Theory of solutions of a superfluid Fermi liquid in a superfluid Bose liquid

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A solution of an isotropic superfluid Fermi liquid in a superfluid Bose liquid is considered in the spirit of the Landau theory of a Fermi liquid. The three-fluid hydrodynamics of such a solution in the limit $\omega\tau \ll 1$ is derived on the basis of the kinetic equation obtained by Betbeder-Matibet and Nozieres for the fermion distribution function. The sound solutions of the hydrodynamic equations are obtained and the temperature dependences of the velocities of the first, second, third and two fourth sounds are investigated at all temperatures. In the high-frequency limit $\omega\tau \gg 1$ two sound solutions of the kinetic equation are obtained, one of which goes over when $T > T_c$ into the zero-sound in a solution of a normal Fermi liquid in a Bose liquid. All the physical quantities are expressed in terms of the Fermi-liquid constants. The question of the critical velocities of a solution of a superfluid Fermi liquid in a superfluid Bose liquid is investigated.

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1. INTRODUCTION

In 1972 a phase transition was discovered experimentally in liquid ^{3}He at a temperature of the order of a few millikelvin[1]. Now there can no longer be any doubt that this transition is the transition of ^{3}He to a superfluid state.

There also exists the possibility of an analogous phase transition in liquid ^{3}He dissolved in superfluid ^{4}He. The point is that a liquid solution of ^{3}He in ^{4}He does not separate into pure components, even at absolute zero temperature and normal pressure, for ^{3}He concentrations up to 6% (when the pressure is raised, the concentration can be increased to 10%). In order that ^{3}He dissolved in ^{4}He become superfluid, Cooper pairing of ^{3}He atoms is necessary. The possibility of pairing depends on the sign of λ—the scattering amplitude for mutual scattering of ^{3}He atoms dissolved in superfluid ^{4}He at small momentum transfers. If $\lambda < 0$, the ^{3}He atoms attract each other and a phase transition of ^{3}He to a superfluid state occurs at a certain temperature T_c. The calculations of λ and T_c are complicated problems. The estimates existing at present mainly lead to an anomalously small value of T_c (cf.[2] and the references therein). We shall not be interested in this question here. In the hope that a solution of a superfluid Fermi liquid in a superfluid Bose liquid will sometime become available to the experimentalists, we shall construct a semi-microscopic theory of such a solution, in the spirit of the Landau Fermi-liquid theory.

A system of hydrodynamic equations for a solution of two superfluid liquids was first obtained from phenomenological considerations in[3] (cf. also[4], where these equations are derived from the equations of motion for the anomalous averages). The propagation of the sounds in such a solution was studied in[5-7]. Unlike the usual hydrodynamics of a superfluid liquid, the hydrodynamics of a solution of two superfluid liquids was found to be a three-velocity hydrodynamics, i.e., in such a solution there are two superfluid flows, with velocities $\mathbf{v}_{s1}$, $\mathbf{v}_{s2}$, and one normal flow, with velocity $\mathbf{v}_n$. As shown in[8,9] the presence of two independent condensate phases and, correspondingly, of two superfluid velocities, follows from the particle-number conservation laws for each of the components of the solution taken separately.

A theory of a solution of a normal Fermi liquid in a superfluid Bose liquid was constructed in the spirit of the Landau Fermi-liquid theory in[10]. On the other hand, the kinetic equation in a pure superfluid Fermi liquid was obtained in[11]. It was found that, in the limit of wave-vectors much smaller than the inverse coherence length and frequencies much smaller than the size of the energy gap, this equation is a scalar equation for the quasi-particle distribution function. A superfluid Fermi liquid can be described in the framework of a two-fluid model, all the thermodynamic quantities of which are expressed in terms of the Fermi-liquid constants of the Landau theory. The system of equations obtained in[11] makes it possible to calculate the first-, second- and fourth-sound velocities in the limit $\omega\tau \ll 1$ and the zero-sound velocity in the limit $\omega\tau \gg 1$ in a superfluid Fermi liquid (cf.[12]).

The purpose of the present article is to construct an analogous theory for a solution of a superfluid Fermi liquid in a superfluid Bose liquid. By means of the kinetic equation and the expression for the energy of the Fermi quasi-particles, the three-fluid hydrodynamics is derived in the hydrodynamic regime $\omega\tau \ll 1$ and the velocities of the first, second, third and two fourth sounds are calculated. In the collisionless regime expressions are obtained for the velocities of the two high-frequency sounds, one of which goes over into the ordinary zero sound in a normal Fermi liquid when $T > T_c$. All physical quantities are expressed in terms of the Fermi-liquid constants.

The following assumptions have been used in the paper. We have assumed that the pairing of ^{3}He atoms dissolved in ^{4}He occurs in the s-state, and have considered the case of an isotropic superfluid liquid. Also, in all the calculations we have made use of the smallness of the ratio T_c/ϵ_F, without stipulating this specifically. In addition, we have neglected the contribution of excitations of the Bose type, such as the phonons (in accordance with Goldstone's theorem, there are two phonon branches in the solution under consideration) and rotons. The contribution of the latter to the thermodynamic quantities is exponentially small for $T \lesssim T_c$, while the contribution of the phonons, as we shall see, becomes important only when $T \ll T_c$. We shall confine ourselves to a qualitative study of this region, although it is also not difficult to do exact calculations using the kinetic equation for the phonons.

2. THE SYSTEM OF EQUATIONS

The system of equations for a mixture of two superfluid liquids consists of the kinetic equation (cf.[11]) for the distribution function $\nu_{\mathbf{k}}$ of the excitations

$$\frac{\partial \nu_{\mathbf{k}}}{\partial t}+\frac{\partial \nu_{\mathbf{k}}}{\partial \mathbf{r}}\frac{\partial E_{\mathbf{k}}}{\partial \mathbf{k}}-\frac{\partial \nu_{\mathbf{k}}}{\partial \mathbf{k}}\frac{\partial E_{\mathbf{k}}}{\partial \mathbf{r}}=I\{\nu\}, \tag{2.1}$$

the two continuity equations

$$\partial \rho_1/\partial t+\nabla \mathbf{j}_1=0, \tag{2.2}$$

$$\partial \rho_2/\partial t+\nabla \mathbf{j}_2=0 \tag{2.3}$$

and the two equations of the superfluid flows (in the approximation linear in the velocities)

$$\partial \mathbf{v}_{s1}/\partial t+\nabla \mu_1=0, \tag{2.4}$$

$$\partial \mathbf{v}_{s2}/\partial t+\nabla \mu_2=0. \tag{2.5}$$

In these equations and everywhere below, the index 1 refers to the Fermi component and the index 2 to the Bose component of the solution.

