

Magnetoacoustic resonance effects in the absorption of Rayleigh sound waves in metals

A. M. Grishin, É. A. Kaner, and Yu. V. Tarasov

Institute of Radiophysics and Electrons, Ukrainian Academy of Sciences

(Submitted June 28, 1975)

Zh. Eksp. Teor. Fiz. 70, 196–213 (January 1976)

The resonance peculiarities of the absorption of Rayleigh sound waves by conduction electrons in metals located in a weak magnetic field parallel to the surface are studied. It is shown that there should be absorption of two types in the propagation of sound waves at an angle with respect to the magnetic field. One of these is due to Doppler-shifted cyclotron resonance for electrons from the limiting points on the Fermi surface, which do not collide with the metal boundary. Another type of oscillation (geometric resonance) is produced by a group of electrons which collide with the surface and drift along it. The shape of these oscillation lines is sensitive to the nature of the electron scattering at the boundary.

PACS numbers: 72.55.+s

1. INTRODUCTION

In recent years, theoretical studies have been carried out by the authors on the propagation of surface sound oscillations (Rayleigh waves) in pure metals at low temperatures (see, for example, ^[1-3]). It has been shown that the absorption and velocity dispersion of surface sound waves possess a number of specific peculiarities in comparison with the case of volume sound oscillations. These peculiarities are due to two circumstances. First, the nonuniform character of Rayleigh waves (their amplitude decreases towards the interior of the metal at a distance of the order of the sound wavelength^[4]) leads to the result that they interact most effectively with electrons moving at the surface. Second, the trajectories of the electrons that collide with the surface in a magnetic field turn out to be more complicated than in the case of an unbounded sample.

In the present work, we study the coefficient of Rayleigh sound wave absorption by conduction electrons in a metal placed in a constant and uniform magnetic field parallel to the metal–vacuum interface. The region of classical magnetoacoustic resonance effects is considered in which the conditions of strong spatial dispersion are satisfied:

$$kl \gg kR \gg 1. \quad (1.1)$$

Here k is the wave number of the surface sound, l the free path length of the conduction electrons, and R the characteristic radius of the electron orbit in a magnetic field. It is shown that when surface waves propagate at an angle to the magnetic field, two types of resonance effects should occur. The first of these is due to the interaction of the sound with the “volume” electrons, which do not collide with the boundaries of the metal. The physical reason for the oscillations of the absorption coefficient in this case is the Doppler-shifted cyclotron resonance on electrons from the vicinity of the limiting points of the Fermi surface. In sound absorption by “surface” electrons, which collide with the boundaries of the metal and which drift along it, resonance oscillations have been discovered of the type of Pippard geometric resonance. The geometric oscillations of the absorption are due to a group of surface electrons which possess an extremal shift along the

sound wave vector and which move on parts of their trajectories, fall in the “skin-layer” of the sound wave on its phase surface. The shape of the lines of these oscillations depends materially on the character of the reflection of the surface electrons from the boundary of the metal.

In the calculation of the absorption coefficient, we have taken into account only the deformation mechanism of electron–phonon interaction. This is explained by the fact that, upon satisfaction of the conditions (1.1), as is shown in the work of one of the authors,^[5] the absorption due to the vortex electric fields is small in comparison with the deformation absorption. The induction part of the absorption also does not exceed the deformation part in order of magnitude. Since all three indicated mechanisms make an additive contribution to the sound absorption, allowance for the electric fields can lead only to a numerical change in the absorption coefficient obtained below.

2. GENERAL RELATIONS

1. We first consider an elastic metallic halfspace (Fig. 1) along the boundary of which a surface sound wave is propagating. The components of the deformation tensor in the Rayleigh wave are determined by the relation

$$u_{ik}(\mathbf{r}, t) = \sum_{\alpha} u_{ik}^{\alpha}(0) \exp[i(\mathbf{k}\mathbf{r} - \omega t) - \kappa_{\alpha} x]. \quad (2.1)$$

Here $u_{ik}^{\alpha}(0)$ is the amplitude of the deformation tensor on the surface $x=0$, $\mathbf{k} = \mathbf{k} \{0, k_y, k_z\}$ is the two-dimensional sound wave vector, ω the frequency, $\kappa_{\alpha} = (k^2 - \omega^2/s_{\alpha}^2)^{1/2}$ is the damping decrement of the sound wave in the interior of the metal, s_{α} is the velocity of longitudinal (s_l) or transverse (s_t) sound. The symbol Σ_{α} indicates that the elastic deformations represent a superposition of the potential ($\alpha=l$) and vortex ($\alpha=t$) sound oscillations (see ^[4]).

Deformation interaction of electrons with sound leads to dissipation of energy of the sound waves. It is convenient to express this in terms of the mean (over a single period of oscillation) damping decrement of the amplitude of the Rayleigh wave^[1]:

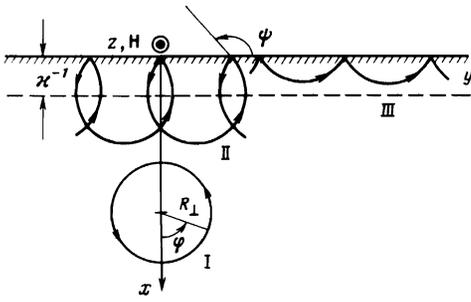


FIG. 1. Possible types of electron trajectories (in the plane $z = \text{const}$) in a magnetic field parallel to the surface of the metal.

$$\Gamma = \frac{1}{4W} \text{Re} \int_0^\infty dx \langle \Lambda_{ik} \chi \rangle \dot{u}_{ik}^* \quad (2.2)$$

Here W is the energy density in the Rayleigh wave, Λ_{ik} is the deformation potential tensor, and $\chi \delta(\varepsilon - \varepsilon_F)$ the density of the nonequilibrium electrons with energy ε . The dot indicates the time derivative, the angular brackets $\langle \dots \rangle$ denote integration over the Fermi surface, the asterisk denotes the complex conjugate, we sum over the twice repeated indices $i, k = 1, 2, 3$.

The nonequilibrium contribution χ to the electron energy is determined by solution of the linearized kinetic equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \nabla + \Omega \frac{\partial}{\partial \varphi} + \nu \right) \chi = \delta \varepsilon = \Lambda_{ik} \dot{u}_{ik} \quad (2.3)$$

in which $\nu = \tau^{-1}$ is the collision frequency of electrons with scatterers in the bulk of the metal, \mathbf{v} is the velocity, $\Omega = eH/mc$ is the cyclotron frequency, $-e$ is the charge, m is the effective mass of the electrons, c is the velocity of light, and φ the azimuthal angle in the plane perpendicular to the magnetic field \mathbf{H} , which is directed along the Oz axis. The boundary conditions to Eq. (2.3) will be discussed below.

