

# Electrohydrodynamic instability in nematic liquid crystals

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Theoretical and experimental investigations have been carried out on the threshold voltage ( $U_t$ ) for electrohydrodynamic (EHD) instability of nematic liquid crystals (NLC) with initial planar and homeotropic orientations. The theoretical dependences of  $U_t$  on the anisotropy  $\Delta\epsilon$  of the dielectric constant and  $\sigma_{||}/\sigma_{\perp}$  of the electrical conductivity and on the coefficients of elasticity and viscosity were obtained by numerical solution of the system of equations describing the NLC, with allowance for boundary conditions. Experimental  $U_t(\Delta\epsilon)$  and  $U_t(\sigma_{||}/\sigma_{\perp})$  curves were measured on doped NLC with constant viscosity and elasticity coefficients. In the region  $\Delta\epsilon \approx 0$  a new form of instability was observed, in the form of a lattice with a large (in comparison with the inverse thickness of the specimen) wave vector. The experimental data are in agreement with calculations that take the boundary conditions into account. The one-dimensional Helfrich model, in the case of homeotropic orientation, does not agree even qualitatively with experiment (in the case of planar orientation, there are only quantitative discrepancies).

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## 1. INTRODUCTION

It is well known<sup>[1]</sup> that in thin layers of nematic liquid crystals (NLC) in an external electric field, electrohydrodynamic (EHD) instability occurs at a certain threshold voltage. Near the threshold, the EHD instability shows up in the form of a periodic pattern of rotational motion of the NLC (Williams domains)<sup>[2]</sup>; and at higher voltages, the motion of the NLC has a turbulent character<sup>[3]</sup>.

A number of papers have been devoted to the theory of EHD instability for the case of low frequencies ( $0 < \omega \ll \omega_0 = 1/\tau_0$ , where  $\tau_0$  is the relaxation time of volume charge). In reference<sup>[4]</sup> the following expression for the threshold voltage was obtained by consideration of the equilibrium of three torques (the dielectric, the elastic, and the frictional torque) for an initial planar orientation of the molecules:

$$U_t^2 = 4\pi^3 K_{33} \left[ \Delta\epsilon \frac{\sigma_{\perp}}{\sigma_{||}} + \epsilon_{||} \frac{\alpha_2}{\eta_2} \left( \frac{\sigma_{\perp}}{\sigma_{||}} - \frac{\epsilon_{||}}{\epsilon_{\perp}} \right) \right]^{-1}. \quad (1)$$

Here  $\epsilon_{||}$ ,  $\epsilon_{\perp}$  and  $\sigma_{||}$ ,  $\sigma_{\perp}$  are the dielectric constants and the electrical conductivities for directions parallel and perpendicular to the director of the NLC;  $\Delta\epsilon = \epsilon_{||} - \epsilon_{\perp}$ ;  $K_{33}$  is the elastic coefficient for bending [bend];  $\eta_2 = \gamma_2/(-\alpha_2 + \alpha_4 + \alpha_5)$ ; and  $\alpha_i$  are the Leslie viscosity coefficients ( $i = 1, 2, \dots, 6$ )<sup>[5]</sup>, Chap. V. In the case of homeotropic orientation<sup>[4]</sup>

$$U_t^2 = 4\pi^3 K_{11} \left[ -\Delta\epsilon \frac{\sigma_{\perp}}{\sigma_{||}} + \epsilon_{||} \frac{\alpha_2}{\eta_1} \left( \frac{\sigma_{||}}{\sigma_{\perp}} - \frac{\epsilon_{||}}{\epsilon_{\perp}} \right) \right]^{-1}, \quad (2)$$

where  $K_{11}$  is the elastic coefficient for transverse bending [splay], and where  $\eta_1 = \gamma_2/(\alpha_3 + \alpha_4 + \alpha_5)$ .

In the derivation of (1) and (2) boundary conditions were disregarded, and these equations give values that are too low as compared with the experimental value of  $U_t$ . A partial allowance for boundary conditions in<sup>[6]</sup>, where an analytic expression for  $U_t$  was found for planar orientation, gives fair agreement of theory with experiment for p-n-methoxybenzylidene-p'-butylaniline (MBBA). A solution of the linearized system of equations describing the NLC with rigorous accounting for boundary conditions was obtained in<sup>[7, 8]</sup>. Numerical estimates in the determination of a set of parameters for p-azoxyanisole and MBBA agree with experimental

data, but no systematic investigation of the influence of the various NLC parameters on the threshold of EHD instability has so far been carried out.

