Permittivity of a nondegenerate collisionless plasma and cyclotron waves in a quantizing magnetic field

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The dielectric tensor of a nondegenerate plasma in a quantizing magnetic field is calculated exactly. Its dependence on the frequency and wave vector is investigated. Allowance is made for the contribution of the spin current, which may greatly exceed the orbital current when the distance between the Landau levels appreciably exceeds the thermal energy. The conditions for transition to the classical limit are considered. They reduce to weakness of the magnetic field and to a sufficiently long wavelength. Finally, the properties of ordinary cyclotron waves propagating transversely are investigated. It is shown that an exact solution leads to significantly new results.

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1. INTRODUCTION

At the present time, quantizing magnetic fields are produced in solid-state plasmas (semiconductors, semimetals). In addition, astrophysical objects have been observed, in which the magnetic field is quantizing even though the plasma temperature is high; we have in mind pulsars in which the magnetic field apparently reaches 10^{13} Oe. This raises the pressing problem of investigating the dielectric constant of a plasma in a quantizing magnetic field, and this is the subject of the present article.

A number of papers directly devoted to this topic have been published in recent years.[1–5] In none of them, however, was the problem solved completely. For a complete solution we need the following: a) Summation over all the Landau quantum numbers, that is, consideration of arbitrary values of $\hbar\omega_c/T$ (ω_c is the cyclotron frequency and T is the temperature in energy units), and not only the extremely large and extremely small values. b) The investigation must be carried out at all values of the parameter $k_\perp^2R^2$ ($k_\perp$ is the wave-vector component perpendicular to the magnetic field and R is the magnetic length), and not only at small values of this parameter. c) The spin current must be taken into account. We show in this article that such an exact solution of the problem leads to a noticeable change in the results obtained in the cited papers, in which only more or less the simplified cases were investigated. The purpose of the present study was therefore a complete investigation of the dielectric constant of a collisionless plasma in a quantizing magnetic field. We shall also demonstrate here, with cyclotron waves as the example, those new effects to which the exact solution leads.

2. PRINCIPAL SYMBOLS AND THEIR RELATIONS

We choose the vector potential of the external magnetic field in the form $(A_0(\mathbf{r}) = (-By, 0, 0)$. We denote by α the aggregate of the orbital quantum numbers $N_\alpha, p_\alpha, y_\alpha = R^2p_x$, where $R^2 = (eB)^{-1}$ and we put $\hbar = c = 1$, while $\sigma = \pm 1$ denotes the spin quantum number. Then the orbital part of the wave function $\psi_\alpha(\mathbf{r})$ and the energy $E_{\alpha\sigma}$ of the stationary state are equal respectively to

$$\psi_\alpha(\mathbf{r}) = \frac{1}{(2\pi R)^{1/2}} \exp\left[i\left(p_\alpha z + \frac{y_\alpha x}{R^2}\right) - \frac{(y-y_\alpha)^2}{2R^2}\right] H_{N_\alpha}\left(\frac{y-y_\alpha}{R}\right),$$

$$E_{\alpha\sigma} = \frac{p_\alpha^2}{2m} + \left(N_\alpha + \frac{\sigma+1}{2}\right)\omega_c, \quad \omega_c = \frac{eB}{m}.$$

Here $H_N(y)$ are Hermite polynomials normalized to unity.

The connection between the average current density $\mathbf{j}(\mathbf{r}, t)$ and the vector potential $\mathbf{A}(\mathbf{r}, t)$ of the wave is given by[6]

$$j^l(\mathbf{r}, t) = -\frac{ne^2}{m} A^l(\mathbf{r}, t) - \int_{-\infty}^{\infty} dt'\, d\mathbf{r}'\, G^{lk}(\mathbf{r}, \mathbf{r}', \tau) A^k(\mathbf{r}', t'), \tag{1}$$

$$G^{lk}(\mathbf{r}, \mathbf{r}', \tau) = -i\theta(\tau)\,\mathrm{Sp}[\rho_0(\hat{j}^l(\mathbf{r}, t)\hat{j}^k(\mathbf{r}', t') - \hat{j}^k(\mathbf{r}', t')\hat{j}^l(\mathbf{r}, t))]. \tag{2}$$