2. We now consider a model of the metal with a spherical Fermi surface. In a magnetic field parallel to the boundary, the trajectories of the electrons are determined by the relation

$$x = X + R_\perp \cos \varphi, \quad y = R_\perp \sin \varphi, \quad z = Rn_\varphi \quad (2.4)$$

Here X is the coordinate of the center of the electron orbit, and $R_\perp = R \sin \theta$ is the radius of that orbit. The angles θ and φ (θ is the polar angle in momentum space, φ varies in the range $[-\pi, \pi]$) are chosen so that

$$n_x = -\sin \theta \sin \varphi, \quad n_y = \sin \theta \cos \varphi, \quad n_z = \cos \theta.$$

The electrons are naturally divided into two groups in accord with the character of the trajectories. The "volume" electrons (trajectory I in Fig. 1) do not collide with the boundaries. In the plane $z = \text{const}$, their trajectories are circles of radius R_\perp and the x coordinate of the centers $X > R_\perp$. The second group of electrons, the "surface" electrons, have trajectories (types II and III in Fig. 1) which touch against the surface $x = 0$, and are incident on the boundary at the grazing angle ψ . For surface electrons, the value of X is connected with ψ by the relation

$$X = -R_\perp \cos \psi, \quad |X| \leq R_\perp \quad (2.5)$$

The boundary condition for the volume electrons is the condition of nonperiodicity: $\chi_V(\varphi) = \chi_V(\varphi + 2\pi)$. Keeping in mind that $\chi(\mathbf{r}, t) = \chi(x) \exp[i(\mathbf{k}, \mathbf{r} - \omega t)]$, we obtain the function $\chi_V(x)$ in the form

$$\chi_V(x) = \mathcal{N} \int_{\varphi - 2\pi}^{\varphi} \frac{d\varphi'}{\Omega} \delta \varepsilon \left(x + R \int_{\varphi}^{\varphi'} n_x d\varphi''; \varphi' \right) \exp \left(\int_{\varphi}^{\varphi'} \gamma d\varphi'' \right) \quad (2.6)$$

$$\mathcal{N} = [1 - \exp(-2\pi \bar{\gamma})]^{-1}, \quad \gamma = \frac{\nu + i(\mathbf{k}\mathbf{v} - \omega)}{\Omega}, \quad \bar{\gamma} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \gamma d\varphi.$$

The distribution function of the surface electrons χ_s depend on the properties of the surface. We shall describe the reflectivity of the boundary phenomenologically in terms of the specularly parameter ρ :

$$\chi_s^{\dagger}(0) = \rho \chi_s^{\dagger}(0) + (1 - \rho) \chi_0 \quad (2.7)$$

On the left side of this relation, we have the density of those electrons which were reflected from the boundary (the arrow \dagger) at an angle $\varphi = -\psi$. It is connected with the density of electrons incident on the surface (arrow \uparrow , $\varphi = \psi$) through the macroscopic "specularity coefficient" ρ ($0 \leq \rho \leq 1$). The second term on the right side of Eq. (2.7) is proportional to the fraction of diffusely reflected particles $1 - \rho$ and contains the constant χ_0 which renormalizes the chemical potential of the electrons emerging from the boundary.

Solving Eq. (2.3) by the method of characteristics and using the condition (2.7), we find

$$\begin{aligned} \chi_s(x) = & \int_{-\varphi}^{\varphi} \frac{d\varphi'}{\Omega} \delta \varepsilon \left(x + R \int_{\varphi}^{\varphi'} n_x d\varphi''; \varphi' \right) \exp \left(\int_{\varphi}^{\varphi'} \gamma d\varphi'' \right) \\ & + [\mathcal{N}(\rho, \psi) - 1] \int_{-\varphi}^{\varphi} \frac{d\varphi'}{\Omega} \delta \varepsilon \left(x + R \int_{\varphi}^{\varphi'} n_x d\varphi''; \varphi' \right) \exp \left(\int_{\varphi}^{\varphi'} \gamma d\varphi'' \right) \\ & + (1 - \rho) \chi_0 \mathcal{N}(\rho, \psi) \exp \left(- \int_{-\varphi}^{\varphi} \gamma d\varphi' \right). \end{aligned} \quad (2.8)$$

The function

$$\mathcal{N}(\rho, \psi) = \left[1 - \rho \exp \left(- \int_{-\varphi}^{\varphi} \gamma d\varphi \right) \right]^{-1} = \sum_{n=0}^{\infty} \rho^n \exp \left(-n \int_{-\varphi}^{\varphi} \gamma d\varphi \right) \quad (2.9)$$

determines the number of cycles of periodic motion of the electrons with a given grazing angle ψ . We note that the use of the single symbol \mathcal{N} in Eqs. (2.6) and (2.8) is not accidental. The trajectories of the volume electrons are exactly the same as those of the surface electrons which execute a complete rotation over the circle ($\psi = \pi$) and are specularly reflected from the boundary of the metal. It is therefore natural that $\mathcal{N} \equiv \mathcal{N}(1, \pi)$.

The constant χ_0 that enters into Eqs. (2.7) and (2.8) should be found from the condition of conservation of the number (energy) of incident and reflected electrons, i. e., the vanishing of the normal component of the electric current \mathbf{j} on the surface:

$$j_x = -e \langle v_x \chi_s \rangle |_{x=0} = 0.$$

The scheme for this calculation is quite simple and we will not give here the explicit but rather cumbersome expression for χ_0 .

3. In conclusion of this section, we shall write out the general formulas for the absorption coefficient Γ .

In the calculation of the integral over x in (2.2), the contributions of the volume and surface electrons are separated. Since the volume electrons correspond to $X > R_L$ and the surface electrons to $|X| < R_L$, it follows that a transition takes place on the characteristic $x_0 = R_L(1 + \cos\varphi)$ from the value χ_V (for $x > x_0$) to the value χ_S (for $0 \leq x < x_0$). In correspondence with this, the coefficient Γ is represented in the form of the sum of the volume and surface components:

$$\Gamma = \Gamma_V + \Gamma_S.$$

The component Γ_V (after integration of the function χ_V over $x > x_0$) takes the form

$$\Gamma_V = \mathcal{F} \frac{kR}{2\pi} \operatorname{Re} \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta}{\nu_\alpha + \nu_\beta} \int_0^\pi d\theta \sin \theta \exp[-(\nu_\alpha + \nu_\beta) R_\perp] \mathcal{M}^\alpha \int_{-\varphi}^\pi d\varphi' \exp[-\bar{\nu}(\varphi - \varphi') - ik_y R_\perp (\sin \varphi - \sin \varphi') - \nu_\alpha R_\perp \cos \varphi - \nu_\beta R_\perp \cos \varphi']. \quad (2.10)$$

Here we have introduced the notation

$$\mathcal{F} = \frac{3}{8} \zeta \frac{N_e \epsilon_F k}{\rho_L v}, \quad B_\alpha = \frac{\Lambda_n u_k^\alpha(0)}{k u_x^\alpha(0) \epsilon_F (\zeta A)^{1/2}}.$$

The quantity \mathcal{F} is the coefficient of collision-free absorption of volume sound, N_e is the concentration of the electrons, ρ_L is the density of the crystal, $u_x^\alpha(0)$ is the amplitude of the x component of the potential mode of the sound field at the boundary, $\zeta = (\Lambda/\epsilon_F)^2$ is the dimensionless constant of electron-phonon interaction, Λ ($\sim \epsilon_F$) is the characteristic value of the deformation potential, and the constant A depends on the relation of the sound velocities, $^{[1]} B_\alpha$ are the components of the dimensionless two-dimensional vector that characterizes the contribution to the interaction of the potential ($\alpha = l$) and vortex ($\alpha = t$) sound modes. In obtaining Eq. (2.10), we have assumed that the factors B_α do not depend on the direction of the electron velocity \mathbf{v} .

