' depend strongly on the interaction with the pomerons. The contribution of P' at low energies s contains terms that decrease in power-law fashion (and can, generally speaking, also oscillate as functions of ln s, depending on the choice of the model). At high energies this contribution goes over into an expression analogous to the usual negative contribution of non-enhanced reggeon branch cuts, but those containing a small cutoff radius and therefore strongly dependent on ln s. This can result in a rather rapid growth of the total cross section even in the experimental energy region. At a momentum transfer $t\neq 0$, a mixed state is produced in the system of two pomerons and its contribution to the angular distribution leads to the appearance of a second maximum at $t\neq 0$. The existence of such a state can therefore explain the known anomalies in the angular distributions of pp scattering at high energies.

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INTRODUCTION

In the theory of complex angular momenta, the $\omega=j=1$ Pomeranchuk pole P is the analog of a nonrelativistic massless excitation. An illustrative confirmation of this property is the fact that the positions $\omega=0$ of all the singularities corresponding to exchange of an arbitrary number of pomerons coincide (at reggeon momenta $\mathbf{k}=0$).[1,2]

For most nonrelativistic physical systems, the appearance of a massless Goldstone excitation is evidence of spontaneous breaking of the continuous symmetry existing in the system.[3] This phenomenon is well known in solid state physics,[4] namely, the onset of zero-gap excitations in a phase transition. It is therefore natural to assume that the existence of a pomeron is also due to an analogous cause, namely spontaneous breaking, at momentum transfers $t<0$, of the symmetry of a certain continuous group characterizing hadron interactions in a vacuum channel of positive signature ($t=-k^2$).

A phenomenological identification of the type of group that can be responsible for the appearance of a pomeron as a Goldstone particle is afforded by the character of the excitations with the aid of which it is customary to describe the vacuum channel. At $t<0$ this channel contains a second vacuum trajectory P' besides the pomeron. It was therefore proposed in[5], henceforth referred to as I, that the Pomeranchuk pole is produced as a Goldstone boson following spontaneous breaking of the symmetry of a system of two interacting reggeons P and P'. This hypothesis, as shown in I, leads to hindrances and constraints on the constants of the reggeon interactions, and makes it possible to find their possible forms when the interactions are expanded in powers of the reggeon momenta $\mathbf{k}_i$.

The traditional representations (see[6]) call for the contribution of P' to the cross sections of the processes to be small and to decrease in power-law fashion with increasing energy s:

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$$s^{-1}\delta A_{P'}\approx g^2 s^{-\Delta} \quad (\Delta\leqslant{}^1/_2). \tag{1}$$

It is customary to try to assume the slope $\alpha'_{P'}$ of the trajectory P' to be equal to the slope of the $\omega-\rho$ trajectory (exchange-degeneracy hypothesis). In the scattering region ($t<0$), however, not much is known concerning this slope.[6,7]

This article considers the properties of P' in the theory where the pomeron is regarded as Goldstone particle in the simplest among the models described in I (Sec. 1). Principal attention is paid to those qualitative features which do not depend on the model and are peculiar to the theory with the Goldstone pomeron. In models of this type, the pomeron is a pole that is either completely stable, or stable at $|t|=k^2=0$ (quasistable). The interactions affect its trajectory little, and its bare (nonrenormalized) characteristics should be identified with the experimental ones ($\alpha'_P\approx 0.3$ GeV^{-2}). The situation is different with the other vacuum pole P'. The bare singularity is a pole at the point $\omega=-\Delta$ ($\Delta>0$), and is strongly distorted by the interaction. The Green's function of P' has poles and a cut in the complex ω plane (Secs. 2, 3, and 4). At relatively low energies $\xi\Delta\lesssim 1$ (formula (26)) ($\xi=\ln s$) the contribution of P' actually contains terms of the type (1) that decrease in power-law fashion with increasing energy. However, both the rate of decrease and the concrete form of these terms depend on the choice of the model.

Under these conditions, the characteristics of the bare pole—the shift Δ and the slope $\alpha'_{P'}$—become simply parameters of the model, having no direct bearing on those experimental characteristics of P' which describe that part of the contribution of the second vacuum singularity which decreases in power-law fashion. Within the framework of the model of Sec. 1 it is natural to assume that

$$\alpha_{P'}'=\alpha_P', \tag{2}$$

i.e., $\alpha'_{P'}\approx 0.3$ GeV^{-2}. Relation (2) is not a mandatory condition: the analysis in I covers also models in which $\alpha'_{P'}$ is not equal to α_P. From the point of view of comparing our model with experiment, however, it is more likely that (2) ($\alpha'_{P'}\approx\alpha_P$) is a good choice. The shift Δ remains purely a fitting parameter.

