Nonlinear resonant instability of a plasma in the field of an ordinary electromagnetic wave

V. V. Vas'kov and A. V. Gurevich

State Radio Research Institute
and P. N. Lebedev Physics Institute, USSR Academy of Sciences
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The energy absorbed by an ordinary wave through its linear transformation by small concentration perturbations δN is determined in the plasma-resonance region. It is demonstrated that if an initial threshold value δN_{thr} is exceeded the concentration perturbations δN are unstable and grow very rapidly. This instability leads to disintegration of a smooth plasma layer near resonance. The threshold δN_{thr} decreases with increasing wave power and with decreasing concentration gradient.

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1. INTRODUCTION

Resonant absorption connected with linear transformation of electro-magnetic waves is determined to a considerable degree by the structure of the plasma in the resonance region (Ginzburg[1], Zheleznyakov[2], Golant and Piliya[3], Erokhin and Moiseev[4]). In fact, very strong energy absorption is possible in the presence of strong plasma concentration gradients N in the vicinity of the resonance $\lambda\mu \sim 1$ (λ is the wavelength, $\mu = N^{-1}|\nabla N|$). To the contrary, in a smooth layer

$$\lambda\mu \ll 1, \qquad (1)$$

the absorption is usually weak. The reason is that the extraordinary wave, which is effectively absorbed at plasma resonance, does not reach the resonance region in a smooth layer.[1)] On the other hand the ordinary wave, which reaches the resonance region, is not absorbed in the latter, since it has a special polarization at which no resonance is excited.[2)] The transformation of the ordinary wave into an extraordinary one is small in the case of normal incidence on the smooth layer.

It is seen that the cause of the absence of resonant effects and of the small absorption of the ordinary wave in the smooth layer is rather specific in character. The situation can be easily changed, for example, if the plasma contains inhomogeneities with a characteristic dimension smaller than or of the order of the wavelength. These inhomogeneities serve, as it were, as resonators excited by the ordinary wave.

It is important that the inhomogeneities can become intensified through absorption of the energy of the ordinary wave. This in turn leads to an increase of the dissipation, which again causes a growth of the inhomogeneities, etc. The resultant instability should lead to a sharp intensification of the inhomogeneous structure of the plasma, i.e., to a disintegration of the smooth plasma layer in the vicinity of the resonance. We shall call this instability "resonant." It develops only in the field of the ordinary wave. It is important also that the resonant instability is nonlinear: effective growth of the concentration perturbations begins only with a certain threshold value δN_{thr}. The value of the threshold decreases with increasing wave intensity and with decreasing plasma concentration gradient.

The phenomenon wherein a smooth plasma layer is destroyed by the action of an ordinary wave was observed in experiments on the perturbation of the upper ionosphere by strong radio waves[6-12]. We therefore consider here the resonant instability for typical conditions of the upper ionosphere. In Sec. 2 we determine the absorption of the ordinary wave in the inhomogeneities, and in Sec. 3 we find the excitation threshold and the characteristic instability growth time. The results of the calculation will be compared with ionosphere experiments.

2. ABSORPTION OF ORDINARY WAVE BY PLASMA CONCENTRATION INHOMOGENEITIES

Let an ordinary electromagnetic wave

$$\mathscr{E} = {}^1/_2[\mathbf{E}e^{i\omega t} + \text{c.c.}] \qquad (2)$$

with frequency ω exceeding the electron gyro frequency $\omega_H = eH/mc$, be normally incident on a layer of a weakly-inhomogeneous plasma (1) situated in a constant magnetic field $\mathbf{H}$. We assume that the plasma contains concentration inhomogeneities δN whose transverse dimension a is small in comparison with the wavelength.

$$a/\lambda \ll 1. \qquad (3)$$

The longitudinal dimension of the inhomogeneities, to the contrary, will be assumed large enough. These inhomogeneities appear in a strong demagnetized plasma, particularly in the upper ionosphere, where the diffusion along the magnetic-field force lines is much larger than across the lines.

