Thermal mechanisms of dissipation in type II superconductors

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We investigate the thermal dissipation mechanism connected with the appearance of temperature gradients in the vortex structure during the passage of a transport current. The viscosity coefficients and the corresponding contribution to the differential resistance ρ_f are calculated. A minimum is observed in ρ_f at a certain temperature t_m. The depth of the minimum and the dependence of t_m on the field strength H are investigated. The results are in sufficiently good agreement with the experimental data.

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The passage of a transport current in type II superconductors is accompanied, as is well known, by energy dissipation processes. The mechanisms of this dissipation is among the pressing problems of the physics of the superconducting state. In the present work, we have studied the thermal mechanism of dissipation, which is connected with the appearance of microscopic temperature gradients in type II superconductors. This mechanism was proposed in the work of Clem,[1] where motion of a single vortex has also been investigated qualitatively as well as the appearance of the thermal mechanism associated with this motion.

The study of the thermal mechanism is of undoubted interest in view of the fact that in a number of experiments[2-5] a minimum has been observed at some temperature t_m in the differential resistance ρ_f. The appearance of this minimum, its depth, and also the dependence of t_m on the magnetic field are explained by the appearance of a thermal dissipation mechanism (see below). The present work is also devoted to a detailed microscopic consideration of the thermal mechanism.

1. The viscosity coefficient. The flow of transport current causes motion of the Abrikosov vortical structure, due to the action of the Lorentz force. At the leading edge of the vortex in this case a transition of the superconducting phase into the normal phase takes place, and at the back wall, the reverse transition. Inasmuch as the entropies of superconducting and the normal phases are not equal, these transitions lead to the appearance of temperature gradients. Thermal currents and the energy dissipation associated with them are developed here. This is the thermal mechanism of dissipation.

The energy dissipation is described by the following formula:

$$W_q=\int\limits_{|r|<\xi}\varkappa^n\frac{(\nabla\tau_n)^2}{T}dr+\int\limits_{|r|>\xi}\varkappa^s\frac{(\nabla\tau_s)^2}{T}dr. \tag{1}$$

It is clear that the first and second terms on the right side of (1) describe the dissipative losses in the vortex core and in the remaining region, respectively. A rigorous derivation of (1) is given in the Appendix. The heat conduction coefficients κ^S and κ^n are known from the microscopic theory of superconductivity (see, for example,[6]) and the theory of a normal metal. The basic problem, as is seen from (1), reduces to the calculation of the microscopic temperature gradients $\nabla\tau_n$ and $\nabla\tau_S$ which arise in the motion of the vortex structure. We note that a similar problem was considered by Andreev and Dzhikaev[7] in a study of a moving filamentary structure and in the intermediate state.

As is well known, the system of vortices in the considered case constitutes a regular triangular lattice, and the vortices move with constant velocity **V** in the superconducting phase (**V** lies in a plane perpendicular to the filaments). Let us find the distribution of the microscopic temperature τ in a set of coordinates connected with the moving vortices. It follows from the symmetry of the problem that the microscopic temperature $\tau(\mathbf{r})$ satisfies the condition (we consider an unbounded sample)

$$\tau(\mathbf{r})=\tau(\mathbf{r}+\mathbf{r}_{ij}),$$

where the set $\{\mathbf{r}_{ij}\}$ describes the position of the normal vortices. Consequently, we need to know the microscopic temperature distribution in the unit cell (the unit cell is a regular hexagon with center on the vortex axis).

We first choose the origin of the coordinates at the center of the cell. The microscopic temperature τ satisfies the Laplace equation $\Delta\tau(\mathbf{r}) = 0$, whose solution we seek in the form (see also[7])

$$\begin{aligned}&\tau_n=-\mathbf{g}_n\mathbf{r} && \text{if}\quad |\mathbf{r}|<\xi,\\ &\tau_s=-\mathbf{g}_s\mathbf{r}+2\mathbf{d}\mathbf{r}/|\mathbf{r}|^2 && \text{if}\quad |\mathbf{r}|>\xi,\end{aligned} \tag{2}$$

where $\mathbf{g}_n$, $\mathbf{g}_S$ and $\mathbf{d}$ are constant vectors.