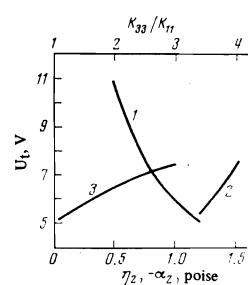
In the present paper, a calculation is made of the dependence of the threshold voltage for EHD instability on the various parameters of the NLC. An experimental investigation is made of the dependence of  $U_t$  on the anisotropy  $\sigma_{||}/\sigma_{\perp}$  of the electrical conductivity and the dielectric anisotropy  $\Delta\epsilon$  for planar and homeotropic orientations. The theoretical dependences  $U_t(\sigma_{||}/\sigma_{\perp})$  and  $U_t(\Delta\epsilon)$  are compared with experimental data.

## 2. RESULTS OF CALCULATIONS (PLANAR ORIENTATION)

Figures 1 and 4–6 (see below) show the results of a calculation of the dependence of the threshold voltage on the coefficients of viscosity and of elasticity, the anisotropy of the electrical conductivity, and the dielectric anisotropy of MBBA; they were obtained on a computer by means of the program<sup>[7]</sup>. The values of the parameters of MBBA used in the calculations were taken from papers<sup>[5–9]</sup> and their references. We determined the viscosity coefficient  $\gamma_1$  experimentally from the relaxation time of the Fredericksz effect for homeotropic orientation of the NLC (the B-effect<sup>[11]</sup>);  $\gamma_1$  at 25°C was found to be 1.1 poise, and not 0.76 poise as follows from<sup>[9]</sup>. Correspondingly, the following viscosity coefficients also differ slightly from the literature values:  $\alpha_2 = -1.12$ ;  $\eta_2 = 1.2$ ;  $\gamma_2 = -1.13$  poise.

We remark that the threshold of EHD instability de-

FIG. 1. Calculated dependences of the threshold voltage for EHD instability on the viscosity coefficients  $\alpha_2$  (curve 1) and  $\eta_2$  (curve 2) and on the elastic-coefficient ratio  $K_{33}/K_{11}$  (curve 3) (planar orientation).  $\epsilon_{||} = 4.70$ ;  $\epsilon_{\perp} = 2.52$ ;  $\sigma_{||}/\sigma_{\perp} = 1.5$ ;  $K_{11} = 0.677 \times 10^{-7}$ ;  $K_{33}/K_{11} = 1.22$  (curves 1, 2);  $\alpha_1 = 0.07$ ;  $\alpha_2 = -1.12$  (curves 2, 3);  $\alpha_3 = -0.012$ ;  $\eta_2 = 1.21$  (curve 3);  $\alpha_4 = 0.84$  (curve 2);  $\alpha_5 = 0.46$  (curve 2). The coefficients of viscosity and of elasticity are everywhere given in poises and in dynes, respectively.



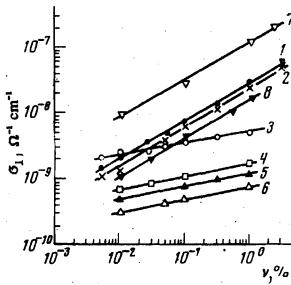


FIG. 2

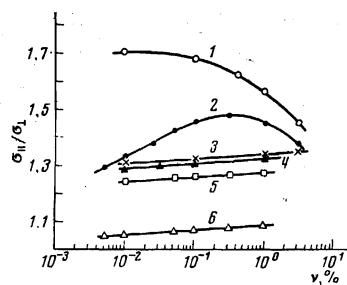


FIG. 3

FIG. 2. Dependence of electrical conductivity  $\sigma_1$  of mixtures A (1-6) and B (7, 8) on impurity concentration ( $T = 25^\circ\text{C}$ ,  $f = 1 \text{ kHz}$ ). Impurities: 1, TBAP; 2 and 8, TBAB; 3 and 7, TCE; 4, FDA; 5, DCDCBQ; 6, TCQM.

