Polarization effects in the interaction between intense light and spin waves

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Polarization effects that appear in the interaction between intense light and spin waves are studied and are due to interference of first- and second-order interactions are studied. The polarization effects that occur during mixed excitation of spin waves by a homogeneous, alternating magnetic field and light, or during excitation of spin waves by nonlinear mixing of two light waves, are also investigated. Some experiments based on the application of the derived relations are discussed.

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1. For a number of magnets—ferrites and orthoferrites—and, in particular, for iron yttrium garnet (YIG), as was first discovered by Pisarev and Le Gall,[1,2] the constants of magneto- and electric dipole interactions of first order turn out to be comparable in magnitude with the magneto-optical interactions of second order. The former interactions are responsible for the Faraday rotation of the plane of polarization, the latter for the effect of double refraction of the light. The interference of these two interactions, as has been noted in [2], leads to the result that the nonlinear interaction of the light waves turns out to be very sensitive to their polarization. As will be shown below, this leads to interesting consequences in such processes as the action of light on the parametric excitation of spin waves and the excitation of spin waves by nonlinear mixing of two light waves.

2. The energy density of a uniaxial magnetodielectric can be expended in a series of the electric field intensity $\mathbf{E}$ and magnetic moment density $\mathbf{M}$:

$$F=\frac{1}{8\pi}(\varepsilon_{ik}E_iE_k+\delta_{ik}H_iH_k)+\mathbf{M}(\mathbf{H}+\alpha\nabla\mathbf{M}+\mathbf{H}^m),\qquad \varepsilon_{ik}=\varepsilon\delta_{ik}+i\zeta e_{ikl}M_l+f_{iklm}M_lM_m. \tag{1}$$

Here ϵ_{ik} is the dielectric tensor, α is the constant of exchange interaction, $\mathbf{H}^m$ is the internal magnetic field, which satisfies the relations curl $\mathbf{H}^m = 0$, $\nabla(\mathbf{H}^m + 4\pi\mathbf{M}) = 0$, e_{ijl} is a tensor that is antisymmetric in all pairs of indices; f_{ijklm} is the tensor of magneto-optic interactions of second order, which is diagonal for cubic crystals; ζ is the specific Faraday rotation. The electric field intensity $\mathbf{E}$ and the magnetic-moment density $\mathbf{M}$ we represent in the form of expansions in the canonical variables A_k and B_q:

$$\mathbf{E}=\sum_{\mathbf{k}}\mathbf{e}_{\mathbf{k}}\alpha_{\mathbf{k}}A_{\mathbf{k}}e^{i\mathbf{k}\mathbf{x}},\quad \alpha_{\mathbf{k}}^2=4\pi ck/\varepsilon^2, \tag{2}$$

$$\mathbf{m}=\sum_{\mathbf{q}}\boldsymbol{\mu}_{\mathbf{q}}(u_{\mathbf{q}}B_{\mathbf{q}}+v_{\mathbf{q}}B_{-\mathbf{q}}^*)e^{i\mathbf{q}\mathbf{x}},\quad \mathbf{m}=\mathbf{M}-\mathbf{M}_0,$$

where $\mathbf{e}_k$ and μ_q are the vectors of polarization of the light and of oscillations of the magnetic moment density with equilibrium value $\mathbf{M}_0$; $\mathbf{k}$ and $\mathbf{q}$ are the wave vectors of the electromagnetic and spin waves; c is the light velocity; u_q and v_q are the standard Goldtein-Primakoff transformation functions.[3] We choose a set of coordinates such that the $\mathbf{M}_0$ is directed along the z axis. The Hamiltonian of the system in these variables takes the form

$$\mathcal{H}=\sum_{\mathbf{k}}\nu_{\mathbf{k}}A_{\mathbf{k}}^*A_{\mathbf{k}}+\sum_{\mathbf{q}}\omega_{\mathbf{q}}B_{\mathbf{q}}B_{\mathbf{q}}^*+\sum_{\mathbf{k}\mathbf{k}_1\mathbf{q}}V_{12}(\mathbf{k}\mathbf{k}_1\mathbf{q})A_{\mathbf{k}}^*A_{\mathbf{k}_1}B_{\mathbf{q}}, \tag{3}$$