On the phase separation boundary, i.e., at $|\mathbf{r}| = \xi$, the conditions

$$\begin{gathered}\tau_n=\tau_s,\\ -\varkappa^n(\nabla\tau_n,\mathbf{n})+(\varkappa^s\nabla\tau_s,\mathbf{n})=Q(\mathbf{V}\mathbf{n});\ \mathbf{n}=\mathbf{r}/|\mathbf{r}|\end{gathered} \tag{3}$$

should be satisfied, where $Q = T(S_n - S_S)$; κ^n and κ^S are the heat conduction coefficients in the normal and superconducting regions. The third necessary condition for the determination of $\mathbf{g}_n$, $\mathbf{g}_S$ and $\mathbf{d}$ is the vanishing of the macroscopic gradient:

$$\frac{1}{S}\int\limits_{|r|<\xi}\nabla\tau_n\,dr+\frac{1}{S}\int\limits_{|r|>\xi}\nabla\tau_s\,dr=0, \tag{4}$$

where the integrals are taken within the limits of the area of the cell S.

Substituting (2) in (4) and (3), we obtain the following set of equations for $\mathbf{g}_n$, $\mathbf{g}_S$ and $\mathbf{d}$:

$$\begin{gathered}\mathbf{g}_n=2\varkappa^s\mathbf{g}_s/(\varkappa^n+\varkappa^s)+Q\mathbf{V}/(\varkappa^n+\varkappa^s),\\ \mathbf{d}=\frac{\xi^2}{2}(\mathbf{g}_s-\mathbf{g}_n)=\frac{\xi^2}{2}\left[\frac{\varkappa^s-\varkappa^n}{\varkappa^s+\varkappa^n}\mathbf{g}_s-\frac{Q\mathbf{V}}{\varkappa^s+\varkappa^n}\right],\\ x_n\mathbf{g}_n+x_s\mathbf{g}_s+\mathbf{D}=0,\end{gathered} \tag{5}$$

where x_n and x_S are the densities of the normal and superconducting regions, respectively. It is easy to see that the vector **D** defined by the equation

$$D = S^{-1}\int\limits_{|r|>\xi} dr\,\nabla\frac{2\mathbf{dr}}{|\mathbf{r}|^2}.$$

vanishes after integration over the cell.

We note (see (2)) that the system of vortices turns out in the given case to be similar to a system of plane dipoles. The relations (5) in this connection can be obtained by another method—with the help of the electrostatic analogy (here one must use the boundary conditions (3) and the Lorentz formula; the quantity τ is similar to the electrostatic potential). The solution of the set (5) is of the form

$$g_n=\left\{1+\frac{2\varkappa^s}{\varkappa^n+\varkappa^s}\frac{x_n}{x_s}\right\}^{-1}\frac{QV}{\varkappa^n+\varkappa^s},$$

$$g_s=-\frac{x_n}{x_s}\left\{1+\frac{2\varkappa^s x_n}{(\varkappa^n+\varkappa^s)x_s}\right\}^{-1}\frac{QV}{\varkappa^n+\varkappa^s}, \tag{6}$$

$$d=-\frac{\xi^2}{2}\left\{1-\frac{(\varkappa^s-\varkappa^n)x_n}{(\varkappa^s+\varkappa^n)x_s}\left[1+\frac{x_n 2\varkappa^s}{x_s(\varkappa^n+\varkappa^s)}\right]^{-1}\right\}\frac{QV}{\varkappa^n+\varkappa^s}.$$

We now proceed to the calculation of the energy (1) released in type-II superconductors and connected with temperature gradients. With account of (2) and (6), we arrive at the following expression for W_q:

$$W_q=\frac{\xi^2 T H_c^2(t)\,[dH_c(t)/dt]^2}{16\pi T_c^2(\varkappa^n+\varkappa^s)^2}\left\{\varkappa^n\left[1+\frac{2\varkappa^s x_n}{(\varkappa^n+\varkappa^s)x_s}\right]^{-2}\right.$$
$$+x_n\varkappa^s\left[x^s\left(1+\frac{2\varkappa^s x_n}{(\varkappa^n+\varkappa^s)x_s}\right)^2\right]^{-1} \tag{7}$$
$$\left.+\varkappa^s[1-x_n]\left[1-\frac{(\varkappa^s-\varkappa^n)x_n}{(\varkappa^s+\varkappa^n)x_s}\left(1+\frac{2\varkappa^s x_n/x_s}{\varkappa^n+\varkappa^s}\right)^{-1}\right]^2\right\}V^2.$$

We have made use of the well-known thermodynamic identity $S_n - S_s = -(H_c(t)/4\pi T_c)\,(dH_c(t)/dt)$, where $H_c(t)$ is the thermodynamic critical field, T_c is the temperature of the superconducting transition at $H = 0$, and $t = T/T_c$ is the relative temperature.

