Concerning the remarks by A. I. Korneev

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The resultant discrepancy consists in the following: In the theory of homogeneous isotropic turbulence one uses Karman's hypothesis of complete self-similarity, according to which the statistical characteristics of the flow (the coupling moments) are expressed in terms of the pulsational energy per unit mass $e = 3b(t)/2$ and the spatial scale $l(t)$. From these follow, in particular, the relations (1) and (2) of A. I. Korneev's note

$$b_d^d(r,t)=b(t)f(\chi),\quad b_d^{nn}(r,t)=b^{3/2}(t)h(\chi)\quad \chi=r/l(t) \tag{1}$$

for the second and third moments. In our article[1] we have asserted that the experiments of Ling et al.[3,4] do not confirm this hypothesis, and we advanced and justified a more general incomplete-self-similarity hypothesis, according to which, in particular, the second and third moment, albeit self-similar

$$b_d^d=b(t)f(\chi),\quad b_d^{nn}=g(t)\varphi(\chi), \tag{2}$$

cannot be represented in terms of the two functions $b(t)$ and $l(t)$ only, since generally speaking $g(t) \neq b^{3/2}(t)$. The difference between the hypotheses of the complete and incomplete self-similarity, which at first glance is insignificant, is in fact of fundamental importance: it leads, in particular, to conservation of the influence of the dimension of the grid in the entire self-similarity region. A. I. Korneev states in his letter that he has demonstrated[2] good agreement between the Karman hypothesis and these experiments as well as others, and that our conclusions can be attributed to our going beyond the limits of the accuracy of the comparison of the theory and experiment.

I do not agree with this for the following reasons: Ling and his co-workers have demonstrated the self-similarity of the correlation function $f(r, \lambda) = b_d^d(r, t)/b_d^d(0, t)$ (λ is the Taylor scale); this is confirmed also by Korneev (see p. 879 of his article[2]). Ling does not cite any primary experimental data for λ, but gives the relation

$$\lambda=\left[\frac{10}{n}\nu(t-t_0)\right]^{1/2} \tag{3}$$

(n and t_0 are the parameters of the power-law approximation, see below), which is the only one that Korneev can use for a comparison with the theory selected by him.

From this we obtain for the second moments

$$b_d^d(r,t)=b_d^d(0,t)f\left[r\left[\frac{10}{n}\nu(t-t_0)\right]^{-1/2}\right]. \tag{4}$$

If we now substitute (4) in the main Karman-Howarth equation that relates the second and third moments, and take into account the fact that the third moments behave like r^3 near $r = 0$, then we obtain rigorously that $b(t) = b_d^d(0, t)$ and $g(t)$ are power-law functions:

$$b(t)=A(t-t_0)^{-n},\quad g(t)=A\nu^{1/2}(t-t_0)^{-n-1/2}. \tag{5}$$

Thus, the power-law character of the functions $b(t)$ and $g(t)$ follows rigorously for homogeneous isotropic turbulence from the assumption that the second moments are self-similar, and from the representation for the Taylor scale (3). Then it is only in the case $n = 1$ that we can assume that $g(t) = b^{3/2}(t)$, i.e., that Karman's hypothesis is valid.

The relation $b_d^d(0, t) = b(t)$ according to the data of Ling et al., with all the accuracy that can be attained with the assumed graphic representation, demonstrates that $b(t)$ is indeed a power-law function of the form (5). (The deviation of the first two points on one of the curves pertaining to active grids is due to the large lengths of the wakes behind the rods of the grid in this case, as is specially stipulated by the authors.) This was noted by Ling and used in our article. The obtained exponents $n = 2.00$, 1.73, and 1.35 differ appreciably from unity, and the deviation of the exponents from unity greatly exceeds the accuracy with which they are determined. It follows therefore that the experiments in question do not confirm the Karman hypothesis. At the same time, this result can agree fully with the hypothesis of incomplete self-similarity advanced by us.

In connection with the question of the accuracy of the comparison of the theory with experiment, I emphasize that the presented reasoning is based in essence only on the approximation (3) of the experimental data with respect to scale, an approximation used also (without due stipulation) by Korneev as an exact representation of the experimental data, and does not introduce any additional error. An indication of the good accuracy of the power-law approximation and formula (3) are contained already in the book of Monin and Yaglom.[5] Let me remark in addition that experimental data by various authors, spanning more than thirty years, were reduced in a recent paper by Gad-el-Hak and Corrs in[6] by using power-law formulas. On the basis of their reduction, they reached the conclusion that the power-law representation of the laws of degeneracy of the second moments is sufficiently accurate, and the exponent turned out to be equal to unity only in one case, at tremendous Reynolds numbers of the grid.

