

necks (see^[6]). In addition, the relation $c_A(\tau) = (R/\tau)^s c_A^0$ assumed above for the Alfvén velocity is valid only on the wave front, but behind the front the magnetic field decreases with time, so that actually it would be necessary to consider nonlinear wave equations, which can be solved only numerically.

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Electrical properties and dependence of the *s-f* exchange integral on atomic volume of single crystals of gadolinium-dysprosium alloys

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An investigation was made of the electrical resistivity of gadolinium-dysprosium alloy single crystals as a function of temperature, magnetic field, and hydrostatic pressure. The temperature dependences of the electrical resistivity had kinks and maxima at the Curie and Néel points and these singularities depended strongly on whether the direction of current flow was parallel to the hexagonal axis or in the basal plane. The magnetic component of the resistivity was determined. The theory of indirect exchange and experimental information on the electrical resistivity and its pressure dependence were used to calculate the *s-f* exchange integral and the effective mass of conduction electrons, and the dependences of these quantities on the atomic volume.

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Rare-earth metals and alloys are characterized by record values of the resistivity at room temperature due to the very large magnetic component of the resistivity at temperatures exceeding the magnetic ordering point. The scattering of conduction electrons by a system of spins which is not in an ideal magnetic order clearly increases with the spin of the scattering atoms (other conditions being constant). The spin is largest for rare-earth metals and particularly for gadolinium, for which we have $S = \frac{7}{2}$. Therefore, anomalies due to the magnetic component of the resistivity are strongest for rare-earth metals and alloys and this makes it easy to study them and compare the experimental and theoretical results. The electrical resistivity of rare-earth metals and alloys is also of interest because the greatest success in the description of electrical properties of transition metals, on the basis of the indirect

exchange theory, has been achieved for these materials.^[1-3] The localization of the 4*f* electrons makes it easier to develop a theory which gives correct values of the exchange parameters (at least in respect of the order of magnitude) and describes, in agreement with experiment, the dependence of the paramagnetic Curie point and electrical resistivity on the atomic constants of rare-earth ions.

The electrical resistivity of pure rare-earth metals has been investigated using polycrystalline and single-crystal samples.^[4-8] However, studies of the electrical properties of rare-earth alloys have only been carried out on polycrystalline samples,^[9-11,20] which is insufficient in view of the strong anisotropy of the properties along the hexagonal axis and in the basal plane. We shall report the results obtained for gadolinium-dys-

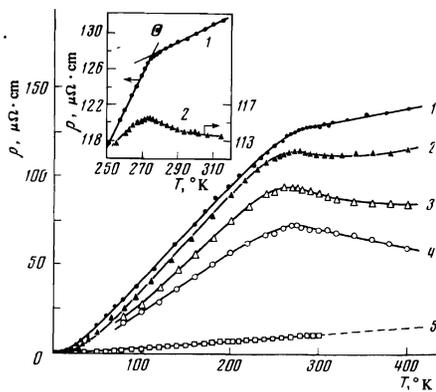


FIG. 1. Temperature dependences of the electrical resistivity of the alloy with 81.7% Gd and 18.3% Dy measured in the basal plane ρ_{\perp} (curve 1) and along the hexagonal axis ρ_{\parallel} (curve 2), and the temperature dependences of the magnetic components of the resistivity $\rho_{\text{mag}}^{\perp}$ (curve 3) and $\rho_{\text{mag}}^{\parallel}$ (curve 4) found by the method described in^[6,7]; curve 5 represents ρ_e of Lu.

prosim alloy single crystals.

These single crystals were prepared as described in^[12]. Measurements were carried out by the conventional potentiometric method using two current and two potential contacts with a $1 \times 1 \times 10$ mm single crystal sample cut by spark machining and then subjected to chemical etching and vacuum annealing for 10 h at 800 °C.