In fields H satisfying the condition $H_{c1} < H \ll H_{c2}$ (H_{c1} and H_{c2} are the lower and upper critical fields, respectively), Eq. (7) is considerably simplified:

$$W_q=\frac{\Phi_0 T H_c^2(t)\,[dH_c(t)/dt]^2 V^2}{32\pi T_c^2 H_{c2}(t)\,(\varkappa^n+\varkappa^s)}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H}{H_{c2}(t)}\right]. \tag{8}$$

The following relations were used (see, for example,[8])

$$\xi^2=\Phi_0/2\pi H_{c2}(t),\quad d^2=2\Phi_0/H\sqrt{3}$$

(Φ_0 is the flux quantum) and use was also made of the equation that follows from them:

$$x_n/x_s=\pi\xi^2/S=0.5H/H_{c2}(t).$$

Further, using the well-known formula $W = \eta(t)V^2$ ($\eta(t)$ is the coefficient of viscosity), we obtain for $\eta_q(t)$ the following expression, which characterizes the investigated temperature mechanism of dissipation:

$$\eta_q(t)=\frac{\Phi_0 T H_c^2(t)\,[dH_c(t)/dt]^2}{32\pi T_c^2 H_{c2}(t)\,(\varkappa^n+\varkappa^s)}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H}{H_{c2}(t)}\right]. \tag{9}$$

2. Minimum of the differential resistance. Dependence of ρ_f on H. In the following, we shall investigate the quantity $\tilde{\eta}_q \equiv \eta_q(t)/\eta(0)$, where $\eta(0) = \alpha\Phi_0 H_{c2}(0)c^2\rho_n$ ($\alpha \approx 1$) is the viscosity coefficient, which corresponds to the ordinary electromagnetic mechanism of dissipation at $T = 0$.[9,10] We then get from (9):

$$\tilde{\eta}_q=a\frac{\varkappa_e^n h_c(t)\,[dh_c(t)/dt]^2}{\varkappa^n+\varkappa^s}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H}{H_{c2}(t)}\right], \tag{10}$$

where $h_c(t) = H_c(t)/H_{c0}$, κ_e^n is the coefficient of electronic thermal conductivity in the normal metal,

$$a=\frac{c^2\rho_n T H_{c0}^2}{64\varkappa_e^n(\pi T_c\varkappa_{GL})^2}.$$

For ordinary superconductors with weak coupling, we get $a_{BCS} = 0.63$, using the relations $4\pi\gamma T_c^2/H_{c0}^2 = 2.115$ $\kappa_{GL} = 0.75\, ce\rho_n\gamma^{1/2}k_B$ (see, for example,[8]) and the Wiedemann-Franz law. In superconductors with strong coupling, the connection of H_{c0} with T_c, and the expression κ_{GL} turn out to be different,[11-13] which must be taken into account in a comparison of the theory with experiment. The difference between a and a_{BCS} can be expressed in terms of the value of the jump in the specific heat β, inasmuch as β is measured directly by experiment. According to[13], we find $a = 0.63\,[1 + 0.5(\beta-\beta_{BCS})]$.

It is easy to see that the function $\eta_q(t)$ has a maximum at some temperature. This maximum is connected with the temperature dependence of the product $h_c(t)[dh_c(t)/dt]^2$. The function $h_c(t)$, which has a maximum at $t = 0$, falls off with increase in temperature and vanishes at $t = 1$. The derivative $dh_c(t)/dt$, on the other hand, is equal to zero at $t = 0$, and increases as $t \to 1$. It is the presence of a maximum in the function $\tilde{\eta}_q(t)$ which leads to the observed minimum minimum ρ_f (see below). The temperature t_m corresponding to the maximum $\tilde{\eta}_q$ can be calculated most easily for superconductors with weak coupling, when $h_c(t)$ can be approximated by the empirical law $h_c(t) = 1 - t^2$. The expression (10) now takes the form

$$\tilde{\eta}_q=2.52\frac{\varkappa_e^n t^2(1-t^2)}{\varkappa^n+\varkappa^s}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H}{H_{c2}(t)}\right]. \tag{11}$$

