Deformation of plasma resonance region in a strong high-frequency field

V. B. Gil'denburg and G. M. Fraĭman

Scientific-Research Radiophysics Institute, Gorkiĭ
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Stationary deformation of the plasma resonance region by ponderomotive forces due to a longitudinal high-frequency field is calculated. It is shown that, as the field amplitude increases, there is a reduction in the energy transformed into the runaway longitudinal wave.

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1. Many physical processes in an inhomogeneous plasma interacting with an alternating electromagnetic field, for example, linear transformation and absorption of waves, parametric instability, and so on, depend on the structure of the plasma resonance region ($\omega_p \approx \omega$) and, therefore, are very sensitive to the nonlinear deformation of this region. The importance of this deformation and the associated nonlinear effects are enhanced by the presence in the resonance region of a sharp maximum of the component of the electric field $\mathbf{E}\exp(i\omega t)$ parallel to the plasma density gradient.[1] Studies of these effects have recently assumed increased importance in connection with the transfer of laser or microwave energy to thermonuclear plasma.

When the amplitudes are not too large, the deformation occurs in a very narrow region (see below), which means that it can be described in a universal fashion by a single one-dimensional model with a potential longitudinal field, and fields outside the resonance region can be ignored. The amplitude of the induction D in the neighborhood of the resonance can, in this model, be regarded as a given constant which, in each specific case, is determined by the parameters of the external field and the structure of the plasma as a whole. So long as the deformations of the density profile are localized in a narrow resonance region, the quantity D (or, at least, its approximate value) can be obtained from the solution of the problem in the linear approximation and without taking into account density perturbations. A solution of this kind is available, for example, for the important case of oblique incidence of a plane wave on an inhomogeneous plasma, with the electric vector $\mathbf{E}$ confined to the plane of incidence.[1]

2. Consider the stationary[1)] strictional deformation of collisionless plasma in which the unperturbed density profile $N_0(x)$ is linear. Suppose that this plasma is placed in a field with given electric induction vector $\mathbf{D} = \mathbf{x}_0 D\exp(i\omega t)$. Assuming that spatial dispersion is weak and the relative density perturbations (but not the density gradient perturbations) are small, we obtain the following equations for the self-consistent stationary state in the region $|\epsilon| \ll 1$:

$$\varepsilon = 1 - N/N_c = \varepsilon_0 + \alpha|E|^2, \qquad (1)$$

$$\delta^2 d^2E/dx^2 + (-x/l + \alpha|E|^2)E = D. \qquad (2)$$

In these expressions, $\epsilon(x)$ is the permittivity, $E(x)$ is the amplitude of the electric field, $\epsilon_0 = 1 - N_0/N_c = -x/l$, $N_c = m\omega^2/4\pi e^2$ is the critical concentration, $\delta^2 = 3T_e/m\omega^2$ is the spatial dispersion parameter, $\alpha = e^2[4m\omega^2(T_e + T_i)]^{-1}$ is the strictional nonlinearity parameter, T_e, T_i are the electron and ion temperatures, respectively, and it is assumed that $\delta \ll l$, $\alpha d^2 \ll 1$. Equation (2) is the quasihydrodynamic approximation to the material equation for a plasma with stationary density perturbations due to the average ponderomotive force. This equation was previously given in[6] for the case $\epsilon_0 = \text{const}$.