Figure 1 shows the temperature dependences of the resistivity of a single crystal^[1] with 81.7% Gd and 18.3% Dy, measured along the hexagonal axis $\rho_{\parallel}(T)$ and in the basal plane $\rho_{\perp}(T)$. The transition from the paramagnetic to the ferromagnetic state is manifested by a sharp change in the slope of the temperature dependence $\rho_{\perp}(T)$ so that, near the Curie point Θ , the curve $\rho_{\perp}(T)$ consists of two almost linear sections whose intersection (found by extrapolation) makes it possible to determine the point Θ . The behavior of $\rho_{\parallel}(T)$ near Θ is different: the $\rho_{\parallel}(T)$ curve (Fig. 1) has a maximum at $T = \Theta$. This resistivity anomaly can be explained by the scattering of conduction electrons near Θ by magnetic clusters.^[13] An inhomogeneous distribution of the Gd and Dy atoms results in some parts of the sample having Curie points differing from the Curie point Θ of the

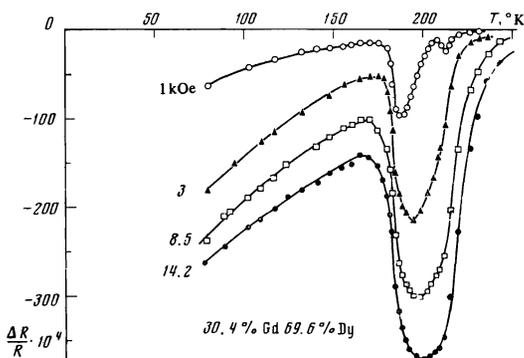


FIG. 2. Temperature dependences of the magnetoresistance of the alloy with 30.4% Gd and 69.6% Dy (the numbers alongside the curves give the field in kilooersted).

bulk of the sample; consequently, magnetic clusters form near Θ . It is not clear what role is played by the anisotropy in the scattering of conduction electrons by magnetic clusters [there are differences between the behavior of $\rho_{\parallel}(T)$ and $\rho_{\perp}(T)$ near Θ].

An increase in the dysprosium content in gadolinium increases the maximum of the negative magnetoresistance at the Curie point, which is in agreement with the hypothesis that the influence of magnetic clusters on the electrical resistivity of alloys is greater than the corresponding influence on pure metals. Similar temperature dependences $\rho_{\parallel}(T)$ and $\rho_{\perp}(T)$ were also obtained for other $\text{Gd}_{1-x}\text{Dy}_x$ alloys in the concentration range $0 < x \leq 0.5$. In this range, the $\text{Gd}_{1-x}\text{Dy}_x$ alloys were ferromagnetic at all temperatures below Θ . Alloys with lower gadolinium concentrations cooled below the point Θ_2 were converted from the paramagnetic to the anti-ferromagnetic state. In the case of pure dysprosium, we found a strong maximum of the dependence $\rho_{\parallel}(T)$ near Θ_2 and this maximum amounted to 10% of the total resistivity. The maximum was due to the intersection of the Fermi surface by superzone boundaries which appeared due to the helicoidal magnetic structure whose periodicity differed from the crystal lattice periodicity.^[14]

Pure Dy and the alloy with 30.4% Gd and 69.6% Dy also exhibited changes in the dependences $\rho_{\parallel}(T)$ and $\rho_{\perp}(T)$ at the transition Θ_1 from helicoidal antiferromagnetism to ferromagnetism, and at the point Θ_2 the dependence $\rho_{\perp}(T)$ of these materials had an anomaly of the same type as the $\text{Gd}_{1-x}\text{Dy}_x$ alloys with $x < 0.5$. The temperature dependence of the weak-field magnetoresistance had two maxima located near Θ_1 and Θ_2 (Fig. 2). If the magnetic field exceeded a certain critical value, these maxima were no longer resolved. The maximum critical field $(H_{\text{cr}})_{\text{max}}$ of the alloy with 30.4% Gd and 69.7% Dy was ~ 2 kOe, whereas, in the case of Dy, it was ~ 11 kOe. The replacement of dysprosium with gadolinium reduced the critical field H_{cr} in which the helicoidal structure was destroyed and for $x < 0.5$ the $\text{Gd}_{1-x}\text{Dy}_x$ alloys became ferromagnetic. The magnetic phase diagram, plotted on the basis of the electrical properties (Fig. 3), was in agreement with the results obtained by other methods.^[11,12]