We note immediately that the very appearance of the maximum of $\tilde{\eta}_q(t)$ is connected with the entropy factor $\sim t^2(1-t^2)$, but its position depends materially on the temperature dependence of the heat conduction coefficients and the value of the field H. The relation $h_c(t) = 1 - t^2$, as is well known, is approximate even for ordinary superconductors with weak coupling. In superconductors with strong coupling (such as the alloy Nb-Zr, in which the thermal dissipation mechanism has been investigated experimentally[5]) the temperature dependence of $h_c(t)$ is quite different from the ordinary parabolic law. This must be taken into account in a detailed comparison of theory with experiment.

According to the theory, the function $h_c(t)$ is of the form[13]

$$h_c(t)\,|_{t\to 0}=(1-\alpha t^2),$$
$$\alpha=1.07\{1-2.3(T_c/\tilde{\omega})^2[\ln(\tilde{\omega}/T_c)+0.2]\}; \tag{12}$$

$$h_c(t)\,|_{t\to 1}=A(1-t),$$
$$A=1.73[1+7.55(T_c/\tilde{\omega})^2\ln(\tilde{\omega}/T_c)]. \tag{12'}$$

Here $\tilde{\omega}$ is the characteristic frequency of the phonon spectrum. With the help of the formula[14] $\Delta(0)/T_c = 1.76[1 + 5.3(T_c/\tilde{\omega})^2\ln(\tilde{\omega}/T_c)]$ the coefficient which determines the $h_c(t)$ dependence can be expressed in terms of the experimentally determined ratio $\Delta(0)/T_c$.

Of greatest convenience to us is the formula in which the character of the $h_c(t)$ dependence is connected with the magnitude of the jump in the specific heat $\beta = (C_s - C_n)/C_n|_{T_c}$, inasmuch as the value of β is very easily measured experimentally:[13]

$$h_c(t)\,|_{t\to 1}=1.45\beta^{1/2}(1-t). \tag{12''}$$

This relation turns out to be valid also for superconductors with strong coupling. This dependence is satisfactorily maintained to temperatures $T_0 \sim T_c/2$. In the calculation of the quantity $g(t) = h_c(t)[dh_c(t)/dt]^2$, which enters into (10), the formulas (12)–(12″) must be used for superconductors with strong coupling. It can be shown that $g(t) = 6.6\beta^{1/2}t^2(1-t)$ is a satisfactory approximation for the function g(t). We then obtain the following expression for $\tilde{\eta}_q(t)$ in the case of superconductors with strong coupling:

$$\tilde{\eta}_q(t) = 4.2\frac{\beta^{1/2}[1+0.5(\beta-\beta_{GL})]\varkappa_c^n t^2(1-t)}{\varkappa^n+\varkappa^s}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H}{H_{c2}(t)}\right] \quad (13)$$

We now proceed to the calculation of the temperature t_m which corresponds to the maximum of $\tilde{\eta}_q(t)$ and consequently to the minimum ρ_f. The value of t_m is determined from the equation

$$d\tilde{\eta}_q(t)/dt=0. \quad (14)$$

With account of (11), we arrive at the following equation, which determines t_m for superconductors with weak coupling:

$$\frac{1}{2}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H}{H_{c2}(0)}\right]=t^2\left[1-\frac{1}{4}P(t)\right]\Big/\left[1-\frac{1}{2}P(t)\right],$$
$$P(t)=tK_s'(t)/(1+K_s(t)), \quad K_s(t)=\varkappa^s/\varkappa^n. \quad (15)$$

In the general case (see (13)), we get the equation

$$\frac{2}{3}\left[1-\frac{\varkappa^s}{\varkappa^n+\varkappa^s}\frac{H/H_{c2}(0)}{1.45\beta^{1/2}}\right]=t\left[1-\frac{1}{3}P(t)\right]\Big/\left[1-\frac{1}{2}P(t)\right]. \quad (15')$$

Equations (15) and (15′) allow us to study the dependence of t_m on the magnetic field. Thus, with account of (15), we easily obtain the relation

$$-\frac{dH}{dt}=\text{const}\,\frac{t[1+K_s(t)][4+4K_s(t)-5tK_s'(t)+t^2K_s''(t)]}{[1+K_s-tK_s'(t)]^2}. \quad (16)$$

If $K_s'(t) < 0$ (this condition is satisfied if the lattice thermal conductivity κ_{pe}^s plays the main role; this quantity, as is well known, increases on decrease in the temperature below T_c) then, as is seen from (16), $dt_m(H)/dH < 0$ ordinarily. We note that κ_{pe}^s in alloys makes a large contribution to the heat flux (see[6]). Consequently, the minimum of ρ_f shifts with increase in field (for $K_s'(t) < 0$) in the direction of lower temperatures, as is observed experimentally (see below).