Since we are interested in states in which the density perturbations are localized mainly in a bounded region, it is natural to take for the boundary conditions for (2) the same radiation ($x \to -\infty$) and boundedness ($x \to +\infty$) conditions for the WKB asymptotic behavior of the plasma wave $E_p = E - D/\epsilon_0$ as in the linear case ($\alpha = 0$):[7]

$$\left(\delta\frac{d}{dx} - i\sqrt{\varepsilon_0}\right)E_p = O\left(\frac{1}{x^2}\right), \quad x \to -\infty, \qquad (3)$$

$$\left(\delta\frac{d}{dx} + \sqrt{|\varepsilon_0|}\right)E_p = O\left(\frac{1}{x^2}\right), \quad x \to +\infty. \qquad (4)$$

Condition (3) means that, in transparent plasma, the field at large distances from the resonance point is the superposition of the "cold" solution D/ϵ_0 and the plasma wave E_p traveling in the direction of decreasing density N. The wave traveling in the opposite direction is absent because there should be strong Landau damping on the plasma periphery ($\epsilon_0 \sim 1$), where the plasma wavelength $\lambda_p \sim \delta/\sqrt{\epsilon_0}$ is of the order of the Debye length. Condition (4) demands that, in the nontransparent region, the plasma wave field should decrease exponentially.

In the absence of spatial dispersion ($\delta = 0$), the two functions $E(x)$ and $\epsilon(x)$ are shown in[8] to be nonsingle-valued and discontinuous for $\epsilon_0(x) < \epsilon_c = -3(\alpha D^2/4)^{1/3}$ (see broken curve in Fig. 2). When $\delta \neq 0$, the solution is continuous and, instead of the coordinate of the point of discontinuity, we can use the coordinate $\tilde{x}$ which characterizes the qualitative change in structure: when $x < \tilde{x}$, the real and imaginary parts of the complex amplitude $E(x)$ are oscillatory, and when $x > \tilde{x}$, they decrease monotonically. The problem of the number of solutions is very difficult. All that can be said with certainty is that the set of solutions is discrete. This follows from the uniqueness of the solution of the linear problem ($\alpha = 0$) and the analyticity of the solution of the Cauchy problem for (2) as a function of the initial conditions and the parameter α. We shall define the "ground state" as the solution $E(x)$, $\epsilon(x)$ which uniformly and continuously transforms to the linear solution in all space as $\alpha \to 0$. It appears that, in addition to this ground state, there are also higher states characterized by large values of $\tilde{x}$, i.e., deeper deformations of the post-critical region.

In terms of the dimensionless variables $\mathscr{E} = (E/D)(\delta/l)^{2/3}$, $z = x(\delta^2 l)^{-1/3}$, Eq. (2) assumes the form

$$d^2\mathscr{E}/dz^2+(-z+\eta|\mathscr{E}|^2)\mathscr{E}=1, \quad \eta=\alpha D^2l^2/\delta^2, \tag{5}$$

from which it is clear that the character of the solution is determined by the single nonlinearity parameter η. When $\eta \ll 1$, the ground-state solution is always close to the linear solution and can be found by perturbation methods. When $\eta \gg 1$, the characteristic scale of oscillations in the field and density for $x \lesssim \tilde{x}$ is $\Lambda = \delta|\epsilon_c|^{-1/2} \sim \delta(\alpha D)^{-1/6}$, i.e., it is lower by a factor of $\eta^{1/6}$ as compared with the corresponding scale in the linear problem, $\Lambda_0 = (\delta^2 l)^{1/3}$.

3. The structure of the ground state was calculated on a computer and the results are shown in Figs. 1—4. Figure 1 shows the permittivity $\epsilon(z)$ for different values of η. Noticeable deformations of $\epsilon(z)$ appear only for $\eta \gtrsim 1$. As η increases, the passage of $\epsilon(x)$ through zero becomes increasingly steeper, the position of the zero approaches the point $x_c = 3l(\alpha D^2/4)^{1/3}$ $[\epsilon_0(x_c) = \epsilon_c]$, and the solution becomes multivalued when $\delta = 0$. This can be illustrated by comparing the solutions for $\delta = 0$ (broken curve) and $\delta \neq 0$, $\eta = 20$ (solid curve) in Fig. 2. The sharp reduction in the ϵ profile in the region of transparency is due to interference between the "cold" solution D/ϵ_0 and the traveling plasma wave. As an illustration, Fig. 3 shows the distributions of the real and imaginary parts of the complex field amplitude for $\eta = 20$ and $\eta = 0$. By analyzing these distributions, one can readily establish the dependence of the amplitude of the runaway plasma wave and the energy flux S carried by it on the parameter η. In the linear approximation $(\eta \ll 1)$,[7]