FIG. 3. Dependence of anisotropy of electrical conductivity ( $\sigma_2/\sigma_1$ ) of mixture A on impurity concentration ( $T = 25^\circ\text{C}$ ,  $f = 1 \text{ kHz}$ ). Impurities: curve 1, TBAB; 2, TBAP; 3, FDA; 4, DCDCBQ; 5, TCQM; 6, TCE.

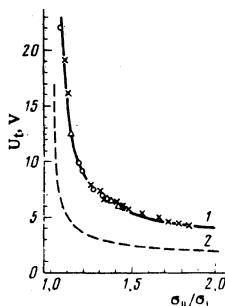


FIG. 4

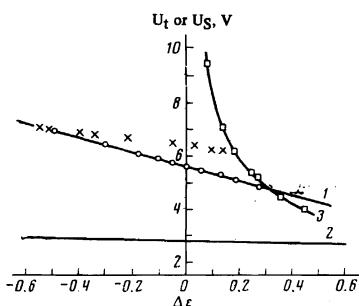


FIG. 5

FIG. 4. Dependence of threshold voltage for EHD instability on anisotropy of electrical conductivity (planar orientation,  $T = 25^\circ\text{C}$ ). Curve 1, calculation by the exact theory:  $\epsilon_{||} = 4.7$ ;  $\epsilon_{\perp} = 5.25$ ;  $K_{11} = 6.77 \times 10^{-7}$ ;  $K_{33}/K_{11} = 1.22$ ;  $\alpha_1 = 0.07$ ;  $\alpha_2 = -1.12$ ;  $\alpha_3 = -0.012$ ;  $\eta_2 = 1.21$ . Curve 2, calculation by equation (1) with the same parameters. Experimental data ( $25^\circ\text{C}$ ): ○, MBBA; X, mixture A; △, mixture B.

FIG. 5. Dependence of threshold voltage for EHD instability ( $U_t$ ) and for S-effect ( $U_S$ ) on dielectric anisotropy (planar orientation,  $T = 25^\circ\text{C}$ ). Curves 1 and 2, calculation of  $U_t$  in accordance with the exact theory and with equation (1) ( $\sigma_2/\sigma_1 = 1.3$ ; other parameters, see under Fig. 4). Experimental data on  $U_t$ : ○, for MBBA; X, for mixture A. Curve 3, calculation of  $U_S$  with  $K_{11} = 6.77 \times 10^{-7}$ ; □, experimental data on  $U_S$  for MBBA.

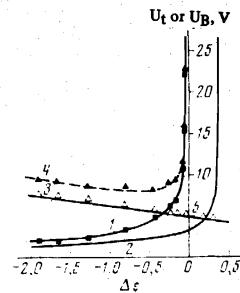
pends most strongly on the Leslie viscosity coefficients  $\alpha_2$ ,  $\alpha_4$ , and  $\alpha_5$ , the elastic coefficient  $K_{11}$  ( $U_t \sim \sqrt{K_{11}}$ ), the ratio  $K_{33}/K_{11}$ , and the anisotropy  $\sigma_2/\sigma_1$  of the electrical conductivity. The viscosity coefficients  $\alpha_1$  and  $\alpha_3$  have little influence on the value of  $U_t$ : with increase of  $\alpha_3$  from  $-0.05$  to  $+0.05$  P,  $U_t$  decreases by 5%; and with change of  $\alpha_1$  from 0 to 0.1 P,  $U_t$  remains practically constant. The calculation shows also that the threshold voltage decreases appreciably on increase of the mean value  $\epsilon = \epsilon_{||} \approx \epsilon_{\perp}$  of the dielectric permittivity. Thus for  $\epsilon_{||} = 4$  we have  $U_t = 6.0$  V, while for  $\epsilon_{||} = 12$ ,  $U_t = 3.25$  V ( $\Delta\epsilon = -0.55$ ; the other parameters are indicated in the caption to Fig. 1).

In Figs. 4-6 (see below), for comparison with the results of the exact calculation, are given also the curves  $U_t(\sigma_2/\sigma_1)$  and  $U_t(\Delta\epsilon)$  constructed according to equations (1) and (2).

