Dipole forces and critical dynamics of anisotropic ferromagnets

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(Submitted May 15, 1975)
Zh. Eksp. Teor. Fiz. 69, 1398–1414 (October 1975)

The influence of dipole forces on the critical dynamics of uniaxial ferromagnets is considered. Like anisotropy, dipole forces suppress the critical fluctuations. Two variants are discussed—weak anisotropy, when, as the Curie point is approached, the suppression first appears as a result of the dipole forces and only after this do the anisotropy forces become operative, and the opposite case, when the anisotropy begins to act first. Expressions are obtained for the characteristic energy of the critical fluctuations and the inverse uniform-relaxation time; these depend in a complicated manner on the energy of the dipole interaction, the anisotropy energy and $\tau=(T-T_c)T_c^{-1}$, the dependences being different in different cases (easy and hard axis) and in different temperature regions. However, in the limit $\tau\to 0$ the temperature dependence of the uniform-relaxation time in the easy direction is the same as for the corresponding susceptibility. This contradicts the recent experiments of Kamleiter and Kötzler with $GdCl_3$. In connection with this it is shown that a weak magnetic field that does not affect the static susceptibility can greatly decrease the uniform-relaxation time in the easy-axis case, owing to the fact that it violates the selection rule forbidding the relaxation of a longitudinal fluctuation with creation of two similar longitudinal fluctuations growing without limit as $\tau\to 0$.

PACS numbers: 75.30.Gn, 75.30.Jy

1. INTRODUCTION

The magnetic-dipole interaction strongly influences the properties of ferromagnets in the paramagnetic phase ($T > T_c$) near the Curie point. If $4\pi\chi < 1$ (χ is the static susceptibility), these forces can be taken into account by perturbation theory and turn out to be important only for the description of phenomena associated with violation of the conservation law for the total spin, e.g., the relaxation of the uniform magnetization and the absorption of long-wave electromagnetic oscillations. But if $4\pi\chi > 1$, then, as Krivoglaz[1] has shown, because of their long range the dipole forces substantially alter the properties of the tensor of the correlations of the magnetization components, and this leads to a change in the critical properties of the ferromagnet. Thus, according to Aharony and Fisher[2], in cubic ferromagnets in this region the values of the critical indices of the static-scaling theory are slightly changed, and, as shown in[3], the dynamics of the long-wave critical fluctuations is completely changed.

The present paper is devoted to an analysis of the dynamics of critical fluctuations in uniaxial ferromagnets. It is found that their dynamical properties depend in an extremely complicated manner on the character of the anisotropy ("easy-axis" or "easy-plane") and on the relative size of the anisotropy energy and the dipole energy (more precisely, on whether the quantity $4\pi\chi_m$ is greater or less than unity, where χ_m is the maximum value of the static susceptibility in the "hard" direction). In the limit $\tau\to 0$ ($\tau=(T-T_c)T_c^{-1}$), the critical damping of the fluctuations of the uniform magnetization in the "easy" direction (this magnetization is the order parameter in our case) has a normal character, i.e., its temperature dependence is the same as that of the inverse susceptibility corresponding to this direction[1)]. This agrees with the well-known prediction of Riedel and Wegner[5]. Recently, the critical properties of the uniaxial ferromagnet $GdCl_3$ have been studied experimentally by Kötzler and co-workers[6,7]. This is a ferromagnet in which the dipole forces are of the same order as the exchange forces ($T_c = 2.2$ K). It was found[6] that the static properties of $GdCl_3$ agree well with the predictions of the logarithmic theory of Larkin and Khmel'nitskiĭ[8], while the critical damping behaves very strangely[7]. In the region of comparatively large τ its temperature dependence is normal and then, with decrease of τ, becomes anomalous. The nature of this phenomenon is studied in the last Section of this paper, where it is shown that a weak magnetic field that does not affect the static susceptibility can lead to anomalous behavior of the critical damping, owing to the fact that it violates the selection rule forbidding the relaxation of a longitudinal fluctuation with creation of two similar longitudinal fluctuations growing without limit as $\tau\to 0$.

2. COMBINED DESCRIPTION OF THE DIPOLE FORCES AND ANISOTROPY

As is well-known, because of the long range of the dipole forces the internal magnetic field $\mathbf{H}_{i\mathbf{k}}$ in a sample does not coincide with the external field $\mathbf{H}_\mathbf{k}$:

$$\mathbf{H}_{i\mathbf{k}}=\mathbf{H}_\mathbf{k}-4\pi(\mathbf{n}\mathbf{M}_\mathbf{k})\mathbf{n},\quad \mathbf{n}=\mathbf{k}k^{-1}, \tag{1}$$

where k is the wave vector and $\mathbf{M}_\mathbf{k}$ is the corresponding Fourier component of the magnetization. Therefore, one distinguishes the magnetic susceptibilities of the body ($\tilde{\chi}$) and of the substance (χ), which are defined by the equalities

$$M_\mathbf{k}^\alpha(\omega)=\tilde{\chi}_{\alpha\beta}(\mathbf{k},\omega)H_\mathbf{k}^\beta(\omega)=\chi_{\alpha\beta}(\mathbf{k},\omega)H_{i\mathbf{k}}^\beta(\omega). \tag{2}$$

By virtue of the equality (1), these two susceptibilities are connected by the relation[2]

$$\tilde{\chi}_{\alpha\beta}(\mathbf{k},\omega)=\chi_{\alpha\beta}(\mathbf{k},\omega)-4\pi\chi_{\alpha\mu}(\mathbf{k},\omega)n_\mu n_\nu\tilde{\chi}_{\nu\beta}(\mathbf{k},\omega). \tag{3}$$

In the limit $k\to 0$ the tensor $n_\mu n_\nu$ goes over into the tensor of the demagnetizing factors $N_{\mu\nu}$ ($N_{\mu\mu}=1$). On the other hand, the energy of interaction of the magnet with an external nonuniform field is described by the formula

$$H'=-g\mu\sum_l \mathbf{S}_l\mathbf{H}(\mathbf{R}_l,t), \tag{4}$$

where $\mathbf{S}_l$ are the operators of the atomic spins and $\mathbf{R}_l$ are the coordinates of the l-th atom' by virtue of this,

$$\tilde{\chi}_{\alpha\beta}(\mathbf{k},\omega)=\frac{\omega_0}{4\pi}G_{\alpha\beta}(\mathbf{k},\omega),$$

$$G_{\alpha\beta}(\mathbf{k},\omega)=i\int_0^\infty dt\,e^{i\omega t}\langle[S_\mathbf{k}^\alpha(t),S_{-\mathbf{k}}^\beta(0)]\rangle,\tag{5}$$

$$\mathbf{S}_\mathbf{k}=N^{-1/2}\sum_l e^{i\mathbf{k}\mathbf{R}_l}\mathbf{S}_l,\quad \omega_0=4\pi(g\mu)^2v_0^{-1},$$

where v_0 is the volume of the unit cell and $G_{\alpha\beta}$ is the retarded spin Green function; according to (3), in the limit $\mathbf{k}\to 0$ the function $G_{\alpha\beta}$ depends on the direction of the vector $\mathbf{k}$. We consider a uniaxial ferromagnet with anisotropy along the z-axis. Its Hamiltonian has the form

$$H=H_e+H_A+H_d,$$

$$H_e=-\frac{1}{2}\sum_{ll'}V_{ll'}\mathbf{S}_l\mathbf{S}_{l'}=-\frac{1}{2}\sum_\mathbf{k}V_\mathbf{k}\mathbf{S}_\mathbf{k}\mathbf{S}_{-\mathbf{k}},$$

$$H_A=-\frac{1}{2}\sum_{ll'}(V_{\parallel ll'}-2\delta_{ll'}\Delta)S_l^zS_{l'}^z=-\frac{1}{2}\sum_\mathbf{k}A_\mathbf{k}S_\mathbf{k}^zS_{-\mathbf{k}}^z,$$

$$H_d=\frac{1}{2}(g\mu)^2\sum_{ll'}[\mathbf{S}_l\mathbf{S}_{l'}R_{ll'}^2-3(\mathbf{R}_{ll'}\mathbf{S}_l)(\mathbf{R}_{ll'}\mathbf{S}_{l'})]R_{ll'}^{-5}\tag{6}$$

$$=\frac{\omega_0}{2}\sum_\mathbf{k}S_\mathbf{k}^\alpha S_{-\mathbf{k}}^\beta\left(n_\alpha n_\beta-\frac{1}{3}\delta_{\alpha\beta}\right).$$

Here H_e is the isotropic exchange Hamiltonian, H_A takes into account the exchange anisotropy and the single-ion anisotropy, and H_d is the dipole energy. As is well-known[9], when we transform to the Fourier representation in H_d an additional contribution to the anisotropy energy arises from the short distances; we have included this in H_A.