$$V_{12}(\mathbf{k}\mathbf{k}_1\mathbf{q})=\frac{\alpha_{\mathbf{k}}\alpha_{\mathbf{k}_1}}{4\pi}\{i\zeta\boldsymbol{\mu}_{\mathbf{q}}[\mathbf{e}_{\mathbf{k}}\times\mathbf{e}_{\mathbf{k}_1}]+2f_{44}M_0[e_{z\mathbf{k}}(\boldsymbol{\mu}_{\mathbf{q}}\mathbf{e}_{\mathbf{k}_1})+e_{z\mathbf{k}_1}(\boldsymbol{\mu}_{\mathbf{q}}\mathbf{e}_{\mathbf{k}})]\},\quad \nu_{\mathbf{k}}=ck/\varepsilon. \tag{4}$$

Here ω_q are the spin wave frequencies, $f_{44} = f_{iziz}$ and the other components of the tensor f_{ijklm} in the uniaxial cubic magnet do not make a contribution to the three-wave interaction processes (the constant f_{44} is determined from the Cotton-Mouton effect [1]). From the expression for the coefficient of interaction of the light with spin waves we see that its first component is asymmetric relative to the substitution $\mathbf{k} \rightleftharpoons \mathbf{k}_1$, as a consequence of the antisymmetric tensor e_{ikl} in the magneto-optical interactions of first order.

The equations of motion for the amplitudes A_k and B_q in the presence of the magnetic field $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}\cos 2\omega_0 t$, where $\mathbf{H}_0$ is the constant and $\mathbf{h}$ the alternating field, directed along $\mathbf{M}_0$ (parallel to the magnetic pump with frequency $2\omega_0$) are of the form

$$i\frac{\partial A_{\mathbf{k}}}{\partial t}=\nu_{\mathbf{k}}A_{\mathbf{k}}+\sum_{\mathbf{k}=\mathbf{k}_1+\mathbf{q}}V_{12}(\mathbf{k}\mathbf{k}_1\mathbf{q})A_{\mathbf{k}_1}B_{\mathbf{q}}, \tag{5}$$

$$i\frac{\partial B_{\mathbf{q}}}{\partial t}=\omega_{\mathbf{q}}B_{\mathbf{q}}+\sum_{\mathbf{q}=\mathbf{k}_1-\mathbf{k}}V_{12}(\mathbf{k}\mathbf{k}_1\mathbf{q})A_{\mathbf{k}}^*A_{\mathbf{k}_1}+\tilde{h}B_{-\mathbf{q}}^*,\qquad \tilde{h}=h\frac{gM_0}{4\omega_{\mathbf{q}}}\left(\alpha q^2+4\pi\frac{q_x^2+q_y^2}{q^2}\right). \tag{6}$$

We shall seek a solution of Eqs. (5)–(6) in the form

$$A_{\mathbf{k}}=a_{\mathbf{k}}(t)e^{-i\nu_{\mathbf{k}}t},\qquad B_{\mathbf{q}}=b_{\mathbf{q}}(t)e^{-i\omega_{\mathbf{q}}t}, \tag{7}$$

where $a_k(t)$ and $b_q(t)$ are slowly varying quantities. The equations for them can be obtained by substituting (7) in (5)–(6) and carrying out time averaging, excluding the rapidly oscillating components, similar to what was done earlier.[5]

3. In the process of parametric excitation of spin waves in parallel pumping, what are excited are the spin waves with $\omega_q = \omega_0$ and with wave vectors $\mathbf{q} \perp \mathbf{M}_0$. The incident light with amplitude a_0 and frequency ν_0 interacts resonantly with the spin waves if the angle between the wave vectors $\mathbf{k}_0$ and $\mathbf{k}_0 \pm \mathbf{q}$ of the incident and scattered light is equal to $\theta = \arcsin(q/2k_0)$, while the polarization vectors are perpendicular to them: $\mathbf{e}_0 \perp \mathbf{e}_1$. The linearized system of equations for the slowly changing amplitudes a_k and b_q of the resonantly interacting waves are of the following form after time averaging (see [5])

$$i\left(\frac{\partial}{\partial t}+\gamma\right)a_+=V_{as}(\mathbf{k}-\mathbf{q},\mathbf{q})a_0b_{-\mathbf{q}},\qquad i\left(\frac{\partial}{\partial t}+\gamma\right)a_-=V_s^*(\mathbf{k}-\mathbf{q},\mathbf{q})a_0b_{\mathbf{q}}^*,$$

$$\left(\frac{\partial}{\partial t}+\Gamma\right)b_{\mathbf{q}}=V_s^*(\mathbf{k}-\mathbf{q},\mathbf{q})a_-^*a_0-\tilde{h}b_{-\mathbf{q}}^*,$$

$$\left(\frac{\partial}{\partial t}+\Gamma\right) b_{-q}=V_{as}^{*}(\mathbf{k}-\mathbf{q},\mathbf{q})\,a_+a_0^{*}-\tilde{h}b_q. \qquad (8)$$