In order that the system (2.1)–(2.5) be closed, it is necessary to express the quasi-particle energy $E_{\mathbf{k}}$, the fluxes $\mathbf{j}_1$ and $\mathbf{j}_2$, and the chemical potentials μ_1 and μ_2 in terms of the basic variables ρ_1, ρ_2, $\mathbf{v}_{s1}$, $\mathbf{v}_{s2}$ and $\nu_{\mathbf{k}}$.

As in[11], we shall assume that the quasi-particles of the superfluid Fermi liquid are formed from quasi-particles of the normal Fermi liquid with a spectrum $\epsilon_{\mathbf{k}}$ that is unaffected by the superfluid transition by virtue of the smallness of T_c/ϵ_F. Therefore, the quasi-particle energy $E_{\mathbf{k}}$ is expressed in terms of $\epsilon_{\mathbf{k}}$ by

$$E_{\mathbf{k}}=\left[\left(\frac{\varepsilon_{\mathbf{k}}+\varepsilon_{-\mathbf{k}}}{2}\right)^2+\Delta^2\right]^{1/2}+\frac{\varepsilon_{\mathbf{k}}-\varepsilon_{-\mathbf{k}}}{2}. \tag{2.6}$$

In the following we shall use the notation

$$\xi_{\mathbf{k}}=(\varepsilon_{\mathbf{k}}+\varepsilon_{-\mathbf{k}})/2. \tag{2.7}$$

The order parameter Δ satisfies the equation

$$1=-\frac{\lambda}{2}\sum_{\mathbf{k}}\frac{1-2\nu_{\mathbf{k}}}{(\xi_{\mathbf{k}}^2+\Delta^2)^{1/2}}. \tag{2.8}$$

To calculate $\epsilon_{\mathbf{k}}$ we shall make use of a result of the paper[10], in which an expression was obtained for the energy H_p of the Fermi excitations in a solution of a normal Fermi liquid in a superfluid Bose liquid in the case when the Bose liquid flows with the superfluid velocity $\mathbf{v}_{s2}$:

$$H_{\mathbf{p}}=\varepsilon_{\mathbf{p}}^0(\rho_2)-m_1\mu_1+\mathbf{p}\mathbf{v}_{s2}\left[1-\frac{m_1}{m^*}\left(1+\frac{F_1}{3}\right)\right]+\sum_{\mathbf{p}'}f_{\mathbf{p}\mathbf{p}'}(n_{\mathbf{p}'}-n_{\mathbf{p}'}^0) \tag{2.9}$$

Here $n_{\mathbf{p}}$ is the distribution function of the normal Fermi quasi-particles and $n_{\mathbf{p}}^0$ is the equilibrium Fermi distribution at T = 0 K. The Fermi-liquid parameters are defined in the usual way:

$$F(\theta)=N_F f(\theta)=\sum_l F_l P_l(\cos\theta).$$

where $N_F = p_F m^*/\pi^2$ is the density of states at the Fermi surface, m^* is the effective mass of a Fermi excitation, defined by the equality $\partial\epsilon^0/\partial \mathbf{p} = \mathbf{p}/m^*$, $\cos\theta = \mathbf{p}\cdot\mathbf{p}/p_F^2$ and the $P_l(\cos\theta)$ are Legendre polynomials.

Inasmuch as the Fermi particles participate in superfluid flow with velocity $\mathbf{v}_{s1}$, it is necessary to define the excitation energy (2.9) in the laboratory coordinate frame but as a function of the momentum $\mathbf{k} = \mathbf{p} - m_1\mathbf{v}_{s1}$, i.e., of the momentum of the excitation in a coordinate frame moving with velocity $\mathbf{v}_{s1}$. We shall also regard the distribution function $n_{\mathbf{p}}$ as a function of k, i.e., we make the replacement $n_{\mathbf{k}+m_1\mathbf{v}_{s1}} = \tilde{n}_{\mathbf{k}}$. Changing to the momentum k in (2.9), we obtain in the approximation linear in $\mathbf{v}_{s1}$ and $\mathbf{v}_{s2}$:

$$\varepsilon_{\mathbf{k}}=H_{\mathbf{k}+m_1\mathbf{v}_{s1}}=\varepsilon_{\mathbf{k}}^0(\rho_2)-\mu_1 m_1+\mathbf{k}\mathbf{w}+\sum_{\mathbf{k}'}f_{\mathbf{k},\mathbf{k}'}(\tilde{n}_{\mathbf{k}'}-n_{\mathbf{k}'}^0); \tag{2.10}$$

$$\mathbf{w}=\mathbf{v}_{s1}\frac{m_1}{m_0^*}+\mathbf{v}_{s2}\left(1-\frac{m_1}{m_0^*}\right),\quad \frac{m^*}{m_0^*}=1+\frac{F_1}{3}. \tag{2.11}$$

The distribution function $\tilde{n}$ of the normal Fermi quasi-particles is expressed in terms of the distribution function ν of the superfluid quasi-particles by means of the Bogolyubov transformation:

$$\tilde{n}_{\mathbf{k}}=\frac{1}{2}\left[\frac{\xi_{\mathbf{k}}}{(\xi_{\mathbf{k}}^2+\Delta^2)^{1/2}}(\nu_{\mathbf{k}}+\nu_{-\mathbf{k}}-1)+(\nu_{\mathbf{k}}-\nu_{-\mathbf{k}}+1)\right]. \tag{2.12}$$