By making use of the diagram technique of Vaks, Larkin and Pikin[10], we can write

$$G_{\alpha\beta}=\Sigma_{\alpha\beta}+\Sigma_{\alpha\mu}U_{\mu\nu}G_{\nu\beta}-\omega_0\Sigma_{\alpha\mu}n_\mu n_\nu G_{\nu\beta},\tag{7}$$

$$U_{\mu\nu}=(V_\mathbf{k}+\omega_0/3)\delta_{\mu\nu}+A_\mathbf{k}z_\mu z_\nu,$$

where $\mathbf{z}$ is the unit vector along the z-axis and Σ is the irreducible part, which cannot be separated into two by cutting only one interaction line. It follows from the relations (7) that

$$G_{\alpha\beta}=G_{0\alpha\beta}-\omega_0G_{0\alpha\mu}n_\mu n_\nu G_{\nu\beta},\tag{8}$$

$$G_{0\alpha\beta}=\Sigma_{\alpha\beta}+\Sigma_{\alpha\mu}U_{\mu\nu}G_{0\nu\beta}.$$

Comparing the first of these equalities with (3), we find that $\omega_0G_0=4\pi\chi$. The diagrams for Σ consist of single-cell blocks joined by more than one interaction line; therefore, the corresponding integrals converge well in the limit $\mathbf{k}=0$, and Σ for $\mathbf{k}\to 0$ does not depend on the direction of the vector $\mathbf{k}$. The same is true for G_0 and χ. Therefore, for sufficiently small $\mathbf{k}$ the irreducible part Σ can be decomposed into parts parallel and perpendicular to the z-axis[2)]. As a result we obtain

$$G_{0\alpha\beta}=G_\parallel z_\alpha z_\beta+G_\perp(\delta_{\alpha\beta}-z_\alpha z_\beta),\tag{9}$$

$$G_\parallel=\Sigma_\parallel(1-U_\parallel\Sigma_\parallel)^{-1},\quad G_\perp=\Sigma_\perp(1-U_\perp\Sigma_\perp)^{-1}.$$

If $G_\parallel$ grows without limit as $T\to T_c$ the z-axis is the easy-magnetization axis, while if $G_\perp$ grows the z-axis is the hard axis. In the region of large τ both functions $G_\parallel$ and $G_\perp$ are growing, and then the growth of one of them ceases. Clearly, two cases are possible: 1) small anisotropy: $\omega_0G_m=4\pi\chi_m\gg 1$; 2) large anisotropy: $\omega_0G_m=4\pi\chi_m\ll 1$, where G_m and χ_m are the maximum values of the corresponding functions in the hard direction. Below, these cases are considered separately. The entire critical region is divided into several regions with different physics, and these must be considered separately. This division into regions is represented in the following scheme, in which the relative sizes of the susceptibilities in the easy-plane case are indicated in brackets:

1. Small anisotropy

Anisotropic dipolar region, AD_1	Dipolar region without anisotropy, D_1	Isotropic exchange region, I_1
$0<\tau<\tau_{DA_1}$	$\tau_{DA_1}<\tau<\tau_D$	$\tau>\tau_D$
$4\pi\chi_\parallel>4\pi\chi_\perp>1$	$\chi_\perp\approx\chi_\parallel\approx\chi$	$\chi_\parallel\approx\chi_\perp=\chi$
$(4\pi\chi_\perp>4\pi\chi_\parallel>1)$	$4\pi\chi>1$	$4\pi\chi\ll 1$

2. Large anisotropy

Anisotropic dipolar region, AD_2	Anisotropic region without dipole forces, A_2	Isotropic exchange region, I_2
$0<\tau<\tau_{DA_2}$	$\tau_{DA_2}<\tau<\tau_{AI}$	$\tau>\tau_{AI}$
$4\pi\chi_\parallel>1;\ 4\pi\chi_\perp<1$	$1>4\pi\chi_\parallel>4\pi\chi_\perp$	$\chi_\perp\approx\chi_\parallel\approx\chi$
$(4\pi\chi_\perp>1;\ 4\pi\chi_\parallel<1)$	$(1>4\pi\chi_\perp>4\pi\chi_\parallel)$	$4\pi\chi\ll 1$

We turn now to the analysis of the tensor $G_{\alpha\beta}$. The first of Eqs. (8) is easily solved and we have

$$G_{\alpha\beta}=G_{0\alpha\beta}-\frac{\omega_0G_{0\alpha\mu}n_\mu n_\nu G_{0\nu\beta}}{1+\omega_0(n_\varphi G_{0\varphi\sigma}n_\sigma)},$$

$$G_{zz}=(1+\omega_0G_\perp n_\perp^2)G_\parallel[1+\omega_0(G_\perp n_\perp^2+G_\parallel n_z^2)]^{-1},$$

$$G_{xx(yy)}=G_\perp[1+\omega_0(G_\perp n_{y(x)}^2+G_\parallel n_z^2)][1+\omega_0(G_\perp n_\perp^2+G_\parallel n_z^2)]^{-1},\tag{10}$$

$$G_{xy}=G_{yx}=-\omega_0G_\perp^2n_xn_y[1+\omega_0(G_\perp n_\perp^2+G_\parallel n_z^2)]^{-1},$$

$$G_{x(y),z}=G_{z,x(y)}=-\omega_0G_\perp G_\parallel n_{x(y)}n_z[1+\omega_0(G_\perp n_\perp^2+G_\parallel n_z^2)]^{-1}.$$

It can be seen from these formulas that, in the easy-axis case, only the fluctuations along the z-axis having momentum in the xy-plane increase without limit[1]; this was used in[8]. In the easy-plane case the fluctuations in the xy-plane grow without limit, but if the momentum is directed, e.g., along the x-axis, the growth of a fluctuation in this direction is limited. However, this limitation is not so strong as in the easy-axis case; the situation does not become logarithmic, and static scaling theory, with slightly changed values of the critical indices, should hold.

We now formulate the principal properties of the static Green functions, which we shall use subsequently. Following Riedel and Wegner[11], we shall assume that there is a scaling property with respect to the anisotropy:

$$G_{\parallel,\perp}(\mathbf{k},\tau,A)=\tau^{-\gamma}\tilde{g}_{\parallel,\perp}\left(\frac{\mathbf{k}}{\varkappa},\frac{A}{T\tau^\varphi}\right)=(\varkappa a)^{-2+\eta}g_{\parallel,\perp}\left(\frac{\mathbf{k}}{\varkappa},\frac{AT_c^{-1}}{(\varkappa a)^{\varphi/\nu}}\right),\tag{11}$$

where $\kappa=\tau^\nu a^{-1}$ is the characteristic momentum of the critical fluctuations, a is a quantity of the order of the lattice constant, $g_{\parallel,\perp}(0,0\sim T_c^{-1}$, $A\ll T_c$ and φ is the critical index for the anisotropy. According to[12], $\varphi\approx 1.25$, i.e., it is smaller than the susceptibility index γ by approximately 0.1. If $T_c\tau^\varphi>A$ the anisotropy can be neglected; otherwise it is important. This means that for $\tau<\tau_A(AT_c^{-1})^{1/\varphi}$ the growth of the critical fluctuations in that hard direction ceases; in this case, the corresponding Green function is equal to

$$G_{\parallel,\perp}^{(\text{тр})}=(\varkappa_0a)^{-2+\eta}g_{\parallel,\perp}^{(\text{тр})}\left(\frac{\mathbf{k}}{\varkappa_0}\right),\quad \varkappa_0=a^{-1}\left(\frac{A}{T_c}\right)^{\nu/\varphi}.\tag{12}$$

In the regions I_1 and D_1 (see the above scheme) the anisotropy is negligibly small, and on going from I_1 to D_1 the critical indices change only slightly[2]. The crossover region is determined by the condition $G_{D_1}(0,0)\approx G_{I_1}(0,0)$. If for these functions we use the Ornstein-Zernike formula (which is legitimate, inasmuch as the Fisher parameter η is very small—below we neglect it), at the boundary we obtain

$$Z_{D_1}(\varkappa_{D_1}^2+k^2)^{-1}=Z_{I_1}(\varkappa_{I_1}^2+k^2)^{-1}.\tag{13}$$

From the equality of these expressions at large k it follows that $Z_{D_1} = Z_{I_1} = Z \sim (T_c^2 a)^{-1}$ (the same comparison shows that Z is also unchanged in the crossover between other regions). Furthermore, from (13) for k = 0 it follows that $\kappa_{D_1} = \kappa_{I_1}$, and therefore the value of the constant a in the definition of κ changes slightly in the crossover between regions: $a_{D_1} = a_{I_1}\tau_{D_1}^{(\nu_{D_1} - \nu_{I_1})}$.