Of fundamental significance for the properties of P' at high energies $\xi\Delta\gg 1$ is its instability—the possibility of decay into a two-pomeron channel (Fig. 1a). The existence of the $P'\to 2P$ transition does not depend on the model or even on the Goldstone character of the pomeron: the second vacuum pole lies on the two-pomeron cut. Therefore those properties of P' that pertain to high energies (Secs. 3 and 4) can be described also purely phenomenologically. The theory of the Goldstone pomeron merely creates automatically the conditions necessary for the realization of these properties. Owing to the instability of P', the asymptotic form of

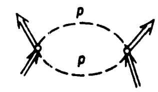

FIG. 2.

the scattering amplitude contains enhanced diagrams of the type of Fig. 1b. The corresponding contribution to the asymptotic form at $k^2=0$ and $\xi\to\infty$ is equal to

$$s^{-1}\delta A_{2P}\approx -g_{P'}{}^2\frac{\lambda}{\Delta(\xi+R_L{}^2/\alpha_P')}, \quad \lambda>0. \tag{3}$$

Formula (3) is formally completely analogous to the usually considered contribution of the non-enhanced branch cuts (Fig. 2), but contains the cutoff radius R_L that arises in the reggeon-interaction vertex (Fig. 1a). In contrast to the large cutoff radius of reggeon-hadron vertices, which is determined by the slope of the diffraction peak ($R_d^2/\alpha'_P\sim 12$ for pp scattering and ~ 7 for πp scattering),[6,7] the radius R_L, insofar as can be judged from the experimental data,[8] is relatively small:

$$R_L{}^2/\alpha_P'\leqslant 1, \tag{4}$$

i.e., the cutoff momenta are $k_L\gtrsim 1-2$ GeV. Therefore (3) depends strongly on ξ already at experimental values of the energy. Since the residue of the second vacuum pole $g^2_{P'}$ is large,[6] this strong dependence leads to a rather rapid increase of the total cross sections ($\delta A_{2P}<0$). The effect due to the instability of P' can be an appreciable fraction of the growth of σ_{tot} observed at high energies.[9]

The second qualitative consequence of the instability of P' (Sec. 4) consists in the appearance, at $k^2\neq 0$, of a bound state of the two interacting pomerons. The appearance of this pole in the two-dimensional (with respect to **k**) problem is ensured by the effective attraction between the pomerons, which is present in the Goldstone-pomeron theory (see Sec. 1). A "bound state" appears at $k^2=0$ from under the two-pomeron cut (Fig. 3) and at fixed k^2 it is located at an exponentially small distance x_0 from the edge of the cut ($\omega=-(1/2)\times\alpha'_P k^2$):

$$x_0=a\exp\{-\eta/k^2\}, \quad \eta=2\Delta/\lambda\alpha_P'. \tag{5}$$

The contribution of the pole x_0 to the elastic-scattering amplitude is also exponentially small realtive to $1/k^2$:

$$s^{-1}\delta A=-b(\eta/k^2)^2\exp\{-\eta/k^2-\alpha_P'k^2\xi/2+x_0\xi\}. \tag{6}$$

Expression (6) has a maximum at

$$\eta/k^2=1+(1+\alpha_P'\xi\eta/2)^{1/2} \tag{7}$$

($x_0\ll 1$). If we choose the parameters λ and Δ such that expression (3) determines approximately the experimental growth of the cross section (Sec. 4), then the maximum of (7) is located in the region $|t|\sim 2$ GeV^2, i.e.,

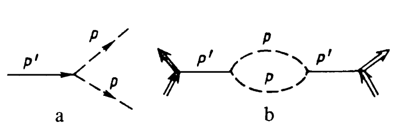

FIG. 1.

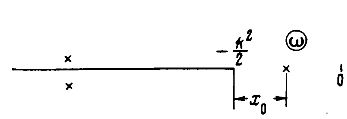

FIG. 3. Poles (×) and cut of the Green's function of P' (31) in the complex ω plane.

near the known maximum of the angular distribution of the pp scattering: $|t_{max}| \sim 1.8 - 1.9$ GeV^2.[10] This circumstance allows us to assume that the pole (5) can play an important role in the formation of the minimum and the maximum of the angular distributions. Expression (6) is, as it were, specially intended for this purpose. The choice of the model parameters and of the hadron form factors makes it possible to attain a qualitative agreement with the experimental picture.[10]

At large momentum transfers, $|t| \sim 1.5 - 3$ GeV^2, the corrections to the considered model are not small, but the very existence of the pole (5) does not depend on the model, being a property of the Goldstone-pomeron theory.