In (8), we have introduced the phenomenological coefficients of damping of the electromagnetic (γ) and spin waves (Γ), V_s and V_{as} are the coefficients of interaction for the Stokes (a_-) and antistokes waves (a_+) with frequencies $\nu_- = \nu_0 - \omega_0$ and $\nu_+ = \nu_0 + \omega_0$. It follows from (8) that the exponential increase in the bare amplitudes b_q and b_{-q} takes place if

$$\tilde{h}^2 \geqslant \Gamma^2 - \frac{1}{\gamma^2} V_s^2 V_{as}^2 |a_0|^4 + \frac{\Gamma}{\gamma}(V_{as}^2 - V_s^2)|a_0|^2. \qquad (9)$$

This is also the condition for mixed parametric excitation of spin waves by light and by the variable magnetic field. It is seen from (9) that the threshold of the mixed excitation depends essentially on the polarization of the incident electromagnetic wave. For waves with $\mathbf{e}_0 \perp \mathbf{M}_0$,

$$V_s = -\frac{\alpha_k\alpha_{k_1}}{4\pi}[\zeta - 2f_{44}M_0], \quad V_{as} = -\frac{\alpha_k\alpha_{k_1}}{4\pi}[\zeta + 2f_{44}M_0], \qquad (10)$$

and for waves with $\mathbf{e}_0 \parallel \mathbf{M}_0$,

$$V_s = \frac{\alpha_k\alpha_{k_1}}{4\pi}[\zeta + 2f_{44}M_0], \quad V_{as} = \frac{\alpha_k\alpha_{k_1}}{4\pi}[\zeta - 2f_{44}M_0]. \qquad (11)$$

If

$$\zeta - 2f_{44}M_0 \ll \zeta, \qquad (12)$$

then the Stokes component will predominate in the scattering at $\mathbf{e}_0 \parallel \mathbf{M}_0$ and the anti-Stokes component at $\mathbf{e}_0 \parallel \mathbf{M}_0$. As a consequence, a decrease occurs in the spin wave excitation threshold at $\mathbf{e}_0 \perp \mathbf{M}_0$:

$$\tilde{h}^2 \geqslant \Gamma^2 - \frac{\Gamma}{\gamma} V_s^2 |a_0|^2, \qquad (13)$$

and at $\mathbf{e}_0 \parallel \mathbf{M}_0$ there is an increase in the threshold:

$$\tilde{h}^2 \geqslant \Gamma^2 + \frac{\Gamma}{\gamma} V_{as}^2 |a_0|^2, \qquad (14)$$

i.e., at $\mathbf{e}_0 \perp \mathbf{M}_0$ the waves that interact in resonant fashion with the light cannot be parametrically excited, and at $\tilde{h} \geq \Gamma$ spin waves will be excited that are resonantly coupled with the pump and nonresonantly with the light. Thus, if the condition (12) is satisfied, one should expect, depending on the polarization of the incident light, a decrease or an increase in the threshold of parametric excitation of the spin waves resonantly coupled with the light. At an light-wave electric field intensity $a_0 \sim 10$ cgs esu, the change that takes place in the value of the threshold is comparable with the threshold itself.

4. Spin waves can be excited by nonlinear mixing of two light waves with frequencies ν_1 and ν_2 and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, such that their differences coincide with the frequency ω and wave vector $\mathbf{q}$ of the spin wave:

$$\nu_1 - \nu_2 = \omega, \ \mathbf{k}_1 - \mathbf{k}_2 = \mathbf{q}. \qquad (15)$$

For moderate constants of interaction of the light with the spin waves and at easily attainable amplitudes of the light (we shall give numerical estimates below), such a method of excitation of the spin waves can be effective enough. Naturally, the interference of the interactions of the Faraday rotation and double refraction should be especially strongly marked in this process.