In the easy-plane case the crossover from D_1 to AD_1 should be characterized by a new anisotropy index φ, different from that calculated in[12]3). However, inasmuch as the dipole forces only weakly alter the values of the static indices[2], it may be supposed that the new value of φ is close to that obtained in[12]. In this case, as before, we shall describe the critical fluctuations in AD_1 by the Ornstein-Zernike formula with the quantity κ replaced by κ_0 in the expression for $G_\parallel$. In the easy-axis case in AD_1 the situation becomes logarithmic for the longitudinal fluctuations[8]. In this case we shall assume that in the xy-plane the fluctuations are described by the formula

$$G_\perp(\mathbf{k}) = Z(k^2+\kappa_0^2)^{-1}, \quad \kappa_0 = a^{-1}(A/T_c)^{\nu/\varphi}, \tag{14}$$

where the quantity κ_0 is, effectively, a parameter of the theory. In this case, by virtue of (10) and (14), for G_{zz} we have

$$G_{zz} = \frac{Z}{k^2+\kappa^2+q_0^2(k^2+\kappa_0^2)n_z^2/(k^2+\kappa_0^2+q_0^2 n_\perp^2)} \approx \frac{Z}{k^2+\kappa^2+\kappa_0^2 n_z^2}, \quad q_0 = (\omega_0 Z)^{1/2} \sim a^{-1}(\omega_0 T_c^{-1})^{1/2}. \tag{15}$$

Here q_0 is the dipolar momentum introduced in[3]; the approximate equality in the right-hand side holds for $k \ll \kappa_0$ and $n_z^2 \ll 1$ (we recall that in the region AD_1 we have $\kappa^2 \ll \kappa_0^2 \ll q_0^2$). It follows from (15) that the characteristic momentum in AD_1 is κ_0; it is precisely this quantity which limits the range of integration in the logarithmic integrals of the perturbation-theory series[8]. In our case the interaction of the critical fluctuations is not small (of the order of T_c). Therefore, for the logarithmic theory to be valid it is necessary that the effective charge be small[8]. According to[8], the scattering amplitude for the fluctuations has the form4)

$$\Gamma(k^2) = \gamma\left(1+3\gamma\ln\frac{\kappa_0^2}{k^2+\kappa^2}\right)^{-1} \approx \left(3\ln\frac{\kappa_0^2}{k^2+\kappa_0^2}\right)^{-1}, \tag{16}$$

where $\gamma \sim (\kappa_0 a)^{-1} \gg 1$. The effective charge is $\Gamma(0)$ and, if $\kappa^2 \ll \kappa_0^2$, is obviously small. Furthermore, according to[8],

$$\kappa^2 Z^{-1} = C'^{-1}\tau\left(1+3\gamma\ln\frac{\Delta}{\tau}\right)^{-1/3} = C^{-1}\tau\left(\kappa_0 a \ln^{-1}\frac{(\kappa_0 a)^2}{\tau}\right)^{1/3} \tag{17}$$

and C is determined from the matching of the functions $G_\parallel$ and $G_0 = Z\kappa_0^{-2}$ at the boundary of the regions AD_1 and D_1:

$$C = \frac{Z}{\kappa_0^2}\tau_{AD_1}\left(\frac{\kappa_0 a}{\ln[(\kappa_0 a)^2/\tau_{AD_1}]}\right)^{1/3} = \frac{Za^2}{(\kappa_0 a)^{2-1/\nu}}\left(\frac{\kappa_0 a}{\ln(\kappa_0 a)^{2-1/\nu}}\right)^{1/3}. \tag{18}$$

Here we have taken into account that $\tau_{AD_1} = (\kappa_0 a)^{1/\nu}$ at the boundary of the regions.

We now discuss the case of large anisotropy. The situation in the regions I_2 and A_2 has effectively already been considered above. Furthermore, since in all regions we now have $\omega_0 G_\perp < 1$ and

$$G_{zz} = Z(\kappa^2+k^2+q_0^2 n_z^2)^{-1}, \tag{19}$$

this means that the characteristic momentum cutoff is q_0. Therefore, in the region AD_2 formulas (17) and (18) hold, if κ_0 is replaced by q_0 in them.

In conclusion, we give formulas, following from the appropriate expression in[8], for the spontaneous magnetization below T_c:

$$\left(\frac{M}{M_0}\right)^2 \approx \frac{3}{4S^2}\frac{(\kappa_0 a)^{1/3} T_c v_0 \kappa_0^3}{\omega_0^2 Z a^2}(-\tau)\left(3\ln\frac{(\kappa_0 a)^2}{-\tau}\right)^{2/3},$$
$$\left(\frac{M}{M_0}\right)^2 \approx \frac{3}{4S^2}\frac{(q_0 a)^{4/3} T_c}{\omega_0}(-\tau)\left(3\ln\frac{(q_0 a)^2}{-\tau}\right)^{2/3}, \tag{20}$$

where M_0 is the saturation magnetization and S is the effective spin ($M_0 = g\mu S v_0^{-1}$). The first of these formulas corresponds to small anisotropy and the second to large anisotropy. In deriving them we have replaced the constant C_+ introduced in[8] by $\omega_0 Z T_c a^2/4\pi$.

3. CRITICAL DYNAMICS IN THE CASE OF SMALL ANISOTROPY

We shall now generalize to the case of small anisotropy the results obtained in[3] pertaining to the critical dynamics of a ferromagnet above the Curie point. As in[3], we shall use the formalism of Kubo formulas, assuming that the quantity that can be determined by them for $k \gtrsim \kappa$, κ_0 is the characteristic energy of the critical fluctuations for the functions $G_\parallel$ and $G_\perp$ in the sense of dynamic-scaling theory, i.e., the energy that sets the scale for the energy dependence of these functions (cf. the paper by Halperin and Hohenberg[13]):

$$G_{\parallel,\perp}(\mathbf{k},\omega) = G_{\parallel,\perp}(\mathbf{k},0)F_{\parallel,\perp}\left(\frac{\omega}{\Gamma_{\mathbf{k},\parallel,\perp}},\mathbf{k}\right), \tag{21}$$

where the scale for momentum dependence of the function $F_{\parallel,\perp}$ is set by the quantity κ or κ_0, depending on whether the easy or hard direction is considered. In the limit $\mathbf{k} \to 0$ the quantities $\Gamma_{0\parallel,\perp}$ describe the uniform relaxation, and

$$G_{\parallel,\perp}(0,\omega) = G_{\parallel,\perp}(0,0)\frac{\Gamma_{0\parallel,\perp}}{-i\omega+\Gamma_{0\parallel,\perp}}. \tag{22}$$

The functions $\Gamma_{\mathbf{k}\parallel,\perp}$ are determined by the relations:

$$\Gamma_{\mathbf{k}\parallel,\perp} = G_{\parallel,\perp}^{-1}(\mathbf{k},0)(i\omega)^{-1}[\Phi_{\parallel,\perp}(\mathbf{k},\omega)-\Phi_{\parallel,\perp}(\mathbf{k},0)]|_{\omega=0},$$
$$\Phi_\parallel(\mathbf{k},\omega) = i\int_0^\infty dt\, e^{i\omega t}\langle[\dot{S}_\mathbf{k}^z(t), \dot{S}_{-\mathbf{k}}^z(0)]\rangle,$$
$$\Phi_\perp(\mathbf{k},\omega) = \frac{i}{2}\int_0^\infty dt\, e^{i\omega t}\langle[\dot{\mathbf{S}}_\mathbf{k}^\perp(t), \dot{\mathbf{S}}_{-\mathbf{k}}^\perp(0)]\rangle, \tag{23}$$

where the dot denotes the time derivative. It follows from (6) that

$$\dot{\mathbf{S}}_\mathbf{k} = (\dot{\mathbf{S}}_\mathbf{k})_e + (\dot{\mathbf{S}}_\mathbf{k})_A + (\dot{\mathbf{S}}_\mathbf{k})_d, \tag{24a}$$

$$(\dot{S}_\mathbf{k}^\alpha)_e = \frac{1}{2N^{1/2}}\sum_{\mathbf{k}_1}(V_{\mathbf{k}_1} - V_{\mathbf{k}_1-\mathbf{k}})\varepsilon_{\mu\alpha\rho}S_{\mathbf{k}_1}^\mu S_{\mathbf{k}-\mathbf{k}_1}^\rho, \tag{24b}$$

$$(\dot{S}_\mathbf{k}^\alpha)_A = \frac{1}{N^{1/2}}\sum_{\mathbf{k}_1}A_{\mathbf{k}_1}S_{\mathbf{k}_1}^z S_{\mathbf{k}-\mathbf{k}_1}^\rho \varepsilon_{z\alpha\rho}, \tag{24c}$$

$$(\dot{S}_\mathbf{k}^\alpha)_d = -\frac{\omega_0}{N^{1/2}}\sum_{\mathbf{k}_1}n_\mu^{(1)}n_\nu^{(1)}S_{\mathbf{k}_1}^\mu S_{\mathbf{k}-\mathbf{k}_1}^\rho \varepsilon_{\nu\alpha\rho}; \quad \mathbf{n}^{(1)} = \mathbf{k}_1/k_1 \tag{24d}$$

and, therefore, we have

$$\Phi_{\parallel,\perp}(\mathbf{k},\omega) = \sum_{ij=e,A,d}\Phi_{\parallel,\perp}^{ij}(\mathbf{k},\omega). \tag{25}$$

