Amplification of ion-acoustic solitons by a beam of charged particles

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The interaction of an ion-acoustic soliton with a monokinetic (one-velocity) charged particle beam is investigated. An integrodifferential equation for the soliton amplitude is obtained in the quasi-hydrodynamic approximation. An effect of transient amplification is described which depends on the initial conditions and which leads to the formation of a stable soliton in the beam-plasma system. Estimates for the effect are presented for a laboratory plasma.

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The possibility of amplifying particle-like waves—solitons—in a beam-plasma system has aroused considerable interest recently. Such processes are usually investigated for electron waves—so-called Langmuir solitons—as well as for charge-density solitons amplified by an electron beam [1-4]. In addition it is of considerable interest to investigate the beam amplification of ion-acoustic solitons, which seem to play an essential role under astrophysical conditions [5] and have recently been observed in laboratory experiments [6]. We note that the interactions of such solitons with a beam may lead to a pulsed acceleration of heavy particles (cf. [7]).

We consider the amplification of an ion-acoustic soliton by a monokinetic beam of charged particles in a non-isothermal plasma. The problem is solved in the quasi-hydrodynamic approximation, when all particles of the beam interact coherently with the wave. As a result we succeed in describing a peculiar effect of "transient" amplification related to the process of establishment of stationary excitations in the beam. The amplitude of the established soliton in the plasma-beam system turns out to be dependent on the whole prehistory of the process.

The initial system of equations has the form[1)]

$$-\frac{\partial^2\varphi}{\partial x^2}=4\pi e\left\{N_0-\frac{e\varphi}{\varkappa T}+\frac{N_0}{2}\left(\frac{e\varphi}{\varkappa T}\right)^2-\rho+\frac{e_s}{e}\rho_s\right\},$$
$$\frac{\partial v}{\partial t}-\frac{e}{M}\frac{\partial\varphi}{\partial x}=-v\frac{\partial v}{\partial x},\quad \frac{\partial\rho}{\partial t}+N_0\frac{\partial v}{\partial x}=-\frac{\partial}{\partial x}(\rho v),\tag{1}$$

$$\frac{\partial v_s}{\partial t}+\frac{e_s}{m_s}\frac{\partial\varphi}{\partial x}+V_0\frac{\partial v_s}{\partial x}=-v_s\frac{\partial v_s}{\partial x},$$
$$\frac{\partial\rho_s}{\partial t}+V_0\frac{\partial\rho_s}{\partial x}+N_s\frac{\partial v_s}{\partial x}=-\frac{\partial}{\partial x}(\rho_s v_s),\tag{2}$$

where v, v_S, ρ, ρ_S are the deviations of the velocities and ion concentrations of the plasma and beam particles, respectively, from the equilibrium values 0, V_0, N_0, N_S; φ is the potential of the electric field; e_S/m, e/M are the specific charges of the beam particles and the plasma ions; in the first equation (1) we have expanded, as usual, the distribution function $\exp(-e\varphi/\kappa T)$ in powers of φ up to the quadratic term (κ is the Boltzmann constant).

First of all we reduce the system (2) to one simplified equation describing the space-time variations of the potential[2)]. We shall assume that $N_S \ll N_0$; then the structure of the nonlinear wave is defined by the underlying plasma and the beam plays the role of an amplifying factor. In addition, in the quasihydrodynamic approximation one may neglect the nonlinearities of the perturbations of the beam (the limits of applicability of this approximation are indicated below). Since we are interested in waves propagating with a velocity close to V_0 it is convenient to change from the variables x, t to $\bar{\xi} = x - V_0 t$, $x' = x$; then the system (2) can be easily solved with respect to the beam variables

$$\rho_s(\bar{\xi},x')=-\frac{e_sN_s}{m_sV_0}\int_0^{x'}\int_0^{x_2'}\varphi_{\bar{\xi}\bar{\xi}}''\,dx_1'\,dx_2,\tag{3}$$

$$v_s=\frac{e_s}{m_sV_0}\int_0^{x'}\varphi_{\bar{\xi}}'\,dx_1.\tag{4}$$

Here it was assumed that $\rho_S(x' = 0) = v_S(x' = 0) = 0$, i.e., that at the input the beam is not modulated.