### 3. EXPERIMENTAL METHOD

The experimental investigations of EHD instability were made on cells of the usual construction, with

FIG. 6. Dependence of threshold voltage for EHD instability ( $U_t$ ) and for B-effect ( $U_B$ ) on dielectric anisotropy (homeotropic and planar orientations,  $T = 25^\circ\text{C}$ , mixture B). Curve 1, calculation of  $U_B$  for  $K_{33} = 8.26 \times 10^{-7}$ ; ■, experimental data on  $U_B$ . Curve 2, calculation of  $U_t$  by equation (2) ( $\sigma_2/\sigma_1 = 1.47$ ,  $\eta_1 = 0.24$ ; other parameters, see under Fig. 4). Curve 3, calculation of  $U_t$  by exact theory for planar orientation ( $\sigma_2/\sigma_1 = 1.47$ ; other parameters, see under Fig. 4); △, experimental data on  $U_t$  (planar orientation).



transparent electrodes of  $\text{SnO}_2$ . The thickness of the NLC layer ( $d = 20 \mu\text{m}$ ) was chosen small enough to achieve good orientation, and at the same time large enough to avoid the specific effects characteristic of thin cells<sup>[10]</sup>. Planar orientation of the NLC was achieved by rubbing the electrodes, homeotropic by treating the electrodes with a hydrophobizing liquid. The frequency of the supply voltage was 20 Hz; this practically eliminates the influence of injection processes and satisfies the condition  $\omega \ll \omega_0$ , as was verified in every experiment by measurement of the frequency dependence of  $U_t$ . The thresholds for EHD instability, for the B-effect, and for the S-effect (the Fredericksz effect for planar orientation of the NLC<sup>[11]</sup>) were recorded on the basis of the change of optical transmission of the cell for a helium-neon laser beam ( $\lambda = 632.8 \text{ nm}$ ), and also visually with the aid of a polarizing microscope. Depending on the angle between the polarization vector  $e$  of the light and the director  $L$  of the NLC, what was recorded was either the threshold for formation of Williams domains ( $e \parallel L$ ) or the threshold for turbulent motion ( $e \perp L$ ). The threshold voltages were determined with accuracy  $\pm 0.1$  V.

The investigations were made on p-n-methoxybenzylidene-p'-butylaniline (MBBA); on a mixture of the azoxy-compounds p-n-butyl-p'-methoxyazoxybenzene and p-n-butyl-p'-heptanoiloxyazoxybenzene in the ratio 2:1 (mixture A), with nematic interval  $-5$  to  $+75^\circ\text{C}$  and  $\Delta\epsilon = -0.40$  ( $25^\circ\text{C}$ ); and on a mixture of MBBA with p-n-ethoxybenzylidene-p'-butylaniline (2:1, mixture B), with nematic interval  $-12$  to  $+52^\circ\text{C}$  and  $\Delta\epsilon = -0.46$  ( $25^\circ\text{C}$ ). The anisotropy of the electrical conductivity was measured by doping the NLC with impurities of three types: ionic-tetrabutylammonium picrate (TBAP) and tetrabutylammonium bromide (TBAB); acceptor-tetracyanoethylene (TCE), tetracyanoquinodimethane (TCQM), and 2,3-dichloro-5,6-dicyanobenzoquinone (DCDCBQ); and donor-p-phenylenediamine (FDA). The electrical conductivity of the initial NLC on alternating current ( $f = 1000 \text{ Hz}$ ) was  $\approx 4 \cdot 10^{-11} (\text{ohm cm})^{-1}$ .

Examples of the dependence of the electrical conductivity  $\sigma_1$  of the NLC and of the anisotropy  $\sigma_2/\sigma_1$  of the electrical conductivity on the concentration of doping impurities,  $\nu$ , are given in Figs. 2 and 3. The  $\sigma_1(\nu)$  and  $\sigma_2/\sigma_1(\nu)$  curves were obtained for all the impurities enumerated above, in all the materials investigated, and were discussed in detail earlier<sup>[11]</sup>. It is important that by doping (with one or two impurities) one can obtain an arbitrary value, prescribed in advance, of  $\sigma_2/\sigma_1$ , within the limits 1.05-1.85 for mixture A, 1.05-1.5 for MBBA and mixture B.