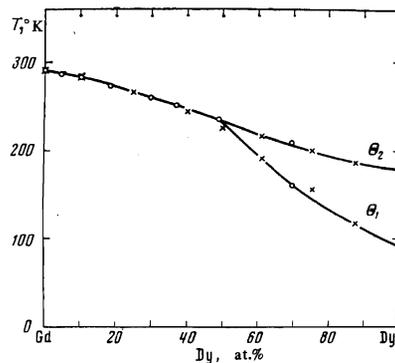


FIG. 3. Dependences of the points Θ_1 and Θ_2 on the concentration of dysprosium in gadolinium-dysprosium alloys (x—taken from^[11]; o—our results).

In accordance with the Matthiessen rule, the resistance of a rare-earth metal can be represented in the form^[7]

$$\rho = \rho_{\text{res}} + \rho_e + \rho_{\text{ph}} + \rho_{\text{mag}} \quad (1)$$

where ρ_{res} is the residual resistivity which is found by extrapolating ρ to 0 °K and which is practically independent of temperature; ρ_e is the resistivity associated with the electron-electron collisions; ρ_{ph} is the resistivity due to scattering by phonons; ρ_{mag} is the resistivity due to scattering by magnetic inhomogeneities.

It is usual to assume that ρ_{res} of nonmagnetic alloys is, firstly, due to scattering by dislocations, grain boundaries, impurity atoms, and other imperfections of the crystal lattice and, secondly, due to the difference between the electrostatic potentials of atoms in the alloy. However, in the case of the gadolinium-dysprosium alloys, ρ_{res} can also be explained by a third contribution to ρ_{res} , which is the disordered distribution, at 0 °K, of magnetic moments of gadolinium and dysprosium atoms differing in respect of the spin and total momentum. This contribution gives rise to a maximum in the dependence of ρ_{res} on the dysprosium concentration (this dependence was deduced from ρ at 4.2 °K), located near 50–70% Dy (Fig. 4). The contribution to ρ_{res} due to the difference between the electrostatic potential of gadolinium and dysprosium atoms should be minimal because of the close similarity of the electron shells of these atoms.

It is very difficult to separate the individual contributions from the total resistivity. The values of ρ_e , ρ_{ph} , and ρ_{mag} of heavy rare-earth metals are found in^[6, 7] by a method based on the use of the values of ρ_e and ρ_{ph} for lutetium. Heavy rare-earth metals differ only slightly in respect of their crystal and electron structures and they include one metal (lutetium) whose ions do not have a magnetic moment so that there is no magnetic term in the electrical resistivity ($\rho_{\text{mag}} = 0$) and this makes it easier to determine ρ_e and ρ_{ph} . The value of ρ_e found for lutetium can be assumed to be the same for all heavy rare-earth metals because of the similarity of their Fermi surfaces. We applied the same method^[6, 7] to find $\rho_{\text{mag}}^{\perp}(T)$ and $\rho_{\text{mag}}^{\parallel}(T)$ of the $\text{Gd}_{1-x}\text{Dy}_x$ alloys. The results obtained for the alloys with 81.7% Gd and 18.3% Dy are plotted in Fig. 1 in a wide temperature range. This figure also includes the temperature dependence of ρ_e for lutetium found by us using the method of Volkenshtein and Dyakin.^[6]

It is worth noting the following features: $\rho_{\text{mag}}^{\perp}$ and

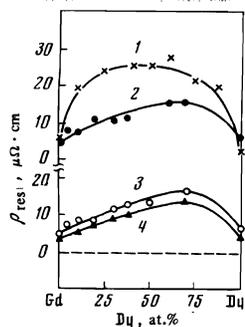


FIG. 4. Dependences on the dysprosium concentration of the residual resistivity ρ_{res} of gadolinium-dysprosium alloys: polycrystalline samples according to^[11] (curve 1) and according to calculations based on Eq. (5) (curve 2); single crystals for current in the basal plane $\rho_{\text{res}}^{\perp}$ (curve 3) and along the hexagonal axis $\rho_{\text{res}}^{\parallel}$ (curve 4).