To estimate the quantities $\Gamma_{\parallel,\perp}$, as in[3, 14] we shall investigate the structure of the diagrams for the functions $\Phi_{\parallel,\perp}^{ij}$ (cf. Fig. 1a). At the boundary of the regions D_1 and AD_1 we have $4\pi\chi \approx 4\pi\chi_m \gg 1$, and, by virtue of (12), we obtain

$$4\pi\chi_m \sim \frac{\omega_0}{T_c(\kappa_0 a)^2} = \frac{\omega_0}{A}\left(\frac{T_c}{A}\right)^{(\gamma-\varphi)/\varphi} \gg 1, \tag{26}$$

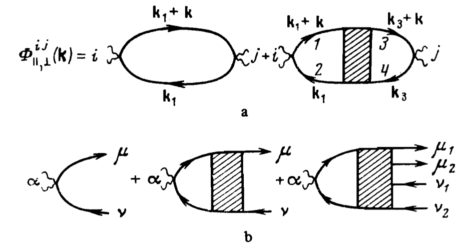

FIG. 1

but γ differs from φ by an amount close to 0.1; therefore, for all reasonable values of the parameters this inequality means that $\omega_0 \gg A$. Therefore, in the region I_1 the anisotropy can be neglected and the critical dynamics is described by the usual theory of exchange dynamic scaling[13], and Huber's result[15, 3] for cubic ferromagnets is valid for the critical damping. The anisotropy can also be neglected in the region D_1. In fact, if this is so, for $k < q_0$ we have, according to[3],

$$\Gamma_{k\parallel}=\Gamma_{k\perp}=T_c(q_0a)^{1/\nu}(\varkappa a)^{z_e-1/\nu}\varphi(k/\varkappa),$$
$$z_e=(5-\eta)/2\approx {}^5/_2. \qquad (27)$$

Using now perturbation theory in the anisotropy and proceeding exactly as in[3], it is not difficult to estimate the quantities

$$\Gamma_{\perp AA}\sim\frac{A^2}{T_c}(\varkappa a)^{1/\nu-3/2}(q_0a)^{-1/\nu},$$
$$\Gamma_{\perp Ad}+\Gamma_{\perp dA}\sim A(\varkappa a)^{1/2}. \qquad (28)$$

In deriving these expressions we have taken into account that, unlike the dipole vertex, the anisotropy vertex does not produce separation of the longitudinal and perpendicular (to the momentum) parts of the critical fluctuations, and have used the principle of coalescence of correlations[16] 5 .

The expressions (28) are small compared with (27) in the whole region D_1, provided that $(4\pi\chi_m)^{1/\nu} \gg (\kappa_0 a)^{2-\varphi/\nu}$; for $2\nu - \varphi \sim 0.1$, this inequality practically always holds. It remains to analyze the region AD_1. For this we note that the anomalous dependence of Γ_k on τ in the dipole region (in the present work—the region D_1) arose in[3] because of the vertices with multiple scattering (Fig. 1b), inasmuch as the bare dipole vertex (24d) is constructed in such a way that only one of the spin factors is responsible for the critical fluctuations perpendicular to the momentum that grow without limit, while the second contains $(n_1 \cdot S_{k_1})$ and is therefore limited. Multiple scatterings lead to intermediate states in which there is no selection of the parts of the fluctuations perpendicular and parallel to the momentum. Therefore, the parts of the Green functions that are perpendicular to the momentum play the main role. In the limit $\tau \to 0$ the corresponding integrals over the intermediate momenta diverge and this leads to an anomalous dependence of Γ_k on τ. In the region AD_1 we must elucidate in an analogous way whether amongst the diagrams for Φ there are some which, in their intermediate states, contain only critical fluctuations that grow without limit. Here, all the problems associated with the analytical continuation of the temperature diagrams, the estimation of the character of the momentum and energy dependences of the vertices, and so on, are solved in exactly the same way as in[3]; we refer the reader to this paper for all the details.

In the easy-axis case the fluctuations along the z-axis grow without limit; it follows immediately from (24) that diagrams without multiple scatterings do not have purely longitudinal intermediate states and, therefore, cannot lead to anomalous behavior. Multiple scatterings also do not lead to purely longitudinal intermediate states. Indeed, all the multiple-scattering vertices are tensors which, like $\Sigma_{\alpha\beta}$, have a definite limit in the limit of zero momenta[6].

The bare vertex is a pseudo-tensor and, therefore, all the vertices with multiple scattering depicted in Fig. 1b are pseudo-tensors, which, obviously, vanish if all the vector indices of the intermediate particles (μ,ν; μ_1, μ_2, ν_1, ν_2, etc.) are equal. For finite momenta of the intermediate particles and finite k, this exclusion is partly lifted, but in this case even powers of the momenta appear in the numerator, leading to the result that the corresponding integral is finite when $\tau \to 0$. In view of what has been said, the characteristic momentum of the intermediate states is the larger of the momenta κ_0 and k. This leads to the result that, in the region AD_1 in the easy-axis case, $\Phi_{\parallel} \approx \Phi_{\perp}$, and this quantity is obtained from the quantity Φ in the region D_1 by replacing κ by κ_0. As a result, using (23) and (27) we find ($\eta = 0$):

$$\Gamma_{k\perp}=T_c(q_0a)^{1/\nu}(\varkappa_0a)^{5/2-1/\nu}\varphi\left(\frac{k}{\varkappa_0}\right), \qquad (29a)$$

$$\Gamma_{k\parallel}=T_c(q_0a)^{1/\nu}(\varkappa_0a)^{5/2-1/\nu}\frac{k^2+\varkappa^2}{k^2+\varkappa_0^2}\varphi\left(\frac{k}{\varkappa_0}\right)$$
$$\approx\omega_0(4\pi\chi_{\parallel}(\mathbf{k}))^{-1}(\varkappa_0a)^{1/2}(4\pi\chi_{\perp}(0))^{1/2\nu}B_{\parallel},$$
$$B_{\parallel}=(T_cZa^2)\varphi(0)\sim 1. \qquad (29b)$$

The approximate equality in the right-hand side of (29b) holds for $k \ll \kappa_0$. For $k \gg \kappa_0$ the quantity $\Gamma_{k\parallel} = \Gamma_{k1}$ coincides with the quantity given by formula (27). We see that, although the temperature dependence of $\Gamma_{0\parallel}$ is normal, its dependence on the anisotropy A and dipole energy ω_0 is extremely complicated.

In the easy-plane case arguments completely analogous to those given above lead to the conclusion that the intermediate states for $\Phi_{\perp}$ are not singular, but there are singular states amongst the intermediate states for $\Phi_{\parallel}$ inasmuch as there are now two critical modes—along the x- and y-axes. As a result, we obtain

$$\Gamma_{k\perp}=G_{\perp}^{-1}(\mathbf{k})(q_0a)^{1/\nu}(\varkappa_0a)^{1/2-1/\nu}\varphi_{\perp}\left(\frac{k}{\varkappa_0}\right)$$
$$\approx\omega_0(4\pi\chi_{\perp}(\mathbf{k}))^{-1}(4\pi\chi_{\parallel}(0))^{1/2\nu}B_{\perp},\quad B_{\perp}=(T_cZa^2)\varphi_{\perp}(0), \qquad (30a)$$

$$\Gamma_{k\parallel}=T_c(q_0a)^{1/\nu}(\varkappa_0a)^{3/2+1/\nu}(\varkappa a)^{1-2/\nu}\varphi_{\parallel}\left(\frac{k}{\varkappa}\right). \qquad (30b)$$

In deriving (30b) we have made the natural assumption that the vertex part leading to an intermediate state containing two critical perpendicular fluctuations behaves as if there were no anisotropy. The approximate equality in the right-hand side of (30a) holds for $k \ll \kappa_0$. We see that the energy of the critical fluctuations in the easy plane is normal in character, while the critical fluctuations in the hard direction are rapidly damped as a result of decay into critical modes in the easy plane.