Taking (3) into account, the system (1) easily reduces to a single integro-differential equation in analogous variables

$$\varphi_{\alpha x}'+\frac{1}{2}\varphi_\alpha\varphi_{\alpha\xi}'+\frac{1}{2}\varphi_{\alpha\xi\xi\xi}'''+\gamma\int_0^x\int_0^{x_1}\varphi_{\alpha\xi\xi\xi}'''\,dx_1\,dx_2,\tag{5}$$

where we have introduced the dimensionless variables

$$\varphi_\alpha=e\varphi/\varkappa T,\ x_\alpha=x'\Omega_0/C_s,\ t_\alpha=\Omega_0 t,$$
$$C_s^2=\varkappa T/M,\ \gamma=\omega_{0s}^2/\Omega_0^2,\ \Omega_0^2=4\pi N_0e^2/M,$$
$$\omega_{0s}^2=4\pi N_se^2/m_s,\ \xi=x_\alpha-t_\alpha.$$

In the sequel we shall omit the index α.

In the linear approximation for monochromatic waves (5) leads, of course, to the well known dispersion relation for ion-acoustic waves in a beam-plasma system [9]. For $\gamma = 0$ the equation (5) coincides with the Korteveg-de Vries equation and has the particular solution of soliton form:

$$\varphi(\xi,x)=A\,\mathrm{ch}^{-2}\sqrt{\frac{A}{6}}\left(\xi-\frac{Ax}{3}\right).\tag{6}$$

Since in our case $\gamma \ll 1$, we shall look for an approximate solution for φ in the form (6), where the amplitude is a function of the slowly varying quantity x and the phase $Ax/3$ is replaced by $\int A dx/3$.

The method for the determination of an approximate equation describing the variation of the wave [10] reduces here to the simple prescription: multiply (5) by $\varphi(\xi, x)$ and integrate with respect to ξ from $-\infty$ to ∞. As a result of this we obtain the following equation for the soliton amplitude:

$$\frac{dA}{dx}=-\frac{\bar{\gamma}}{A^{1/2}}\int_{-\infty}^{\infty}\varphi(\xi,x)\int_0^x dx_1\int_0^{x_1}dx_2\,\varphi_{\xi\xi\xi}'''(\xi,x_2)\,d\xi,$$
$$\varphi=A\,\mathrm{ch}^{-2}\sqrt{\frac{A}{6}}\left(\xi+\delta x-\frac{1}{3}\int_0^x A\,dx\right),\quad \bar{\gamma}\approx 0.5\gamma,\quad \delta=\frac{V_0}{C_s}-1.\tag{7}$$

For the initial stage of the process, when the variations of the soliton amplitude are small, the solution (7) has the form

$$A(x)-A_0\approx-\bar{\gamma}0.03x^3\sqrt{A_0}(A_0/3-\delta),\quad A_0\equiv A(0).$$

Consequently, for the natural condition $\delta > A_0/3$ there occurs an amplification of the soliton which ab initio has a nonlinear character. Moreover, analyzing the right-hand side of (7), it is easy to show that the solution of (7) is a stationary soliton with arbitrary A = const (such solitons are discussed in more detail below). From these limiting cases one can see the general character of the process: a sudden amplification followed by the approach of A to a (nonuniversal) constant. However, detailed quantitative results could only be obtained by means of a computer.

Some results of the computations are shown in Figs. 1–3. Depending on the velocity of the beam the soliton is either amplified or attenuated and in both cases saturation is reached. As synchronism between the beam and the soliton is approached the amplification factor increases and within the framework of the quasihydrodynamic approximation it can attain considerable values. As the difference $\delta - A/3$ decreases further it becomes necessary to take into account kinetic effects (particle capture), and extrapolation to this region gives reason to believe that the amplification may be considerably larger.

In analyzing the results obtained here the question may arise as to why already in the linear approximation in the beam variables the amplification process stops and the soliton amplitude becomes constant. We note, first of all, that this final stage of the evolution of the initial disturbance can be described starting directly from Eqs. (1) and (2) by looking for solutions which depend on the single running coordinate $\xi_0 = x - ut$, where u is a constant velocity (cf. [11]). This solution satisfies the equation

$$\varphi''_{\xi_0\xi_0}+\varphi+{}^1/_2\varphi^2-\rho_\delta-\beta\rho_{\delta_s}=0,\qquad \rho_\delta=\frac{u_\delta-(u_\delta^2+2\varphi)^{1/2}}{(u_\delta^2+2\varphi)^{1/2}},\tag{8}$$

$$=\frac{\varphi}{((\delta^2+\varphi)^{1/2}+|\delta|)(\delta^2+\varphi)^{1/2}},\tag{9}$$

where

$$\rho_\delta=\rho/N_0,\ \rho_{\delta_s}=\rho_s/N_s,$$
$$u_\delta=u/C_s,\ \delta=u/V_0-1,$$
$$\beta=N_s/N_0\ll1$$

(for simplicity we have assumed that $e_S/m_S = |e|/M$).