Strong doping with electrically conducting impurities

( $\nu > 10^{-2}\%$ ) leads to a change of the dielectric properties of the NLC. Therefore in the investigation of the threshold characteristics, we restricted ourselves to moderate doping ( $\nu < 10^{-2}\%$ ). For measurement of  $\Delta\epsilon$  at a prescribed ratio  $\sigma_{||}/\sigma_{\perp}$ , we used as additives (p'-cyanophenyl ester of p-n-heptylbenzoic acid (CEHBA),  $\Delta\epsilon \approx +20$ ) or large negative (p-n-amyl-p'-ethoxy- $\alpha$ -cyanostilbene (AECS),  $\Delta\epsilon \approx -6$ ) dielectric anisotropy. (In individual cases the addition of AECS reached 50%; here there was a change of  $\sigma_{||}/\sigma_{\perp}$  and of other parameters.)

We considered it extremely important, in the investigation of the dependence on  $\Delta\epsilon$  and  $\sigma_{||}/\sigma_{\perp}$ , to maintain invariant all the viscosity and elasticity coefficients of the NLC. Experimentally, however, it was possible to monitor only two of these:  $K_{33}$  and  $\gamma_1$ . For this purpose on the doped mixtures with known values of  $\Delta\epsilon$  careful determinations were made of the threshold and relaxation time of the B-effect, and also of the relaxation time of dynamic light scattering (in the case  $\Delta\epsilon < 0$ ). It can be definitely stated that for the concentrations of all doping impurities used by us, the coefficients  $K_{33}$  and  $\gamma_1$  remained constant within the accuracy of the experiment. This provides a basis for supposing that the other components of the viscosity and elasticity tensors also remained practically unchanged on doping.

#### 4. RESULTS OF INVESTIGATION OF THRESHOLD OF EHD INSTABILITY

##### A. Planar Orientation

The experimental dependence of the threshold voltage  $U_t$  for domain instability on the anisotropy  $\sigma_{||}/\sigma_{\perp}$  of the electrical conductivity is shown in Fig. 4. The points on the graph correspond to results of measurements on MBBA, mixture A, and mixture B, for constant dielectric and visco-elastic properties of each of the materials. It is evident that the experimental data, within the limits of experimental error, are described well by the exact theory and are significantly higher than the threshold according to Helfrich. The points fall on a single curve, constructed for MBBA, apparently in consequence of the accidental compensation of the influence of different NLC parameters determining  $U_t$  when  $\Delta\epsilon \approx -0.5$ ; see Fig. 5.

Figure 5 gives the experimental dependence of  $U_t$  on the value of the dielectric anisotropy  $\Delta\epsilon$  for MBBA and for mixture A. Here the anisotropy of the electrical conductivity was maintained constant:  $\sigma_{||}/\sigma_{\perp} = 1.3$ . As is evident from the figure, the experimental  $U_t(\Delta\epsilon)$  dependence for MBBA is in quantitative agreement with the exact theory. For mixture A the  $U_t(\Delta\epsilon)$  dependence is weaker<sup>[12]</sup>; this is apparently explained by a certain difference between the viscosity and elasticity coefficients of these two materials.

The exact theory for planar orientation of the NLC predicts no singularities for the  $U_t(\Delta\epsilon)$  dependence in the range  $\Delta\epsilon \geq 0$ . Experimentally we observed EHD instability for positive dielectric anisotropy<sup>[1]</sup> up to  $\Delta\epsilon_1$ , where  $\Delta\epsilon_1$  is determined by the threshold of the S-effect (Fig. 5, curve 3) for the given NLC:

$$U_s = \pi \sqrt{4\pi K_{33}/\Delta\epsilon}. \quad (3)$$

For MBBA at  $\sigma_{||}/\sigma_{\perp} = 1.3$  we have  $\Delta\epsilon_1 \approx +0.3$ . In the region of small positive  $\Delta\epsilon \lesssim 0.1$ , with increase of voltage there are observed successively stationary

Williams domains, and then turbulent motion of the NLC. For  $0.1 \lesssim \Delta\epsilon < \epsilon_1$  there are only Williams domains, because before turbulence is reached there occurs a reorientation of the NLC (S-effect) to a quasi-homoeotropic structure. In the range  $\Delta\epsilon > \Delta\epsilon_1$ , with increase of voltage there occurs only reorientation of the NLC; stationary EHD instability does not occur because of the stabilizing role of the dielectric moment. In this case there is observed only nonstationary motion of the liquid during the process of reorientation of the molecules; it disappears at the instant when the director settles along the direction of the field.