4. CRITICAL DYNAMICS IN THE CASE OF LARGE ANISOTROPY

Now the maximum value of the susceptibility in the hard direction is small, and instead of (26) we have the opposite inequality. In practice, this means that $\omega_0 \ll A$. In the region I_2 both the dipole forces and the anisotropy can be taken into account by perturbation theory; therefore, for the critical damping we obtain a generalization of Huber's formulas[15, 9]:

$$\Gamma_{0\parallel}\sim\frac{\omega_0^2}{T_c(\varkappa a)^{3/2}}\approx\frac{\omega_0^2}{T_c\tau},$$

$$\Gamma_{0\perp}\sim\frac{A^2}{T_c(\varkappa a)^{3/2}}\approx\frac{A^2}{T_c\tau}. \quad (31)$$

In the region A_2 in the easy-axis case, because $G_\perp$ is bounded, momenta of the order of the larger of the quantities k and κ_0 are important in all the integrals. In this case, if we assume that $2\nu=\varphi$ and $\eta=0$, the exchange forces and anisotropy give equal contributions to the energy, and

$$\Gamma_{k\parallel}=T_c(ka)^2\frac{(\varkappa^2+k^2)a^2}{(\varkappa_0a)^{1/2}}f_\parallel\left(\frac{k}{\varkappa_0}\right)+\frac{\omega_0^2(k^2+\varkappa^2)a^2}{T_c(\varkappa_0a)^{7/2}}\varphi_\parallel\left(\frac{k}{\varkappa_0}\right), \quad (32a)$$

$$\Gamma_{k\perp}=T_c(ka)^2(\varkappa_0a)^{1/2}f_\perp\left(\frac{k}{\varkappa_0}\right)+\frac{A^2}{T_c(\varkappa_0a)^{3/2}}\varphi_\perp\left(\frac{k}{\varkappa_0}\right). \quad (32b)$$

The first terms in these formulas describe the exchange contribution to $\Gamma_{\parallel,\perp}$, and the second term in (32a) corresponds to the uniform relaxation along the z-axis; it must be taken into account if $k<q_0$; $\varphi_\perp(k)$ falls off for $k\gg\kappa_0$, and $f_\parallel$ and $f_\perp$ are such that for $k\gg\kappa_0$ we obtain $\Gamma_{k\parallel}=\Gamma_{k\perp}\sim T_c(ka)^{5/2}$, as in the absence of anisotropy.

In the easy-plane case in A_2 there arises a situation analogous to that which is obtained in the region AD_1: the energy $\Gamma_\perp$ has a normal form while $\Gamma_\parallel$ grows with decreasing τ, viz:

$$\Gamma_{k\perp}=T_c(ka)^2\frac{(\varkappa^2+k^2)a^2}{(\varkappa_0a)^{1/2}}h_\perp\left(\frac{k}{\varkappa_0}\right)+\frac{A^2(k^2+\varkappa^2)a^2}{T_c(\varkappa_0a)^{7/2}}\psi_\perp\left(\frac{k}{\varkappa_0}\right), \quad (33a)$$

$$\Gamma_{k\parallel}=T_c(ka)^2(\varkappa_0^2+k^2)a^2\frac{(\varkappa_0a)^{3/2}}{(\varkappa a)^3}h_\parallel\left(\frac{k}{\varkappa}\right)h_\perp^{-1}\left(\frac{k}{\varkappa_0}\right)+\frac{\omega_0^2}{T_c}(\varkappa_0^2+k^2)a^2\frac{(\varkappa_0a)^{3/2}}{(\varkappa a)^5}\psi_\parallel\left(\frac{k}{\varkappa}\right). \quad (33b)$$

Here the functions $h_{\parallel,\perp}$ at large arguments are such that for $k\gg\kappa_0$ we have $\Gamma_{k\parallel}=\Gamma_{k\perp}\sim T_c(ka)^{5/2}$; the functions $\psi_{\parallel,\perp}$ fall off for large arguments and can be neglected.

In the anisotropy-dipole region AD_2 in the easy-axis case, as above, momenta of the order of k and κ_0 are important in the calculation of $\Phi_{\parallel,\perp}$. As a result, in the crossover from region A_2 to AD_2 the transverse energy (32b) is practically unchanged, while in (32a) it is necessary to substitute for κ^2 the quantity given by the Larkin-Khmel'nitskiĭ theory (cf. Sec. 2):

$$\varkappa^2=\tau a^{-2}(q_0a)^{2-1/\nu}\left(\frac{\ln(q_0a)^{2-1/\nu}}{\ln(q_0a)^2\tau^{-1}}\right)^{1/3}. \quad (34)$$

In the hard-axis case and in the region AD_2 formulas (33a) and (33b) remain valid, with a certain change in the form of the functions h and ψ.

5. CRITICAL DAMPING IN $GdCl_3$. THE ROLE OF THE MAGNETIC FIELD

Kötzler and co-workers have studied the critical properties of the ferromagnet $GdCl_3$[6, 7, 18] ($T_C\approx2.2$ K, easy-axis anisotropy). It was found that its static properties agree with the predictions of the Larkin-Khmel'nitskiĭ[8] theory (the logarithmic dependences distinguishing this theory from the Landau theory were not detected). According to[7], for $\tau<0.4$ we have

$$4\pi\chi_\parallel\approx1/\tau, \quad 4\pi\chi_\perp\approx0.9(\tau+0.24)^{-1}, \quad (35)$$

so that in practice we are concerned with small anisotropy and the entire critical region is the region AD_1. However, the temperature dependence of the critical absorption was found to be extremely strange:

$$\Gamma_{0\parallel}[\sec^{-1}]=1.9\cdot10^{10}\tau\sim\chi_\parallel^{-1}, \quad 0.4>\tau>0.03,$$

$$\Gamma_{0\parallel}[\sec^{-1}]=3.5\cdot10^{9}\tau^{1/2}\sim\chi_\parallel^{-1/2}; \quad 0.03>\tau>0.004, \quad (36)$$

i.e., for $\tau<0.03$ the damping is anomalous. It was shown above that normal behavior of $\Gamma_{0\parallel}$ is a consequence of the pseudo-vectorial properties of the bare vertices. Therefore, the anomalous dependence of $\Gamma_{0\parallel}$ should be associated with a pseudo-vectorial perturbation. We shall show that a magnetic field can lead to the observed phenomenon, i.e., to the result that for sufficiently small τ we have $\Gamma_{0\parallel}\sim\sqrt{\tau}$ while, at the same time in this temperature region, the static properties remain independent of the field.

It must be emphasized, however, that in the derivation of formulas (29) for the critical damping it was necessary to know both the static and dynamical behavior of the Green functions and vertices at momenta greater than κ_0. This behavior has a scale-invariant character with known indices, if the dipolar-scaling region D_1 exists. In exactly the same way, we can clarify the question of the influence of an external field only by taking the scaling properties at large momenta into account. But for $GdCl_3$ the region D_1 is absent; therefore, the formulas obtained below are not strictly applicable to it. We can only suppose that estimates made with their help do not lead to gross errors, and, moreover, the qualitative result is certainly correct: if there is a region of fields in which $\Gamma_{k\parallel}$ is anomalous, then in this region $\Gamma_{0\parallel}\sim\tau^{1/2}$.