If we expand the exponential in (2.10), which contains the oscillating functions φ and φ' , in a double Fourier series, and carry out integration over φ and φ' , we obtain

$$\Gamma_V = \mathcal{F} \frac{\pi}{2} kR \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta}{\nu_\alpha + \nu_\beta} \times \sum_{n=-\infty}^{\infty} \int_0^\pi d\theta \sin \theta D \left(n + k_y R \cos \theta - \frac{\omega}{\Omega} \right) M_n(\nu_\alpha R_\perp) M_n(\nu_\beta R_\perp), \quad (2.11)$$

$$D(n + k_y R \cos \theta - \omega/\Omega) = \pi^{-1} \operatorname{Re} (\bar{\nu} + in)^{-1}.$$

The "matrix elements"

$$M_n(\nu_\alpha R_\perp) = \frac{1}{2\pi} \int_{-\pi}^\pi d\varphi \exp[-\nu_\alpha R_\perp (1 + \cos \varphi) + ik_y R_\perp \sin \varphi - in\varphi] = \exp(-\nu_\alpha R_\perp) \left(\frac{k_y - \nu_\alpha}{k_y + \nu_\alpha} \right)^{n/2} J_n(\sqrt{k_y^2 - \nu_\alpha^2} R_\perp) \quad (2.12)$$

describe the interaction of the electron with the field of the inhomogeneous Rayleigh wave, and $J_n(q)$ are the Bessel functions.

It is not difficult to establish the fact that the absorption (2.11) does not depend on the sign of k_z and therefore we shall set $k_z > 0$ everywhere in what follows.

The initial expression for the absorption Γ_S is obtained by substitution of $\chi = \chi_S$ in Eq. (2.2) and integra-

tion over x in the interval $0 < x < x_0$. Changing the order of integration over x and φ , and transforming from integration over x to integration over ψ with the help of the formulas (2.4) and (2.5), we obtain

$$\Gamma_S = -\mathcal{F} \frac{(kR)^2}{4\pi} \operatorname{Re} \sum_{\alpha, \beta} B_\alpha B_\beta \int_0^\pi d\theta \sin \theta \int_0^\pi d\psi n_x(\psi) \times \int_{-\varphi}^\varphi d\varphi \exp \left[-(\nu_\alpha + \nu_\beta) R \int_{-\varphi}^\varphi n_x d\varphi'' \right] \int_{-\varphi}^\varphi d\varphi' \exp \left[\int_{-\varphi}^\varphi d\varphi'' (\gamma - \nu_\alpha R n_x) \right] - \rho \mathcal{F} \pi (kR)^2 \sum_{\alpha, \beta} B_\alpha B_\beta \operatorname{Re} \int_0^\pi d\theta \sin \theta \int_0^\pi d\psi n_x(\psi) \mathcal{M}^\alpha(\rho, \psi) \times \mathcal{M}(\nu_\alpha R_\perp, \psi) \mathcal{M}(\nu_\beta R_\perp, \psi) - (1-\rho) \mathcal{F} \pi \sum_{\alpha, \beta} B_\alpha B_\beta \operatorname{Re} (\mathcal{L}_\alpha \mathcal{L}_\beta G^{-1}). \quad (2.13)$$

In this formula, the matrix elements

$$\mathcal{M}(\nu_\alpha R_\perp, \psi) = \frac{1}{2\pi} \int_{-\varphi}^\varphi d\varphi \exp \left[- \int_{-\varphi}^\varphi d\varphi' (\gamma + \nu_\alpha R n_x) \right] \quad (2.14)$$

describe the interaction of the surface electrons with the sound on one turn of the trajectories of type II and III. The quantities \mathcal{L}_α and G have the following form

$$\mathcal{L}_\alpha = kR \int_0^\pi d\theta \sin \theta \int_0^\pi d\psi n_x(\psi) \mathcal{M}(\nu_\alpha R_\perp, \psi) \mathcal{M}^\alpha(\rho, \psi), \quad (2.15)$$

$$G = \int_0^\pi d\theta \sin \theta \int_0^\pi d\psi n_x(\psi) \frac{\mathcal{M}^\alpha(\rho, \psi)}{\mathcal{M}^\alpha(1, \psi)}.$$

3. ABSORPTION BY VOLUME ELECTRONS

1. The volume part of the absorption (2.11) was studied in the work of the authors^[2] in the case in which the sound wave was propagated perpendicular to the magnetic field ($k_z = 0$). In such a geometry,

$$\Gamma_V = \mathcal{F} \frac{\pi}{2} \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta}{\nu_\alpha + \nu_\beta} \sum_{n=-\infty}^{\infty} D \left(n - \frac{\omega}{\Omega} \right) T_n^{\alpha\beta}, \quad (3.1)$$

$$T_n^{\alpha\beta} = kR \int_0^\pi d\theta \sin \theta M_n(\nu_\alpha R_\perp) M_n(\nu_\beta R_\perp).$$

It is easy to estimate the quantity $T_n^{\alpha\beta}$ by using the asymptotic expansion of the matrix elements. The maximum values of the functions M_n are achieved in that range of the parameters in which the argument of the Bessel functions entering into Eq. (2.12) turns out to be less than the index. The principal term of the asymptotic matrix elements at large values of the index n has the following form^[6]:

$$M_n(\nu_\alpha R_\perp) = \frac{(-1)^n}{(2\pi)^{1/2} [n^2 - (k_y^2 - \nu_\alpha^2) R_\perp^2]^{1/4}} \times \left(\frac{k_y - \nu_\alpha}{k_y + \nu_\alpha} \right)^{n/2} \exp \left\{ -\nu_\alpha R_\perp + [n^2 - (k_y^2 - \nu_\alpha^2) R_\perp^2]^{1/2} - |n| \operatorname{Arth} \left[1 - \frac{(k_y^2 - \nu_\alpha^2) R_\perp^2}{n^2} \right]^{1/2} \right\}. \quad (3.2)$$

Using Eq. (3.2), we obtain the following asymptotic expression for the quantity $T_n^{\alpha\beta}$ for $1 \ll |n| \ll |k_y R|$:

$$T_n^{\alpha\beta} = \sqrt{\frac{2}{\pi}} \frac{(kR)^{1/2} |n|^{1/2}}{|k_y R|^{1/2}} \left(\frac{q_\alpha q_\beta}{q_\alpha + q_\beta} \right)^{1/2} C_n(k_y), \quad q_\alpha = \frac{k}{\nu_\alpha}, \quad C_n(k_y) = \begin{cases} 1, & n \operatorname{sgn} k_y < 0 \\ \left(\frac{k_y - \nu_\alpha}{k_y + \nu_\alpha} \right)^n \left(\frac{k_y - \nu_\beta}{k_y + \nu_\beta} \right)^n, & n \operatorname{sgn} k_y > 0. \end{cases} \quad (3.3)$$

Formula (3.3) gives the correct order of magnitude of the result even for $|n| \approx 1$. To estimate the component with $n=0$, we must set $|n|=1$ and $C_n=1$ in Eq. (3.3).