In the picture under consideration, the angular distributions of the scattering of different hadrons at high energies should be similar to the pp-scattering angular distributions. The differences can appear only via the hadron form factors of P and P' (and the contributions of the non-enhanced diagrams).

In Sec. 5, an expression is derived for the three-reggeon limit of the total cross sections.[11] It is unusual in that there is no three-pomeron vertex at $k^2 = 0$ in the Goldstone-pomeron theory.

Other ("relativistic") variants of the approach to the pomeronas a Goldstone particle are proposed in[12,13]. From our point of view, the consequences of these variants are much farther from the real world of hadrons than in the model described here.

1. MODEL LAGRANGIAN OF THE INTERACTION OF P AND P'

The interaction of a massless particle cannot be perfectly arbitrary. It must satisfy a certain system of conditions that are called upon to cancel out the infrared singularities and do not allow the interaction to cause an increase in the mass.[3]

For interacting reggeons P and P', where P is analogous to a nonrelativistic zero-gap excitation and P' to excitation with a gap Δ, the simplest Lagrangian satisfying the necessary conditions (see I) is

$$L = -\frac{1}{2}\left(\psi_i^+ \frac{\partial \psi_i}{\partial \xi} - \frac{\partial \psi_i^+}{\partial \xi}\psi_i\right) - \alpha' \nabla_\rho \psi_i^+ \nabla_\rho \psi_i - \Delta\psi_1^+\psi_1 + \sqrt{\lambda\Delta}\,(\psi_1^+\psi_2^2 - \psi_1\psi_2^{+2}) + \lambda\psi_i^{+2}\psi_k^2. \quad (8)$$

Here $\psi_i(\xi, \rho)$ and $\psi_i^+(\xi, \rho)$ ($i = 1, 2$) are the operators of the creation and annihilation of the reggeons P' ($i = 1$) and P ($i = 2$). They depend on the imaginary time ξ, which is canonically conjugate to the angular momentum $\omega = j - 1$, and on the two-dimensional vector ρ of the impact parameters.

Formula (8) appears when the symmetry of the Lagrangian that is invariant to the rotation of the axes

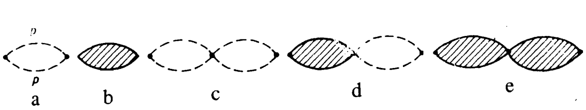

FIG. 4.

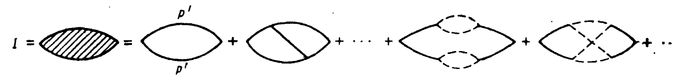

FIG. 5.

$i = 1, 2$ is spontaneously broken. This formula does not contain interaction that might displace the pomeron from the point $\omega = 0$, and leaves it absolutely stable. The connection of Δ and λ with the coefficient of the non-Hermitian three-reggeon vertex in (8) leads to cancellation of the infrared divergences (see I).

It must be noted that the signs of λ and Δ in (8) are fixed: if $\Delta > 0$ (unitary pole satisfying the Froissart formula), then also $\lambda > 0$ (see I). This is the essential difference between the Goldstone-pomeron theory and the usual Regge theory.[2] The fact that the constants in (8) have definite signs leads to an effective attraction in a system of two pomerons, and to formation of their "bound state" (Sec. 4).

More general forms of the Goldstone pomeron interaction (see I) have similar properties. These forms include many-particle vertices ($n > 4$) and vertices that contain $\nabla\psi$ and $\nabla\psi^+$.[1)] The principal difference from (8) is that the interactions with the gradients make the pomeron quasi-stable with a small width proportional to a power of its momentum (k^2 or k^4). This circumstance has little effect on the qualitative properties of the theory, and the latter can be investigated with the Lagrangian (8) as an example.

The Feynman diagrams for the interactions (8) make it possible to calculate the partial waves $f(\omega, k^2)$, which determine the asymptotic form of the imaginary part of the amplitudes of the processes:

$$\frac{A_1(s,t)}{s} = \int_{-i\infty+l_0}^{+i\infty+l_0} \frac{d\omega}{2\pi i} e^{\omega\xi} f(\omega, k^2). \quad (9)$$

2. THE GREEN'S FUNCTION OF P'

The vertex (Fig. 1a) of the Lagrangian (8) leads to instability of the state P'. If P and P' were particles, then P' could not appear in the asymptotic states of the theory and the s-matrix would describe only processes with P.[3] The Regge theory is a theory with external sources (scattered hadrons) that can emit both stable reggeons P and unstable ones P'. Therefore in the Regge theory P' not only organizes the pomeron interaction, but also plays an independent role. Since the Goldstone pomeron is stable (or almost stable), its properties are determined entirely by the bare trajectory and the residues. The interaction between the reggeons appears in fact only in the properties of P', and this leads to the appearance of a number of qualitative effects described in the Introduction.