We now discuss in somewhat more detail the conditions under which an external field affects the properties of the system. Regarding the interaction (4) with an external uniform and constant field as a perturbation, we obtain (compare with, e.g.,[16])

$$\delta G_{\alpha\beta}^{-1}(\mathbf{k},0)=-\Gamma_{\alpha\beta\mu\nu}(\mathbf{k},\mathbf{k},0,0)(G_{\mu\rho}(0,0)g\mu H_\rho)(G_{\nu\gamma}(0,0)g\mu H_\gamma). \quad (37)$$

By virtue of the equalities (2), this expression does not depend on the shape of the solid. It follows from the relation (3) that $\delta G^{-1}=\delta G_0^{-1}$, and the field does not affect the longitudinal susceptibility if

$$G_\parallel(0)\Gamma_{zzzz}(0)(G_{zz}g\mu H_z)^2=G_\parallel(0)\Gamma_{zzzz}(0)(G_\parallel(0)g\mu H_i)^2<1,$$
$$G_{zz}(0)=G_\parallel(0)[1+\omega_0N_{zz}G_\parallel(0)]^{-1}. \quad (38)$$

If we take into account the correspondences indicated in footnote [4], it follows from[8] that

$$\Gamma_{zzzz}(0)\sim16\pi\varkappa_0\left(3T_cv_0Z^2\ln\frac{\varkappa_0^2}{\varkappa^2}\right)^{-1}. \quad (39)$$

In this formula κ_0 ensures matching with the scaling region, where the four-point function is proportional to κ. The fact that the magnetic field appears quadratically in formula (38) is a consequence of the fact that it is odd under time reversal. At finite frequencies this odd parity is cancelled by the frequency dependence and for $\omega\neq0$ we have

$$\delta G_{\alpha\beta}(\mathbf{k},\omega)=G_{\alpha\mu}(\mathbf{k},\omega)F_{\mu\nu\gamma}(\mathbf{k},\omega,0,0)\omega G_{\nu\beta}(\mathbf{k},\omega)(G_{\gamma\gamma'}(0,0)g\mu H_{\gamma'}). \quad (40)$$

Here $\omega F_{\mu\nu\gamma}$ is the three-particle vertex depicted in Fig. 2a; $F(\omega)$ is finite at zero and is an analytic function of ω with the property $F_{\mu\nu\gamma}(\omega)=F_{\mu\nu\gamma}(-\omega^*)$, which follows from the time-reversal symmetry. In addition, this amplitude is symmetric to interchange of the particles 1 and 2 in Fig. 2a, but, inasmuch as this interchange is accompanied by the replacement $\omega\to-\omega$, $F_{\mu\nu\gamma}$ is antisymmetric in μ and ν.

Up to the present time, odd dynamic vertices have

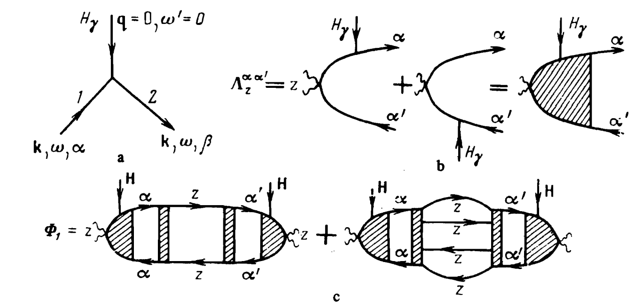

FIG. 2

not been considered in the literature. It is shown below that we need the function $F_{\mu\nu\gamma}$ for $k \gtrsim \kappa_0$ and $\gamma = z$. We now try to analyze the quantity ωF. First suppose that the momentum q in Fig. 2 is large ($k \sim q \gg \kappa_0$). In this region of momenta the dynamic- and static-scaling theories are valid and from the unitarity estimate of[19] we find that ωF has dimensions $k^{3/2}$. If $q \ll k$, then, in accordance with the general idea of coalescence of correlations[16], $\omega F \sim k^x q^{(3/2)-x}$; obviously, for $q < \kappa_0$ the quantity q must be replaced by κ_0. Thus, for $q = 0$ and $k \gtrsim \kappa_0$ we have

$$\omega F_{\mu\nu\gamma}(\mathbf{k}, \omega, 0, 0) = T_c \frac{\omega}{\Gamma_k}(ka)^x (q_0 a)^{3/2-x} f_{\mu\nu\gamma}\left(\frac{\mathbf{k}}{\varkappa_0}, \frac{\omega}{\Gamma_k}\right) \quad (41)$$

Here, in accordance with dynamic scaling, we have used the energy $\Gamma_{\mathbf{k}}$ to set the scale for ω. The index x is treated below as a parameter of the theory. In the static theory a large momentum is always separated out in the form of the factor $k^{(1/\nu)-1}$; if this property is universal, then $x = (1/\nu) - 1 \approx 1/2$.

The correction (40) leads to intermediate states with critical fluctuations with the same polarization α (Fig. 2b); as a result there arises a correction Φ_1, quadratic in the field, to the Kubo function $\Phi_{\parallel}$ with purely longitudinal intermediate states. These states arise from that component of the pseudo-tensorial vertex (Fig. 2c) with all indices equal to z. Obviously, this vertex is constructed from $H_{i\alpha}$, z_β and $\delta_{\alpha\beta}$, and, therefore, the component under consideration is proportional to H_{iz}. According to the general analytic-continuation formula for Φ[3], by virtue of (24d) the singular contribution to Φ_1 has the form

$$\Phi_1 = \frac{T_c^4 v_0^2}{(2\pi)^6 i} \int d\mathbf{k}_1\, d\mathbf{k}_3 \prod_{i=1}^{4} \frac{dx_i \Delta_i}{\pi x_1} \Lambda_z^{\alpha_1\alpha_1}(\mathbf{k}, \mathbf{k}_1, x_1, x_2)$$
$$\times \Lambda_z^{\alpha_1\alpha_2}(\mathbf{k}, \mathbf{k}_3, x_3, x_4) \frac{\partial}{\partial\omega} W_{\alpha_1\alpha_2}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_3, x_i, \omega)\,|_{\omega=0}, \quad (42)$$

$$\Lambda_z^{\alpha\alpha'}(\mathbf{k}, \mathbf{k}_1, x_1, x_2) = -\omega_0 n_{1\mu} \varepsilon_{\mu z\rho} [\delta G_{\rho\alpha}(\mathbf{k}+\mathbf{k}_1, x_1)(\mathbf{n}_1 G(\mathbf{k}_1, x_2))_{\alpha'}$$
$$+ G_{\rho\alpha}(\mathbf{k}+\mathbf{k}_1, x_1)(\mathbf{n}_1 \delta G(\mathbf{k}_1, x_2))_{\alpha'}].$$

Here Λ is the correction to the bare vertex (24d) (Fig. 2b), W is the vertex part with longitudinal intermediate states (Fig. 2c), x_i are the energy variables and Δ_i are the discontinuities with respect to them. We must now estimate the transition amplitudes $\Gamma_{\alpha z}$ depicted in Fig. 2c. First we shall consider a two-particle intermediate state. The singular contribution arises from the region of momenta $k_2 \ll \kappa_0$. But, as we shall see below, $k_{1,3} \gtrsim \kappa_0$. Therefore, it is natural to make use of the principle of coalescence of correlations[16]. The dependence on k_1 is separated out immediately: $\Gamma_{\alpha z}(k_1, k_2) = T_c (k_1 a)^{(1/\nu)-1} \rho(\kappa_0, k_2)$. Furthermore, for $k_1 \sim k_2 \sim \kappa_0$ we have $\Gamma_{\alpha z} \sim \kappa_0$; therefore, if $k_2 \sim \kappa_0$, $\rho \sim \kappa_0^{2-1/\nu}$. For $k_2 \ll \kappa_0$ the theory is logarithmic. In[16] it is shown that, for a small coupling constant γ, if only one of the momenta (k_2) is small, the dimensionless four-point function is proportional to $\lambda[\lambda \ln(\kappa_0^2/k_2^2)]^{-1/3} = \lambda^{2/3}[\ln(\kappa_0^2/k_2^2)]^{-1/3}$. If, following the logic of[8], we assume that this formula is also valid for $\lambda \sim (\kappa_0 a)^{-1} \gg 1$, then, for $k_1 \gtrsim \kappa_0$ and $k_2 \ll \kappa_0$, we have

$$\Gamma_{\alpha z}(\mathbf{k}_1, \mathbf{k}_2) \sim T_c (k_1 a)^{1/\nu-1} (\varkappa_0 a)^{2-1/\nu-2/3} \left(\ln \frac{\varkappa_0^2}{k_2^2}\right)^{-1/3}$$
$$\approx T_c (k_1 a)^{1/2} (\varkappa_0 a)^{-1/6} \left(\ln \frac{\varkappa_0^2}{k_2^2}\right)^{-1/3}. \quad (43)$$

We stress once more that in this expression the power of κ_0 is not determined sufficiently reliably. In (42) the integration over k_1 and k_2 is performed over the two regions $k_{1,3} \ll \kappa_0$ and $k_{1,3} \gtrsim \kappa_0$. The contribution of the first region can become anomalously large if there is a nonintegrable singularity for $k_{1,3} \ll \kappa_0$. In this respect, the terms containing the greatest number of singular functions G_{zz} and $G_{\alpha z}$ (cf. (10)) are suspect. The most singular term contains $F_{\alpha\beta z}$ for $\alpha \neq z$, $\beta \neq z$. The corresponding integral diverges only if F is proportional to k^{-2}. But there is no reason for such a divergence, inasmuch as the linear power of ω in (41) is made dimensionless by the energy of "its mode," i.e., by $\Gamma_{\mathbf{k}\perp}$. In the second region, $G_{\alpha\beta} \approx G_0(\delta_{\alpha\beta} - n_\alpha n_\beta)$, $(\mathbf{n}G) \approx \omega_0^{-1}\mathbf{n}$, and formula (41) is valid for F.