In the range of low frequencies $\omega \ll \nu$ the quantity ω/Ω in Eq. (3.1) can be neglected and only the component with $n=0$ remains in the sum over n . The absorption (3.1) in this case has a collisional character and is equal to

$$\Gamma_\nu = \mathcal{F} \frac{\pi}{2} \Omega \tau \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta^*}{\kappa_\alpha + \kappa_\beta} T_0^{\alpha\beta} \sim \mathcal{F} \frac{\Omega \tau}{kR}. \quad (3.4)$$

In the opposite limiting case of high frequencies $\omega \gg \nu$, the argument of the D functions in (3.1) vanishes upon satisfaction of the condition $\omega = n\Omega$ (n is an integer). The absorption in this case undergoes periodic oscillations as a function of H^{-1} , due to the acoustical cyclotron resonance (ACR) and, close to the resonance maximum, is described by the formula

$$\Gamma_\nu = \mathcal{F} \frac{\pi}{2} D \left(n - \frac{\omega}{\Omega} \right) \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta^*}{\kappa_\alpha + \kappa_\beta} T_n^{\alpha\beta}. \quad (3.5)$$

The effect of the nonuniform character of the surface sound wave appears in the decrease in the amplitude of the ACR relative to the case of volume sound. This decrease is connected with the fact that only those electrons absorb the sound more effectively which do not leave the "skin layer" of the sound wave in the time of free flight. Such electrons are concentrated in the vicinity of the limiting points of the Fermi surface. Their relative number is determined by the value of $T_n^{\alpha\beta}$ in (3.5).

The surface character of the Rayleigh wave leads also to the appearance of a dependence of Γ_ν on the sign of k_y , which is described by the quantity $C_n(k_y)$ from (3.3). Using the scheme shown in Fig. 2, we can note that for $\mathbf{k} \perp \mathbf{H}$ all the resonance electrons have a smaller velocity relative to the sound wave on the upper portions of their trajectories if $k_y < 0$. It is this which explains why the absorption (3.5) at $k_y < 0$ is larger than at $k_y > 0$.

2. We now consider the situation in which the vectors \mathbf{k} and \mathbf{H} are oriented at arbitrary angles. The characteristic width of the D functions near their maxima is determined by the quantity

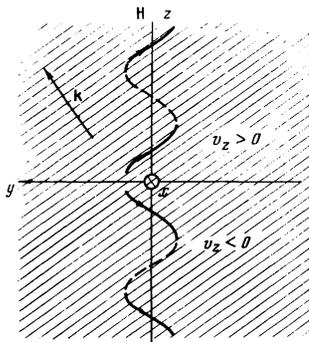


FIG. 2. Effectiveness of the interaction of volume electrons with the Rayleigh wave (view of the metal from above). The solid line denotes the upper portions of the electron trajectories, the dashed lines, the lower portions.

$$\Delta(\cos \theta) \sim (k_z l)^{-1}. \quad (3.6)$$

If the angle between \mathbf{k} and \mathbf{H} is such that the parameter $k_z l \ll 1$, the D functions can be assumed to be independent of the momentum. The result for the absorption in this case turns out to be the same as for $\mathbf{k} \perp \mathbf{H}$.

Upon satisfaction of the inequality

$$k_z l \gg 1 \quad (3.7)$$

the interval (3.6) is small in comparison with the interval of integration over θ . However, the matrix elements in Eq. (2.11) are also sharply peaked functions and their width under certain cases can compete with the width of the D functions. Using the asymptotic form (3.2), we can establish the fact that in the range $0 < \theta < \pi$ the functions M_n have at $|n| < |k_y R|$ two maxima whose locations are determined by the equality

$$\sin \theta_n = |n/k_y R|. \quad (3.8)$$

The characteristic width of the matrix elements near these maxima is of the order of

$$\Delta(\cos \theta) \sim (kR)^{-1/2} |n|^{3/2} / |k_y R|^{1/2}. \quad (3.9)$$

If $|n| > |k_y R|$, then the matrix elements have a single maximum in the range $0 < \theta < \pi$, at $\theta = \pi/2$, the width of which is

$$\Delta(\cos \theta) \sim |nk_y/k|^{-1/2}. \quad (3.9')$$

We consider the behavior of Γ_ν in the region of (3.7) as a function of the parameter $k_z R$, assuming that its value changes due to change in the angle between the vectors \mathbf{k} and \mathbf{H} .

Low frequencies, $\omega \ll \nu$. In the region

$$\nu/\Omega \ll k_z R \ll 1$$

the maxima of the D functions do not fall in the range of integration over θ in (2.11). This enables us to remove the D function from the integral at the points of maximum value of the matrix elements. The absorption in this case is essentially determined by the component with $n=0$,

$$\Gamma_\nu = \mathcal{F} \frac{\pi}{2} D(k_z R) \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta^*}{\kappa_\alpha + \kappa_\beta} T_0^{\alpha\beta} \sim \mathcal{F} \frac{\Omega \tau}{kR} (k_z l)^{-2}, \quad (3.10)$$

and turns out to be smaller by a factor $(k_z l)^2$ than for $k_z = 0$.

When the quantity $k_z R$ reaches the value $k_z R = 1$, the maxima of the D functions in the components with $n = \pm 1$ begin to enter the range of integration over θ . This corresponds to the appearance near the limiting points of the Fermi surface of two groups of electrons, which satisfy the condition of Doppler-shifted cyclotron resonance $k_z v_F = \pm \Omega$. At the moment of superposition of the maxima of the D functions and matrix elements, the absorption exhibits a sharp spike. A similar increase in the absorption takes place each time $k_z R$ reaches the next integer value. The characteristic feature of the low-frequency situation is that two components with $n = \pm k_z R$ are resonant simultaneously. This leads to the result that the dependence of $T_n^{\alpha\beta}$ on the sign of k_y does not appear in the absorption.

The shape of the resonant spikes of absorption can be

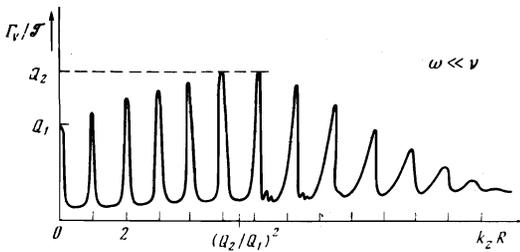


FIG. 3. Schematic diagram of the absorption curve Γ_V in the low-frequency region, $Q_1 = \Omega\tau/kR$, $Q_2 = (\Omega\tau)^{4/5}/(kR)^{3/5}$. The region $k_z R \sim (Q_2/Q_1)^2$ corresponds to the transition from the collisional absorption to collision-free.

different, depending on the relations between the characteristic widths of the D functions and the corresponding spike of the matrix elements. If the matrix elements are sharper than the D functions, the absorption turns out to be collisional and is described in the vicinity of the resonance spike, which appears at $k_z R = |n|$, by the expression

$$\Gamma_V = \mathcal{F} \frac{\pi}{4} D(|n| - k_z R) \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta^*}{\nu_\alpha + \nu_\beta} (T_{|n|}^{\alpha\beta} + T_{-|n|}^{\alpha\beta}). \quad (3.11)$$

The width of the collisional resonance peaks is $\Delta(k_z R) \sim \nu/\Omega$, and their shape is Lorentzian. The absorption (3.11) has at maximum the order

$$\mathcal{F} \frac{(kR)^{3/5}}{|k_y R|^{1/2}} \Omega\tau |n|^{3/5} \quad (3.12)$$

and increases with the number n (Fig. 3). Such an oscillation picture can occur upon satisfaction of the inequality

$$k_z R \ll |k_y R| / (kR)^{1/5} (\Omega\tau)^{2/5}. \quad (3.13)$$

It expresses the fact that the width of the matrix elements (3.9) for $|n| \approx k_z R$ is less than the width of the D functions (3.6).