The coefficient of viscosity of the moving vortices and the differential resistivity ρ_f are connected by the relation[15]

$$\eta(t)=\Phi_0 H/c^2\rho_f. \quad (17)$$

We represent $\eta(t)$ in the form

$$\eta(t)=\eta_0+\eta_q(t), \quad (18)$$

where $\eta_0 = \eta(0) + \eta_0(t)$ corresponds to the ordinary electromagnetic mechanism of dissipation[9,10,16] (the expression for $\eta(0) \equiv \eta_0$ (T = 0) was given above) η_q is the viscosity coefficient for the thermal mechanism.

With account of (17) and (18),the expression for ρ_f/ρ_n can be written in the form

$$\frac{\rho_f}{\rho_n}=\frac{H}{H_{c2}(0)}\frac{1}{1+\tilde{\eta}_0+\tilde{\eta}_q}, \quad (19)$$

where $\tilde{\eta}_0 = \eta(t)/\eta(0)$; $\tilde{\eta}_q$ is determined above (see (10)). It is seen that the experimentally observed minimum (see below) can be connected with the maximum $\tilde{\eta}_q$.

3. Comparison with experiment. In a number of experimental works[2-5], the minimum of the differential resistance ρ_f has been observed. The study of the thermal mechanism of dissipation (see above) permits us to explain the appearance of this minimum. With the help of Eqs. (10), (15), (15′), (16), we can make a detailed comparison of theory with experimental data. The works of Gonacharov et al.[5] are especially interesting in this sense; they obtained the temperature dependence of ρ_f for the alloy Nb_{20}–Zr_{80} and studied in detail the singularities of ρ_f associated with the appearance of the thermal dissipation mechanism.

As is seen from (10), (15), (16), the value of the temperature t_m depends significantly on the character of the dependence of κ^s(T) and κ^n(T) on the temperature. The κ^s(T) dependence for the alloy Nb-Zr was studied in[5] and is a function that increases with decrease in temperature below[1)] T_c. This means that in this case the principal role is played by the lattice thermal conductivity (increase in the coefficient of thermal conductivity with decrease in the temperature below T_c is typical of the phonon thermal conductivity κ_{pe}^s; see, for example,[6]). In the range of temperatures where κ^s increases with decrease in temperature, the function κ^s(T) for the alloy Nb-Zr is approximated with sufficient accuracy by the function $K_s(t) = \kappa^s/\kappa^n = 0.2te^{-1.8/t}$.

In the calculation of t_m, we begin with Eq. (15′). We first determine t_m as $H \to 0$. The problem reduces to the solution of the equation

$$t(1-P(t)/3)(1-P(t)/2)=2/3.$$

The function P(t) is defined in (15). After simple calculations, we find $t_m(H \to 0) = 0.7$. This result is in satisfactory agreement with the experimental data of[5] ($t_m^{exp}(H \to \theta) = 0.68$).

We note that the function $\tilde{\eta}_0$ can affect the location of the minimum (see (19)); however, the rather good agreement of the maximum $\tilde{\eta}_q^{theor}$ and minimum ρ_f^{exp} indicates that $\tilde{\eta}_0$ depends rather weakly on the temperature at temperatures corresponding to t_m.

We now consider the question of the dependence of the location of the minimum on H. It was shown above (see (16)) that the value of t_m shifts to lower temperatures with increase in H. This conclusion agrees with the experimental data. A more detailed comparison can be made with the help of (15′). It is seen that the value of t_m depends on the form of the function $K_s(t)$ and the value of β (some contribution can be made by $\tilde{\eta}_0$). The measurements of κ^s and κ^n obtained for the alloy Nb-Zr were used in the analysis. If $\beta = \beta_{BCS} = 1.43$, then, for example, H = 8.34 kOe corresponds to $t_m = 0.66$ ($t_m^{exp} = 0.61$[5]), and for H = 24 kOe, $t_m = 0.59$ ($t_m^{exp} = 0.50$[5]). It is seen that the theory agrees with the experimental data, but for a more detailed comparison, measurements of β should be carried out.