By virtue of dynamic scaling, the integration over x_i for $k_{1,3} \geq \kappa_0$ does not alter the momentum dependence of the integrand. This enables us, as in[3], to estimate the integrals over k_1 and k_3 by truncating them, where necessary, at the upper limit by the momentum q_0, since for $k > q_0$ we have $nG \sim k^{-2}$. To estimate the integral over k_2, as in[3] we can make use of the pole expression

$$G_{zz}(\mathbf{k}, \omega) \approx G_{zz}(\mathbf{k}, 0)\Gamma_{zz}(\mathbf{k})[-i\omega + \Gamma_{zz}(\mathbf{k})]^{-1},$$
$$\Gamma_{zz}(\mathbf{k}) = G_{zz}^{-1}(\mathbf{k}, 0)\Phi_{\parallel} = G_{zz}^{-1}(\mathbf{k}, 0) G_{\parallel k}(0)\Gamma_{\mathbf{k}\parallel}, \quad (44)$$

and the contribution of the two-particle state has the form

$$N_2(\mathbf{k}, \varkappa) \sim \frac{T_c^3 v_0}{(2\pi)^3} \int d\mathbf{k}_2 \left(\ln \frac{\varkappa_0^2}{k_2^2}\right)^{-2/3} \frac{1}{\pi} \int dx \left(\frac{\mathrm{Im}\, G_{zz}(\mathbf{k}_2, x)}{x}\right)^2$$
$$\sim (\varkappa_0 a)^{-1} T_c \int_{\max(k,\varkappa)} \frac{dk_2}{k_2 \Gamma_{\mathbf{k}_2\parallel}} \left(\ln \frac{\varkappa_0^2}{k_2^2}\right)^{-2/3}. \quad (45)$$

It is not difficult to convince oneself that the many-particle longitudinal states are nonsingular, and, therefore,

$$\Phi_1(\mathbf{k}) = \varphi_1 (g\mu H_z G_{zz}(0,0))^2 A_x T_c \int_{\max(k,\varkappa)} \frac{dk_2}{k_2 \Gamma_{\mathbf{k}_2\parallel}} \left(\ln \frac{\varkappa_0^2}{k_2^2}\right)^{-2/3},$$
$$A_x = (q_0 a)^{2x+2/\nu-4} (\varkappa_0 a)^{14/3-2/\nu-2x}, \quad x > -1/\nu+2,$$
$$A_x = (\varkappa_0 a)^{2/3} \ln^2(q_0^2/\varkappa_0^2), \quad x \approx -1/\nu+2, \quad (46)$$
$$A_x = (\varkappa_0 a)^{2/3}, \quad x < -1/\nu+2,$$

where $\varphi_1 \sim 1$. As a result, taking (29a) into account, for $\kappa \ll k \ll \kappa_0$ we obtain an equation for $\Gamma_{\mathbf{k}\parallel}$:

$$G_{\mathbf{k}\parallel}\Gamma_{\mathbf{k}\parallel} = \Phi_{\parallel} + \Phi_1(\mathbf{k}),$$
$$\Phi_{\parallel} = \varphi_0 (q_0 a)^{1/\nu} (\varkappa_0 a)^{1/2-1/\nu} \quad (47)$$

(where $\varphi_0 \sim 1$), which is easily solved; we have

$$\Gamma_{\mathbf{k}\parallel} = G_{\mathbf{k}\parallel}^{-1} \Phi_{\parallel} \left\{1 + (g\mu H_z G_{zz})^2 (\varkappa_0 a) \left(\ln \frac{\varkappa_0^2}{k^2}\right)^{-1} \frac{\varphi_1}{\varphi_0^2} T_c G_{\mathbf{k}\parallel} C_x\right\}^{1/2}, \quad (48)$$
$$C_x = (q_0 a)^{-1} (\varkappa_0 a)^{-1/3} \left(\ln \frac{\varkappa_0^2}{k^2}\right)^{1/3} \begin{cases} (\varkappa_0/q_0)^{3-2x}, & x > -1/\nu+2 \approx 1/2 \\ (\varkappa_0/q_0)^{2/\nu-1} \ln^2(q_0^2/\varkappa_0^2); & x \approx -1/\nu+2 \approx 1/2. \\ (\varkappa_0/q_0)^{2/\nu-1} \sim (4\pi\chi_m)^{-1}, & x < -1/\nu+2 \approx 1/2 \end{cases}$$

In this expression k must be replaced by κ, if $k < \kappa$. The formula with the logarithm squared holds for $|x - 1/2| \ll 1$ and, in particular, for the "universal" $x = \nu^{-1} - 1$. For $k < \kappa$ the factor multiplying C_x coincides in order of magnitude with the left-hand side of the condition (38). Therefore, if $C_x \gg 1$, there indeed exists a region of magnetic fields leading to anomalous critical damping but not affecting the static susceptibility. In this region,

$$\Gamma_{0\|}=\omega_0\left(\frac{T_c}{\omega_0}\varphi_1(4\pi\chi_\|)^{-1}\right)^{1/3}(\varkappa_0 a)^{1/3}\left(\ln\frac{\varkappa_0^2}{\varkappa^2}\right)^{-1/3}(g\mu H_z G_{zz})^2 \sim \omega_0\left(\frac{T_c}{\omega_0}(4\pi\chi_\|)^{-1}\varphi_1\right)^{1/3}(\varkappa_0 a)^{1/3}(g\mu H_z G_{zz}). \quad (49)$$

It follows from (38) that, if $\omega_0 G_\| N_{zz} = 4\pi\chi_\| N_{zz} \gg 1$, we indeed have $\Gamma_{0\|} \sim \sqrt{\tau}$.

For a direct comparison of the formulas obtained with experiment it is necessary to take two circumstances into account: first, we must take into account the dependence of all quantities on the spin S, which is important for large spins (for $GdCl_3$ we have $S = 7/2 \approx 4$) and, secondly, we must try to separate out the large numerical factors analogous to the 16π in (39). The dependence on the spin arises from the fact that $Z \sim S(S+1) \approx S^2$. As a result, using, e.g., the unitarity power[15, 19] for the vertices, we obtain $\Gamma_n \sim S^{-n}$; $\Gamma_4 \sim S^{-4}$ and $\omega F \sim S^{-3}$. Furthermore, the factor 16π in (39) is not a chance factor; it is necessary, e.g., for the numerical agreement of the unitarity estimates[16] for Γ^4. Its appearance can best be seen by means of $(4-\epsilon)$-theory (cf., e.g.,[17]). In exactly the same way, there should also be a large numerical factor of the order of $(16\pi)^{1/2}$ in the expression for ωF. If we take S and these factors explicitly into account in the estimates given above, we obtain

$$\varphi_0 = S(16\pi)^{1/2}\psi_0, \quad \varphi_1 = (16\pi)^2 S^{-2}\psi_1, \quad \varphi_1\varphi_0^{-2} = 16\pi S^{-4}\psi_1\psi_0^{-2}, \quad \psi_1\psi_0^{-2} \sim 1. \quad (50)$$

Thus, in (47) a large factor completely analogous to that in (38) and (39) is separated out explicitly. We now give certain estimates for $GdCl_3$. For $GdCl_3$ we have $\omega_0 = 6.8 \times 10^9$ sec^{-1}; and from the estimate $(\kappa_0 a)^2 \sim S^2\omega_0(4\pi\chi_m T_c)^{-1}$ and (35) it follows that $(\kappa_0 a)^2 \sim 0.1$ and $q_0 \approx 2\kappa_0$. In the critical-absorption experiments of[7] a sample with $N_{zz} = 0.03$ was used, so that in the entire anomalous region ($\tau < 0.03$), in accordance with (35), $4\pi\chi_\| N_{zz} > 1$ and $g\mu G_{zz} H_z \approx g\mu H_z(\omega_0 N_{zz})^{-1}$.