$$S=S_0={}^1/_8\omega lD^2. \tag{6}$$

When $\eta > 2$, numerical calculations (see Fig. 4) show that, to a good approximation,

$$S/S_0=0.63/\sqrt{\eta}, \tag{7}$$

and hence $S = 0.79\omega\delta D/\sqrt{\alpha}$, i.e., the efficiency of transformation of energy into the plasma wave decreases with increasing η and ceases to depend on l.

We note that the result $S \sim 1/\sqrt{\eta}$ can be obtained directly from simple model representations by considering the generation of a plasma wave across the discontinuity of the "cold" ($\delta = 0$) distribution $\epsilon(x)$ at the point $x = x_c = |\epsilon_c|l$. This distribution (dashed curve in Fig. 2) gives a qualitative approximation to the actual distribution $\epsilon(x)$ averaged over the scale of fine oscillations. The reduction in the amplitude of the plasma wave excited across the discontinuity, which occurs as η increases, is then simply a consequence of the increase in $|\epsilon|$ on either side of the discontinuity and the associated reduction in the gradient of the average "cold" field D/ϵ. In point of fact, if we write the expressions for the field in the neighborhood of the discontinuity in the form

$$\begin{aligned} &E=D/\varepsilon_++A\exp[i\varepsilon_+^{1/2}(x-x_c)/l], \quad x<x_c \\ &E=D/\varepsilon_-+B\exp[-|\varepsilon_-|^{1/2}(x-x_c)/l], \quad x>x_c \end{aligned} \tag{8}$$

where $\epsilon_+ = |\epsilon_c|/3$, $\epsilon_- = {}^2/_3\epsilon_c$ on either side of the discontinuity,[8] and match up the values of E and dE/dx at $x = x_c$, we obtain the following expressions for the amplitude A of the runaway plasma wave and the energy flux $S = \omega\delta\epsilon_+^{1/2}|A|^2/8\pi$ carried by it:

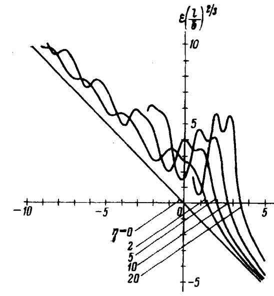

FIG. 1

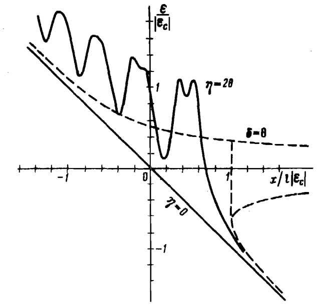

FIG. 2

FIG. 1. Stationary profiles of plasma permittivity $\epsilon(x) = 1 - N(x)/N_c$ for different values of the nonlinearity parameter η (ground state solution).

FIG. 2. Comparison of the ground state solution in the case of strong nonlinearity ($\eta = 20$) with the solution obtained without taking into account the excitation of the plasma wave ($\delta = 0$, broken curve).