Let us determine the energy flux, due to linear transformation of an ordinary into a longitudinal (extraordinary) wave, into the electron component of the plasma on the concentration perturbation (3). Within the framework of perturbation theory, the potential φ of the excited longitudinal oscillations is determined from the wave equation

$$\frac{\partial}{\partial x_i}\varepsilon_{ij}(\omega)\frac{\partial}{\partial x_j}\varphi = -4\pi\rho, \quad i,j=1,2,3,$$

where ϵ_{ij} is the dielectric tensor of the plasma[1], and the charge density ρ is connected with the polarization of the concentration perturbation δN in the electric field of the incident wave:

$$\rho = \frac{1}{8\pi(1-u)}\{\mathbf{E}_\perp + i(u)^{1/2}[\mathbf{E}_\perp\times\mathbf{h}]\}\nabla\delta v, \qquad (4)$$

$$\delta v = 4\pi e^2\delta N/m\omega^2, \quad \mathbf{E}_\perp \perp \mathbf{H}, \quad \mathbf{h} = \mathbf{H}/H.$$

Here and below we use the standard notation[1]:

$$v = 4\pi e^2 N/m\omega^2, \quad u = (\omega_H/\omega)^2,$$

and omit small perturbation gradients δv along the magnetic field.

We introduce next an orthogonal coordinate system with axis $z = x_3$ directed against the concentration gradient in the layer, i.e., parallel to the propagation of the incident wave. (Under the conditions of the ionosphere, the z axis is directed downward.) The axis $y = x_2$ lies in the (**zH**) plane perpendicular to ∇N, and the axis $x = x_1$ is orthogonal to the wave propagation plane (**zH**) (see Fig. 1). We measure the coordinate z from the plane of reflection of the ordinary wave $v(z = 0) = 1$. In this system, the tensor $\epsilon_{ij} = \epsilon_{ij}(z)$ does not depend on x or y. Therefore the Fourier transform $\varphi(k_x, k_y, z)$ of the potential φ with respect to the coordinates x and y satisfies the equation

$$\left\{-\varepsilon_{33}\frac{d^2}{dz^2}+\left(2ik_y\varepsilon_{23}-\frac{d\varepsilon_{33}}{dz}\right)\frac{d}{dz}+k_x^2\varepsilon_{11}+k_y^2\varepsilon_{22}\right.$$
$$\left.+ik_y\frac{d\varepsilon_{23}}{dz}-ik_x\frac{d\varepsilon_{13}}{dz}\right\}\varphi=4\pi\rho(k_x,k_y,z), \quad (5)$$

where we have used the following symmetry properties of the dielectric constant $\epsilon_{23} = \epsilon_{32}$, $\epsilon_{13} = -\epsilon_{31}$, and $\epsilon_{12} = -\epsilon_{21}$.

We consider the physical picture of the excitation of longitudinal oscillations by the incident wave. The dispersion equation $\epsilon_{ij}k_ik_j = 0$, which describes the propagation of longitudinal waves with the thermal motion of the electrons neglected, has a well-known solution corresponding to the high-frequency branch of the longitudinal oscillations:

$$\omega^2={}^1/_2[\omega_0^2+\omega_H^2+[(\omega_0^2+\omega_H^2)^2-4\omega_0^2\omega_H^2\cos^2\theta]^{1/2}],$$
$$\omega_0^2=4\pi e^2N/m. \quad (6)$$

Here θ is the angle between the wave vector **k** of the longitudinal oscillations and the magnetic field **H**. According to Fig. 1 we have

$$\cos^2\theta=(k_y\sin\alpha+k_z\cos\alpha)^2/(k_x^2+k_y^2+k_z^2), \quad (7)$$

where α is the angle between the z axis and **H**.

It follows from (6) and (7) that the group-velocity vector component $V_z = \partial\omega/\partial k_z$, parallel to the z axis is given by

$$V_z=-\frac{\omega}{2}\frac{uv}{2-u-v}\frac{\partial\cos^2\theta}{\partial k_z},$$
$$\frac{\partial\cos^2\theta}{\partial k_z}=2\cos^2\alpha\frac{(k_y\,\mathrm{tg}\,\alpha+k_z)(k_x^2+k_y^2-k_zk_y\,\mathrm{tg}\,\alpha)}{(k_x^2+k_y^2+k_z^2)^2}. \quad (8)$$

The dependence of $\cos^2\theta$ on k_z is shown in Fig. 2 for $k_y > 0$. At the points $(k_z)_1 = -k_y\tan\alpha$ and $(k_z)_2 = (k_x^2 + k_y^2)/k_y\tan\alpha$ at which the group velocity V_z of (8) vanishes, the quantity $\cos^2\theta$ reaches respectively its minimum and maximum values:

$$\cos^2\theta_1=0,\quad \cos^2\theta_2=(k_x^2\cos^2\alpha+k_y^2)/(k_x^2+k_y^2)\geqslant\cos^2\alpha.$$

The resonance point $|k_z| \to \infty$ corresponds to $\cos^2\theta_r = \cos^2\alpha$. The arrows in Fig. 2 show the direction of the group velocity V_z (downward along the z axis).