Eqs. (2.6)–(2.8), (2.10) and (2.12) enable us to obtain, in the linear approximation, the relation between the deviation $\delta E_{\mathbf{k}}$ of the Fermi quasi-particle energy from its equilibrium value and $\delta\mu_1$, $\delta\rho_2$, $\delta\nu_{\mathbf{k}}$, $\mathbf{v}_{s1}$ and $\mathbf{v}_{s2}$:

$$\delta E_{\mathbf{k}}=-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\frac{m_1\delta\mu_1-(\partial\varepsilon_{\mathbf{k}}^0/\partial\rho_2)\delta\rho_2}{1+F_0\chi}+\mathbf{k}\mathbf{w}+\sum_{\mathbf{k}'}g_{\mathbf{k},\mathbf{k}'}\frac{\delta\nu_{\mathbf{k}'}+\delta\nu_{-\mathbf{k}'}}{2}$$
$$+\sum_{\mathbf{k}'}f_{\mathbf{k},\mathbf{k}'}\frac{\delta\nu_{\mathbf{k}'}-\delta\nu_{-\mathbf{k}'}}{2}-\frac{\Delta^2}{N_F\chi E_{\mathbf{k}}}\sum_{\mathbf{k}'}\frac{\delta\nu_{\mathbf{k}'}}{E_{\mathbf{k}'}}. \tag{2.13}$$

Here,

$$g_{\mathbf{k},\mathbf{k}'}=\frac{\xi_{\mathbf{k}}\xi_{\mathbf{k}'}}{E_{\mathbf{k}}E_{\mathbf{k}'}}\sum_l\frac{f_l P_l(\cos\theta)}{1+F_l\chi/(2l+1)}, \tag{2.14}$$

$$\chi=\frac{1}{2N_F}\sum_{\mathbf{k}}\frac{\Delta^2}{E_{\mathbf{k}}^3}\operatorname{th}\frac{E_{\mathbf{k}}}{2T}, \tag{2.15}$$

where χ varies from 1 to 0 as the temperature varies from 0 to T_c.

The variation $\delta\mu_1$ of the chemical potential can be expressed in terms of the basic variables by means of the relation

$$\delta\rho_1=m_1\sum_{\mathbf{k}}\delta\tilde{n}_{\mathbf{k}}=\frac{m_1}{1+F_0\chi}\left[\sum_{\mathbf{k}}\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\delta\nu_{\mathbf{k}}+N_F\chi\left(m_1\delta\mu_1-\frac{\partial\varepsilon_F}{\partial\rho_2}\delta\rho_2\right)\right]. \tag{2.16}$$

For the total flux j and the Fermi-particle flux $\mathbf{j}_1$ we have the following expressions:

$$\mathbf{j}=\mathbf{j}_1+\mathbf{j}_2=\rho_1\mathbf{v}_{s1}+\rho_2\mathbf{v}_{s2}+\sum_{\mathbf{k}}\mathbf{k}\tilde{n}_{\mathbf{k}},\quad \mathbf{j}_1=\sum_{\mathbf{k}}\frac{\partial\varepsilon_{\mathbf{k}}}{\partial\mathbf{k}}\tilde{n}_{\mathbf{k}},$$

which, in the basic variables, are written in the form

$$\mathbf{j}=\rho_1\mathbf{v}_{s1}+\rho_2\mathbf{v}_{s2}+\sum_{\mathbf{k}}\mathbf{k}\delta\nu_{\mathbf{k}}, \tag{2.17}$$

$$\mathbf{j}_1=\rho_1\mathbf{w}+\frac{m_1}{m^*}\sum_{\mathbf{k}}\mathbf{k}\delta\nu_{\mathbf{k}}. \tag{2.18}$$

It remains for us to find an expression for $\delta\mu_2$. Since $\delta\mu_2$ does not depend on $\mathbf{v}_{s1}$, $\mathbf{v}_{s2}$ in the linear approximation, to calculate it it is sufficient to make use of the expression for the total energy of the system for $\mathbf{v}_{s1} = \mathbf{v}_{s2} = 0$:

$$E=E_0(\rho_2)+\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}^0(\rho_2)(n_{\mathbf{k}}-n_{\mathbf{k}}^0)+\frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'}f_{\mathbf{k},\mathbf{k}'}(n_{\mathbf{k}}-n_{\mathbf{k}}^0)(n_{\mathbf{k}'}-n_{\mathbf{k}'}^0).$$

Hence

$$\delta\mu_2=\delta\left(\frac{\partial E}{\partial\rho_2}\right)\approx\frac{\partial^2E_0}{\partial\rho_2^2}\delta\rho_2+\frac{1}{m_1}\frac{\partial\varepsilon_F}{\partial\rho_2}\delta\rho_1+\frac{1}{v_F}\frac{\partial v_F}{\partial\rho_2}\sum_{\mathbf{k}}\frac{\xi_{\mathbf{k}}^2}{E_{\mathbf{k}}}\delta\nu_{\mathbf{k}}. \tag{2.19}$$

Thus, Eqs. (2.1)–(2.5) together with (2.13) and (2.16)–(2.19) form a closed system of equations for the basic variables.

3. THE HYDRODYNAMIC EQUATIONS

In the low-frequency regime $\omega\tau \ll 1$ the quasi-particle distribution function $\nu_{\mathbf{k}}(\mathbf{r},t)$ in the vicinity of any point is the local equilibrium distribution function, i.e., it depends on the local energy, the temperature and the velocity $\mathbf{v}_n$ of the normal component. Therefore for $\delta\nu_{\mathbf{k}}$ we can immediately write

$$\delta\nu_{\mathbf{k}}=\frac{\partial\nu_{\mathbf{k}}^0}{\partial E_{\mathbf{k}}}\left(\delta E_{\mathbf{k}}-\mathbf{k}\mathbf{v}_n-E_{\mathbf{k}}\frac{\delta T}{T}\right), \tag{3.1}$$

$$\nu_{\mathbf{k}}^0=\frac{1}{2}\left(1-\operatorname{th}\frac{E_{\mathbf{k}}}{2T}\right). \tag{3.2}$$

Since $\delta\nu_{\mathbf{k}}$ has only two spherical harmonics in the hydrodynamic limit (the others are small in the parameter $\omega\tau \ll 1$), the expression for $\delta E_{\mathbf{k}}$ is substantially simplified, since it now also contains only the two spherical harmonics of the function $f = f_0 + f_1\cos\theta$ and the others vanish on integration over the angles. Using (2.13)–(2.16), we obtain

$$\begin{aligned}\delta E_{\mathbf{k}}=&-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\left(m_1\delta\mu_1-\frac{\partial\varepsilon_F}{\partial\rho_2}\delta\rho_2-\frac{f_0}{m_1}\delta\rho_1\right)\\&+\mathbf{k}\left(\mathbf{w}+\frac{F_1m_1}{3\rho_1m^*}\sum_{\mathbf{k}'}\mathbf{k}'\delta\nu_{\mathbf{k}'}\right)-\frac{\Delta^2}{N_F\gamma E_{\mathbf{k}}}\sum_{\mathbf{k}'}\frac{\delta\nu_{\mathbf{k}'}}{E_{\mathbf{k}'}}.\end{aligned} \tag{3.3}$$

The expressions (3.1)–(3.3) enables us to change from the initial set of basic variables ρ_1, ρ_2, $\nu_{\mathbf{k}}$, $\mathbf{v}_{s1}$, $\mathbf{v}_{s2}$ to the set of hydrodynamic variables ρ_1, ρ_2, $\mathbf{v}_{s1}$, $\mathbf{v}_{s2}$, $\mathbf{v}_n$, T.