Let the magnet be subjected to two beams of light with amplitudes a_1 and a_2, polarizations $\mathbf{e}_1$ and $\mathbf{e}_2$, and frequencies and wave vectors satisfying the conditions (15). Neglecting the change in the amplitudes of the light waves in the interaction process and omitting the equations for them, we represent only the equation for the amplitude of the spin wave, which follows from (8):

$$\left(\frac{\partial}{\partial t}+\Gamma\right) b_q = V_{12}(\mathbf{k}_1\mathbf{k}_2\mathbf{q})\,a_1a_2^{*}e^{i\Delta\omega t},$$

$$\Delta\omega = \nu_1(\mathbf{k}_1) - \nu_2(\mathbf{k}_2) - \omega(\mathbf{k}_1 - \mathbf{k}_2), \qquad (16)$$

where $\Delta\omega$ is the frequency shift. This equation possesses the stationary stable solution

$$b_q = \frac{V_{12}(\mathbf{k}_1\mathbf{k}_2\mathbf{q})}{\Gamma \pm i\Delta\omega} a_1a_2^{*}, \qquad (17)$$

or, in the variables of the electric field intensity and the magnetic moment,

$$m_q = \frac{u_q V_{12}(\mathbf{k}_1\mathbf{k}_2\mathbf{q})}{\alpha_{k_1}\alpha_{k_2}(\Gamma \pm i\Delta\omega)} E_1E_2^{*}. \qquad (18)$$

It is clear from (4) that the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ should be mutually perpendicular, so that the interaction coefficient can be maximal. But, if $\mathbf{e}_1 \perp \mathbf{M}_0$ and $\mathbf{e}_2 \parallel \mathbf{M}_0$, then the Faraday rotation adds up with the double-refraction interaction and the excitation of the magneto-acoustic wave will be maximal. If $\mathbf{e}_1 \parallel \mathbf{M}_0$ and $\mathbf{e}_2 \perp \mathbf{M}_2$, a difference will appear in V_{12} in the coefficients of these interactions and the excitation will be minimal. Thus, for YIG, where the constants of the Faraday rotation and double refraction can be almost equal[2] at $\mathbf{e}_1 \parallel \mathbf{M}_0$ and $\mathbf{e}_2 \perp \mathbf{M}_0$, there will be practically no excitation of the spin waves by nonlinear mixing of the light. Substituting the numerical values of the constants for YIG[3] in (18), we find that the oscillations of the magnetic moment with $m \sim 10^{-2}\,M_0$ should be observed for such amplitudes of light that $E_1E_2 \leq 1$ (cgs esu)^2, if $\mathbf{e}_1 \perp \mathbf{M}_0$ and $\mathbf{e}_2 \parallel \mathbf{M}_0$.

5. We now discuss briefly the experimental consequences of the results that have been obtained.

The sensitivity of the magneto-optical interactions to the orientation of the polarization vector of the incident light makes it possible to investigate more completely the details of the effect of light on the parametric excitation of spin waves, both experimentally[4] and theoretically.[5] In particular, it was shown in [5] that the light has a significant effect on the excitation threshold of spin waves via interaction with the elastic oscillations. Inasmuch as this interaction is insensitive to the direction of the polarization vector of the incident light, the contribution of purely magneto-optical interactions can be separated by varying the polarization of the incident light. This would allow us to carry out a more detailed comparison of theory with experiment and to separate the contributions of the various mechanisms of the effect of light on the parametric excitation of the spin waves.

The possibility mentioned above of the excitation of intense spin waves with the required frequency and wave vector by nonlinear mixing of two light waves is an attractive one. We discuss here only the single possibility of using this process for the investigation of the nonlinear relaxation of spin waves. As is seen from (17), the amplitude of the excited spin wave is proportional to the product of the amplitudes of the light waves and inversely proportional to the sum of the attenuation coefficients of the spin wave Γ and the detuning $\Delta\omega$. The directions of the incident light beams ($\mathbf{k}_1$ and $\mathbf{k}_2$) can be so chosen that the detuning vanishes. In this case the amplitude of the spin wave m_q reaches a maximum value and its magnitude is determined only by the value of the damping coefficient. For parametric excitation of spin waves in YIG[6] (and recently also in antiferromagnets[7]), the jumps in the absorption of the pump power has been observed experimentally, a possible explanation of which, proposed in [6,8], is that the mechanism of absorption of spin waves saturates at sufficiently large amplitudes and thus the attenuation coefficient depends significantly on the amplitude and falls off with its increase, in any case at small amplitudes. In the parametric excitation, a large number of spin waves with approximately equal increments come into play simultaneously, and it is therefore difficult to estimate the contribution of the collective interactions of the parametrically excited spin waves with one another and with the field of the pump in the jumps of absorped pump power. If one succeeds in establishing the excitation of a spin wave by nonlinear mixing of light waves, then in this case, the excitation of other spin waves is excluded and the dependence of the attenuation coefficient on the amplitude is determined from the relation (18):