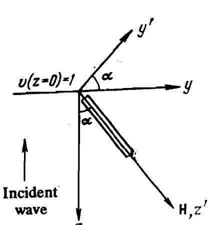

FIG. 1

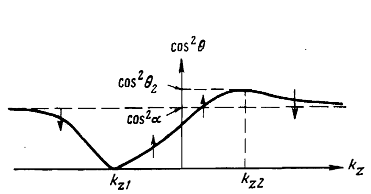

FIG. 2

Thus, at fixed values of k_x and k_y the longitudinal wave moves between the lower and upper reflection points v_1 and v_2, as shown in Fig. 3. The values of $v_{1,2}$ are given by

$$v_1=1-u,\quad v_2=(1-u)(k_x^2+k_y^2)/[k_x^2+k_y^2-u(k_x^2\cos^2\alpha+k_y^2)]. \quad (9)$$

On its downward path the wave passes through resonance in the region where the dimensionless concentration $v = 4\pi e^2N/m\omega^2$ is equal to

$$v_r=\frac{1-u}{1-u\cos^2\alpha}=1-\frac{u\sin^2\alpha}{1-u\cos^2\alpha}. \quad (10)$$

As is well known[1-4], when passing through a resonance point (a pole of the refractive index) the longitudinal wave is almost completely damped and transfers its energy to the plasma electrons. This energy dissipation occurs at an arbitrarily low frequency ν_e of collisions between the electrons and heavy particles.

To calculate the energy flux absorbed in the plasma in the limit of small ν_e, we find the solution of Eq. (5) in the resonance region. We assume that in the vicinity of the resonance the electron concentration in the layer varies linearly

$$v=1-\mu z,\quad \mu=-\frac{1}{N}\frac{dN}{dz}. \quad (11)$$

Introducing the dimensionless variables

$$k_1=\frac{k_x}{\mu},\quad k_2=\frac{k_y}{\mu},\quad \xi=\frac{2K_1}{1-u\cos^2\alpha}(v-v_r).$$
$$K_1=[k_1^2+k_2^2-u(k_1^2\cos^2\alpha+k_2^2)]^{1/2}, \quad (12)$$

we can reduce, using the substitution $\varphi = F\exp\{\eta_1\xi\}$, Eq. (5) to an equation for the confluent hypergeometric function

$$\left\{\xi\frac{d^2}{d\xi^2}+(\gamma-\xi)\frac{d}{d\xi}-\beta\right\}F=q\equiv\frac{4\pi\rho(k_x,k_y,\xi)}{\mu^2}\frac{1-u}{2K_1}\exp\{-\eta_1\xi\}, \quad (13)$$

$$F=\varphi(k_x,k_y,\xi)\exp\{-\eta_1\xi\}.$$

We have introduced here the notation

$$\beta=\beta_1-i\beta_2,\quad \gamma=1-2i\beta_2,$$
$$\beta_1=\frac{u(1-u)}{2K_1(1-u\cos^2\alpha)^2}[K_1^2\cos^2\alpha-k_2^2\sin^2\alpha]+\frac{1}{2}\left(1+\frac{k_1(u)^{1/2}\sin\alpha}{K_1}\right)$$
$$\beta_2=k_2\sin\alpha\cos\alpha\frac{u(1-u)}{(1-u\cos^2\alpha)^2},\quad \eta_1=-\frac{1}{2}+\frac{ik_2u\sin\alpha\cos\alpha}{2K_1}. \quad (14)$$

In deriving these equations we have neglected the small collision damping of the high-frequency longitudinal oscillations.

The homogeneous equation (13) has two independent solutions

$$F_1(\xi)=G(\beta,\gamma,\xi)\equiv\xi^{1-\gamma}G(\beta-\gamma+1,2-\gamma;\xi),$$
$$F_2(\xi)=e^{\xi}(e^{-i\pi}\xi)^{1-\gamma}G(1-\beta,2-\gamma;e^{-i\pi}\xi),\quad \arg\xi=0 \text{ for } \xi>0, \quad (15)$$

which have the following asymptotic forms: $F_1(\xi\to\infty)$

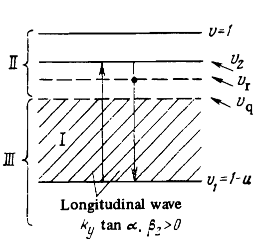

FIG. 3. Excitation of longitudinal plasma oscillations in the region of reflection of the ordinary wave: I—region of excitation of longitudinal plasma wave, II—region of quasi-transverse propagation of ordinary wave, III—region of quasi-longitudinal propagation of wave.