The behavior of a solution of two superfluid liquids in the low-frequency regime is described by a system of six hydrodynamic equations, of which four are Eqs. (2.2)–(2.5) and the other two—the equations for the momentum $\mathbf{j}$ and the entropy density S—are obtained in the standard way from the kinetic equation (2.1), with the aid of (3.1):

$$\frac{\partial\mathbf{j}}{\partial t}+\nabla p=0,\qquad \frac{\partial S}{\partial t}+\nabla(S\mathbf{v}_n)=0. \tag{3.4}$$

The pressure p is determined from the thermodynamic identity

$$\delta p=\rho_1\delta\mu_1+\rho_2\delta\mu_2+S\delta T, \tag{3.5}$$

and μ_1, μ_2 and S are functions of ρ_1, ρ_2 and T:

$$S=-\frac{1}{3T}\sum_{\mathbf{k}}\mathbf{k}\frac{\partial\xi_{\mathbf{k}}}{\partial\mathbf{k}}\xi_{\mathbf{k}}\frac{\partial\nu_{\mathbf{k}}^0}{\partial E_{\mathbf{k}}}=-\frac{N_F}{T}\int_{-\infty}^{\infty}d\xi\cdot\xi^2\frac{\partial\nu^0}{\partial E}; \tag{3.6}$$

$$\begin{aligned}\delta\mu_1&=\frac{\partial\mu_1}{\partial\rho_1}\delta\rho_1+\frac{\partial\mu_1}{\partial\rho_2}\delta\rho_2+\frac{\partial\mu_1}{\partial T}\delta T,\\ \delta\mu_2&=\frac{\partial\mu_2}{\partial\rho_1}\delta\rho_1+\frac{\partial\mu_2}{\partial\rho_2}\delta\rho_2+\frac{\partial\mu_2}{\partial T}\delta T.\end{aligned} \tag{3.7}$$

We shall write out expressions for the derivatives of the chemical potentials with respect to the thermodynamic variables, which are obtained from (2.16) and (2.19) using (3.1):

$$\begin{aligned}&\frac{\partial\mu_1}{\partial\rho_1}=\frac{1+F_0}{m_1^2N_F},\quad \frac{\partial\mu_2}{\partial\rho_2}=\frac{\partial^2E_0}{\partial\rho_2^2},\quad \frac{\partial\mu_1}{\partial\rho_2}=\frac{\partial\mu_2}{\partial\rho_1}=\frac{1}{m_1}\frac{\partial\varepsilon_F}{\partial\rho_2},\\ &\frac{\partial\mu_1}{\partial T}=-\frac{\partial S}{\partial\rho_1}\approx-\frac{S}{N_F}\frac{\partial N_F}{\partial\rho_1},\quad \frac{\partial\mu_2}{\partial T}=-\frac{\partial S}{\partial\rho_2}\approx-\frac{S}{N_F}\frac{\partial N_F}{\partial\rho_2}.\end{aligned} \tag{3.8}$$

Of the quantities appearing in the system of hydrodynamic equations, it only remains for us to obtain expressions for the currents $\mathbf{j}_1$ and $\mathbf{j}_2$. We first find the momentum carried by the excitations:

$$\mathbf{P}=\sum_{\mathbf{k}}\mathbf{k}\delta\nu_{\mathbf{k}}=\sum_{\mathbf{k}}\mathbf{k}\frac{\partial\nu_{\mathbf{k}}^0}{\partial E_{\mathbf{k}}}\left(\mathbf{k}\mathbf{w}-\mathbf{k}\mathbf{v}_n+\frac{F_1m_1}{3\rho_1m^*}\mathbf{k}\mathbf{P}\right).$$

Hence

$$\mathbf{P}=(\mathbf{v}_n-\mathbf{w})\frac{\rho_1m^*}{m_1}\frac{\varphi}{1+\varphi F_1/3}, \tag{3.9}$$

$$\varphi=-\frac{m_1}{3\rho_1m^*}\sum_{\mathbf{k}}k^2\frac{\partial\nu_{\mathbf{k}}^0}{\partial E_{\mathbf{k}}}=-\int_{-\infty}^{\infty}d\xi\frac{\partial\nu^0}{\partial E}; \tag{3.10}$$

φ varies from 0 to 1 as T varies from 0 K to T_c.

Substituting (3.9) into (2.17) and (2.18), we obtain

$$\mathbf{j}_1=\rho_{s1}\mathbf{v}_{s1}+\tilde\rho_s\mathbf{v}_{s2}+\rho_{n1}\mathbf{v}_n,\quad \mathbf{j}_2=\tilde\rho_s\mathbf{v}_{s1}+\rho_{s2}\mathbf{v}_{s2}+\rho_{n2}\mathbf{v}_n. \tag{3.11}$$

The coefficients multiplying the velocities satisfy the relations

$$\rho_{s1}+\tilde\rho_s+\rho_{n1}=\rho_1=m_1N_1,\quad \rho_{s2}+\tilde\rho_s+\rho_{n2}=\rho_2=m_2N_2 \tag{3.12}$$

and are equal to

$$\begin{aligned}&\rho_{s1}=\frac{m_1^2}{m_0^*}N_{s1},\quad \tilde\rho_s=\frac{m_1(m_0^*-m_1)}{m_0^*}N_{s1},\\ &\rho_{n1}=m_1(N_1-N_{s1}),\quad \rho_{n2}=(m_0^*-m_1)(N_1-N_{s1});\end{aligned} \tag{3.13}$$