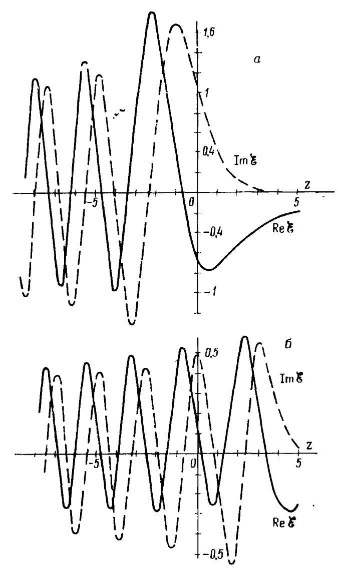

FIG. 3

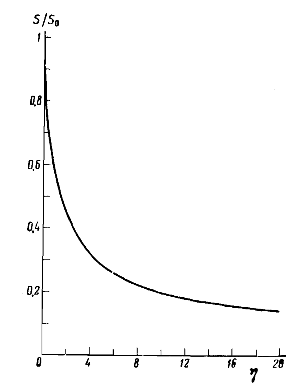

FIG. 4

FIG. 3. Distribution of the real and imaginary parts of the field amplitude $\mathscr{E}$: a—in the absence of nonlinearity ($\eta = 0$), b—when $\eta = 20$.

FIG. 4. Power transferred to the runaway plasma wave as a function of the nonlinearity parameter η.

$S=\omega\delta\varepsilon_+^{1/2}|A|^2/8\pi$:

$$A=\frac{D}{\varepsilon_+}\left(1-i\left(\frac{\varepsilon_+}{|\varepsilon_-|}\right)^{1/2}\right)=3\frac{D}{|\varepsilon_c|}\left(1-\frac{i}{2^{1/2}}\right), \tag{9}$$

$$S=\frac{\omega\delta D^2}{8\pi\varepsilon_+^{3/2}}\left(1+\frac{\varepsilon_+}{|\varepsilon_-|}\right)=\frac{3}{\pi}\frac{S_0}{\eta^{1/2}}. \tag{10}$$

4. The function $S(\eta)$ found above can be used to determine the relative reduction in the transformation coefficient R in the case of oblique incidence of a transverse wave with electric vector parallel to the plane of incidence: $R/R_0 \approx S/S_0 \sim \eta^{-1/2}$. The induction D on which η depends can be expressed in terms of the amplitude E_0 of the incident wave in vacuum and, in particular, for $kl \gg 1$ ($k = \omega/c$) and the optimum of incidence $\theta \approx (kl)^{-1/3}$ (when $R_0 \approx 0.5$), we have $D \approx E_0/(2\pi kl)^{1/2}$.[1] Thus, linear transformation (excitation of runaway plasma wave) is a low-efficiency process[2)] from the point of view of the transfer of the energy of external radiation to plasma at high energy densities. It is probably inferior to other loss mecha-

nisms which we have not considered here and which are due to electron collisions or collisionless damping directly in the region of strong nonlinearity (where $\epsilon \sim |\epsilon_c|$).

In conclusion, we summarize the arguments which suggest that the above stationary state is stable (at least within the framework of the one-dimensional model) and that the initial unperturbed structure should eventually approach this state. Thus, firstly, stability is favored by the possibility of free transport of the energy associated with the perturbations out of the resonance region and, secondly, small-scale modulation of the density profile N(x) can be regarded as a stabilizing factor, since it is precisely this modulation that is capable of limiting the development of parametric instability in the initially homogeneous or weakly inhomogeneous plasma.[10] Finally, out of all the possible stationary states, the ground state should exhibit "maximum stability", since it requires the smallest expenditure of energy for its establishment, and is characterized by the minimum values of stored and dissipated energy.

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[1]Since all the possible stationary states of the system must be investigated first, we shall ignore such important questions as the establishment of these states and their stability. The initial stages of the establishment process have recently been investigated experimentally [2,3] and numerically on a computer. [4] The regular stationary model which we shall consider can be looked upon as an alternative to models based on the idea of stationary turbulence. [5]

[2]These results are very different from some of the qualitative predictions in [9] which suggest that the coordinate of the transition through plasma resonance is continuously displaced into the interior of the plasma (well beyond the point x_c), and the transformation coefficient R increases continuously (up to values $R \sim 1$). In our opinion, the initial assumptions concerning the dynamics of the transition region, which are adopted in [9], cannot be regarded as justified.

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