Here $n, m, e = |e|$ are respectively the concentration, mass, and absolute value of the electron charge, $\tau = t - t'$, $\theta(\tau)$ is the step function, $\hat{\mathbf{j}}(\mathbf{r}, t)$ is the current-density operator in the Heisenberg representation, the Hamiltonian being

$$\hat{H} = \sum_{\alpha,\sigma} E_{\alpha\sigma} a_{\alpha\sigma}^+ a_{\alpha\sigma},$$

and ρ_0 the equilibrium density matrix. The corresponding Schrödinger current operator is

$$\hat{j}^k(\mathbf{r}) = \sum_{\alpha,\sigma,\beta,\sigma'} j^k_{\alpha,\sigma,\beta,\sigma'}(\mathbf{r}) a_{\alpha\sigma}^+ a_{\beta\sigma'}.$$

Substitution in (2) yields

$$G^{lk}(\mathbf{r}, \mathbf{r}', \tau) = -i\theta(\tau)[g^{lk}(\mathbf{r}, \mathbf{r}', \tau) - \tilde{g}^{lk}(\mathbf{r}, \mathbf{r}', \tau)] = -i\theta(\tau) \sum_{\alpha,\sigma,\beta,\sigma'} (f_{\alpha\sigma} - f_{\beta\sigma'})$$
$$\times j^l_{\alpha,\sigma,\beta,\sigma'}(\mathbf{r}) j^k_{\beta,\sigma',\alpha,\sigma}(\mathbf{r}') \exp[i(E_{\alpha\sigma} - E_{\beta\sigma'})\tau]. \tag{3}$$

In this expression $f_{\alpha\sigma}$ is the equilibrium distribution function.

We shall henceforth use the quantities $j^\pm = j^x \pm ij^y$ and j^z (and correspondingly the functions $g^{+-}(\mathbf{r}, \mathbf{r}', \tau$, etc.), which can be reduced, by using the recurrence relations for the Hermite polynomials, to the form

$$j^+_{\alpha,\sigma,\beta,\sigma'}(\mathbf{r})=j^+_{\alpha,\beta,\sigma}(\mathbf{r})\delta_{\sigma,\sigma'}-i\frac{e}{m}\Sigma^+_{\sigma,\sigma'}\frac{\partial}{\partial z}(\psi_\alpha^*(\mathbf{r})\psi_\beta(\mathbf{r})),$$

$$j^z_{\alpha,\sigma,\beta,\sigma'}(\mathbf{r})=\frac{e}{2m}\left[(p_\alpha+p_\beta)\delta_{\sigma,\sigma'}+\Sigma^x_{\sigma,\sigma'}\frac{\partial}{\partial y}-\Sigma^y_{\sigma,\sigma'}\frac{\partial}{\partial x}\right]\psi_\alpha^*(\mathbf{r})\psi_\beta(\mathbf{r}),$$

$$j^-_{\alpha,\sigma,\beta,\sigma'}(\mathbf{r})=(j^+_{\beta,\sigma',\alpha,\sigma}(\mathbf{r}))^*.$$

Here $\Sigma_{\sigma,\sigma'}$ is the matrix element of the Pauli-matrix vector.

Introducing the notation

$$\psi_{N\pm1}(\mathbf{r})=\psi_{p,\,y_\alpha,\,N\pm1}(\mathbf{r}),$$

we can write

$$j_{\alpha,\beta,-1}(\mathbf{r})=\frac{e}{mR}[2(N_\beta+1)]^{\prime\prime}\psi_\alpha^*(\mathbf{r})\psi_{N_\beta+1}(\mathbf{r}),$$

$$j^+_{\alpha,\beta,-1}(\mathbf{r})=\frac{e}{mR}(2N_\alpha)^{\prime\prime}\psi^*_{N_\alpha-1}(\mathbf{r})\psi_\beta(\mathbf{r}).$$