N_{s1} has the meaning of the density of superfluid Fermi particles and is equal to

$$N_{s1}=N_1(1-\varphi)/(1+\varphi F_1/3). \tag{3.14}$$

It can be seen from the expressions (3.11) that the flux of each component is a linear combination of the two superfluid velocities. This fact, unremarked in the papers[3-7] and first pointed out by Andreev and Bashkin[13], does not contradict, as might appear at first sight, the well-known statement of Landau and Pomeranchuk[14] that impurity atoms are not dragged along by the superfluid flow of particles of the other component. In fact, for $T > T_c$, according to the statement of Landau and Pomeranchuk, the fermions move only with the normal velocity $\mathbf{j}_1 = \rho_1\mathbf{v}_n$. The bosons take part both in the superfluid flow with velocity $\mathbf{v}_{s2}$ and in the normal flow, since part of their mass $(m_0^* - m_1)$ forms the "fur coat" of the Fermi particles:

$$\mathbf{j}_2=[\rho_2-(m_0^*-m_1)N_1]\mathbf{v}_{s2}+(m_0^*-m_1)N_1\mathbf{v}_n.$$

If $\mathbf{j}_1 = 0$, then $\mathbf{j}_2 = (\rho_2 - (m_0^* - m_1)N_1)\mathbf{v}_{s2}$.

Now suppose that $T < T_c$; we require that the flux of Fermi particles be equal to zero:

$$\mathbf{j}_1=\rho_{s1}\mathbf{v}_{s1}+\tilde\rho_s\mathbf{v}_{s2}+\rho_{n1}\mathbf{v}_n=0,$$

and then

$$\mathbf{j}_2=\left(\rho_{s2}-\frac{\tilde\rho_s^2}{\rho_{s1}}\right)\mathbf{v}_{s2}+\left(\rho_{n2}-\frac{\tilde\rho_s\rho_{n1}}{\rho_{s1}}\right)\mathbf{v}_n.$$

Using the expressions (3.13), we obtain the same result for $\mathbf{j}_2$ as when $T > T_c$:

$$\mathbf{j}_2=[\rho_2-(m_0^*-m_1)N_1]\mathbf{v}_{s2}. \tag{3.15}$$

This means that Fermi particles are not dragged along by the superfluid flux of Bose particles, and vice versa.

We note that it is possible to obtain expressions for $\tilde\rho_s$, ρ_{s1} and ρ_{s2} appearing in (3.11) without turning to a microscopic treatment, by requiring that the following conditions be fulfilled: 1) for $\mathbf{j}_1 = 0$ the flux of Bose particles does not depend on the presence of superfluidity of the Fermi component and is given by the expression (3.15), i.e., that part of the boson density that does not appear in the "jacket" of the Fermi particles takes part in this flux; 2) the mass conservation law (3.12) should be fulfilled; 3) ρ_{n1} is equal to the density of normal fermions $m_1(N_1 - N_{s1})$; 4) ρ_{n2} is equal to the mass

$(m_0^* - m_1)(N_1 - N_{s1})$ of the "boson-jacket" moving together with the normal fermions.

4. HYDRODYNAMIC SOUNDS

We shall seek a solution of the system (2.2)–(2.5), (3.4) in the form of a plane wave in which all quantities vary like $\exp[-i\omega(t-x/u)]$, where u is the sound velocity. In this case the hydrodynamic equations are written in the form

$$u\delta\rho_1=\rho_{s1}v_{s1}+\tilde{\rho}_s v_{s2}+\rho_{n1}v_n, \quad u\delta\rho_2=\tilde{\rho}_s v_{s1}+\rho_{s2}v_{s2}+\rho_{n2}v_n,$$
$$uv_{s1}=\delta\mu_1,\ uv_{s2}=\delta\mu_2, \quad u\delta S=Sv_n, \tag{4.1}$$
$$u[(\rho_{s1}+\tilde{\rho}_s)v_{s1}+(\rho_{s2}+\tilde{\rho}_s)v_{s2}+\rho_n v_n]=\delta p,$$

where $\rho_n = \rho_{n1} + \rho_{n2}$. The expressions for δp, $\delta\mu_1$ and $\delta\mu_2$ are given by formulas (3.5)–(3.7).

Eliminating the velocities v_{s1}, v_{s2} and v_n from (4.1), we obtain a system of three linear homogeneous equations for $\delta\rho_1$, $\delta\rho_2$ and δT. The condition for compatibility of these equations is that their determinant be equal to zero, and this condition will be the dispersion equation for the sound velocity. It is a third-order algebraic equation for u^2. We shall find it convenient to write the dispersion equation immediately in a form in which it can be seen easily that it factorizes (to within terms that are small in the parameter T/ϵ_F compared with the terms appearing in the product) into a product of a quadratic and a linear equation:

$$\det\begin{vmatrix} A_{1i}\dfrac{\partial\mu_i}{\partial\rho_1}-u^2 & A_{2i}\dfrac{\partial\mu_i}{\partial\rho_1} & \dfrac{A_{i1}}{S}\dfrac{\partial S}{\partial\rho_i}-\dfrac{\rho_{n1}}{\rho_n} \\ A_{1i}\dfrac{\partial\mu_i}{\partial\rho_2} & A_{2i}\dfrac{\partial\mu_i}{\partial\rho_2}-u^2 & \dfrac{A_{i2}}{S}\dfrac{\partial S}{\partial\rho_i}-\dfrac{\rho_{n2}}{\rho_n} \\ B_1 & B_2 & B_3 \end{vmatrix}=0,$$
$$B_k=\left(\frac{\rho_n}{S}\frac{\partial S}{\partial\rho_k}-(A^{-1})_{ki}\rho_{ni}\right)u^2 \quad (k=1,2), \tag{4.2}$$
$$B_3=1-\frac{\rho_n}{S^2}\frac{\partial S}{\partial T}u^2-(A^{-1})_{ik}\frac{\rho_{ni}\rho_{nk}}{\rho_n}.$$