$= \xi^{-\beta}$, $F_2(\xi \to -\infty) = e^{\xi}(-\xi)^{\beta-\gamma}$. Here $G(\beta, \gamma; \xi)$ is a confluent hypergeometric function of the second kind[13]. With the aid of the functions $F_{1,2}$ we can easily obtain for the inhomogeneous equation (13) a solution that decreases at infinity (as $\xi \to \pm\infty$

$$F(\xi)=-F_1(\xi)\int_{-\infty}^{\xi}\frac{q(\xi')F_2(\xi')}{\xi'\Delta(\xi')}d\xi'+F_2(\xi)\int_{\infty}^{\xi}\frac{q(\xi')F_1(\xi')}{\xi'\Delta(\xi')}d\xi', \tag{16}$$

where $\Delta = F_1 dF_2/d\xi - F_2 dF_1/d\xi$ is the Wronskian of the solutions (15):

$$\Delta=C_0e^{\xi}\xi^{-\gamma},$$
$$C_0=(\gamma-1)\Gamma(1-\gamma)\Gamma(\gamma-1)\left\{\frac{e^{i\pi\gamma}}{\Gamma(\beta-\gamma+1)\Gamma(\gamma-\beta)}+\frac{1}{\Gamma(\beta)\Gamma(1-\beta)}\right\}. \tag{17}$$

Here $\Gamma(x)$ is the Euler gamma function. In the derivation of (17) we took into account the rule for going around the singularities of the functions $F_{1,2}(\xi)$ at the point $\xi = 0$, namely $\xi = \xi + i\epsilon$, $\epsilon \to 0$.

In the case $\nu_e \to 0$, energy absorption takes place at the resonance point (10).

The energy flux incident on the pole $\xi = 0$ of the longitudinal wave is equal to $P(k_x, k_y) = WV_z$, where

$$W=\frac{k^2}{4\pi}\omega\frac{\partial\tilde\varepsilon}{\partial\omega}|\varphi_{\rm inc}|^2,\qquad \tilde\varepsilon=\frac{\varepsilon_{ij}k_ik_j}{k^2}$$

is the energy density of the incident wave, and

$$V_z=-\frac{\partial\tilde\varepsilon}{\partial k_z}\Big/\frac{\partial\tilde\varepsilon}{\partial\omega}$$

is the projection of the group velocity on the z axis and is defined in (8).

Using (15), (16), and the expressions for the quantities

$$\frac{\partial\tilde\varepsilon}{\partial k_z}=-\frac{2\beta_2}{v_pk^2},\qquad G(\beta,\gamma;\xi)=\frac{\Gamma(\gamma-1)}{\Gamma(\beta)}\xi^{1-\gamma}+\frac{\Gamma(1-\gamma)}{\Gamma(\beta-\gamma+1)}$$

in the resonance region $|k_z| \to \infty$, $|\xi| \to 0$, we obtain

$$P(k_x,k_y)=\frac{\omega\mu}{2\pi}\frac{|\beta_2|}{v_p}|\varphi_{\rm inc}(\xi\to0)|^2;$$

$$\varphi_{\rm inc}(\xi\to0)=-\frac{\Gamma(\gamma-1)}{\Gamma(\beta)}C_0^{-1}(e^{-i\pi}\xi)^{1-\gamma}\left\{\int_0^{\infty}q(-\xi')G(1-\beta,2-\gamma;\xi')d\xi'\right.$$
$$\left.-\frac{\Gamma(\beta)}{\Gamma(\gamma-\beta)}\int_0^{\infty}q(\xi')e^{-\xi'}G(\beta-\gamma+1,2-\gamma,\xi')d\xi'\right\}, \tag{18}$$
$$\arg\xi=0,\quad \beta_2>0,$$
$$\arg\xi=\pi,\quad \beta_2<0.$$

The constant C_0 from (17), which enters in this expression, can be reduced with the aid of the definition (14) and the known relations for the gamma functions to the form

$$C_0=-\exp[\pi(\beta_2-i\beta_1)]. \tag{19}$$

The general expression (18) greatly simplifies in the case

$$(\mu a)^2\ll{}^1/_2\sin^2\alpha<(u)^{1/2}=\omega_H/\omega<{}^1/_2, \tag{20}$$

which is just the case which will be considered from now on. First, we recognize that in the region of the quasi-transverse propagation of the ordinary pump wave $\xi > \xi_q = \xi(v_q)$, where

$$v_q=1-(u)^{1/2}\sin^2\alpha/2\cos\alpha, \tag{21}$$

the value of the source $q \sim \rho$, which enters in the right-hand side of (13) and (18), is close to zero, inasmuch as in this region the ordinary wave is polarized along the magnetic field **H** (**E** ∥ **H**). In the considered case (20), the lower limit of the quasi-transverse propagation $v = v_q$ lies between the resonance point v_r from (10) and the lower limit of the region of excitation of the longitudinal oscillations $v_l = 1 - u$ as shown in Fig. 3.