The exact Green's function of the reggeon P' is

$$G^{-1} = \omega + k^2 + \Delta - \Sigma(\omega, k^2). \quad (10)$$

Here and below $\alpha' k^2 \to k^2$. The irreducible self-energy part of Σ is a sum of the contributions of all the possible diagrams of the type of Figs. 4 and 5. In the model

(8) it is then possible to sum not only diagrams with exchange of pomerons, but also with exchange of many P' (Fig. 5). It is obvious that allowance for these contributions, which correspond to rapidly decreasing terms of the amplitudes, depends on the character of the model, and is therefore useful only for estimates. Summing all the diagrams, we obtain only a model representation of the power-law contributions to the diagrams and of the possible effect of the contributions on the principal terms of the asymptotic form. The solvable model (8) offers a useful opportunity for this purpose.

We denote by $\mathscr{I}(\omega, k^2)$ that part of the P' self-energy which does not include diagrams with two-pomeron exchange and is not divided into two parts by the four-reggeon vertex λ (Fig. 4e). The total self-energy part of Σ is then

$$\Sigma(\omega, k^2) = \Delta \frac{\mathscr{I}(\omega, k^2) + J(\omega, k^2)}{1 - \mathscr{I}(\omega, k^2) - J(\omega, k^2)}. \tag{11}$$

Here $J(\omega, k^2)$ is the contribution of the two-pomeron loop:

$$J(\omega, k^2) = 2\lambda \int \frac{d^2k'\, d\omega'}{(2\pi)^2 2\pi i} G_P(\omega', k') G_P(\omega-\omega', k-k') = \frac{\lambda}{4\pi} \ln \frac{L}{\omega + k^2/2},$$

$$G_P^{-1}(\omega, \mathbf{k}) = \omega + k^2. \tag{12}$$

The quantity L is connected with the cutoff momentum k_{max} of the integral in (12):

$$L = 2k^2_{max}$$

i.e., with the radius of the form factor of the vertex of Fig. 1a. In accord with the statements made in the Introduction, $k^2_{max} \sim 1 (\alpha'_P \tilde{k}^2_{max} \sim 1)$, and therefore $L \sim 2$. The quantity $\mathscr{I}(\omega, k^2)$ contains contributions of the diagrams of Fig. 5 and is a singular function of ω with singularities $|\omega| \sim \Delta$ corresponding to asymptotic amplitudes that decrease in power-law fashion. It includes also contributions of many-pomeron ($n \geq 4$) exchange diagrams (Fig. 5) that are singular at small ω and k^2:

$$\mathscr{I}(\omega, k^2) = \frac{\lambda}{4\pi} \ln \frac{L}{2\Delta + \omega + k^2/2} + \ldots + \frac{3}{2} \left(\frac{\lambda}{4\pi}\right)^3 \frac{(\omega + k^2/4)^2}{\Delta^2} \ln\left(\omega + \frac{k^2}{4}\right). \tag{13}$$

Substituting (11) in (10), we obtain an exact Green's function for P':

$$G^{-1}(\omega, k^2) = \omega + k^2 + \frac{\Delta}{1 - \mathscr{I}(\omega, k^2) - J(\omega, k^2)}. \tag{14}$$

Formula (14) can also be derived by successively summing all the diagrams Green's function G itself. This derivation is of interest because it is easy to trace in it those cancellations of the infrared divergences at small ω and k^2 which are characteristic of the theory with a Goldstone particle (see I). In the diagrams for G, the cancellations take place in all the higher orders in ω, except for the diagram of Fig. 1b. This property of G can be easily traced in formula (14).

3. CONTRIBUTION OF P' TO THE TOTAL CROSS SECTIONS

We consider in this section the function (14) at $k^2 = 0$. In addition to the branch point $\omega = 0$, the function $G(\omega)$ has in the complex ω plane poles corresponding to amplitude contributions that decrease in power-law fashion. When calculating the exact positions of these poles, which lie far from the point $\omega = 0$, the dependence of the function $\mathscr{I}(\omega)$ on ω is appreciable. However, only the very fact of their existence is of qualitative (non-model) character, and this property can be traced by assuming $\mathscr{I}(\omega)$ to be constant, a procedure that is in fact valid at small ω (see (13)).