3. CALCULATION PROCEDURE AND RESULT

Since the expressions are cumbersome, we shall calculate in detail only the quantity g^{++}, confining ourselves in its terms (diagonal in the spin indices σ and σ') only to the term g^{++}_{+1} corresponding to $\sigma=+1$. The integration of

$$j^+_{\alpha,\beta,+1}(\mathbf{r})j^+_{\beta,\alpha,+1}(\mathbf{r}')$$

with respect to y_α and y_β using the formula[7]

$$\int_{-\infty}^{\infty}\exp(-x^2+\alpha x)H_m(x+y)H_n(x+z)\,dx$$
$$=\left(\frac{2^n m!}{2^m n!}\right)^{1/2}\left(\frac{\alpha}{2}+z\right)^{n-m}\exp\left(\frac{\alpha^2}{4}\right)L_m^{n-m}\left(-2\left(y+\frac{\alpha}{2}\right)\left(z+\frac{\alpha}{2}\right)\right)$$

at $m\leqslant n$, where $L^n_m(x)$ are Laguerre polynomials, leads to the expression

$$g^{++}_{p_\alpha,p_\beta,N_\alpha,N_\beta}=-\frac{e^2[i(x-x')-(y-y')]^2}{(2\pi)^2m^2R^4}$$
$$\times\exp[i(p_\beta-p_\alpha)(z-z')-\rho^2/2]L_{N_\alpha}^{\ 1}(\rho^2/2)L_{N_\beta}^{\ 1}(\rho^2/2),$$
$$\rho^2=R^{-2}[(x-x')^2+(y-y')^2].$$

The next step is summation over N_β. We introduce the quantity

$$g^{++}_{p_\alpha,p_\beta,N_\alpha}=\sum_{N_\beta}g^{++}_{p_\alpha,p_\beta,N_\alpha,N_\beta}\exp(-iN_\beta\omega_c\tau).$$

By virtue of the identity[7]

$$\sum_{n=0}^{\infty}a^nL_n^\alpha(x)=(1-a)^{-1-\alpha}\exp[xa(a-1)^{-1}],\quad |a|<1 \qquad (4)$$

this quantity is equal to

$$g^{++}_{p_\alpha,p_\beta,N_\alpha}=-\frac{e^2[i(x-x')-(y-y')]^2}{(2\pi)^2m^2R^4}\exp[i(p_\beta-p_\alpha)(z-z')]F(\rho).$$
$$F(\rho)=[1-\exp(-i\omega_c\tau)]^{-2}\exp\left[-\frac{\rho^2}{4}\left(1-i\,\mathrm{ctg}\frac{\omega_c\tau}{2}\right)\right]L^1_{N_\alpha}\left(\frac{\rho^2}{2}\right). \qquad (5)$$

Although in our case we have

$$|a|=|\exp(-i\omega_c\tau)|=1,$$

a subsequent Fourier transformation with respect to $\mathbf{r}-\mathbf{r}'$ leads, as we shall see, to a result that confirms the validity of the summation carried out in (5). The Fourier transform of expression (5) with respect to the variables $x-x'$ and $y-y'$ can be expressed in terms of the derivative $\partial F(k_\perp)/\partial k_\perp$ of the Fourier transform of the function $F(\rho)$:

$$\frac{\partial F(k_\perp)}{\partial k_\perp}=\frac{\partial}{\partial k_\perp}\int\exp(ik_xx+ik_yy)F\left(\frac{x^2+y^2}{2R^2}\right)dx\,dy$$
$$=2\pi R^2\int_0^\infty F(\rho)\frac{\partial}{\partial k_\perp}J_0(k_\perp R\rho)\rho d\rho=2\pi R^2k_\perp$$
$$\times\exp\left\{-\frac{k_\perp^2R^2}{2}[1-\exp(-i\omega_c\tau)]\right\}L^1_{N_\alpha}(k_\perp^2R^2(1-\cos\omega_c\tau)), \qquad (6)$$

where $k_\perp^2=k_x^2+k_y^2$, and $J_0(x)$ is a Bessel function. As a result we obtain after taking the Fourier transform with respect to $z-z'$

$$g^{++}_{+1}(\mathbf{k},\tau)=-\frac{e^2k_\perp^{-2}}{2\pi m^2R^6k_\perp}\sum_{N_\alpha,p_\alpha}f_{\alpha,+1}$$
$$\times\exp\left[-i\left(\frac{p_\alpha k_z}{m}+\frac{k_z^2}{2m}\right)\tau\right]\frac{\partial}{\partial k_\perp}\left(\frac{1}{k_\perp}\frac{\partial F(k_\perp)}{\partial k_\perp}\right).$$