In the region of the quasi-longitudinal propagation $\xi < \xi_q < 0$, the general expression (4) for the density ρ of the induced charges takes the form

$$\rho(k_x,k_y,z)=\exp\{ik_y\operatorname{tg}\alpha z\}\left[\exp\left\{i\frac{\omega}{c}(\varepsilon)^{1/2}(z-z_r)\right\}\right.$$
$$\left.+\exp\left\{i\psi-i\frac{\omega}{c}(\varepsilon)^{1/2}(z-z_r)\right\}\right]\rho_1(k_x,k_y), \tag{22}$$
$$\rho_1(k_x,k_y)=-i\frac{k_x+ik_y}{1+(u)^{1/2}}\frac{\delta v(k_x,k_y)}{4\pi}\frac{E_0}{2^{1/2}\varepsilon^{1/4}}$$

Here E_0 is the amplitude of the plane wave incident on the plasma layer, with components $E_x = E_0/2^{1/2}\epsilon^{1/4}$ and $E_y = iE_x$; the first and second terms in the square brackets correspond to the waves incident and reflected from the plane $v = 1$, and ψ is the phase shift of the reflected wave at the resonance point $z_r = (1 - v_r)/\mu$. Further, $\delta v(k_x, k_y)$ is the Fourier transform of the dimensionless concentration perturbation $\delta v(x, y')$ from (4) in the primed coordinate system connected with the perturbation (see Fig. 1), and the dielectric constant ϵ in the region of excitation of the longitudinal waves $v_l = 1 - u < v < v_q$ is approximately equal to $\epsilon \approx (u)^{1/2}$. (We have taken into account here the fact that $\cos\alpha = 1$ and $u \ll 1$ according to (20).) In the derivation of (22) it was also recognized that perturbation $\delta v(x, y')$ is strongly elongated along the magnetic field and at the characteristic dimension of the problem does not depend on the coordinate $z' \parallel$ **H**.

According to (22), the expression for the source q in (13) and (18) can be represented in the form

$$q=q_1\{\exp[\eta^{(+)}\xi]+\exp[i\psi+\eta^{(-)}\xi]\}\Theta(\xi_q-\xi),$$
$$\eta^{(\pm)}=\frac{1}{2}-\frac{ik_2\operatorname{tg}\alpha}{2K_1}\pm i\frac{\omega}{c\mu}u^{1/4}\frac{u\cos^2\alpha-1}{2K_1}, \tag{23}$$
$$\Theta(x)=1,\quad x>0,$$
$$\Theta(x)=0,\quad x<0,$$

where q_1 does not depend on the coordinate

$$q_1=\frac{4\pi\rho_1}{\mu^2}\frac{1-u}{2K_1}=-i\frac{1-(u)^{1/2}}{2K_1\mu^2}(k_x+ik_y)\delta v(k_x,k_y)\frac{E_0}{2^{1/2}u^{1/8}}, \tag{24}$$

and the ($\pm$) signs correspond to terms generated by the incident and reflected pump waves.

When substituting (23) in the definition (18) of $\varphi_{\rm inc}$, we recognize that under the condition (20) the quantity ξ_q is much smaller in absolute value than the lower limit of the region of excitation of the longitudinal waves $\xi_l(v = 1 - u)$. Putting in this case $\xi_q = 0$ in (23), we obtain[14]

$$\varphi_{\rm inc}(\xi\to0)=-\frac{\Gamma(\gamma-1)\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta+1)}C_0^{-1}(e^{-i\pi}\xi)^{1-\gamma}q_1\{F(1,\gamma,\gamma-\beta+1,1-\eta^{(+)})$$
$$+e^{i\psi}F(1,\gamma,\gamma-\beta+1,1-\eta^{(-)})\}, \tag{25}$$

where $F(\alpha, \beta, \gamma, z)$ is a hypergeometric function.