Upon further increase in $k_z R$, when

$$k_z R \gg |k_y R| / (kR)^{1/5} (\Omega\tau)^{2/5}, \quad (3.14)$$

the D functions become sharper than the matrix elements. This means that in the vicinity of (3.14) the D functions can be replaced by δ functions, by formally setting $\nu = 0$. The resonance absorption in this case becomes collision-free and is described by the formula

$$\Gamma_V = \frac{k}{k_z} \mathcal{F} \frac{\pi}{2} \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta^*}{\nu_\alpha + \nu_\beta} \sum_{|n| \leq k_z R} M_n \left[\nu_\alpha R \left(1 - \frac{n^2}{(k_z R)^2} \right)^{1/2} \right] \times M_n \left[\nu_\beta R \left(1 - \frac{n^2}{(k_z R)^2} \right)^{1/2} \right]. \quad (3.15)$$

It should be remarked that in the low-frequency case, the parameter $(Q_2/Q_1)^2 = (kR)^{4/5} (\Omega\tau)^{-3/5}$, which describes the region of transition of the absorption from collisional to collision-free in the scale of $k_z R$, really does not become larger in comparison with unity. This means that for $\omega \ll \nu$, the volume absorption, even for not very large $k_z R$, becomes collision-free and is described by the expression (3.15).

The properties of the collision-free resonance peaks

can be analyzed by using the asymptotic form (3.2) for M_n . The maximum value

$$\Gamma_{V \max} \sim \mathcal{F} kR/n^2 \quad (3.16)$$

of the absorption spike, corresponding to a component with numbers $\pm |n|$ is reached at $k_z R = |n| + \delta$. The quantity

$$\delta = \frac{1}{2} \frac{|n|^3}{(kR)^2 - n^2}$$

in the region of resolvability of the resonance spikes (see the inequality (3.18)) is relatively small: $\delta \ll |n|$.

The characteristic width of the collision-free peaks of absorption turns out to be of the order of

$$\Delta(k_z R) \sim (kR)^{1/5} (\delta/|n|)^{1/4}. \quad (3.17)$$

Beginning with the values

$$k_z R \gg |k_y R| / (kR)^{1/5} \quad (3.18)$$

the width (3.17) becomes greater than the distance between the neighboring peaks. This leads to the result that the absorption coefficient becomes a smooth function of the parameter $k_z R$.

In contrast with the collisional region (3.13), the peaks of the collision-free absorption have an asymmetric shape (see Fig. 3). The right-hand exponentially decreasing wing of the peak is cut up by the oscillations of geometric resonance, which result from the oscillating asymptotic form of the Bessel functions in M_n (they are shown only in the middle part of the curve on Fig. 3). In the absorption of volume sound, a combination of such geometric resonances led to aperiodic changes in the absorption coefficient.^[7] In Rayleigh sound, these oscillations have an exponentially small amplitude.

The value of the absorption in the region (3.18) can be found by replacing the summation over n in Eq. (3.15) by integration. An important role in the integral over n is assumed by the large number of components $\Delta n \sim (k_z/k)^{5/2} (kR)^{1/2}$ localized near

$$n_0 = -k_z R k_y / k. \quad (3.19)$$

Using the asymptotic form (3.2), we find after integration over n ,

$$\Gamma_{V \text{ mon}} = \mathcal{F} \frac{\pi}{4} \frac{(k_z R)^{3/5}}{kR} \sum_{\alpha, \beta} B_\alpha B_\beta^* \left(\frac{q_\alpha q_\beta}{q_\alpha + q_\beta} \right)^{1/2}. \quad (3.20)$$

The fact that, out of all the components in the sum over n of (3.15), the terms with $n \approx n_0$ turn out to be singled out is due to the coincidence of the maxima of the D functions and of the matrix elements for these components. Actually, if we substitute the value of θ_n from (3.8) in the argument of the D functions, then this argument vanishes for $n = n_0$. It is not difficult also to establish the fact that for electrons with polar angle θ_n on the upper part of their trajectories ($\varphi = \pm \pi$) the phase relation $\mathbf{k} \cdot \mathbf{v}|_{\varphi = \pm \pi} = 0$ is satisfied. These electrons move in the skin layer along the front of the sound wave (see Fig. 2) and absorb its energy most effectively.

High frequencies, $\omega \gg \nu$. In the previous section, it was established that the appearance of resonance max-

ima in Γ_V is connected with the circumstance that new resonance groups of electrons appear on the Fermi surface near the limiting points. In the quasistationary region ($\omega \ll \nu$) such groups appear simultaneously at both limiting points. At high frequencies, under conditions of resonance absorption, it is necessary to take into account the contribution ω/Ω . It leads to the result that, upon a change in the parameter $k_z R$, new resonances appear at the alternating limiting points. The oscillation peaks of the absorption $\Gamma_V(k_z R)$ are split by a value ω/Ω . If $\omega/\Omega > 1$, then the splitting of a given maximum is greater than the distance between the neighboring low-frequency peaks.

We now consider in more detail the dependence of Γ_V on $k_z R$ for $\omega > \nu$. From a comparison of the widths of the functions D and M_n , it is not difficult to establish the fact that the boundary between the collisional and collision-free absorption is located at

$$k_z R \sim \zeta = \frac{|k_y R|}{(kR)^{1/2} (\Omega \tau)^{1/2}} \quad (3.21)$$

just as for sound of low frequency. Thus, upon satisfaction of the condition (3.13), the absorption is described by the formula

$$\Gamma_V = \tau \sum_{\alpha, \beta} k \frac{B_\alpha B_\beta}{z_\alpha + z_\beta} \sum_{n, \omega} T_n^{\alpha\beta} \left[D \left(n + k_z R - \frac{\omega}{\Omega} \right) + D \left(n - k_z R - \frac{\omega}{\Omega} \right) \right] \quad (3.22)$$

It is seen that, for $k_z R = |n - \omega/\Omega|$, sharp absorption peaks appear which have a symmetric shape with width $\Delta(k_z R) \sim \nu/\Omega$, just as in the previous case.

It is clear from the scheme shown in Fig. 4 that it is convenient to regard the system of resonance spikes as a superposition of two equidistant series: the first with $n > \omega/\Omega$ and the second with $n < \omega/\Omega$. The amplitude of the peaks in these two series behaves in essentially different fashion. If $k_y < 0$, then the amplitude of the spikes in the first series increase with number as $n^{1/2}$. The amplitude of the peaks of the second series initially falls off in proportion to $n^{1/2}$ and at distances $k_z R > \omega/\Omega$ begins to decrease exponentially (see Fig. 5). If $k_y > 0$, then the spikes of the first series lose amplitude exponentially with the number. The growth of the spikes of the second series is at first exponential and then according to the law $|n|^{1/2}$ at $k_z R > \omega/\Omega$. Naturally, one can speak of the two subsystems of spikes only in the case in which the two neighboring peaks are separated by a distance that exceeds their total width, i.e., $\Delta > \nu/\Omega$ (Δ is the fractional part of the quantity ω/Ω).

The fact that the amplitudes of the spikes in both series are determined by the direction of propagation of the sound wave relative to H (the sign of k_y) is explained in the following way. It is seen from Fig. 2 that the resonance electrons with $v_z < 0$ (the first series of spikes—the dashed envelope) move in phase with the sound wave at $k_y > 0$ on the lower part of their trajec-



FIG. 4.