The validity of the indicated summation over N_β becomes clear if it is noted that when a small imaginary increment $-i\delta$ is added to ω_c, expression (6) is continuous and finite as $\delta\to0$. In the investigation that follows we confine ourselves to nondegenerate plasma, for which

$$f_{\alpha,+1}\sim\exp[-T^{-1}(N_\alpha\omega_c+p_\alpha^2/2m)].$$

Here T is the temperature in energy units. The summation with respect to N_α yields according to (4)

$$\sum_{N_\alpha=0}^{\infty}\exp\left(-\frac{N_\alpha\omega_c}{T}\right)L^1_{N_\alpha}(x)=\left[1-\exp\left(-\frac{\omega_c}{T}\right)\right]^{-2}\exp\left\{x\left[1-\exp\left(\frac{\omega_c}{T}\right)\right]^{-1}\right\}.$$

We use for the obtained functions of $\omega_c\tau$ the series expansion

$$\exp\left[\frac{k_\perp^2R^2}{2}(-i\sin\omega_c\tau+\mathrm{cth}\,\xi\cos\omega_c\tau)\right]$$
$$=\sum_{s=-\infty}^{\infty}\exp(-is\omega_c\tau+s\xi)I_s(\tfrac{1}{2}k_\perp^2R^2\,\mathrm{sh}^{-1}\xi),$$

where $I_s(x)$ is a Bessel function of imaginary argument and $\xi=\omega_c/2T$. Noting that

$$\tilde{g}^{ik}(\mathbf{k},\tau)=g^{ki}(-\mathbf{k},-\tau).$$

and taking the Fourier transform of the function G^{++} (3) with respect to τ, we obtain after integrating with respect to p_α

$$G^{++}(\mathbf{k},\omega)=2G^{++}_{+1}(\mathbf{k},\omega)=\frac{-ne^2k_\perp^{-2}}{m^2R^2vk_\perp|k_z|\mathrm{sh}\,2\xi}\frac{\partial}{\partial k_\perp}\sum_{s=-\infty}^{\infty}A_sB_s. \qquad (7)$$

The functions A_s and $B_s^\pm$, which enter, as we shall see, in all the components of the tensor G^{ik}, take the form

$$A_s=\exp\left(-\frac{k_\perp^2R^2}{2}\mathrm{cth}\,\xi\right)I_s\left(\frac{k_\perp^2R^2}{2\sinh\xi}\right),$$
$$B_s^\pm=\frac{J_+(\beta_s^-)}{\beta_s^-}e^{s\xi}\pm\frac{J_+(\beta_s^+)}{\beta_s^+}e^{-s\xi}. \qquad (8)$$

The function $J_+(\beta)$, which was investigated in detail by Silin and Rukhadze,[8] is equal to

$$J_+(\beta)=\beta\exp\left(-\frac{\beta^2}{2}\right)\int_{i\infty}^{\beta}\exp\left(\frac{\tau^2}{2}\right)d\tau,$$
$$\beta_s^\mp=\frac{\omega-s\omega_c}{v|k_z|}\pm\frac{|k_z|}{2mv},\quad mv^2=T. \qquad (9)$$

We write now without calculations the remaining components of the tensor $G^{ik}(\mathbf{k}, \omega)$. We have

$$G^{+-}(\mathbf{k}, \omega) = G^{-+*}(\mathbf{k}, -\omega) = -\frac{2ne^2\omega_c}{mv|k_z|} e^{-\xi} \tanh \xi$$

$$\times \sum_{s=-\infty}^{\infty} B_s^- \left(\frac{\partial}{\partial \xi} - 1\right) \{A_{s+1}(1-e^{-2\xi})^{-1}\} + k_z^2 S^+,$$

$$G^{+z}(\mathbf{k}, \omega) = \frac{ne^2 k_+ k_z}{m^2 \operatorname{sh} 2\xi} \sum_{s=-\infty}^{\infty} \left\{ \frac{m}{k_z^2}(A_s - A_{s+1}\cosh \xi)\,[e^{s\xi}[1-J_+(\beta_s^-)]\right.$$

$$\left. - e^{-s\xi}[1-J_+(\beta_s^+)]] + \frac{1}{2v|k_z|}(A_s + A_{s+1}\cosh \xi) B_s^+ \right\} + k_+ k_z S^+,$$