Let us simplify the derived expression. Using the properties of hypergeometric functions, we obtain

$$F(1,\gamma,\gamma-\beta+1,1-\eta^{(\pm)})=\frac{\beta-\gamma}{\beta}F(1,\gamma,\beta+1,\eta^{(\pm)})$$
$$+\left(\frac{1-\eta^{(\pm)}}{\eta^{(\pm)}}\right)^{\beta}(1-\eta^{(\pm)})^{-\gamma}\frac{\Gamma(\gamma-\beta+1)\Gamma(\beta)}{\Gamma(\gamma)}. \tag{26}$$

We now recognize that the imposed conditions (1), (3), and (20) ensure satisfaction of the inequalities

$$\operatorname{Re}\eta^{(\pm)}={}^1/_2\gg|\operatorname{Im}\eta^{(\pm)}|,\quad \operatorname{Re}\beta=\beta_1\sim1/\mu a\gg1,\quad |\gamma\eta^{(\pm)}/\beta|\ll1,$$

from which it follows primarily that in the right-hand side of (26) we have $F(1, \gamma, \beta + 1; \eta^{(\pm)}) \approx 1$. Further, using the properties $\gamma = 1 + 2i \operatorname{Im} \beta = 1 - 2i\beta_2$, $1 - \eta^{(\pm)} = (\eta^{(\pm)})^*$ and the known relations for the gamma functions, we can easily verify that the first term in the right-hand side of (26) is small (of the order of $(\mu a)^{1/2}$ of the second), and reduce (25) to the form

$$\varphi_{\rm inc}(\xi\to 0) = q_1\Gamma(-2i\beta_2)\exp\{\pi(\beta_2+i\beta_1)\}\xi^{2i\beta_2}$$
$$\times\left[\frac{\exp\{i\arg(\eta^{(+)})^{-2\beta}\}}{1-\eta^{(+)}} + \frac{\exp\{i\arg(\eta^{(-)})^{-2\beta}+i\psi\}}{1-\eta^{(-)}}\right], \tag{27}$$
$$\arg\xi = 0, \quad \beta_2 > 0,$$
$$\arg\xi = \pi, \quad \beta_2 < 0.$$

In the derivation of (27) we have used expression (19) for the constant C_0.

It follows from (27) that the amplitude of the longitudinal wave incident on the hole decreases by a factor $\exp(-\pi|\beta_2|)$, i.e., if $|\beta_2| \sim (\mu a)^{-1}\sin\alpha \gg 1$ then the wave is practically completely absorbed in the resonance region.[1-4] With the aid of (27) and (24) it is easy to find the intensity of the wave incident on the pole

$$|\varphi_{\rm inc}(\xi\to 0)|^2 = \frac{\pi v_2|E_0|^2}{4|\beta_2|\mu^2}|\delta v(k_x, k_y)|^2\frac{f(u)}{8}[|\eta^{(+)}|^{-2} + |\eta^{(-)}|^{-2}], \tag{28}$$
$$f(u) = u^{-1/4}(1-(u)^{1/2})/(1+(u)^{1/2}).$$

We have left out from (28) the crossing term $\sim e^{i\varphi}/\eta^{(+)}(\eta^{(-)})^*$, since it vanishes after averaging over the difference of the phases ψ between the incident and reflected pump waves. The parameter v_2 is defined in (9).

Substituting (28) in the definition of the energy flux (18) and recognize that under condition (20) we have $v_2/v_r|2\eta^{(\pm)}|^2 \approx 1$, we obtain

$$P(k_x, k_y) = {}^1/_8|E_0|^2(\omega/\mu)|\delta v(k_x, k_y)|^2 f(u). \tag{29}$$

It follows from (29) that the total energy flux dissipated in the electron component of the plasma as a result of linear transformation of the ordinary wave by the concentration perturbation δv is given by[3)]

$$P = 4\pi^2\int P(k_x, k_y)\,dk_x\,dk_y = |E_0|^2\frac{\omega}{\mu}\frac{f(u)}{8}\int[\delta v(x, y')]^2 dx\,dy'. \tag{30}$$

(We recall that the perturbation v is defined in a coordinate system with axes x and $y' \perp \mathbf{H}$, see Fig. 1.)