Summation is performed over repeated indices. The matrix

$$A=\begin{pmatrix} \rho_{s1}+\dfrac{\rho_{n1}^2}{\rho_n} & \tilde{\rho}_s+\dfrac{\rho_{n1}\rho_{n2}}{\rho_n} \\ \tilde{\rho}_s+\dfrac{\rho_{n1}\rho_{n2}}{\rho_n} & \rho_{s2}+\dfrac{\rho_{n2}^2}{\rho_n} \end{pmatrix}. \tag{4.3}$$

The elements of the first two rows of the determinant (4.2) are of the same order in the temperature, and the first two elements of the third row, for any T, are small compared with the last element of this row by virtue of the smallness of T/ϵ_F. They can, therefore, be neglected, and Eq. (4.2) decomposes into two equations:

$$\det\left(A_{il}\frac{\partial\mu_l}{\partial\rho_k}-u^2\delta_{ik}\right)=0, \tag{4.4}$$

$$u^2=\frac{S^2}{\rho_n}\frac{\partial T}{\partial S}\left(1-(A^{-1})_{ik}\frac{\rho_{ni}\rho_{nk}}{\rho_n}\right). \tag{4.5}$$

Equation (4.4) determines the sound velocities u_1 and u_2:

$$u_{1,2}^2=\frac{1}{2}A_{ik}\frac{\partial\mu_i}{\partial\rho_k}\pm\left[\frac{1}{4}\left(A_{ik}\frac{\partial\mu_i}{\partial\rho_k}\right)^2-\det\left(A_{ik}\frac{\partial\mu_i}{\partial\rho_k}\right)\right]^{1/2}. \tag{4.6}$$

Coupled oscillations of the densities ρ_1 and ρ_2 propagate with the velocities u_1 and u_2. In the limit of low concentration we have

$$u_1^2=\rho_2\,\partial\mu_2/\partial\rho_2, \tag{4.7}$$

$$u_2^2=\frac{\rho_{s1}\rho_n+\rho_{n1}^2}{\rho_n}\frac{\det(\partial\mu_i/\partial\rho_k)}{\partial\mu_2/\partial\rho_2}=\frac{v_F^2}{3}\left(1+\frac{F_1}{3}\right)(1+\tilde{F}_0), \tag{4.8}$$

where $\tilde{F}_0$ is defined in the same way as in[10]:

$$\tilde{F}_0=F_0-3\alpha^2\frac{u_1^2\rho_1 m^*}{v_F^2\rho_2 m_1}, \quad \alpha=\frac{m_1\rho_2}{m^* u_1^2}\frac{\partial\mu_1}{\partial\rho_2}. \tag{4.9}$$

The expressions (4.7) and (4.8) coincide with those for the velocities of the first and second sound in a solution of a normal Fermi liquid in a superfluid Bose liquid[10]. Thus, we see that, in the approximation $T/\epsilon_F \ll 1$ under consideration, the transition of the Fermi liquid to the superfluid state has no effect on these sounds. The first sound corresponds to oscillations of the density of the Bose component of the solution and the second to density oscillations of the Fermi component. We emphasize that this separation is possible only in the limit $\rho_1 \ll \rho_2$, when $u_1 \gg u_2$.

Equation (4.5) gives the velocity of propagation of the temperature oscillations:

$$u_3^2=\frac{\rho_{s1}\rho_{s2}-\tilde{\rho}_s^2}{\rho_n\rho_1\rho_2-\rho(\rho_{n1}\rho_{n2}+\rho_n\tilde{\rho}_s)}S^2\frac{\partial T}{\partial S}. \tag{4.10}$$

In the case when we take into account only the contribution of the Fermi excitations to the normal densities, by making use of the formulas (3.13) and (3.14) we can rewrite this expression in the form

$$u_3^2=\frac{N_{s1}S^2\,\partial T/\partial S}{N_1(N_1-N_{s1})m_0^*}=\frac{m_1(1-\varphi)}{m^*\varphi\rho_1}S^2\frac{\partial T}{\partial S}.$$

The velocity u_3 corresponds to the velocity of second sound in the case of a one-component superfluid liquid. Of course, the oscillations of the temperature and densities are coupled, but this coupling is small by virtue of the smallness in the parameter T/ϵ_F of the terms that we discarded in the determinant. This fact is well-known in the case of an ordinary one-component superfluid liquid, in which the decoupling of the first- and second-sound waves occurs by virtue of the small magnitude of the thermal-broadening coefficient.

It follows from (4.10) that, as $T \to T_c$, u_3 goes to zero like

$$u_3^2\sim v_F^2\frac{N_{s1}}{N_1}\left(\frac{T_c}{\epsilon_F}\right)^2$$

At low temperatures we have $u_3^2 \sim v_F^2(T/\epsilon_F)^2$. Finally, in the region of extremely low temperatures, when the entropy $S_{ph} \sim (T/u_2)^3$ associated with the phonons of the sound propagating with velocity u_2 ($u_2 \ll u_1$ for $\rho_1 \ll \rho^2$) becomes greater than the entropy $S_F \sim m_1 p_F T^{-1/2}\Delta^{3/2}e^{-\Delta/T}$ of the Fermi excitations, we have $u_3^2 \approx u_2^2/3$.

The propagation of sounds in a solution of two superfluid liquids was first studied in[5]. However, inasmuch as the contribution of the second-sound quanta to the entropy was not taken into account in this paper (only the contribution from the first sound was considered), the expressions obtained in[5] do not apply in the phonon region of temperatures. For higher temperatures and $\rho_1 \ll \rho_2$, they can be obtained from our results if we put $\tilde{\rho}_s$ equal to zero in (4.7), (4.8) and (4.10) (cf. also[13]).

A specific property of a superfluid liquid is the presence of fourth sound, which consists of oscillations of the superfluid density in conditions when the normal component is at rest: $v_n = 0$, i.e., when the depth of penetration of the viscosity wave is greater than the dimensions of the vessel. In the case of a solution of two superfluid liquids there will naturally be two fourth sounds. Using (3.5)–(3.7), from Eqs. (4.1) we obtain for $v_n = 0$ a system of three linear homogeneous equations. The condition for compatibility of this system gives us the dispersion equation

$$u^4 - u^2 A_{ik}^s \frac{\partial \mu_k}{\partial \rho_i} + \det\left(A_{il}^s \frac{\partial \mu_l}{\partial \rho_k} \right) = 0. \qquad (4.11)$$

Here,

$$A^s = \begin{pmatrix} \rho_{s1} & \tilde{\rho}_s \\ \tilde{\rho}_s & \rho_{s2} \end{pmatrix}.$$

It can be seen from Eqs. (4.11) and (4.4) that when $T \to 0$ the velocities u_4' and u_4'' of the two fourth sounds coincide, respectively, with the velocities u_1 and u_2 of the first and second sounds. An analogous result was obtained in[6] without taking terms of the type $\tilde{\rho}_S$ into account when $\rho_1 \ll \rho_2$. As $T \to T_c$ the velocity u_4'' vanishes, as we should expect (cf. (4.11)), and the velocity u_4' goes over into the velocity of fourth sound in a solution of a normal Fermi liquid in a superfluid Bose liquid.