The principal role assumed by $\mathscr{I}(\omega)$ in that case is the renormalization of the bare constants of the theory:

$$\Delta \to \Delta/(1-\mathscr{I}) \quad \lambda \to \lambda/(1-\mathscr{I}). \tag{15}$$

We can therefore consider for simplicity a simplified equation for the poles of (14):

$$\omega + \Delta/[1 - J(\omega)] = 0. \tag{16}$$

Substituting in (16)

$$\omega = \rho \exp\{\pm i(\pi - \varphi)\}, \tag{17}$$

we obtain for the positions of the poles the following system of equations for ρ and φ:

$$\rho^2 = \frac{\Delta^2}{\mathscr{L}_\rho^2 + (\pi-\varphi)^2 \lambda_0^2}, \quad |\sin\varphi| = \frac{(\pi-\varphi)\lambda_0}{\Delta}\rho, \tag{18}$$

where

$$\mathscr{L}_\rho \equiv 1 - \lambda_0 \ln \frac{L}{\rho}, \quad \lambda_0 \equiv \frac{\lambda}{4\pi}.$$

The second equation of (18) has the obvious solution $\varphi = \pi (\omega > 0)$. If the first equation, which reduces at positive ω to

$$\omega \mathscr{L}_\omega + \Delta = 0 \quad (\mathscr{L}_\omega \equiv 1 - \lambda_0 \ln(L/\omega)), \tag{19}$$

were also to have a solution, then the Green's function $G(\omega)$ would have an anti-unitary pole. Equation (19) has no solution if the following condition is satisfied:

$$\frac{\Delta}{\lambda_0 L} \exp\left(\frac{1+\lambda_0}{\lambda_0}\right) > 1. \tag{20}$$

We shall therefore assume that the parameters of the problem satisfy the condition that forbids anti-Froissart poles (see also Sec. 4). Since the cutoff L is a finite quantity in the reggeon problem, the condition (20) is easy to satisfy ($\lambda_0 < 1$).

Even when the condition (20) is satisfied, the system (18) has two complex-conjugate roots, which correspond to contributions to the amplitude (9) that decrease in power-law fashion. It is easiest to trace these solutions by perturbation theory. At $\lambda_0 \ll 1$ the system has the solution

$$\rho \approx \Delta, \quad \varphi \approx \pm \pi \lambda_0. \tag{21}$$

At larger values of λ_0 the roots are always in the left-hand (unitary) ω half-plane and reach its boundary ($\varphi = \pi/2$) only outside the region (20), when

$$\frac{\Delta}{\lambda_0 L} \exp\left(\frac{1+\lambda_0}{\lambda_0}\right) = \frac{\pi}{2e} < 1. \tag{22}$$

Their position depends strongly on the chosen model, and therefore an exact determination of the roots (18) is of no particular meaning. The very fact of the existence of such necessarily unitary poles is of interest.

At sufficiently small λ, the contributions made by them to the amplitude are positive and equal to

$$s^{-1}\delta_{P'}A_1(s)\approx 2g_{P'}{}^2 s^{-\Delta}. \quad (23)$$

The total contribution of the Green's function of P' includes also the branch-point $\omega=0$. At small ω, with (15) taken into account, the discontinuity on the two-pomeron cut from (14) is

$$\Delta G(\omega)=-\frac{\Delta\lambda_0}{\mathscr{L}_{-\omega}{}^2+\pi^2\lambda_0{}^2}\left[\left(\omega+\frac{\Delta\mathscr{L}_{-\omega}}{\mathscr{L}_{-\omega}{}^2+\pi^2\lambda_0{}^2}\right)^2+\frac{\pi^2\lambda_0{}^2\Delta^2}{[\mathscr{L}_{-\omega}{}^2+\pi^2\lambda_0{}^2]^2}\right]^{-1}. \quad (24)$$

The integral (9) of $\Delta G(\omega)$

$$s^{-1}\delta_{2P}A_1(s)=g_{P'}{}^2\int\limits_{-\infty}^{0}d\omega\, e^{\omega\xi}\Delta G(\omega) \quad (25)$$

is a complicated function of ξ. Its qualitative behavior is determined by the quantity

$$\chi=\frac{\xi\Delta|\mathscr{L}_{\omega_0}|}{\mathscr{L}_{\omega_0}{}^2+\pi^2\lambda_0{}^2}, \quad (26)$$

where ω_0 is an effective parameter, $\omega_0\sim 1/\xi$. At $\chi\gg 1$, the integral of (25) goes over into the usual formula for the contribution of a branch cut:

$$s^{-1}\delta_{2P}A_1(s)\to -g_{P'}{}^2\lambda/\Delta\xi. \quad (27)$$

At $\chi\sim 1$, the integral of (25) depends essentially on the energy, including also the power-law dependences. Let us consider again $\lambda_0\ll 1$, just as in (21). The main contribution to (25) comes at $\chi<1$ from the reggeon "resonance" of formula (24). The resonant contribution is negative and is equal to half the contribution of the poles (23):

$$s^{-1}\delta_{2P}A_1(s)\approx -g_{P'}{}^2 s^{-\Delta}. \quad (28)$$

At $\lambda_0\ll 1$ the nonresonant part of (25) is smaller than (28) if $\chi<1$, but at $\lambda_0\lesssim 1$ it becomes large and practically independent of energy in this region.

As already mentioned several times, the concrete forms of all these quantities, which describe the behavior of the amplitudes at relatively low energies, depend essentially on the model. At the same time, the appearance of the relation (27) in the contribution of P' at high energies does not depend on the model (see the Introduction). The qualitative effect, which determines (27), of the instability of P' can be responsible to a considerable degree for the observed growth of the total cross sections.[9] For $\chi\gg 1$ we have

$$\sigma_{\rm tot}\approx g_P{}^2-g_{P'}{}^2\lambda/\Delta\xi. \quad (29)$$

It is not particularly difficult to select the parameters in such a way that formula (29) describes the growth of the total cross sections of pp scattering in the observed energy interval ($\sigma_{pp}(\infty)\approx 50$ mb). The "pole" character of the contribution (27) (Fig. 1b) ensures a large value of the second term in (29) and a strong dependence on ξ [2)] even at energy values used in experiments. It is precisely this property which distinguishes (29) from the usually considered contribution of non-enhanced branch cuts.[14]

4. POMERON BOUND STATE. ANGULAR DISTRIBUTIONS OF HADRON SCATTERING

We consider now formula (14) at $k^2\neq 0$. We are interested in the region

$$x=\omega+k^2/2\ll 1. \quad (30)$$

The function $\mathscr{I}(\omega, k^2)$ can be considered, as before, constant, since by virtue of (13) it depends little on k^2 at small $k^2<1$. Then Eq. (14) can be rewritten in a simpler form

$$G^{-1}(\omega, k^2)=x+k^2/2+\Delta/\mathscr{L}_x \quad (\mathscr{L}_x\equiv 1-\lambda_0\ln(L/x)). \quad (31)$$

The equations for the roots of (31) are now ($x=\rho e^{i\varphi}$)

$$\frac{k^2}{2}+\rho\cos\varphi+\frac{\Delta\mathscr{L}_\rho}{\mathscr{L}_\rho{}^2+\varphi^2\lambda_0{}^2}=0,$$
$$\rho\sin\varphi+\frac{\Delta\lambda\varphi}{\mathscr{L}_\rho{}^2+\varphi^2\lambda_0{}^2}=0. \quad (32)$$

In the vicinity of $x=0$, the system (32) has a solution ($x_0\ll 1$)

$$\varphi=0,\quad \rho\approx x_0=L\exp\left\{-\frac{1}{\lambda_0}-\frac{2\Delta}{\lambda_0 k^2}\right\}. \quad (33)$$

It is easy to show (e.g., at $k^2\ll 1$) that (33) is the only solution of (32) at small x on the first sheet of $\ln x$ in (31). At large $|x|\sim\Delta+k^2/2$ the equations in (32) have two complex-conjugate roots that are continuations of (21) to the value $k^2\neq 0$.

The contribution to the amplitude (9) from the pole x_0 is given by

$$s^{-1}\delta_{x_0}A_1(s,t)=\frac{g_{P'}{}^2(k^2)}{\Phi'(x_0)}\exp\left\{-\frac{k^2\xi}{2}+x_0\xi\right\}, \quad (34)$$

where

$$\Phi'(x_0)=1-\frac{\Delta}{L\lambda_0}\left(\frac{\lambda_0 k^2}{2\Delta}\right)^2\exp\left\{\frac{2\Delta}{\lambda_0 k^2}+\frac{1}{\lambda_0}\right\}. \quad (35)$$

We now find the condition under which the pole x_0 remains in the left-hand ω half-plane at all values of k^2, i.e., it remains a unitary pole. From (32) at $\varphi=0$ it is obvious that the pole x_0 reaches the point $\omega=0$ ($x_0=k^2/2$) if the equation

$$k^2[1-\lambda_0\ln(2L/k^2)]+\Delta=0 \quad (36)$$

can have a solution. Equation (36) is perfectly analogous to (19), and the condition under which there are no solutions of (36) is

$$\frac{\Delta}{2\lambda_0 L}\exp\left(\frac{1+\lambda_0}{\lambda_0}\right)>1. \quad (37)$$

Just as in (20), the condition (37) is easily satisfied in the region $\lambda_0<1$.