TABLE 1. Magnetic component of electrical resistivity of gadolinium-dysprosium alloys.

	Composition, at.% Dy	Gd	4.4	10.4	18.3	29.6	37.2	49.1	69.6	Dy
1	$\rho_{\text{mag}}^{\parallel}, \mu\Omega \cdot \text{cm}$	108	99	98	90	80	76	—	76	52
2		112	100	93	94	86	79	—	63	53
3	$\rho_{\text{mag}}^{\perp}, \mu\Omega \cdot \text{cm}$	102	107	107	99	89	85	75	74	59
4		102	114	104	103	91	87	78	77	61

$\rho_{\text{mag}}^{\parallel}$ decrease in the temperature range $T < \Theta$, in agreement with the theoretical investigations,^[1–3] and this is attributed to a reduction in the scattering by magnetic disorder because of an increase in the magnetic order below Θ . The fall of $\rho_{\text{mag}}^{\perp}$ and $\rho_{\text{mag}}^{\parallel}$ at temperatures $T > \Theta$ does not agree with theoretical predictions that the scattering of conduction electrons by magnetic disorder in the paramagnetic region $T > \Theta$ should be practically independent of temperature.^[8] It follows from our analysis of the experimental data for Dy by the method of Volkenshtein *et al.*^[6, 7] and from the data for Dy, Tb, Ho, Er, and Tm^[7] that $\rho_{\text{mag}}^{\parallel}$ and $\rho_{\text{mag}}^{\perp}$ indeed vary slowly with temperature at $T > \Theta$, but, in the case of Gd and $\text{Gd}_{1-x}\text{Dy}_x$, the temperature dependences $\rho_{\text{mag}}^{\parallel}(T)$ and $\rho_{\text{mag}}^{\perp}(T)$ are stronger. This can clearly be explained by the fact that the Fermi surface changes from dysprosium to gadolinium in the $\text{Gd}_{1-x}\text{Dy}_x$ alloys and this alters the values of $\rho_e + \rho_{\text{ph}}$ for these alloys compared with lutetium and, therefore, it results in an overestimate of the contributions of $\rho_e + \rho_{\text{ph}}$ calculated for this case by the method used in^[6, 7].

However, ρ_{mag} can be determined sufficiently accurately in the paramagnetic region by a different method. On the one hand, above the Debye temperature the value of ρ_{ph} varies linearly with temperature. On the other, the data for lutetium (Fig. 1) show that ρ_e of heavy rare-earth metals represents only 8–14% of the total resistivity and, in a wide range of temperatures, this term also depends linearly on temperature. Moreover, ρ_{\perp} at $T > \Theta$ is a linear function of temperature because of the linear rise of the phonon contribution to the resistivity above the Debye temperature. Therefore, extrapolating the linear part of ρ_{\perp} at $T > \Theta$ to 0 °K, we can find the values of $\rho_{\text{mag}}^{\perp}$ at $T > \Theta$ in the paramagnetic region. The value of $\rho_{\text{mag}}^{\perp}$ found by this method is less accurate because, for some of the $\text{Gd}_{1-x}\text{Dy}_x$ alloys, the dependence $\rho_{\parallel}(T)$ is nonlinear at $T > \Theta$ near Θ .

Naturally, this approach is only approximate because it ignores the nonlinear dependence $\rho_{\text{ph}}(T)$ at low temperatures ($T \leq \Theta_D/5$). However, this error is only slight because extrapolation is made from the range of temperatures where the resistivity is an order of magnitude higher than the resistivity in this low-temperature range. Table 1 gives the values of $\rho_{\text{mag}}^{\parallel}$ and $\rho_{\text{mag}}^{\perp}$ found by extrapolation to $T = 0$ °K of the dependence $\rho(T) - \rho_{\text{res}}$ from the paramagnetic region (rows 1 and 3 in Table 1), and the values of the same quantities found by extrapolation of $\rho_{\text{mag}}(T)$ from the paramagnetic region to $T = 0$ °K by the method of Volkenshtein *et al.*^[6, 7] (rows 2 and 4). We can see that, in the case of Dy, these values agree within the limits of the experimental error (the Fermi surface of Dy is similar to that of Lu)

whereas, in the case of the $Gd_{1-x}Dy_x$ alloys, there are discrepancies of $\sim 5-8\%$.