$$G^{zz}(\mathbf{k}, \omega) = -\frac{ne^2}{m} \sum_{s=-\infty}^{\infty} A_s \left\{ e^{s\xi}[1-J_+(\beta_s^-)] \left[\frac{m(\omega - s\omega_c)}{k_z^2} + \frac{1}{2}\right]\right.$$

$$\left. - e^{-s\xi}[1-J_+(\beta_s^+)] \left[\frac{m(\omega - s\omega_c)}{k_z^2} - \frac{1}{2}\right] - \frac{k_z}{4mv} B_s^- \right\} + \frac{k_\perp^2}{4}(S^+ + S^-),$$

$$G^{--}(\mathbf{k}, \omega) = \frac{k_-^2}{k_+^2} G^{++}(\mathbf{k}, \omega), \quad G^{z\pm}(\mathbf{k}, \omega) = G^{\pm z}(\mathbf{k}, \omega).$$

$$G^{-z}(\mathbf{k}, \omega) = G^{+z*}(\mathbf{k}, -\omega).$$

$$S^\pm = \frac{ne^2}{2m^2 v |k_z| \operatorname{ch} \xi} \sum_{s=-\infty}^{\infty} A_{s\pm 1} B_s^-. \tag{10}$$

The components of the dielectric tensor $\varepsilon_{ij}(\mathbf{k}, \omega)$ are connected with the component of $G^{ij}(\mathbf{k}, \omega)$ by the relation

$$\varepsilon^{ij}(\mathbf{k}, \omega) = \left(1 - \frac{\Omega_p^2}{\omega^2}\right)\delta_{ij} - \frac{4\pi G^{ij}(\mathbf{k}, \omega)}{\omega^2} \tag{11}$$

(Ω_p is the plasma frequency).

4. CLASSICAL LIMITING CASE

It is obvious that the classical limit is reached under the condition $\omega_c \ll T$. In this the function $e^{s\xi}$ becomes noticeably different from unity only at values $s \gtrsim \xi^{-1}$. The classical theory becomes applicable if at these values of s the quantity

$$A_s \approx A_s^{(cl)}(k_\perp^2 v^2/\omega_c^2) = \exp(-k_\perp^2 v^2/\omega_c^2) I_s(k_\perp^2 v^2/\omega_c^2)$$

turns out to be negligibly small. At $s \gg 1$ the asymptotic form of the function $A_s^{(cl)}$ is such[9] that it decreases exponentially at $s > k_\perp v \omega_c^{-1}$, from which we obtain a second condition for the transition to the classical limit: $k^2(mT)^{-1} \ll 1$.

To calculate B_s^- in the case when both conditions are satisfied, we expand $e^{s\xi}$ and $\beta_s^{-1} J_+(\beta_s)$ in powers of $s\xi$ and $|k_z|(mv)^{-1}$. We then obtain

$$B_s^- \approx B_s^{(cl)} = \frac{\omega}{T} \frac{J_+(\beta_s)}{\beta_s} \frac{|k_z|}{mv}, \quad \beta_s = \frac{\omega - s\omega_c}{v|k_z|}.$$

Substituting $A_s^{(cl)}$ and $B_s^{(cl)}$ in (7), and recognizing that

$$\sum_{s=-\infty}^{\infty} I_s(x) = e^x,$$

we obtain

$$\varepsilon^{++} = \varepsilon^{xx} - \varepsilon^{yy} + i(\varepsilon^{xy} + \varepsilon^{yx}) = 2\frac{\Omega_p^2}{\omega} \frac{k_+^2 v^2}{\omega_c^2} \sum_{s=-\infty}^{\infty} A_s'^{(cl)} \frac{J_+(\beta_s)}{\omega - s\omega_c},$$

which agrees with the result of Ginzburg and Rukhadze.[10] Here

$$A_s'^{(cl)} = \partial A_s^{(cl)}(x)/\partial x.$$

5. ORDINARY ELECTRON CYCLOTRON WAVES TRANSVERSELY PROPAGATING IN QUANTIZING MAGNETIC FIELD

Such waves were considered by Korneev and Starostin[5] in the limiting case when

$$k_\perp^2 R^2 \ll \hbar\omega_c/mc^2, \quad \hbar\omega_c \gg T$$

(we write out here explicitly the constants $\hbar$ and c). We, to the contrary, consider arbitrary values of the two indicated parameters, retaining $k_z = 0$. We shall see that this leads to new qualitative results.