The pole (33) arises in a two-dimensional (in terms of k) system of two pomerons as the consequence of the effective attraction between them, described by the Lagrangian (8) for the Goldstone pomeron. A non-analytic spectrum connected with the instability of the excitations, of the form $\exp(-a/k^2)$, is known in solid-state theory problems.[15] In the considered relativistic problem, such a non-analytic spectrum and the contributions (34) and (35) to the amplitude do not contradict the analytic properties of the amplitudes with respect to the momentum transfer, inasmuch as on going over to the complex k^2 plane the singularity of x_0 as $k^2 \to 0$ vanishes on the infinitely remote sheets of the logarithmic cut in (31).

We consider now the contribution of this cut to the scattering amplitude at $k^2 \neq 0$. The discontinuity of the Green's function on the cut is

$$\Delta G(\omega, \mathbf{k}) = -\frac{\Delta\lambda_0}{\mathscr{L}_{-x}^2+\pi^2\lambda_0^2}\left[\left(x+\frac{k^2}{2}+\frac{\Delta\mathscr{L}_{-x}}{\mathscr{L}_{-x}^2+\pi^2\lambda_0^2}\right)^2+\frac{\pi^2\lambda_0^2\Delta^2}{(\mathscr{L}_{-x}^2+\pi^2\lambda_0^2)^2}\right]^{-1}. \tag{38}$$

Expression (38) makes two types of contribution to the amplitude (9) at $\xi \gg 1$. First, the usual contribution of the simple two-pomeron branch cut, which can be represented approximately in the form

$$s^{-1}\delta A_{2P} \approx -g_{P'}^2(k^2)\frac{\lambda}{\Delta\xi}\exp\left\{-\frac{k^2\xi}{2}\right\}. \tag{39}$$

Second, the contribution from the quasi-resonant region near $x=0$ ($|x| \sim x_0$), which is equal to (34) in order of magnitude and in sign.

We can choose the parameters of (34) and (35) such that the pole x_0 makes an appreciable contribution to the scattering amplitude at momenta $k^2 \sim (2\Delta/\lambda_0)$, becoming comparable with the contribution of the pomeron and of the branch cut (39). At the energies at which this takes place (the contribution of the pomeron is larger at small ξ), anomalies can arise in the angular distributions. Parameter values such as $g_{P'}^2(k^2) \approx g_P^2(k^2)$, $\lambda_0 \approx \Delta \approx 0.4 - 0.5$, and $L = 2k_{\max}^2 \approx 2$ also reconcile quite well formula (29) with the total pp-interaction cross section, starting with energies $E_{\rm lab} \sim 100$ GeV.

Taking into account the contributions of P and P' at high energies (neglecting the non-enhanced diagrams), the imaginary part of the scattering amplitude can be approximated by the formula

$$s^{-1}A_1(s,t) = a(k^2)\left[\exp(-k^2\xi) - \frac{\lambda}{\Delta\xi}\exp\left(-\frac{k^2\xi}{2}\right) - \frac{2}{|\Phi'(x_0)|}\exp\left(-\frac{k^2\xi}{2}+x_0\xi\right)\right], \tag{40}$$

$$\Phi'(x_0) < 0.$$

For simplicity we have set in (40) the form factors of P and P' equal to each other. The factor 2 in the last term of (40) takes into account the contribution of the quasi-resonant region in (38).

The real part of the amplitude can be estimated in the usual manner (see, e.g., [7,14]):

$$s^{-1}\operatorname{Re} A(s,t) \approx \frac{\pi}{2}\frac{\partial}{\partial\xi}[s^{-1}\operatorname{Im} A(s,t)]. \tag{41}$$

The angular distributions described by formulas (40) and (41) agree qualitatively with the experimental picture.[10] The presence of a pole contribution in (34) makes a quantitative reconciliation much more feasible. The maximum and minimum fall in the experimental region ($0.4 \lesssim k^2 = \alpha_P' \bar{k}^2 \lesssim 0.6$).