It follows from the indirect exchange theory^[1-3] that the magnetic contribution to the resistivity due to the disorder of spins in the paramagnetic region at temperatures $T > \Theta$ or $T > \Theta_2$ is

$$\rho_{\text{mag}} = \frac{3\pi}{8\hbar e^2} \frac{\Gamma^2 m^*}{VE_f} (g-1)^2 J(J+1), \quad (2)$$

and the paramagnetic Curie point is

$$\Theta_p = \frac{3\pi Z^2}{4k} \frac{\Gamma^2}{V^2 E_f} (g-1)^2 J(J+1) \sum_{n \neq m} F(2K_f, R_{nm}). \quad (3)$$

Here, Γ is the integral of the exchange interaction between the spin of a conduction electron and the spin of a $4f$ electron; E_f is the Fermi energy; K_f is the wave vector; m^* is the effective mass of conduction electrons; V is the atomic volume; R_{nm} is the distance from a site n to a site m ; Z is the number of conduction electrons; F is the oscillatory Ruderman-Kittel function.

We used Eqs. (2) and (3) to determine the exchange integral Γ and effective mass of conduction electrons m^* substituting our experimental values of $\rho_{\text{mag}}^{\perp} = 102 \mu\Omega \cdot \text{cm}$, $\Theta_p = 317^\circ \text{K}$ for Gd and $\rho_{\text{mag}}^{\perp} = 59 \mu\Omega \cdot \text{cm}$, $\Theta_p = 169^\circ \text{K}$ for Dy, as well as the calculated^[2] sum $\sum F(2K_f, R_{nm}) = 68 \cdot 10^{-4}$; we also substituted $Z = 3$. In this way, we obtained the following values for Gd: $m^* = 2.68 m_e$, $\Gamma = 5.98 \text{ eV} \cdot \text{\AA}^3$, $\Gamma/V = 0.183 \text{ eV}$; the corresponding values for Dy were $m^* = 2.96 m_e$, $\Gamma = 6.0 \text{ eV} \cdot \text{\AA}^3$, $\Gamma/V = 0.190 \text{ eV}$; these values were generally in agreement with the results published earlier^[2] and they indicated that the exchange integral Γ did not change greatly from gadolinium to dysprosium.

In the case of a binary alloy, the magnetic resistivity ρ_{mag} can be represented in the form^[15]

$$\rho_{\text{mag}} = \frac{3\pi m^*}{8\hbar e^2 V} \{x\Gamma_a^2 G_a + (1-x)\Gamma_b^2 G_b - x(1-x)[\Gamma_a S_a - \Gamma_b S_b]^2\}, \quad (4)$$

where the indices a and b show that the exchange integral Γ , spin S , and de Gennes factor $G = (g-1)^2 J(J+1)$ apply to the two components of an alloy $A_x B_{1-x}$. The formula (4) ignores the anisotropy and the effects of superzone boundaries, which are important in the case of $\rho_{\text{mag}}^{\parallel}$ and, therefore, the above formula applies—strictly speaking—only to the composition dependence of $\rho_{\text{mag}}^{\perp}$. We used the calculated values of Γ and m^* in deriving the theoretical dependence of $\rho_{\text{mag}}^{\perp}$ on the composition. The values of the Gd and Dy spins differed only slightly, so that the third term in the brackets did not exceed 2% of the total value of $\rho_{\text{mag}}^{\perp}$. Consequently, the theoretical dependence of $\rho_{\text{mag}}^{\perp}$ on the dysprosium concentration (dashed line in Fig. 5) was almost linear and described satisfactorily the experimental results. However, in the case of alloys containing less than 20% dysprosium, the value of $\rho_{\text{mag}}^{\perp}$ was somewhat higher than predicted theoretically. The resistivity of the polycrystalline material was calculated from ρ_{\parallel} and ρ_{\perp} using

$$\rho = \frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp}. \quad (5)$$

The results obtained (Fig. 5) were found to be in satisfactory agreement with the results of measurements on