For $x \approx 1/2$ we have $C_x \approx 2$ and, taking into account the 1/3 in (39), we may expect anomalous damping to exist in the region $10\tau_H > \tau > \tau_H$, where τ_H is the value of τ at which the magnetic field begins to affect the static susceptibility. The experiments of[7] were performed down to $\tau \sim 3 \times 10^{-3}$, and the field was not observed to affect $\chi_\|$. This makes it possible to estimate an upper bound for the field that could have been present in these experiments. Taking (35), (38) and (39) into account and neglecting the logarithmic factor, we obtain

$$H_z[\text{Oe}] \lesssim \frac{\omega_0 N_{zz} S^2}{g\mu}\left(\frac{\omega_0\tau}{16 T_c \varkappa_0 a}\right)^{1/2} = 3.2\sqrt{\tau}|_{\tau=3\cdot10^{-3}} \approx 0.7, \quad (51)$$

Thus, there could have been an external field of the order of 1 Oe at the sample. We shall now see what anomalous damping such a field will lead to. It follows from the cited values of q_0 and κ_0 and (36) and (50) that $\psi_{0\exp} \approx 0.06$; such a value of ψ_0 is not too small, inasmuch as integrals over k_1 and k_3 with factors $(2\pi)^{-3}$ (cf. (42)) appear in the expression for $\Phi_\|$ (cf.[3]). Therefore, there is a factor π^{-2} in the expression for ψ_0 (we recall that to estimate $\Phi_\|$[3] it is necessary to extract a root). Taking (50) into account, we must suppose that $\psi_1 = 4 \times 10^{-3} d$ ($d \sim 1$). Substitution of the parameter values found into (49) gives for the anomalous critical damping $\Gamma_{0\|} = 3.3 \times 10^9 H(\tau d)^{1/2}$, where H is in Oersteds.

As reported to the author by Dr. Kötzler, his experiments were performed in the earth's magnetic field ($H = 0.34$ Oe), so that there is an unexpectedly good agreement for such rough estimates between the obtained value of $\Gamma_{0\|}$ and the experimental value (36).

We now discuss briefly the case of large anisotropy. From (32a) for $k < \kappa_0$ we obtain

$$\Phi_\| = (\varkappa_0 a)^{1/3}[\varphi_0^{(1)}(k/\varkappa_0)^2 + \varphi_0^{(2)}(q_0/\varkappa_0)^4], \quad (52)$$

and it can be shown that $\varphi_0^{(1)} = S\varphi_0^{(1)}$ and $\varphi_0^{(2)} = 16\pi S^5\varphi_0^{(2)}$. Since there is now no suppression of the fluctuations parallel to the momentum, the integrals over $k_{1,3}$ in (42) contain Green functions in threes and, for any x, from (41) momenta $k_{1,3} \sim \kappa_0$ are important. As a result,

$$\Phi_1 = \varphi_1(g\mu H_z G_{zz})^2\left(\frac{q_0}{\varkappa_0}\right)^4(\varkappa_0 a)^{2/\nu-1} \times \begin{cases} T_c\int\limits_{\max(k,\varkappa)}^{\infty} dk' k'^{2-2/\nu}\Gamma_{k'\|}^{-1}, & A_2 \\ (q_0 a)^{-4/3} T_c \int\limits_{\max(k,\varkappa)}^{q_0} dk' k'^{-1}\Gamma_{k'\|}^{-1}\left(\ln\frac{q_0^2}{k'^2}\right)^{-2/3}, & AD_2 \end{cases} \quad (53)$$

Here $\varphi_1 = (16\pi)^2 S^2\psi_1$, and the regions in which the corresponding expressions are valid are indicated on the right. The factor $(q_0 a)^{-4/3}$ in the region AD_2 arose because now formula (43), with κ_0 replaced by q_0, holds for $\Gamma_{\alpha z}$. In (53), in both cases, the integrand is approximately equal to $(k'\Gamma_{k'\|})^{-1}$ and the corresponding integrals are of the order of $\Gamma_{\|\max(k,\kappa)}^{-1}$. Therefore, taking (38) into account, we can write the conditions for the critical damping ($k = 0$) to be anomalous in the form

$$G_{0\|}^{-1}\Gamma_{zzzz}^{-1} \gg (g\mu H_z G_{zz})^2 \gg \Phi_{\|0}^2 G_{0\|}^{-1} B^{-1}, \quad (54)$$

where B is the coefficient in (53) multiplying the product of the integral and $(g\mu H_z G_{zz})^2$. In the region A_2 we find for the anomalous damping

$$\Gamma_{0\|} = \omega_0 4\pi\chi_\perp(\varkappa_0 a)\left[\frac{T_c}{\omega_0}\varphi_1(4\pi\chi_\|)^{-1}\right]^{1/2}\frac{g\mu H_z}{\omega_0}4\pi\chi_\|, \quad (55a)$$

$$T_c\Gamma_{zzzz}^{-1} \sim S^4(16\pi\varkappa a)^{-1} \gg \left(\frac{g\mu H_z}{\omega_0}\right)^2\frac{T_c}{\omega_0}(4\pi\chi_\|)^3 \gg \frac{(\varphi_0^{(2)})^2}{\varphi_1\varkappa_0 a}(4\pi\chi_\perp)^2. \quad (55b)$$

Here $4\pi\chi_\perp = q_0^2/\kappa_0^2 \ll 1$ is the maximum value of the transverse susceptibility. In addition, we have used the estimate $\Gamma_{zzzz} \sim 16\pi T_c(\kappa a)S^{-4}$ and have taken into account that in A_2 demagnetization effects are small and, therefore, $G_{zz} \approx G_\| = \omega_0^{-1}4\pi\chi_\|$. The inequality (55b) is the condition for which anomalous damping occurs. The analogous formulas in the region AD_2 have the form

$$\Gamma_{0\|} = \omega_0 4\pi\chi_\perp(\varkappa_0 a)(q_0 a)^{-2/3}\left[\frac{T_c}{\omega_0}\varphi_1(4\pi\chi_\|)^{-1}\right]^{1/2}\frac{g\mu H_z}{\omega_0}\frac{4\pi\chi_\|}{1+4\pi\chi_\| N_{zz}}, \quad (56a)$$

$$T_c\Gamma_{zzzz}^{-1} \sim \frac{S^4\ln(q_0^2/\varkappa^2)}{16\pi q_0 a} \gg \frac{T_c}{\omega_0}4\pi\chi_\|\left(\frac{g\mu H_z\cdot 4\pi\chi_\|}{1+4\pi\chi_\| N_{zz}}\right)^2 \gg \frac{(\varphi_0^{(2)})^2}{\varphi_1(\varkappa_0 a)}(4\pi\chi_\perp)^2(q_0 a)^{2/3}. \quad (56b)$$

Here, for Γ_{zzzz} we have made use of the estimate (39) with κ_0 replaced by q_0. Thus we see that in the case of strong anisotropy a magnetic field can lead to anomalous critical damping while not affecting the static susceptibility. We shall not discuss here the influence of the field on the dynamics at finite k, since an experimental check of the corresponding results seems difficult.

In conclusion, the author expresses his gratitude to Dr. Kötzler for sending material on the experiments with $GdCl_3$ and for an interesting letter discussing the problems of the critical dynamics, and to V. A. Ruban, who raised the question of the role of the magnetic field.

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1)From the Kubo formulas, the dynamical properties of a physical quantity are determined by the product of the inverse susceptibility corresponding to this quantity with the Kubo function of the time derivatives of this quantity. According to van Hove [4], the critical dynamics is called normal if the Kubo function is finite as $T \to T_c$, and anomalous otherwise.

2)If we neglect the dipole forces, this is also true for finite k. When they are taken into account, the tensor properties of the internal Green functions depend, by virtue of (8), on their momenta, and this leads to terms proportional to $k_\alpha k_\beta$ and $(k \cdot z)(k_\alpha z_\beta + k_\beta z_\alpha)$ in the expression for Σ. However, these terms are unimportant for the estimates below.

3)We shall not distinguish the values of the indices in the different regions, since it will always be clear which region is being discussed.

4)There is the following correspondence between our parameters and those introduced in [8]: $s = Z^{-1}$, $m = \kappa^2 Z^{-1}$, $\lambda = \kappa_0^2 Z^{-1} \sim (\kappa_0 a)^2 T_c$, $b \sim T_c$, $\gamma \sim (\kappa_0 a)^{-1} \gg 1$, $\Lambda \sim (\kappa_0 a)^2 T_c$, $\mu - \mu_0 \sim \tau$.

5)The estimate of the vertices with multiple scattering (Fig. 1b) carried out [3] in the calculation of Γ_0 in the exchange region showed that multiple scatterings give a contribution of the same order as that given by the bare vertices. In principle, these contributions could cancel, as, e.g., in the case of the Ward identity. In our case, however, because of the pseudo-vectorial character of the vertices, there is no cancelation. In $(4 - \epsilon)$-theory it is easy to convince oneself of this by means of the formulas from Ginzburg's paper [17].

6)These tensors are not symmetric under permutation of the tensor indices. There is symmetry only under simultaneous permutation of the tensor indices and the corresponding energies. Inasmuch as, after analytic continuation (cf. [3], Appendix II), the energies of the lines 1 and 2, and also of 3 and 4, (Fig. 1a) appear asymmetrically, the whole diagram with multiple scattering is nonzero.

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Translated by P. J. Shepherd
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