This can raise the question of whether it is possible to have in the considered theory an analogous mechanism that leads to formation of a second maximum at $k^2 \neq 0$, etc. It would be natural to relate this mechanism with the appearance of poles that are connected with many-pomeron interactions and cuts. It is perfectly clear that the appearance of such many-pomeron bound states is an extremely unlikely event. The infrared divergences are cancelled out in the higher order of the theory with a Goldstone particle, and therefore the many-pomeron amplitudes cannot be large. For the model (8), the influence of the many-pomeron exchanges on the Green's function of P' is via the quantity $\mathscr{I}(\omega, k^2)$ (13). Typical diagrams for them are shown in Fig. 5. The value of their singular part is estimated in formula (13) and turns out to be small and insufficient for the production of a new pole. A similar situation obtains also for the more complicated models of I. It is precisely because of the smallness of the many-pomeron exchanges that the pole x_0 (33), which is a "resonance" with respect to the many-pomeron cuts with $n \geq 3$, has a small imaginary part and can be regarded as a real pole.

The up-to-date theoretical explanation of the appearance of maxima and minima in the angular distributions of hadron scattering is based in the interference of the Pomeranchuk pole with non-enhanced-branch-cut contributions that decrease slowly and have alternating signs. The appearance of the anomalies x in this explanation are not the result of a qualitative phenomenon, as is the case in the presence of the pole x_0 (33), but to a considerable degree of a fortuitous combination of the numerical factors.

For different hadrons, the angular-distribution picture corresponding to this hypothesis can differ greatly from the case of pp scattering. On the other hand, if the principal role is played by the pole x_0, then the angular distributions of the hadrons differ only in the form factors of P and P'. The appearance of maxima and minima at approximately the same points relative to t becomes practically inevitable for all hadron scatterings.

In concluding this section, we note that the more complicated forms of the interaction energies from I, while affecting the quantitative characteristics of the pole (33) and its contribution to the amplitude (especially at large k^2), do not change the fact of its existence and its qualitative properties.

5. THE THREE-REGGEON LIMIT

In view of the absence of a three pomeron vertex (identically at $k^2 = 0$, see I), the three-reggeon limit[11] of the cross sections is ensured in the theory of the Goldstone pomeron mainly by the properties of the Green's function of P'. If we disregard the non-enhanced reggeon diagrams, which can be much smaller than the "enhanced" ones connected with P', then the asymptotic form of the cross sections in the three-reggeon limit is determined by the diagrams of Fig. 6.

The partial wave corresponding to the sum of the diagrams of Fig. 6 is equal to

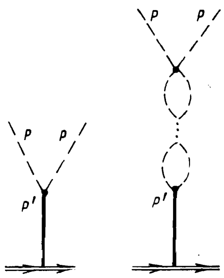

FIG. 6.

$$f=-g_{P'}(\lambda_0\Delta)^{1/2}\left\{G(\omega,\mathbf{k})+\frac{J(\omega,\mathbf{k})}{1-J(\omega,\mathbf{k})}G(\omega,\mathbf{k})\right\}. \tag{42}$$

Substituting (14) in (42), we obtain at $k^2=0$

$$f=-g_{P'}\frac{(\lambda_0\Delta)^{1/2}}{\omega[1-J(\omega)]+\Delta}. \tag{43}$$

The asymptotic form of the total cross section for the interaction of the pomeron with a particle is

$$\bar{\sigma}(\xi)=-\int\limits_{-\infty}^{0}\frac{g_{P'}(\lambda_0{}^3\Delta)^{1/2}\omega e^{\omega\xi}\,d\omega}{|\omega[1-J(\omega)]+\Delta|^2}\to g_{P'}\frac{(\lambda_0{}^3\Delta)^{1/2}}{(\Delta\xi)^2}. \tag{44}$$

Formula (44) shows that the total cross section σ_{tot} decreases in the three-reggeon limit with increasing s

$$\Delta\sigma_{tot}\sim\int\limits_{m^2}^{s}\frac{dM^2}{M^2}\frac{\bar{\sigma}(\ln M^2)}{\ln(s/M^2)}. \tag{45}$$

At lower energies it is necessary to add to the integral of (44) the contribution of the poles (21), which decreases in power-law fashion with energy. If their contribution to the total cross section is positive, then the contribution to the cross section (44) has a negative sign ($g_{P'}>0$). Since $\bar{\sigma}$ must remain positive at all energies, in realistic models this condition can impose constraints on the parameters of the theory (e.g., an upper bound for the value of λ).

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1) The nonlocal character of the reggeon interaction, viz., the dependence of the vertices on the reggeon momenta, leads to the appearance of a natural cutoff in the reggeon theory, just as in solid-state theory. Therefore the renormalizability property is not of the same fundamental significance for the Regge theory as for modern relativistic field theories.

2) $\xi=\ln(s/s_0)$, with s_0 unknown. We choose $\alpha'_P s_0\sim 1$, and then the vacuum reggeons have only one dimensional parameter α'_P.

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