5. HIGH-FREQUENCY SOUNDS

In the high-frequency region $\omega\tau \gg 1$ we can neglect the collision integral in the kinetic equation (2.1). Going over to the $\mathbf{q},\omega$-representation in (2.1), we obtain the following expression for the deviation $\delta\nu_{\mathbf{k}}(\mathbf{q},\omega)$ of the distribution function from its equilibrium value:

$$\delta\nu = -\frac{\mathbf{q}\,\partial E/\partial \mathbf{k}}{\omega - \mathbf{q}\,\partial E/\partial \mathbf{k}} \frac{\partial \nu^0}{\partial E} \delta E. \qquad (5.1)$$

For simplicity we shall assume that the Fermi-liquid function $F(\theta)$ contains only the two first harmonics, i.e., $F(\theta) = F_0 + F_1 \cos\theta$. Then the expression (3.3) holds for δE. By means of Eqs. (2.2)–(2.5) and (2.16)–(2.19) we eliminate the variables $\delta\rho_1$, $\delta\rho_2$, $\delta\mu_1$, v_{s1} and v_{s2} from (3.3) and obtain the following dependence of δE on $\delta\nu$:

$$\delta E_k = \frac{1}{N_F \chi}\left[\frac{\xi_k}{E_k} - \gamma\left(\frac{\xi_k}{E_k} - \frac{N_F}{N_1} u \chi(\mathbf{kn})\right)\right] \sum_{k'} \frac{\xi_{k'}}{E_{k'}} \delta\nu_{k'} + \frac{1}{N_1}\left[\gamma u \left(\frac{\xi_k}{E_k} - \frac{N_F}{N_1} u\chi(\mathbf{kn})\right) - \frac{1}{m^*}(\mathbf{kn})\right] \sum_{k'} (\mathbf{k}'\mathbf{n}) \delta\nu_{k'}, \qquad (5.2)$$

where

$$\mathbf{n} = \mathbf{q}/q, \quad u = \omega/q,$$

$$\gamma^{-1} = (1-\chi) - \chi (u^2 - u_1^2(0))(u^2 - u_2^2(0)) \times \left\{\frac{1}{3} v_F^2 \left(1 + \frac{F_1}{3}\right)\left[u^2 - \frac{\partial \mu_2}{\partial \rho_2}(\rho_{s2}(0) - \tilde{\rho}_s^2(0)/\rho_{s1}(0))\right]\right\}^{-1}. \qquad (5.3)$$

Here, $u_1(0)$, $u_2(0)$, $\tilde{\rho}_S(0)$, $\rho_{S1}(0)$ and $\rho_{S2}(0)$ are the values of the first- and second-sound velocities and superfluid densities at $T = 0$.

The condition for compatibility of Eqs. (5.1) and (5.2) gives the dispersion equation for u:

$$(\eta_1^2 - \eta_2\eta_3)(\zeta_2\zeta_3 - \zeta_1^2) + \eta_2\zeta_2 + \eta_3\zeta_3 - 2\eta_1\zeta_1 = 1, \qquad (5.4)$$

where

$$\eta_1 = \frac{1}{2}\int_{-\infty}^{\infty} d\xi \int_{-1}^{1} d\mu \frac{\partial \nu^0}{\partial E} \frac{(\xi\mu/E)^2}{u/v_F - \mu\xi/E}, \quad \eta_2 = \frac{1}{2}\int_{-\infty}^{\infty} d\xi \int_{-1}^{1} d\mu \frac{\partial \nu^0}{\partial E} \frac{\xi\mu^2/E}{u/v_F - \mu\xi/E}, \qquad (5.5)$$

$$\eta_3 = \frac{1}{2}\int_{-\infty}^{\infty} d\xi \int_{-1}^{1} d\mu \frac{\partial \nu^0}{\partial E} \frac{\xi^2\mu/E^2}{u/v_F - \mu\xi/E};$$

$$\zeta_1 = 3\gamma \frac{u}{v_F}, \quad \zeta_2 = 3\left(1 + 3\gamma \frac{u^2}{v_F^2}\right), \quad \zeta_3 = \frac{1}{\chi}(\gamma - 1). \qquad (5.6)$$

We shall first consider Eq. (5.4) for $T \to 0$. All the η_i tend to zero, inasmuch as they contain the rapidly decreasing function $e^{-\Delta/T}$, and, therefore, Eq. (5.4) can be satisfied only when $\gamma \to \infty$. Since we shall have $\chi \to 1$ as $T \to 0$, the function γ, as can be seen from (5.3), becomes infinite when $u = u_1(0)$ and when $u = u_2(0)$. Consequently, for $T \to 0$ there are two high-frequency sounds, the velocities of which coincide with the first- and second-sound velocities.

We shall elucidate how the velocities of these high-frequency sounds vary as the temperature is raised. For simplicity, we shall consider the case of a low concentration of the Fermi component ($\rho_1 \ll \rho_2$). It is easy to show that in the limit of low concentration the velocity of one of the sounds coincides with u_1. This fact is explained by the fact that in a pure superfluid Bose liquid the velocities of the high-frequency sound and of the hydrodynamic sound coincide to within terms $(T/m_2\mu_2)^4$, which, by virtue of the smallness of $T_c/m_2\mu_2$, we do not take into account.