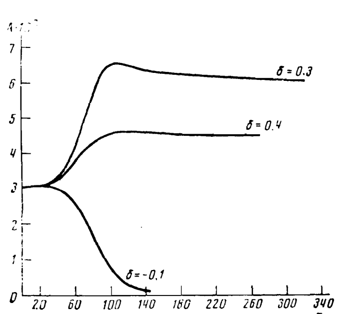

FIG. 1

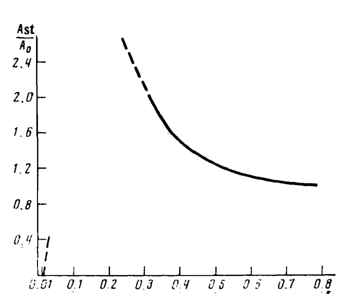

FIG. 2

FIG. 1. The dependence of the soliton amplitude on the coordinate x for different synchronism parameters δ; $\bar{\gamma}=0.01$.

FIG. 2. The dependence of the amplification coefficient A_{st}/A_0 (A_{st} is the stationary value of the amplitude, $A_0=0.03$ is the initial soliton amplitude) on the synchronism parameter δ; $\bar{\gamma}=0.01$.

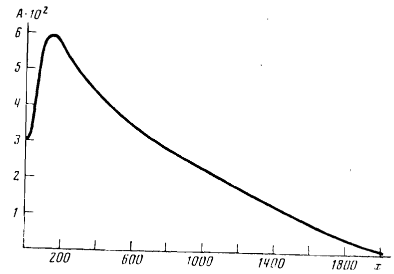

FIG. 3. The variation of the soliton amplitude A as a function of the coordinate x in the presence of damping (an additional term $-\nu A$ in (7)); the damping coefficient $\nu=10^{-3}$; $\bar{\gamma}=0.01$; $\delta=0.3$.

The equation (8) has a solution in the form of an isolated wave—a soliton—which for small φ coincides with (6), and according to (9) yields for ρ_S small values, as indicated above. Consequently there exist nontransient pulses with constant amplitude. If however the initial disturbance of the beam does not correspond to the stationary solution (9), there will be energy exchange between the beam and the potential wave, leading to amplification or attenuation of the wave.

In the linear approximation the excitations of the beam can be classified into natural excitations, traveling with velocity V_0, and forced excitations, corresponding to (9). Over a time of the order $A^{-1/2}(\delta - A/3)$ the natural wave travels out of the region occupied by the pulse and there remains the stationary wave (9), the form of which does not change. Thus, the amplification has a transient character and lasts for a limited time. In fact, the amplification occurs on account of a linear mechanism, the nonlinearity being due only to the character of the amplified pulses (the dependence of their duration on the amplitude). It is for this reason that the asymptotic value of the soliton amplitude is not a universal quantity (as it is in the self-oscillation regime) but depends on the prehistory of the process. If one takes damping into account the asymptotic value of the soliton amplitude will always be zero, however over finite intervals a considerable amplification may be maintained.

In conclusion we estimate the magnitude of the amplification for a plasma system with the following parameters: $N_0 \sim 10^8\ \mathrm{cm}^{-3}$; $N_S/N_0 \sim 2\times10^{-2}$; $T \sim 10^4$ K, $|e|/M = |e_S|/m_S$ (ion beam), $V_0 \sim 1.3C_S$ ($\delta \sim 0.3$), $C_S \sim 10^6$ cm/s; then for an initial amplitude of the soliton of $\sim 3\times10^{-2}$ V there will occur an amplification over a distance of ~10 cm to an amplitude $\sim 6.6\times10^{-2}$ V. At the same time the width of the soliton decreases from 1 to 0.7 cm. For these parameters the beam current is ~1 μA/cm² and the power ~1 μW/cm².

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1)We assume that $T \gg T_i$ (T, T_i are the temperatures of the electrons and ions); in the sequel we assume that $T_i \simeq 0$. In this approximation the particles captured by wave are not taken into account, which is valid if $m_S(V_0-u)^2/2 > e_S\varphi$, where u is the velocity of the soliton (cf. [8]). One may neglect the thermal spread in the beam if $(N_S/N_0)^{1/3}(V_0/v_{T_S}) \gg 1$ (where v_{T_S} is the thermal velocity of the beam particles).

2)The Fourier method which was used with success in the analysis of amplification of Langmuir solitons [1] is not effective in this case owing to the nonresonant character of the process, as well as due to the smallness of the dispersion (in the stationary "videosoliton" considered here all Fourier harmonics propagate with the same velocity).

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