According to[5] (see also[11]) the dispersion relation for these waves follows from the equality

$$c^2 k^2 \omega^{-2} = \varepsilon_{zz}.$$

In our case it takes the form

$$\left(\frac{ck}{\omega}\right)^2 = 1 - \frac{\Omega_p^2}{\omega^2} + \frac{T\Omega_p^2}{\hbar\omega^2} \exp(-X\cosh\xi)$$

$$\cdot \sum_{s=-\infty}^{\infty} \frac{2\sinh s\xi}{s\omega_c - \omega}[I_s(X) + \xi \tanh \xi X I_s'(X)];$$

$$X = \frac{k_\perp^2 R^2}{2\operatorname{sh}\xi}, \quad I_s'(X) = \frac{\partial I_s(X)}{\partial X}. \tag{12}$$

The terms containing $I_s(X)$ and $I_s'(X)$ result respectivetively from allowance for the orbital and spin currents. Their ratio $\varepsilon_{sp}/\varepsilon_{orb}$ in both limiting cases $X \ll 1$ and $X \gg 1$ amounts to (in a quantizing magnetic field)

$$\varepsilon_{sp}/\varepsilon_{orb} = s\xi\,(X \ll 1) = \xi X \quad (X \gg 1). \tag{13}$$

We see therefore that at $\xi > 1$ the ratio $\varepsilon_{sp}/\varepsilon_{orb}$ is also larger than unity, is proportional to ξ, and in the former case increases with increasing number of the harmonic. It is further seen from (12) that the quantum effects become considerable not at the customarily employed inequality $\xi \gg 1$, but under the weaker condition $e^\xi \gg 1$. In this case $\tanh\xi \approx 1$, and this is precisely the approximation used by us to derive (13).

Of course, the conclusion that $\varepsilon_{sp} > \varepsilon_{orb}$ in a quantizing field is relative only if the effective electron mass m^* coincides with the mass m of the free electron. If $m^* < m$, however, then $\varepsilon_{sp}/\varepsilon_{orb}$ decreases in a ratio $(m^*/m)^2$.

The solution of (12) has a relatively simple form in the immediate vicinity of the resonant frequency $s\omega_c$ $(s = 1, 2, 3, \ldots)$. It is convenient to write it in the form

$$\Delta_s = \frac{s\omega_c - \omega}{s\omega_c} = \frac{\sinh s\xi}{s\xi} \frac{Y\exp(-X\operatorname{ch}\xi)}{2X\sinh\xi + Y}[I_s(X) + \xi\tanh\xi X I_s'(X)]. \tag{14}$$

Here $Y = 4\pi n\hbar\omega_c/B^2$.

This solution is valid under two conditions. First, the right-hand side of (14) must be much smaller than unity; second, as we see, the right-hand side of (12) diverges at $\omega = s\omega_c$. Silin and Rukhadze[8] have shown that this divergence vanishes when account is taken of the relativistic corrections, so that the solution (14) is valid only if $\Delta_s \gg v^2/c^2$. It follows therefore that the right-hand side of (14) must satisfy the double inequality

$$\frac{v^2}{c^2} \ll \frac{\sinh s\xi}{s\xi} \frac{Y \exp(-X \cosh\xi)}{2X \sinh\xi + Y} [I_s(X) + \xi \tanh\xi X I_s'(X)] \ll 1. \quad (15)$$

The dispersion relation (14) becomes much simpler in the limiting case $X \ll 1$. We note that in spite of the assumed smallness of X we obtain a result that differs significantly from the conclusions of Korneev and Starostin.[5] The reason is that we shall use the asymptotic value of the function $I_s(X)$, and not the expansion of $\exp(-X\cosh\xi)$. Since

$$I_s(X) \approx X^s/2^s s! \qquad (X \ll 1).$$

it follows that, putting $\sinh\xi \approx \cosh\xi \approx (1/2)e^\xi$ we get

$$\Delta_s = \frac{Y(1+s\xi)}{2^{s-1}s!\, s\xi} \frac{Z^s e^{-Z/2}}{Z+Y}, \qquad Z = Xe^\xi. \quad (16)$$