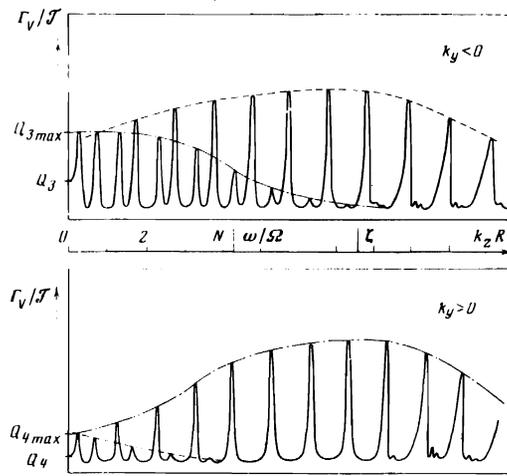


FIG. 5. Schematic dependence of the absorption Γ_V on the mutual orientation of the vectors \mathbf{k} and \mathbf{H} in the case $\omega \gg \nu$, $Q_3 = N^{1/2}(kR)^{-1}D(\Delta)$, $Q_{3\max} = N^{1/2}(kR)^{-1}\Omega\tau$, $Q_4 = Q_3|(q+1)/(q-1)|^{-2N}$, N is the integral and Δ the fractional part of the quantity ω/Ω , $q = k/z > 1$.

tories. These parts are far removed from the surface of the metal, and the amplitude of the sound wave there is small. For electrons with $v_z > 0$ (the second series, dot-dash envelope) the situation is more favorable. They move in the phase plane of the wave on the upper part of the trajectories, where the amplitude of the sound field is large. Therefore, for $k_y > 0$, the peaks of the second series are higher than those of the first. For a change in the sign of k_y , the situation changes to the opposite.

With further increase in the parameter $k_z R$, when $k_z R > \zeta$, the absorption becomes collision-free. In this region, the amplitude and shape of the peaks change according to the same laws (3.16) and (3.17) as for the low-frequency oscillations. The expression (3.20) for $\Gamma_{V\text{mon}}$ also remains valid in the region (3.18), where the spatial dispersion leads to a broadening of the resonance peaks.

3. It is also of interest to study the peculiarities of the volume absorption as a function of the quantity H at fixed orientation. The resonance singularities of Γ_V as a function of the parameter $k_z R$, which now changes because of the magnetic field, are described by the same formulas as in the case of a change of k_z/k . The picture of the absorption of low-frequency sound, represented in the third drawing, remains the same as before in this case. As to the high-frequency sound, since the relation between $k_z R$ and ω/Ω is given only by the orientation and is fixed, the following picture will be observed here. For $k_z < \omega/\nu$, (i.e., $k_z R < \omega/\Omega$, see Fig. 4) only the components with $n > \omega/\Omega$ will remain resonant with increase in $k_z R$. Consequently, only a single subsystem of resonances should be observed on the plot of $\Gamma_V(k_z R)$ in this case. If $k_z > \omega/\nu$, a second subsystem appears; however, the spikes of the subsystem with $n > \omega/\Omega$ will be more frequent, because of the fact that the quantity ω/Ω increases simultaneously with $k_z R$.

4. ABSORPTION BY SURFACE ELECTRONS

The contribution of surface electrons to the damping of Rayleigh sound is described by the expression (2.13). The components in (2.13) will be numbered by the indices 1, 2, 3 in order of their location.

1. In the region of strong spatial dispersion (1.1), the characteristic dimensions of the trajectories of the surface electrons significantly exceed the depth of penetration of the sound wave. Therefore, most of such electrons effectively interact with the sound over a small portion of its trajectories near the points of collision with the boundaries of the metal. Exceptions are the so-called grazing electrons, whose trajectories (type III) are contained entirely in the sound skin layer. However, as will be shown, the contribution to the absorption from such electrons cannot be taken into account.

Because of the presence of the factor

$$\exp\left(-\kappa R \int_{-\psi}^{\psi} n_x d\varphi'\right)$$

in the integrals over φ and φ' and the matrix elements $\mathcal{M}(\kappa R_{\perp}, \psi)$ in (2.13), the principal contribution to the asymptotic form of Γ_s is made by the ends of the intervals of integration. This is valid for not too small angles ψ :

$$\psi \gg (\kappa R_{\perp})^{-1/2}, \quad (4.1)$$

which correspond to trajectories of electrons which penetrate from the surface to a depth much greater than the thickness of the skin layer. This contribution for the integrals over φ and φ' from Γ_{s1} is the following:

$$\int_{-\psi}^{\psi} d\varphi \int_{-\psi}^{\psi} d\varphi' \dots = -\frac{1}{(\kappa_{\alpha} + \kappa_{\beta}) R n_x(\psi)} \{ [\gamma - \kappa_{\alpha} R n_x(\psi)]^{-1} + [\gamma - \kappa_{\beta} R n_x(\psi)]^{-1} \} + \exp\left(-\int_{-\psi}^{\psi} \gamma d\varphi\right) [\gamma + \kappa_{\alpha} R n_x(\psi)]^{-1} [\gamma + \kappa_{\beta} R n_x(\psi)]^{-1}. \quad (4.2)$$

Similarly, the asymptotic form of the matrix elements entering in Γ_{s2} and Γ_{s3} has the form

$$\mathcal{M}(\kappa R_{\perp}, \psi) = \frac{1}{2\pi} \left\{ [\gamma - \kappa_{\alpha} R n_x(\psi)]^{-1} - \exp\left(-\int_{-\psi}^{\psi} \gamma d\varphi\right) [\gamma + \kappa_{\alpha} R n_x(\psi)]^{-1} \right\}. \quad (4.3)$$

Substituting (4.2) and (4.3) in (2.13), we can represent the quantity Γ_s in the following form:

$$\Gamma_s = \Gamma_s(0) + \Delta\Gamma_s(H).$$

The quantity $\Gamma_s(0)$ is the absorption for $H=0$. It is determined by those components with asymptotic forms (4.2) and (4.3) which do not contain the factor

$$\exp\left(-\int_{-\psi}^{\psi} \gamma d\varphi\right). \quad (4.4)$$

Since all the electrons collide with the boundaries of the metal at $H=0$, i.e., they are surface electrons, the value of $\Gamma_s(0)$ is equal to the total absorption coefficient of Rayleigh sound in the absence of a magnetic field $\Gamma(0)$:

$$\Gamma(0) = \Gamma_s(0) = \mathcal{F} \sum_{\alpha, \beta} \operatorname{Re}(B_{\alpha} B_{\beta}^*) \left(\frac{k}{\kappa_{\alpha} + \kappa_{\beta}} + \rho \frac{k}{\kappa_{\beta} - \kappa_{\alpha}} \right) q_{\alpha} \frac{\operatorname{arctg}(q_{\alpha}^2 - 1)}{(q_{\alpha}^2 - 1)^{1/2}}$$

$$+ (1-\rho) \mathcal{F} \sum_{\alpha, \beta} B_{\alpha} B_{\beta}^* \frac{q_{\alpha} q_{\beta}}{(q_{\beta} + 1)(q_{\alpha} + 1)} \sim \mathcal{F}. \quad (4.5)$$