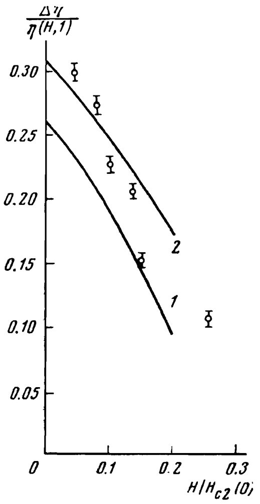
The depth of the minimum of ρ_f is also measured experimentally. The figure shows the dependence of the relative depth of the minimum on the magnetic field for the alloy Nb-Zr,[5] determined from the experimental dependence of ρ_f(t) from the formula of[5], which can be obtained from the relations (17)–(19):

$$\Delta\eta/\eta(H,1)=\rho_f(H,1)/\rho_f(H,t_m)-1 \quad (20)$$

(ρ_f(H,1) is the value of ρ_f in the field H at T = 1°K). Using (19), we can write the expression (20) in the form

$$\frac{\Delta\eta}{\eta(H,1)}=\frac{\tilde{\eta}_q(H,t_m)-\tilde{\eta}_q(H,1)}{\tilde{\eta}_q(H,1)+\eta_0}. \quad (21)$$

The curve $\Delta\eta/\eta$(H,1) is constructed in this same figure. It is calculated from (21) with $\tilde{\eta}_q(t)$ determined from (15′). The result also depends on whether the investigated material is a superconductor with weak coupling or whether it is necessary to take into account the ef-



Dependence of the relative depth of the minimum of ρ_f, described by the relation $\Delta\rho/\rho(t_m) = \Delta\eta/\eta(H, 1)$, on the magnetic field. The points are the experimental data of [5]; the solid curves are theoretical: $1-\beta = \beta_{BCS} = 1.4$; $2-\beta = 2$.

fects of strong coupling in it. This is determined directly by the value of the jump in the specific heat β. We have carried out a comparison for two values of β: 1) $\beta = \beta_{BCS} = 1.43$; 2) $\beta = 2$. It is seen that the theory on the whole adequately agrees with the experiment, but a detailed comparison is possible after measurement of β.

In a quantitative investigation of the singularities in the behavior of ρ_f, associated with the thermal dissipation mechanism, we have used data on the measurement of the thermal conductivity κ^S of the superconducting alloy Nb-Zr. In this connection, we note that a comparison of the theory with experiment can be made even without measurement of the dependence $\kappa^S(T)$, inasmuch as the character of the change of both the electron (κ_e^S) and the lattice (κ_p^S) thermal conductivities is known from the microscopic theory of superconductivity (see[6]). However, for the corresponding calculation, it is necessary to know the values of $\kappa_e^n(T_c)$ and $\kappa_p^n(T_c)$ (in the theory of superconductivity, the form of the functions $\kappa_e^S(T)/\kappa_e^n(T)$ and $\kappa_p^S(T)/\kappa_p^n(T_c)$ is known), i.e., a separation of the electron and lattice contributions to the thermal flux in the normal phase should be made (see[17]). For this purpose, the measurement of the conductivity should be carried out along with the measurement of the thermal conductivity. Use of the Wiedemann-Franz law allows us to separate the electron contribution to the thermal conductivity, i.e., to determine $\kappa_e^n(T_c)$.

It is thus necessary to know the values of $\kappa_e^n(T_c)$, $\kappa_p^n(T_c)$ and β for comparison of theory with experiment. In this connection, the setting up of the corresponding experiments both in the alloy Nb-Zr and in other superconducting alloys is of interest.

In conclusion, we express our sincere gratitude to I. N. Goncharov, G. L. Dorofeev and I. S. Khukharev for numerous discussions pertaining to the experimental situation, and also for acquainting us with the results of[5] before its publication, and A. F. Andreev, B. T. Geĭlikman, M. I. Kaganov and I. M. Lifshitz for interesting discussions.