The maximum of this expression occurs at

$$Z = Z_s = X_s e^\xi = [{}^1/_4(Y+2-2s)^2 + 2sY]^{1/2} - {}^1/_2(Y+2-2s). \quad (17)$$

The condition $X_s \ll 1$ is easily satisfied. In the limiting case $Y \ll 1$, the condition $X_s \ll 1$ is automatically satisfied for the harmonic $s=1$, and at $s>1$ it reduces to the requirement $e^\xi \gg 2(s-1)$.

We write now the maximum value of Δ_s at $Y \ll 1$:

$$(\Delta_s)_{max} = \frac{Y}{4s\xi} e^{-(s-1)}(1+s\xi) \frac{(s-1)^{s-1}}{s!}. \quad (18)$$

As $s=1$, the last factor must be replaced by unity. In[5], this maximum could not be obtained, since the authors confined themselves there to excessively small values of X. We therefore compare our results with the classical result obtained by Akhiezer *et al.*[11] The qualitative differences are the following: a) $(\Delta_s)_{max}$ decreases exponentially with increasing number of the harmonic. b) The value of Z_s increases linearly with increasing number of the harmonic. c) Δ_s decreases exponentially with increasing Z at $Z > Z_s$.

[1]P. S. Zyryanov and V. P. Kalashnikov, Zh. Eksp. Teor. Fiz. **41**, 1119 (1961) [Sov. Phys.-JETP **14**, 799 (1962)].

[2]L. E. Gurevich and R. G. Tarkhanyan, Fiz. Tekh. Poluprovodn. **3**, 1139 (1969) [Sov. Phys. Semicond. **3**, 962 (1969)].

[3]V. Arunsalam, J. Math. Phys. **10**, 1305 (1969).

[4]V. Canuto and J. Ventura, Astrophys. Space Sci. **18**, 104 (1972).

[5]V. V. Korneev and A. N. Starostin, Zh. Eksp. Teor. Fiz. **63**, 930 (1972) [Sov. Phys.-JETP **36**, 487 (1973)].

[6]D. N. Zubarev, Neravnovesnaya statisticheskaya termodinamika (Nonequilibrium Statistical Thermodynamics), Nauka, 1971.

[7]I. S. Gradshteĭn and I. M. Ryzhik, Tablitsi integralov, summ, ryadov i proizvedeniĭ (Tables of Integrals, Sums, Series, and Products), Nauka, 1971 [Academic, 1966].

[8]V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media), Gosatomizdat, 1961. Functions A. Erdelyi, Higher Transcendental, Vol. 2, McGraw, 1954.

[9]V. L. Ginzburg and A. A. Rukhadze, Volny v magnitoaktivnoĭ plazme (Waves in Magnetoactive Plasma), Nauka, 1970.

[10]A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Elektrodinamika plazmy (Plasma Electrodynamics), Nauka, 1974.

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Effect of Penning collisions between optically oriented Rb and He atoms on electron density in plasma

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The effect of mutual spin orientation of Rb atoms and metastable (2^3S_1) He atoms on electron density in plasma, due to the dependence of the free-electron yield during Penning collisions between Rb atoms and metastable He atoms on their mutual spin orientation, has been observed experimentally and investigated.

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The effect of optical orientation of atoms on electron density in plasma was described in[1], where the discovery of the variation in the electrical conductivity of helium plasma during optical orientation of metastable helium atoms was reported and investigated. Once it was established that the total spin was conserved in ionizing (Penning) collisions between metastable helium atoms,[2] the change in the electron density under the influence of optical orientation was attributed to the dependence of the free-electron yield during Penning collisions between the metastable orthohelium atoms on their mutual spin orientation.[3–5] Conservation of total spin should cause the mutual spin orientation to affect the free-electron yield not only in the case of collisions of metastable helium atoms with one another, but also in the case of collisions between metastable helium atoms and alkali metal atoms.[6] This phenomenon is of considerable interest for the investigation of the spin dependence of Penning collisions since, in contrast to the case of collisions between metastable helium atoms with one another, it offers the possibility of independent variation of the spin orientation of the