The quantity $\Delta\Gamma_s(H)$ is represented in the form of a sum of three components. Two of them, $\Delta\Gamma_{s1}(H)$ and $\Delta\Gamma_{s2}(H)$, after substitution of the expressions (4.2) and (4.3) in the corresponding components of Eq. (2.13) and some transformations, take the form

$$\Delta\Gamma_{s1}(H) + \Delta\Gamma_{s2}(H) = \rho \mathcal{F} \frac{kR}{2\pi} \sum_{\alpha, \beta} k \frac{\operatorname{Re}(B_{\alpha} B_{\beta}^*)}{\kappa_{\beta} - \kappa_{\alpha}} \operatorname{Re} \int_0^{\pi} d\theta \sin \theta \times \int_0^{\psi} d\psi \exp\left(-\int_{-\psi}^{\psi} \gamma d\varphi\right) \mathcal{N}(\rho, \psi) \left[\frac{(\kappa_{\beta} - \kappa_{\alpha}) / (\kappa_{\beta} + \kappa_{\alpha}) + \rho}{\gamma - \kappa_{\alpha} R n_x} - \frac{(\kappa_{\beta} - \kappa_{\alpha}) / (\kappa_{\beta} + \kappa_{\alpha}) + \rho^{-1}}{\gamma + \kappa_{\alpha} R n_x} \right]. \quad (4.6)$$

The third component, $\Delta\Gamma_{s3}(H)$, is obtained as follows. Using the asymptotic form (4.3), we can represent the quantities \mathcal{L}_{α} and G (see (2.15)) in the form

$$\mathcal{L}_{\alpha} = \mathcal{L}_{\alpha}(0) + \Delta\mathcal{L}_{\alpha}(H), \quad G = G(0) + \Delta G(H); \quad (4.7)$$

$$\mathcal{L}_{\alpha}(0) = -\frac{q_{\alpha}}{q_{\alpha} + 1}, \quad G(0) = -\pi,$$

$$\Delta\mathcal{L}_{\alpha}(H) = \frac{kR}{2\pi} \int_0^{\pi} d\theta \sin \theta \int_0^{\psi} d\psi n_x(\psi) \exp\left(-\int_{-\psi}^{\psi} \gamma d\varphi\right) \times \mathcal{N}(\rho, \psi) \left(\frac{\rho}{\gamma - \kappa_{\alpha} R n_x} - \frac{1}{\gamma + \kappa_{\alpha} R n_x} \right),$$

$$\Delta G(H) = -(1-\rho) \int_0^{\pi} d\theta \sin \theta \int_0^{\psi} d\psi n_x(\psi) \exp\left(-\int_{-\psi}^{\psi} \gamma d\varphi\right) \mathcal{N}(\rho, \psi). \quad (4.8)$$

The presence in Eqs. (4.8) of the oscillating exponential (4.4) leads to the result that $|\Delta\mathcal{L}_{\alpha}(H)| \ll |\mathcal{L}_{\alpha}(0)|$ and $|\Delta G(H)| \ll |G(0)|$. This allows us to keep in $\Delta\Gamma_{s3}(H)$ [by substituting the quantities \mathcal{L}_{α} and G in the form (4.7) in Eqs. (2.13)] only the terms of first order in $\Delta\mathcal{L}_{\alpha}(H)$ and $\Delta G(H)$. The value of $\Delta\Gamma_{s3}(H)$ is thereby represented by the relation

$$\Delta\Gamma_{s3}(H) = -2(1-\rho) \mathcal{F} \sum_{\alpha, \beta} \operatorname{Re}(B_{\alpha} B_{\beta}^*) \frac{q_{\alpha} q_{\beta}}{(q_{\alpha} + 1)(q_{\beta} + 1)} \times \operatorname{Re} \left[\frac{q_{\alpha} + 1}{q_{\alpha}} \Delta\mathcal{L}_{\alpha}(H) - \frac{1}{2\pi} \Delta G(H) \right]. \quad (4.9)$$

2. We proceed to the asymptotic calculation of $\Delta\Gamma_s(H)$. As is seen from Eqs. (4.6) and (4.9), the quantity $\Delta\Gamma_s(H)$ is determined by the factor $\mathcal{N}(\rho, \psi)$ (see (2.9)), which depends on the character of the scattering of electrons by the boundary of the metal. The presence in the expansion of $\mathcal{N}(\rho, \psi)$ of the rapidly-oscillating exponent

$$\exp\left(-n \int_{-\psi}^{\psi} \gamma d\varphi\right)$$

leads to the result that the principal contribution to the integral over ψ is made by the stationary-phase points

$$k\nu(\psi, \theta) = \omega. \quad (4.10)$$

In addition to the oscillating exponentials, Eqs. (4.6) and (4.9), the factors contain

$$(\gamma \pm \kappa R n_x)^{-1}, \quad (4.11)$$

These factors are equal in magnitude to the relative "lifetime" of the electron in the skin layer. In the calculation of the asymptotic form, these factors can be assumed to be smooth functions in comparison with the oscillating exponent if the point of stationary phase falls in the allowed region of angles (4.1).

The calculation of integrals over θ is also carried out by the stationary phase method. The point of stationarity in the integrals over θ is determined by the equation

$$\frac{\partial}{\partial \theta} \int_{-\psi}^{\psi} \gamma d\varphi = 0, \quad (4.12)$$

which is the condition that the displacement of the electron along the wave vector of the sound be extremal in the time between successive collisions.

The set of equations (4.1) and (4.12) determines the points of stationary phase of the exponential (4.4) in the two-dimensional region $0 < \psi, \theta < \pi$. There can be two such points in the region of integration. One of them is $\psi = 0$, and θ is a solution of Eq. (4.10). This point corresponds to grazing electrons for which the condition (4.1) is violated; therefore, the contribution from it will be discussed below.

The second solution is the point (ψ_0, θ_0) , $\psi_0 \neq 0$ and corresponds to electrons with maximum displacement along the direction of propagation of sound. This solution exists for not all values of the parameters. It is simplest to demonstrate this in the case of low-frequency sound $\omega \ll \nu$. Excluding the angle θ from Eqs. (4.10) and (4.12), we obtain an equation for the determination of ψ_0 :

$$f(\psi_0) = \psi_0^{-1} \sin \psi_0 \cos \psi_0 = -k_x^2/k_y^2. \quad (4.13)$$

It is seen from this that the solution ψ_0 , if it exists, lies in the interval $\pi/2 < \psi_0 < \pi$, which corresponds to electrons with the x coordinate of the center in the depth of the metal. However, if the ratio k_x^2/k_y^2 exceeds the value

$$\Xi = \max |f(\psi)| \approx 0.217, \quad \pi/2 < \psi < \pi,$$

the quantity (4.13) will not have a solution.

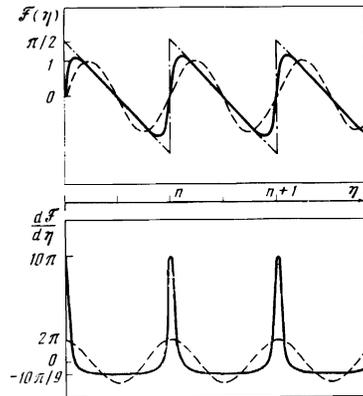


FIG. 6. Shape of the lines of resonance oscillations of sound absorption by surface electrons and their derivative as a function of the inverse magnetic field. The dot-dash lines correspond to $\bar{\rho} = 1$, the solid lines to $\bar{\rho} = 0.8$, the dashed lines to $\bar{\rho} = 0$; n is an integer, positive or negative.