Specifying by way of example δv in the form of a Gaussian function with a characteristic dimension a,

$$\delta v = \delta v_1 \exp\{-(x^2+y'^2)/a^2\},$$

we obtain

$$P = \pi a^2\frac{|E_0|^2}{8\pi}\frac{\omega}{\mu}(\delta v_1)^2\frac{\pi}{2}f(u). \tag{31}$$

Thus, the dissipated energy turns out to be proportional to the square of the relative perturbation of the concentration $4\pi e^2\delta N/m\omega^2$, to the intensity $|E_0|^2$ of the wave incident on the plasma, and to the characteristic dimension $1/\mu$ over which the change of the concentration in the layer takes place.

It follows from (27) that as $\nu_e \to 0$ the energy of the longitudinal waves is dissipated in a collisionless plasma in the resonance region (10) over distances $\sim u/\mu$ from the axis of the inhomogeneity in which the excitation of the longitudinal oscillations takes place.

The picture of the absorption of longitudinal waves in the presence of noticeable damping at $\nu_e \neq 0$ is significantly different. Indeed, it can be shown that the transformation of the transverse pump wave into a

$${}^2/_9(1-u)^2(\omega_H{}^3a^2/c^3\mu)^2/u\cos^2\alpha \ll 1$$

longitudinal wave on an inhomogeneity with a small characteristic dimension takes place in the region of the upper hybrid resonance $v = v_1 = 1 - u$. In this region the group velocity of the excited longitudinal oscillations $\mathbf{V} = \partial\omega/\partial\mathbf{k}$ vanishes (see (16) and (7)). Therefore at not too small values of ν_e, the linear absorption of the longitudinal waves turns out to be decisive in the immediate vicinity of their excitation.

Using the expression for the Joule heating of the plasma electrons,[15]

$$\tilde{P} = \int\frac{\nu_e}{2\pi}|\nabla\varphi|^2 v\varphi_p\,dx\,dy'\,dz'$$

(φ_p is the polarization factor) we can find the energy flux dissipated in the volume of the inhomogeneity. In first-order approximation in the small quantity ν_e, we obtain

$$\tilde{P} = P\frac{\nu_e}{\omega}(1+u)[(\mu a)^2u(1-u)\cos^2\alpha]^{-1/3}C_1.$$

The total energy flux P transformed into longitudinal plasma oscillations, which enters in this expression, is defined in (31), while the numerical factor is $C_1 \sim 1$. Under the conditions of the upper atmosphere at $a \sim 1$ m, $\mu \sim 10^{-5}$ m^{-1}, and $\nu_e/\omega \sim 10^{-5}$, an appreciable fraction of the longitudinal-wave energy is absorbed as a result of Joule heating directly in the inhomogeneity.

We emphasize that the large energy dissipation in the inhomogeneity is due to the fact that in the region of the upper hybrid resonance the group velocity of the excited longitudinal waves is not only small in magnitude but is also directed along the inhomogeneity axis.

3. INSTABILITY OF SMALL INITIAL PLASMA PERTURBATIONS

It was shown above that the presence of inhomogeneities in a smooth plasma layer leads to an effective dissipation in them of the energy of the ordinary wave. If the dissipated energy contributes to an incident of the initial homogeneities, then this process can generate an instability that leads to disintegration of the smooth plasma layer.

We consider a strongly magnetized plasma in which the transport processes are determined by diffusion and by the electronic thermal conductivity along the magnetic-field force lines. The perturbations of the electron temperature δT_e and of the concentration δN in such a plasma are described by the equation[15]:

$$\frac{\partial\delta N}{\partial t} - D_a\frac{\partial^2}{\partial z'^2}\delta N - k_T\frac{N}{T_e}\frac{\partial^2}{\partial z'^2}\delta T_e = -\tau_N^{-1}\left(\delta N - \gamma_1\delta T_e\frac{N}{T_e}\right)$$
$$\frac{\partial\delta T_e}{\partial t} - \frac{\varkappa_e}{N}\frac{\partial^2}{\partial z'^2}\delta T_e = -\tau_T^{-1}\delta T_e + \frac{2W_1}{3N}. \tag{32}$$

Here D_a is the coefficient of longitudinal ambipolar diffusion, $k_T \sim 1$ is the thermodiffusion ratio, κ_e is the coefficient of longitudinal thermal conductivity of the electrons, τ_N is the electron lifetime, γ_1 is the coefficient of displacement of the ionization equilibrium $\tau_T = 1/\delta\nu_e\varphi_T$ is the time of relaxation of the electron temperature T_e as a result of the collisions with the ions and the neutral molecules (ν_e is the collision frequency, δ is the relative fraction of the energy lost by the electron in one collision, and $\varphi_T \sim 1$ is the nonisothermy factor).