From the theory it follows, as was mentioned above, that  $U_t$  can be significantly decreased by increasing the mean dielectric permittivity  $\bar{\epsilon}$  and decreasing the elastic coefficients of the NLC. By addition to mixture A of NLC's with large positive (a three-component mixture of p'cyanophenyl esters of p-n-alkylbenzoic acids,  $\Delta\epsilon \approx +22$ ) and large negative (p-n-amyl-p'-ethoxy- $\alpha$ -cyanostilbene,  $\Delta\epsilon \approx -6$ ) dielectric anisotropy, we obtained a material with  $\epsilon_{||} = 9.2$ ,  $\epsilon_{\perp} = 9.37$  and with elastic coefficient  $K_{11} \approx 5 \times 10^{-8}$  dyn ( $T = 25^\circ\text{C}$ ). In fact, the threshold of domain instability for this NLC ( $\sigma_{||}/\sigma_{\perp} = 1.26$ ) was found to be 2.5 V; this corroborates the predictions of the theory.

##### B. Homeotropic Orientation

In this case we were interested in the problem of the interconnection between the threshold for EHD instability and the threshold of the B-effect over a wide range of values of  $\Delta\epsilon$ . Figure 6 gives experimental values of the threshold  $U_B$  of the B-effect for the doped mixture B; they coincide with the calculated curve 1, obtained from the formula

$$U_B = \pi \sqrt{4\pi K_{33}/\Delta\epsilon} \quad (4)$$

with  $K_{33} = 8.26 \cdot 10^{-7}$  dyn. Curve 2 is constructed on the basis of equation (2) and predicts the occurrence of EHD instability at voltages below the threshold of the B-effect. Numerical calculation with allowance for boundary conditions in the Meyerhofer approximation<sup>[6]</sup>, in which the wave vector of the deformations in the field direction ( $z$ ) is assumed equal to  $k_z = \pi/d$ , shows that the threshold for EHD instability coincides with the threshold of the B-effect in the range  $-2 \leq \Delta\epsilon < \Delta\epsilon_2$ , where  $|\Delta\epsilon_2| \approx \frac{1}{2}(K_{33}/K_{11})(\sigma_{||}/\sigma_{\perp})(\alpha_3/\alpha_2)^2$  ( $\approx 3 \cdot 10^{-4}$  for MBBA), and in the range  $\Delta\epsilon_2 < \Delta\epsilon \lesssim 0$  we have  $U_t < U_B$ . In particular, for  $\Delta\epsilon = 0$ ,  $U_t = 242$  V; here deformations should occur with wave vector  $k_z \ll k_x = 14\pi/d$ . It can be shown that the Meyerhofer approximation is rigorous under the conditions  $k_x d \rightarrow \infty$  and  $|\Delta\epsilon| \gtrsim |\alpha_3/\alpha_2| \bar{\epsilon}$  ( $\gtrsim 0.1$  for MBBA). Since at  $\Delta\epsilon = 0$  the second condition is not satisfied, at this point we carried out a numerical calculation of  $U_t$  according to the exact program<sup>[7b]</sup> and obtained the value<sup>[2]</sup>  $U_t = 86.4$  V at  $k_x = 14\pi/d$ . It should be noted that the value of  $U_t$  at  $\Delta\epsilon \gtrsim 0$  is strongly dependent on the value of  $\alpha_3$ .

Figure 6 gives, for comparison, the two experimental dependences of  $U_t$  on  $\Delta\epsilon$  for mixture B with initial planar and homeotropic orientations. For the planar orientation the experimental points are close to the theoretical curve 3. In the case of initial homeotropic orientation, stationary EHD instability occurs at voltages substantially above the threshold of the B-effect; this is, from a quasiplanar orientation. Its threshold somewhat exceeds  $U_t$  for planar orientation

(apparently because of a drop of part of the external voltage at the homeotropic layers next to the electrodes). In the region of small  $\Delta\epsilon$  ( $-0.1 \lesssim \Delta\epsilon < 0$ ),  $U_t$  coincides with the threshold of the B-effect.