$$\Gamma=\frac{u_{\mathbf{q}}V_{12}(\mathbf{k}_1\mathbf{k}_2\mathbf{q})}{\alpha_{\mathbf{k}_1}\alpha_{\mathbf{k}_2}m_{\mathbf{q}}}E_1E_2^*, \qquad (19)$$

which is valid also for nonlinear damping of the waves. The amplitude of the spin wave m_q can be measured from the change in the amplitude of the longitudinal magnetization:

$$\Delta M_z=M_z-M_0=-m_{\mathbf{q}}^2/M_0.$$

The departure of the relation (19) from a constant value is equal to a nonlinear contribution to the attenuation coefficient.

The intense light can produce a number of phonon phenomena (for example, heating of the sample, excitation of low-frequency elastic oscillations and so on), which determine the departure of Γ from a constant value. The sensitivity of the interaction coefficient V_{12} to a change in the polarization of the light waves allows a separation of the nonlinear contributions to Γ from the background phenomena. In fact, the value of the numerator in the right side of (19) can change without change in the intensity of the light waves, through a change in the directions of the polarization vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. Here the amplitude of the excited spin wave should change in proportion to V_{12}, while the background phenomena, which are insensitive to a change in the light polarization, will remain unchanged.

[1]R. Pisarev, I. Siniĭ and G. Smolenskiĭ, ZhETF Pis. Red. 9, 112 (1969) [JETP Lett. 9, 64 (1969)].

[2]B. Desomiere and H. Le Gall, Intern. Conf. of Magnetism, Grenoble, France, September, 1970. J. de Phys. Suppl. 32, C1-119 (1971).

[3]A. I. Akhiezer, V. G. Bar'yakhtar and S. V. Peletminskiĭ, Spinovye volny (Spin Waves) Nauka, 1967.

[4]H. Le Gall, J. P. Jamet and V. Cagan, Solid State Comm. 7, 27 (1969); H. Le Gall, J. P. Jamet, B. Desomière, J. Appl. Phys. 42, 1480 (1971); A. A. Solomko and V. I. Maĭstrenko, ZhETF Pis. Red. 14, 36 (1971) [JETP Lett. 14, 23 (1971)]; V. U. Agartanov, V. B. Antipov, V. V. Kolpakov and E. M. Fedorin, Fiz. Tverd. Tela 14, 2446 (1972) [Sov. Phys.-Solid State 14, 2116 (1973)].

[5]A. S. Bakaĭ, and G. G. Sergeeva, Zh. Eskp. Teor. Fiz. 65, 1677 (1973) [Sov. Phys.-JETP 38, 840 (1974)]; A. S. Bakaĭ, Fiz. Tverd. Tela 12, 2384 (1970 [Sov. Phys.-Solid State 12, 1903 (1971)].

[6]H. Le Gall, B. Lemaire and D. Sere, Solid. State Comm. 5, 919 (1967).

[7]L. A. Prozorova, A. S. Borovik-Romanov, ZhETF Pis. Red. 10, 316 (1969) [JETP Lett. 10, 201 (1969)]; L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teor. Fiz. 67, 1952 (1974) [Sov. Phys.-JETP 40, 970 (1975)].

[8]V. S. L'vov, O vzaimodeĭstvii parametricheski vozbuzhdennykh spinovykh voln s tepolvymi (The Interaction of Parametrically Excited Spin Waves with Thermal Waves). Preprint IYaF 69-72, Novosibirsk, 1972; V. S. L'vov and M. I. Shirokov, Zh. Eksp. Teor. Fiz. 67, 1932 (1974) [Sov. Phys.-JETP 40, 960 (1975)].

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