APPENDIX

We now carry out a rigorous derivation of Eq. (1). The irreversible losses that are associated with the temperature gradient per unit time are determined by the expression

$$W_q=\int_S dr\,\mathbf{q}(\mathbf{r})\frac{\nabla\tau}{T}, \tag{A.1}$$

where $\mathbf{q}(\mathbf{r})$ is the thermal flux vector, arising from the gradient $\nabla\tau$, S is the area of the elementary cell (W_q corresponds to the energy in the calculation over unit length of the vortex). A contribution to the thermal flux $\mathbf{q}(\mathbf{r})$ will be made by both the electrons and phonons. The total thermal flux will be equal to

$$\mathbf{q}(\mathbf{r})=\mathbf{q}_e(\mathbf{r})+\mathbf{q}_p(\mathbf{r}).$$

We consider the energy losses associated with $\mathbf{q}_e(\mathbf{r})$:

$$W_q^e=\int_S dr\,\mathbf{q}_e(\mathbf{r})\frac{\nabla\tau}{T},\quad \mathbf{q}_e(\mathbf{r})=\langle|\hat{\mathbf{q}}_e(\mathbf{r})|\rangle, \tag{A.2}$$

where $\hat{\mathbf{q}}_e$ is the operator of the electron thermal flux, which can be rewritten in the form[18]

$$\hat{\mathbf{q}}_e(\mathbf{r})=\hat{L}_e\psi^+(\mathbf{r}')\psi(\mathbf{r})|_{r'=r},\qquad \hat{L}_e=\left[\frac{\nabla\nabla'}{2m}-\mu+V\right]\frac{\nabla-\nabla'}{2mi}. \tag{A.3}$$

We transform to the Fermi amplitudes in (A.3); these describe the ground state of the type-II superconductor (see[8]):

$$\psi(\mathbf{r},\uparrow)=\sum_n\left[u_n(\mathbf{r})\gamma_{n\uparrow}-v_n^*(\mathbf{r})\gamma_{n\downarrow}^+\right],\qquad \psi(\mathbf{r},\downarrow)=\sum_n\left[u_n(\mathbf{r})\gamma_{n\downarrow}+v_n^*(\mathbf{r})\gamma_{n\uparrow}^+\right]. \tag{A.4}$$

We then get the following expression for W_q^e:

$$W_q^e=\int dr\sum_n \hat{L}_eF_{\epsilon_n}(\mathbf{r},\mathbf{r}')|_{r'=r}\frac{\nabla\tau}{T}\langle\gamma_n^+\gamma_n\rangle,\qquad F_{\epsilon_n}=u_n^*(r')u_n(r)+v_n(r')v_n^*(r). \tag{A.5}$$

The function $F_{\epsilon n}(\mathbf{r},\mathbf{r}')$ in the case $\epsilon_n < \Delta$ agrees with the exponential law for $|\mathbf{r}| > \xi$ ($\sim e^{-|\mathbf{r}|/\xi}$, see, for example,[8]). The "step" model of Bardeen and Stephen[9] is a sufficiently good approximation for $\Delta(\mathbf{r})$:

$$\Delta(\mathbf{r})=0 \text{ for } |\mathbf{r}|<\xi;\quad \Delta(\mathbf{r})=\Delta_0 \text{ for } |\mathbf{r}|>\xi.$$

With account of the above, we arrive at the following expression for W_q^e:

$$W_q^e=\int_{|r|<\xi} dr\,\varkappa_e^n\frac{(\nabla\tau_n)^2}{T}+\int_{|r|>\xi} dr\,\varkappa_e^s\frac{(\nabla\tau_s)^2}{T}.$$

The problem of the irreversible losses associated with $q_p(\mathbf{r})$ can be considered in similar fashion. As a result, we obtain Eq. (1) for the complete dissipation function.

Note added in proof (May 23, 1975). As $T \to T_c$, the quantity η_q is small: $\eta_q \sim \Delta^4$ (see[1] and (7)). A calculation has been carried out by Kopnin (this issue, preceding article) with accuracy $\sim\Delta^2$, which led to $\eta_q = 0$. Furthermore, the phonon contribution to the mechanism, which plays (see above) the fundamental role) was not considered by him.

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1)The measurements of the thermal conductivity of the alloy Nb-Zr were carried out by L. G. Mezhov-Deglin and coworkers.

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