On passage through the point  $\Delta\epsilon = 0$  into the region of positive values of  $\Delta\epsilon$ , the character of EHD instability for initial homeotropic orientation changes. In a narrow range of  $\Delta\epsilon$  near  $\Delta\epsilon = 0$ , where the values of  $U_B$  and  $U_S$  calculated from equations (3) and (4) exceed  $\approx 200$  V, the instability manifests itself in the form of a lattice structure with threshold  $80-100$  V and wave vector  $k_x \approx 5\pi/d$ , which increases somewhat with the voltage. The presence of a lattice structure with a large wave vector is in qualitative agreement with the predictions of the exact theory. In the low-voltage range, where the occurrence of instability is predicted by Helfrich's one-dimensional model (Fig. 6, curve 2), no instabilities are observed in defect-free homeotropically oriented specimens (on defects of orientation, there are nucleated at voltages of  $30-50$  V, and then propagated throughout the whole specimen, instabilities not connected with the intrinsic homeotropic structure). At sufficiently large positive  $\Delta\epsilon$ , beyond the limits of the region indicated above, the initial homeotropic orientation of the NLC is stable at arbitrary voltages.

It is easy to understand why, for  $\Delta\epsilon \approx 0$ , the one-dimensional theory predicts, for homeotropic orientation, low values of  $U_t$  that are not realized experimentally. In Helfrich's model<sup>[4]</sup>, instability occurs exclusively because of the destabilizing torque  $\alpha_3 \partial v_z / \partial x$  ( $v_z$  is the velocity of the liquid in the field direction; the velocity component  $v_x$  is disregarded). In a two-dimensional theory, the same destabilizing torque is opposed by a stabilizing torque  $-\alpha_2 \partial v_x / \partial z$ . Since  $|\alpha_2| \gg |\alpha_3|$ , instability cannot arise for  $k_x \approx k_z$ . A necessary condition for the occurrence of instability is  $\partial v_z / \partial x \geq (\alpha_2 / \alpha_3) (\partial v_x / \partial z)$ ; that is,  $k_x \approx k_z \sqrt{\alpha_2 / \alpha_3} \approx (\pi/d) \sqrt{\alpha_2 / \alpha_3} \approx 10\pi/d$ . Consequently, for  $\Delta\epsilon \approx 0$  and homeotropic orientation only long-wave-vector instability can occur, as is observed experimentally. Here the  $U_t$  obtained is quite high.

## 5. CONCLUSION

The experimental data confirm the theory of the threshold for EHD instability based on the Helfrich model<sup>[4]</sup> with allowance for boundary conditions (the two-dimensional model<sup>[7,8]</sup>). The one-dimensional model in the case of planar orientation gives much too low values of the threshold, and in the case of homeotropic orientation predicts an instability not observable experimentally, with a relatively low threshold in the range of small  $\Delta\epsilon$  (positive and negative).

For the case of planar orientation, the exact theory predicts (and this is confirmed experimentally) the presence of stationary periodic motions, with  $\sigma_{||}/\sigma_{\perp} > 1$ , over a wide range of  $\Delta\epsilon$  from large negative values to a positive value  $\Delta\epsilon_1$  determined by the S-effect; the point  $\Delta\epsilon = 0$  is not different in any respect.

For homeotropic orientation of the NLC, EHD instability occurs over the whole range  $\Delta\epsilon < 0$  and in the range of small positive  $\Delta\epsilon$ ; the point  $\Delta\epsilon = 0$  again is

not different in any respect. In the small- $\Delta\epsilon$  range, the instability occurs in the form of a fine lattice, whose wave vector can be calculated from the three-dimensional model. In the region of large negative  $\Delta\epsilon$ , with increase of voltage first the B-effect occurs, and then EHD instability with a threshold slightly exceeding that for planar orientation.

In order to obtain low threshold voltages for EHD instability, it is necessary in all cases to increase the values of  $\sigma_{||}/\sigma_{\perp}$ ,  $\bar{\epsilon}$ , and  $|\alpha_2|$  and to decrease the elastic coefficients  $K_{11}$  and  $K_{33}$ . For planar orientation one must choose  $\Delta\epsilon \approx 0$ , and for homeotropic  $\Delta\epsilon \approx -0.3$ .

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- <sup>1</sup>)Domain instability for  $\Delta\epsilon > 0$  was observed also in [13-15], but the dependence of  $U_t$  on  $\Delta\epsilon$  under constancy of the other parameters was not investigated.  
<sup>2</sup>)The low value given in [8],  $U_t = 23$  V at  $\Delta\epsilon = 0$ , is apparently erroneous. Our calculation for the same parameters gave  $U_t = 121.5$  V and  $k = 12.3 \pi/d$ .

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