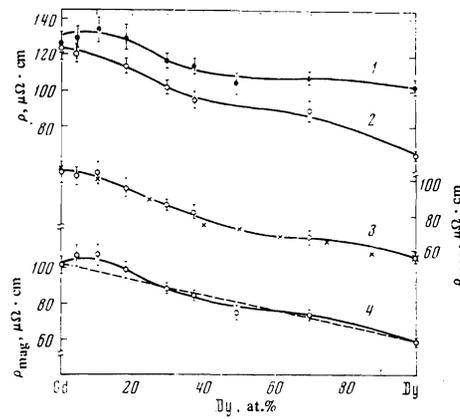


FIG. 5. Dependences, on the dysprosium concentration, of the electrical resistivity $\rho - \rho_{\text{res}}$ (curve 1 represents the current in the basal plane and curve 2—along the hexagonal axis) and of the magnetic component of the resistivity ρ_{mag} in the paramagnetic state of $Gd_{1-x}Dy_x$ alloys [curve 3 represents polycrystalline samples: \times —results from^[11], \circ —calculated from Eq. (5); curve 4 is the magnetic resistivity of a single crystal for current in the basal plane, $\rho_{\text{mag}}^{\perp}$].

polycrystalline gadolinium-dysprosium alloys.^[11]

It follows from our results that the indirect exchange theory describes satisfactorily the composition and temperature dependences of the magnetic resistivity of rare-earth alloys when the current flows in the basal plane and it also makes it possible to find—from the Curie temperature and electrical resistivity—the s - f exchange integral and the effective mass of conduction electrons, which are in agreement with the results obtained by other methods.^[16] However, when the current flows along the hexagonal axis, the theory fails to explain the resistivity anomaly near Θ_2 and the anisotropy of the magnetic component of the resistivity. The theoretical model can give a full description of the properties of rare-earth metals and alloys only if it allows for the anisotropy of the s - f exchange integral, for the effective mass of conduction electrons, and for the strong magnetic anisotropy of hexagonal crystals which exerts a considerable influence on the magnetic and electrical properties below and above the magnetic ordering temperature.

We also investigated the influence of pressures up to $15 \times 10^3 \text{ kg/cm}^2$ on the resistivity of $Gd_{1-x}Dy_x$ single crystals with the aim of determining the dependences of the s - f exchange integral and the effective mass m^* on the atomic volume. The published investigations of the pressure dependence of the resistivity were confined to single crystals of pure rare-earth metals.^[17,18] Hydrostatic pressure was produced in an aviation-spirit filled bomb by a pressure generator developed at the All-Union Scientific-Research Institute of Physicotechnical and Radio Engineering Measurements. This pressure was measured with a manganin manometer to within 0.5%. The measurements were carried out in the temperature range $293-430^\circ \text{K}$, when the samples were in the paramagnetic state.

It was established that the resistivity of all the alloys decreased linearly with rising pressure and the value of

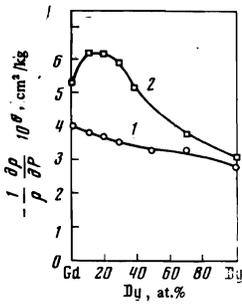


FIG. 6. Pressure-induced change in the electrical resistivity $\rho^{-1} \partial \rho / \partial P$ of gadolinium-dysprosium alloy single crystals in the paramagnetic state at 300°K determined by measurements at right-angles to the hexagonal c axis (curve 1) and along the c axis (curve 2).

$\rho^{-1} \partial \rho / \partial P$ for the current in the basal plane depended weakly on temperature in the range $T > \Theta_2$ or $T > \Theta$, whereas, along the hexagonal axis, the temperature dependence of the pressure coefficient was stronger. Figure 6 shows the relative value of the pressure-induced change in the resistivity of the paramagnetic state as a function of the dysprosium concentration in the alloys.