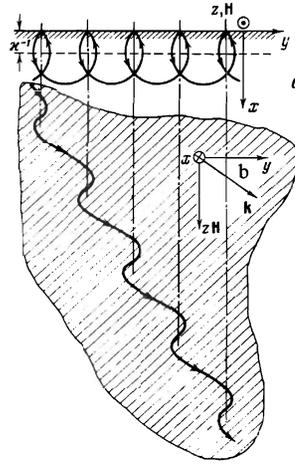


FIG. 7. Characteristic trajectory of resonance electrons which enter into the oscillations of the absorption (4.14): a —in the plane $z = \text{const}$, b —in the plane of the boundary of the metal.

Thus, the set of Eqs. (4.10) and (4.12) can be solved relative to (ψ_0, θ_0) in a limited interval of angles between \mathbf{k} and \mathbf{H} . In the case considered of a spherical Fermi surface, the maximum angle of departure of the vector \mathbf{k} from the direction perpendicular to \mathbf{H} , when the point (ψ_0, θ_0) still exists, is equal to 25° .

We now consider the case in which the points (ψ_0, θ_0) exists. Calculation of the integrals over ψ and θ gives¹⁾

$$\begin{aligned} \Delta \Gamma_s(H, \psi_0, \theta_0) &= \mathcal{F}(2k_y R)^{-1} \exp\left(-\frac{2\psi_0 \nu}{\Omega}\right) \mathcal{F}(\eta) \\ &\times \sum_{\alpha, \beta} B_\alpha B_\beta \frac{q_\alpha q_\beta}{\sin^2 \psi_0} \left\{ \frac{(1+\rho)^2}{2} + (1-\rho) \sin \theta_0 \sin \psi_0 \right. \\ &\times \left. \frac{(1+\rho)(2+q_\alpha+q_\beta) + 2(1-\rho) \sin \theta_0 \sin \psi_0}{(q_\alpha+1)(q_\beta+1)} \right\}, \\ \eta = \eta(\psi_0, \theta_0) &= \frac{1}{2\pi} \int_{-\psi_0}^{\psi_0} \frac{d\varphi}{\Omega} [k\nu(\varphi, \theta_0) - \omega]. \end{aligned} \quad (4.14)$$

The function

$$\mathcal{F}(\eta) = \bar{\rho}^{-1} \arctg \frac{\bar{\rho} \sin 2\pi\eta}{1 - \bar{\rho} \cos 2\pi\eta} \quad (4.15)$$

describes the absorption-coefficient oscillations that are periodic in the parameter η . The quantity $\bar{\rho}$ in Eq. (4.15) is the renormalized specular coefficient:

$$\bar{\rho} = \bar{\rho}(\psi_0) = \rho \exp\left(-\int_{-\psi_0}^{\psi_0} \frac{\nu}{\Omega} d\varphi\right). \quad (4.16)$$

Figure 6 shows the function $\mathcal{F}(\eta)$ and its derivative $d\mathcal{F}/d\eta$ for various values of the parameter $\bar{\rho}$. The sensitivity of the shape of the resonance curves to the quantity $\bar{\rho}$ makes the measurement of the absorption coefficient of surface sound a convenient method of direct determination of the specular coefficient ρ . The change in $\bar{\rho}$ manifests itself most strongly in the derivative $d\mathcal{F}/d\eta$, which reaches at $\eta = n$ (n an integer) a maximum value equal to $2\pi(1 - \bar{\rho})^{-1}$.

The occurrence of oscillations of the absorption (4.14) is explained in the following way. The quantity $\mathcal{N}(\rho, \psi)$ reaches a maximum for $\eta(\psi, \theta) = n$, i. e., for

$$\overline{k\nu} - \omega = n\Omega(\psi). \quad (4.17)$$

The bar indicates averaging over a single cycle of the motion of the surface electron with given grazing angle ψ , $\bar{\Omega}(\psi) = \Omega\pi/\psi$ is the frequency of this periodic motion. The relation (4.17) is no other than the condition of Doppler-shifted cyclotron resonance for the surface electron. The Doppler shift of the frequency in (4.17) is due to the drift of such an electron along the surface.

The decrease in the absorption amplitude (4.14) in comparison with $\Gamma(0)$ is due to two circumstances. One of them is connected with the condition (4.10) of in-phase motion of the electron with the surface wave. The other is the choice of electrons with extremal shift along the wave vector \mathbf{k} (the condition (4.12)). Each of these mechanisms of selection has an "effectiveness" $|k_y R|^{-1/2}$, as a result of which the relative amplitude of the oscillations (4.14) turns out to be of the order of $(k_y R)^{-1}$.

Thus, the reason for the resonance oscillations (4.14) turns out to be the Doppler-shifted cyclotron resonance sound absorption of a small group of electrons. The characteristic trajectory of the resonance electrons for specular reflection from the metal boundary is shown in Fig. 7.

It must be emphasized that the constancy of the quantity ρ , which is used in obtaining Eq. (4.14), is not strictly necessary. It is sufficient that the specular coefficient be a relatively smooth function of the angle of incidence of the electrons on the surface of the metal. This conclusion follows from the fact that the oscillations described above are due to a small group of electrons. The quantity ρ corresponding to these electrons also enters into (4.15). The angle of incidence of the resonance electrons on the surface of the metal (we denote it by ϑ) is connected with ψ_0 and θ_0 by the relation

$$\sin \vartheta = \sin \psi_0 \sin \theta_0$$

and can vary upon change of the mutual orientation of

the vectors \mathbf{k} and \mathbf{H} (for a spherical Fermi surface, within the limits $28^\circ < \vartheta < 90^\circ$).

3. It was noted above that, along with the solution (ψ_0, θ_0) of Eqs. (4.10) and (4.12), there exists a solution with $\psi = 0$, corresponding to electrons glancing inside the skin layer. It is quite evident that the contribution of such electrons to the absorption is a nonoscillating one, since the change in the phase of the Rayleigh wave over one cycle of its motion is proportional to ψ and does not have a definite, non-zero value. Since the coefficient $\Gamma_s(0)$ is determined by all the values of ψ , the monotonic contribution due to the glancing electrons will obviously be small with smallness of the corresponding angles ψ and will not exceed $\Gamma_s(0) (kR)^{-1/2}$ (the quantity $\varphi \sim (kR)^{-1/2}$ is the fraction of the glancing electrons whose trajectories fit inside the skin layer). Thus, we can neglect the absorption due to the glancing electrons.

¹⁾In the particular case $\mathbf{k} \perp \mathbf{H}$, this result was published by the authors in a short communication.^[3]

¹A. M. Grishman and É. A. Kaner, Zh. Eksp. Teor. Fiz. **63**, 2304 (1972) [Sov. Phys.-JETP **36**, 1217 (1973)].

²A. M. Grishman and Yu. V. Tarasov, Zh. Eksp. Teor. Fiz. **65**, 1571 (1973) [Sov. Phys.-JETP **38**, 783 (1974)].

³A. M. Grishin, É. A. Kaner and Yu. V. Tarasov, Solid State Comm. **16**, 425 (1975).

⁴Lord Rayleigh, Proc. London Math. Soc. **17**, 4 (1885); L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) Nauka, 1965, Sec. 24.

⁵É. A. Kaner, Zh. Eksp. Teor. Fiz. **43**, 216 (1962) [Sov. Phys.-JETP **16**, 154 (1963)].

⁶H. Bateman and A. Erdelyi, Higher Transcendental Functions (Russian translation) **2**, Nauka, 1966.

⁷É. A. Kaner and V. I. Fal'ko, Phys. Stat. Sol. **22**, 319 (1967).

Translated by R. T. Beyer