As to Korneev's reduction of the experimental data, I have some doubts concerning its reliability. Indeed, take for example his comparison[2] of the distribution of the third moments with Stewart's experiments (see Fig. 8 of[2]). At first glance, this figure gives the impression that the experimental points taken from Stewart's corresponding plot lie, with a certain scatter, near the solid line corresponding to that chosen by Korneev for a comparison of the theory. If, however, we plot Korneev's curve directly on Stewart's plot (Fig. 5 of Steart's article[7]), then this impression is significantly altered (see the figure, where the solid line is Korneev's curve). Indeed, the difference between the plot of Korneev and that shown in the figure is instructive. On Stewart's plot we see not a scatter but a systematic deviation of the curves for various instants of time, and Stewart writes ([7], p. 152) that "there is a definite tendency of the maximum to increase with increasing degeneracy time."

The conclusion obtained in our paper[1] for the third-order moments lead to the law

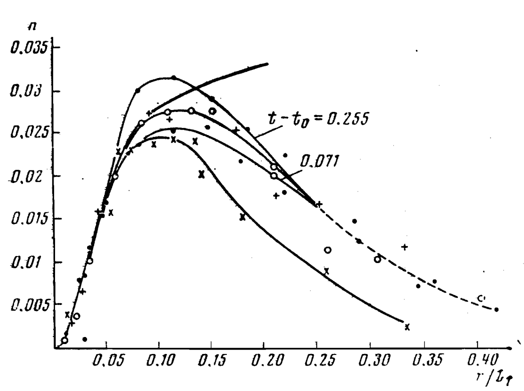
$$b_d^{nn}(r, t)/b^{3/2}(t)=\varphi(\chi)g(t)/b^{3/2}(t). \tag{6}$$

Since the abscissas of the points of the maxima for different instants of time differ little, we can assume χ for them to be the same, and for the maxima of the function $h = b_d^{nn}(r, t)/b^{3/2}(t)$ we obtain the following increase with time:

$$h\sim(t-t_0)^{(n-1)/2}, \tag{7}$$

in full agreement with Stewart's plot presented by us, which was used by Korneev incompletely, and does not agree with the hypothesis of complete self-similarity, which if valid would require the maxima for different times to coincide.

Equally doubtful is Korneev's reduction of the experimental results of Ling et al.[3,4] In fact, a comparison based on the dependence of the scale λ on the time constitutes a comparison of two theoretical formulas, formula (3) and formula (4.7) of[2]: on Fig. 14 of[2], the experimental points are calculated by Korneev by using in fact a power-law approximation, which is not mentioned in[2]. The comparison is terminated at $\nu(t + t^*) = \lambda^{*2}$. This limit is invariably larger than the lower limits indicated in the table, so that there seem to be no visible grounds for terminating the comparison. In fact, at $\nu(t + t^*) < \lambda^{*2}$ the theoretical formula (4.7) used by Korneev leads to a result that is contrary to nature—the scale λ decreases with time, so that the discrepancy with formula (3), which gives a reasonable monotonic increase of the scale with time, becomes not only quantitative but also qualitative—I can see no other reason for terminating the comparison. Further, without going into the details of the comparison of the experiment with the theoretical relations given by Korneev for $b(t)$ and in the estimate of the deviations, which certainly exceed the deviations from the power laws, I note only that certain points that fit splendidly on lines corresponding to power laws turn out to be simply imaginary on Korneev's plots, owing to the fact that his parameter t^* is negative. The foregoing leads me to assume that Korneev's comparison of the theoretical calculations with the experimental data is insufficiently accurate and complete, something that is inadmissible when a large number of fitting parameters (four!) is used.



Finally, Korneev's remark (1) is based on a misunderstanding. The main conclusion of our paper is based on comparing with experiment not the quantity b_1/b (in our notation $g^{2/3}(t)/b$), but the laws of degeneracy of the energies and of the scale $b(t)$ and $l(t)$, which are determined with an accuracy that is sufficient to demonstrate that the experimental results do not agree with Karman's hypothesis.

The foregoing leaves me convinced that the correctness of our paper is not affected by Korneev's remarks. To the contrary, I have serious doubts concerning Korneev's reduction of the experimental data and his conclusions.

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