In accordance with Eq. (1), the change in the resistivity under pressure consists of four components:

$$\frac{\partial \rho}{\partial P} = \frac{\partial \rho_{\text{res}}}{\partial P} + \frac{\partial \rho_{\text{ph}}}{\partial P} + \frac{\partial \rho_e}{\partial P} + \frac{\partial \rho_{\text{mag}}}{\partial P}. \quad (6)$$

We shall analyze the experimental data on the basis of the indirect exchange theory because only this theory gives quantitative estimates of the exchange parameters. We shall consider the effects in the basal plane in which the anisotropy is negligible.

The rare-earth metals Gd and Dy, and their alloys are characterized by very high values of ρ_{mag} (Fig. 5) in the paramagnetic state so that

$$\rho_{\text{mag}} \gg \rho_{\text{re}} \quad \frac{\partial \rho_{\text{mag}}}{\partial P} \gg \frac{\partial \rho_{\text{res}}}{\partial P},$$

and

$$\rho_{\text{mag}} \gg \rho_{\text{ph}} + \rho_e \quad \frac{\partial \rho_{\text{mag}}}{\partial P} > \frac{\partial \rho_{\text{ph}}}{\partial P} + \frac{\partial \rho_e}{\partial P}.$$

Using Eqs. (2) and (3), and bearing in mind that the sum $\Sigma F(2K_r R_{\text{em}})$ for the isotropic case is independent of volume,^[10] we can find by differentiation the relative change in Θ_p and in the magnetic component of the resistivity ρ_{mag} in the paramagnetic state due to the application of pressure:

$$-\frac{1}{\kappa} \left(\frac{1}{\Theta_p} \frac{\partial \Theta_p}{\partial P} \right) = \frac{4}{3} + \left(\frac{\partial \log \Gamma}{\partial \log V} \right)_{\text{H.T.}} + \left(\frac{\partial \log m^*}{\partial \log V} \right)_{\text{H.T.}}, \quad (7)$$

$$-\frac{1}{\kappa} \left(\frac{1}{\rho_{\text{mag}}} \frac{\partial \rho_{\text{mag}}}{\partial P} \right) = -\frac{1}{3} + 2 \left(\frac{\partial \log \Gamma}{\partial \log V} \right)_{\text{H.T.}} + 2 \left(\frac{\partial \log m^*}{\partial \log V} \right)_{\text{H.T.}}, \quad (8)$$

where κ is the compressibility ($\kappa = 2.7 \times 10^{-6} \text{ cm}^2/\text{kg}$).

Thus, the changes in the s - f exchange integrals and in the effective mass of conduction electrons under pressure and the corresponding changes in the atomic volume can be calculated from Eqs. (7) and (8) having found experimentally $\Theta_p^{-1} \partial \Theta_p / \partial P$ and $\rho_{\text{mag}}^{-1} \partial \rho_{\text{mag}} / \partial P$.^[17] We carried out such calculations for the basal plane because only the results obtained for this plane were described satisfactorily by the indirect exchange theory.^[1-3] The values of $\Theta_p^{-1} \partial \Theta_p / \partial P \approx \Theta_2^{-1} \partial \Theta_2 / \partial P$ were taken

from^[20] and those of $\rho_{\text{mag}}^{-1} \partial \rho_{\text{mag}} / \partial P$ from our measurements (Fig. 6). The calculated values of $\partial \log \Gamma / \partial \log V$ and $\partial \log m^* / \partial \log V$ are plotted in Fig. 7.

The continuous curves apply to the case when the value of $(\rho_{\text{mag}}^{-1} \partial \rho_{\text{mag}} / \partial P)_1$ is found from $\rho^{-1} \partial \rho / \partial P$ ignoring the pressure-induced changes in ρ_e and ρ_{ph} , i. e., assuming that

$$\frac{\partial \rho_{\text{mag}}}{\partial P} \gg \frac{\partial \rho_{\text{ph}}}{\partial P} + \frac{\partial \rho_e}{\partial P}.$$