In order to elucidate the temperature dependence of the second high-frequency sound, we make use of the fact that its velocity $u \sim v_F \ll u_1 \approx (\rho_2 \partial\mu_2/\partial\rho_2)^{1/2}$. Then from (5.3) we obtain for γ the expression

$$\gamma^{-1} \approx 1 - \chi - \chi \frac{u^2 - {}^1/_3 v_F^2 (1 + F_1/3)(1 + F_0)}{{}^1/_3 v_F^2 (1 + F_1/3)}$$

and, substituting this into (5.4), we have, after straightforward transformations, the following dispersion equation:

$$F_0 + \frac{s^2 F_1}{1 + F_1/3} = \frac{(\eta_3 + \chi)s^2 + (\eta_2 - {}^1/_3) - 2\eta_1 s}{\eta_1^2 - (\eta_2 - {}^1/_3)(\eta_3 + \chi)}, \qquad (5.7)$$

where $s = u/v_F$.

Inasmuch as the quantities η_i depend on u in a complicated way (cf. (5.5)), it is difficult to obtain the dependence $u(T)$ in the whole temperature region $0 \le T \le T_c$. Therefore, we shall consider only the limiting cases. For $e^{-\Delta/T} \ll 1$ the velocity u, as we have already seen, tends to $u_2(0)$. In the other limiting case ($T \to T_c$), the right-hand side of Eq. (5.7) becomes equal to $w^{-1}(s)$, where

$$w(s) = \frac{s}{2} \ln\frac{s+1}{s-1} - 1,$$

and, therefore, Eq. (5.7) goes over into the equation for ordinary zero sound in a solution of a normal Fermi liquid in a superfluid Bose liquid (cf.[10]):

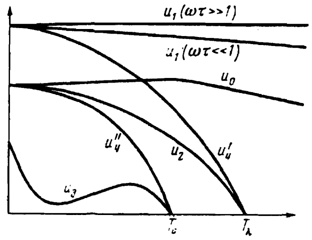
$$1 - w(s)\left(F_0 + \frac{s^2 F_1}{1 + F_1/3}\right) = 0. \qquad (5.8)$$

Thus, as the temperature is raised from 0 to T_c the velocity of the second high-frequency sound varies from u_2 to the zero-sound velocity u_0, if the latter sound exists (i.e., if Eq. (5.8) has a solution). An analogous result was obtained in[12] for a pure superfluid Fermi liquid.

The qualitative temperature dependences of the sound velocities obtained in this paper are given in the Figure. The velocity of the second high-frequency sound is denoted by u_0.

6. CRITICAL VELOCITIES

In deriving the hydrodynamic equations we used the fact that the deviation $\delta\Delta$ of the order parameter from its equilibrium value is small, i.e., $\delta\Delta/\Delta \ll 1$. To fulfill this condition we assumed that the velocities of the relative motions in the solution ($v_{s1} - v_n$ and $v_{s2} - v_n$) are small compared with Δ/p_F. It is also possible, however, to obtain hydrodynamic equations for the case when the relative velocities in the solution are comparable with Δ/p_F. For this it is necessary to write equations



not for $\mathbf{v}_{s1}$, $\mathbf{v}_{s2}$ and $\mathbf{v}_n$ but for their deviations $\delta\mathbf{v}_{si} = \mathbf{v}_{si} - \mathbf{v}_{si}^0$ and $\delta\mathbf{v}_n = \mathbf{v}_n - \mathbf{v}_n^0$ from their equilibrium values, in which case, in order to fulfil the condition $\delta\Delta/\Delta \ll 1$, we should require that $\delta\mathbf{v}_{si} \ll \mathbf{v}_{si}^0$ and $\delta\mathbf{v}_n \ll \mathbf{v}_n^0$. Then all the coefficients in the hydrodynamic equations will depend on $\mathbf{v}_{s1}^0 - \mathbf{v}_n^0$ and $\mathbf{v}_{s2}^0 - \mathbf{v}_n^0$. This dependence is important for the quantities ρ_{si}, ρ_{ni}, $\tilde{\rho}_s$ and S directly related to the order parameter Δ, and unimportant for the other quantities by virtue of the smallness of $\Delta/p_F \ll v_F$.

The dependence of the order parameter Δ on the velocities can be found by substituting the equilibrium distribution function of the Fermi quasi-particles:

$$\nu_k = \left[\exp\frac{E_k - \mathbf{k}\mathbf{v}_n}{T} + 1\right]^{-1}$$

into Eq. (2.8) for Δ. Using the expression (2.6), (2.10) for the energy we obtain the following equation for the equilibrium Δ:

$$\Delta = \frac{|\lambda|}{4}\sum_{\mathbf{k}} \frac{\Delta}{(\xi_k^2+\Delta^2)^{1/2}} \operatorname{th} \frac{(\xi_k^2+\Delta^2)^{1/2} + \mathbf{k}(\mathbf{w}_0 - \mathbf{v}_n^0)}{2T}. \qquad (6.1)$$

where

$$\mathbf{w}^0 = \frac{m_1}{m_0^*}\mathbf{v}_{s1}^0 + \frac{m_0^* - m_1}{m_0^*}\mathbf{v}_{s2}^0.$$

Thus, Δ depends on a linear combination of the velocities:

$$\mathbf{v} = \frac{m_1}{m_0^*}(\mathbf{v}_{s1}^0 - \mathbf{v}_n^0) + \frac{m_0^* - m_1}{m_0^*}(\mathbf{v}_{s2}^0 - \mathbf{v}_n^0). \qquad (6.2)$$

It can be seen from Eq. (6.1) that $\Delta(T,v)$ vanishes at a certain critical velocity $v = v_c(T)$. Here, $v_c(T)$ has the same temperature dependence as the critical velocity of a pure superconductor, varying from $e\Delta(0,0)/2p_F$ at $T = 0$ to zero at $T = T_c$.

As the order parameter Δ vanishes, so too do ρ_{s1} and $\tilde{\rho}_s$, i.e., the superfluidity of the Fermi component in the solution disappears.

We note that there is also a region of velocities

$$v_{c1}(T) < v < v_c(T),$$

in which the energy gap in the Fermi-excitation spectrum is equal to zero while the order parameter Δ is nonzero. In this region the superfluidity of the Fermi component is not destroyed. This case corresponds to gapless superconductivity in superconductors. The quantity $v_{c1}(T)$ is determined from the equation

$$\Delta(T, v_{c1}(T)) = p_F v_{c1}(T)$$

and varies from $\Delta(0,0)/p_F$ at $T = 0$ to zero at $T = T_c$.

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73