The dashed curves in Fig. 7 are the values of $\partial \log \Gamma / \partial \log V$ and $\partial \log m^* / \partial \log V$ calculated for the case when $(\rho_{\text{mag}}^{-1} \partial \rho_{\text{mag}} / \partial P)_1$ is found allowing for the pressure-induced change in $\rho_{\text{ph}} + \rho_e$:

$$\frac{\partial \rho_{\text{mag}}}{\partial P} = \frac{\partial \rho}{\partial P} - \frac{\partial \rho_{\text{ph}}}{\partial P} - \frac{\partial \rho_e}{\partial P}.$$

We took $\partial \rho_{\text{ph}} / \partial P + \partial \rho_e / \partial P$ to be the value of $(\partial \rho / \partial P)_1$ for an yttrium single crystal which is paramagnetic throughout the investigated temperature range and has electron and crystal structures similar to those of gadolinium and dysprosium but has $\rho_{\text{mag}} = 0$. Lutetium has the same properties.

Our measurements indicated that the pressure-induced changes in the resistivity of yttrium and lutetium, from which we could estimate $\partial \rho_{\text{ph}} / \partial P + \partial \rho_e / \partial P$ for heavy rare-earth metals were only about half the values of $\rho^{-1} \partial \rho / \partial P$ for Gd and Dy.

In fact, according to our results, the value of $\rho^{-1} \partial \rho / \partial P_0$ for yttrium at $T = 295^\circ \text{K}$ was -1.98×10^{-6} when the current was perpendicular to the hexagonal axis and -2.02×10^{-6} when the current was parallel to this axis; in the case of polycrystalline lutetium, the value was -1.65×10^{-6} .

It was clear that allowance was necessary for $\partial \rho_{\text{ph}} / \partial P + \partial \rho_e / \partial P$, which had not been made in the calculation of $\partial \log \Gamma / \partial \log V$ and $\partial \log m^* / \partial \log V$ in the case of pure rare-earth metals.^[17] This considerably altered the values of $\partial \log \Gamma / \partial \log V$ and particularly of $\partial \log m^* / \partial \log V$ (Fig. 7).

Thus, our calculations yield the following results, which indicate that changes occur in the energy spectrum of conduction electrons under pressure.

1. The s - f exchange interaction integral increases

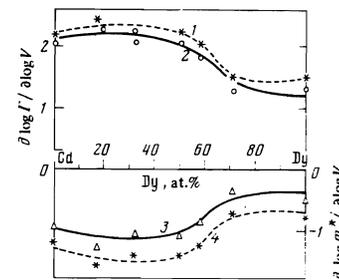


FIG. 7. Dependence of the s - f exchange integral $\partial \log \Gamma / \partial \log V$ and of the effective mass of conduction electrons $\partial \log m^* / \partial \log V$ on the atomic volume V of gadolinium-dysprosium alloys.

with increasing atomic volume V of Dy proportionally to $V^{1.5}$, whereas, in the case of Gd, it increases proportionally to $V^{2.1}$; in the case of $Gd_{1-x}Dy_x$ alloys, this exchange integral Γ is proportional to V^n , where $2.2 \geq n \geq 1.5$.

2. The effective mass of conduction electrons m^* decreases with increasing atomic volume of $Gd_{1-x}Dy_x$ alloys; in the case of Gd and $Gd_{1-x}Dy_x$, we find that $m^* \propto V^{-k}$, where $1.2 \leq k \leq 1.6$ in the range $x < 0.6$ and this dependence is much stronger than for Dy, which is characterized by $m^* \propto V^{-0.8}$. The different changes in the indirect exchange integral and the effective mass of the conduction electrons observed when pressure is applied to gadolinium and dysprosium and to gadolinium-dysprosium alloys may be due to the features of the Fermi surface topology of gadolinium which distinguish it from the corresponding surface of dysprosium.^[21,22] According to the theory given in^[23], we can also assume that the change in the topology of the Fermi surface from gadolinium to dysprosium gives rise to helical magnetic structures (Fig. 3) in $Gd_{1-x}Dy_x$ alloys with $x > 0.5$.

We shall conclude by thanking Prof. K. P. Belov for discussing our results, and E. M. Savitskii and V. F. Terekhova for their advice and supplying alloy single crystals.

¹⁾Here and later, the compositions will be specified in atomic percent.

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