The z' axis is directed as before along the magnetic field $\mathbf{H}$. Furthermore, W_1 is the power dissipated in a unit volume and serves as the source of the perturbations. In our case, according to (30),

$$W_1=\frac{P}{S}\delta(z'-z_1')=\frac{f(u)}{8}\frac{\omega}{\mu}|E_0|^2\overline{\delta v^2}\delta(z'-z_1'),$$
$$\overline{\delta v^2}=S^{-1}\int[\delta v(x,y')]^2dx\,dy',\qquad S=\int dx\,dy'. \tag{33}$$

It is assumed here that the entire energy of the longitudinal waves is absorbed in the inhomogeneity in the region of the upper hybrid resonance $z_1' = u/\mu\cos\alpha$ (see Fig. 1); S is the area of the inhomogeneity cross section perpendicular to the magnetic field.

It is convenient to write down the solution of (33) in the form[16]

$$\frac{\delta N}{N}=\int_{-\infty}^{t}dt''\int\frac{dz''}{L_T}G(z'-z'',t-t'')\frac{W_1(z'',t'')\tau_T}{{}^3/_2NT_e}, \tag{34}$$

where the Green's function G(z, t) is given by

$$G(z,t-t'')=\frac{L_T}{\pi^{1/2}\tau_T\tau_N}\int_{t''}^{t}\frac{dt'}{b_1^{1/2}}\exp\left\{-\frac{t-t'}{\tau_N}-\frac{t'-t''}{\tau_T}-\frac{z^2}{b_1}\right\}$$
$$\times\left[\gamma_1+\frac{4L_N^2k_T}{b_1}\left(-\frac{1}{2}+\frac{z^2}{b_1}\right)\right], \tag{35}$$
$$b_1=4D_a(t-t')+4D_T(t'-t''),$$
$$L_N=(D_a\tau_N)^{1/2},\qquad L_T=(D_T\tau_T)^{1/2},\qquad D_T=\varkappa_e/N.$$

Here L_N and L_T are the characteristic diffusion and electron thermal conductivity lengths.[15]

Let $\delta N_0(x, y')$ be the initial concentration perturbation, i.e., independent of the source W_1 (33). Then the total perturbation is $\delta N(x, y') = \delta N_0(x, y') + \delta N_E(x, y')$, where δN_E is the additional perturbation generated by the interaction with the ordinary pump wave. For the relative concentration perturbation $\delta v = \delta N/N = \delta N 4\pi e^2/m\omega^2$ we have the analogous expression

$$\delta v=\delta v_0+\delta v_E. \tag{36}$$

Substituting (33), (36) in (34) and taking into account the slow character of the considered processes (see expression (42) below for the instability increment). We arrive at the integral equation

$$\delta v_E=\frac{\pi f(u)}{\mu L_T}\frac{\omega}{\nu_e}\frac{|E_0|^2}{E_p^2\varphi_T}\int_0^t G(0,t-t'')\overline{(\delta v_0+\delta v_E)^2}dt'', \tag{37}$$

which describes the time variation of the initial perturbation δv_0 following the application of the pump wave electric field at the instant $t = 0$. Here $E_p^2 = 3mT_e\delta\omega^2/e^2$ is the intensity of the characteristic plasma field.[15]

Continuing the analysis, we consider for simplicity an individual inhomogeneity and assume the perturbation distribution in it to be Gaussian (see (31); $S = \pi a^2$). In this case Eq. (37) reduces to the following equation for the concentration perturbation at the maximum of the inhomogeneity:

$$\delta v_E=K\int_0^t G(0,t-t'')[\delta v_0+\delta v_E(t'')]^2dt'',$$
$$K=\frac{\pi}{2}f(u)\frac{\omega}{\nu_e}\frac{1}{\mu L_T}\frac{|E_0|^2}{E_p^2\varphi_T}, \tag{38}$$

where the function f(u) and the parameter L_T are defined in (28) and (35).

We consider next the stationary solution of this equation. Recognizing that the stationary perturbation $\delta v_s = \delta v_0 + \delta v_E$ does not depend on the time, we obtain

$$(\delta v_s-\delta v_0)=KG_0(\delta v_s)^2. \tag{39}$$

The stationary value of the Green's function (35) is[16]

$$G_0=\int_0^\infty G(0,t)dt=-\frac{k_TL_N-\gamma_1L_T}{2(L_N+L_T)}. \tag{40}$$

Introducing further the concept of threshold perturbation

$$\delta v_{\rm thr}=\frac{1}{4G_0K}=-\frac{\mu L_T}{\pi f(u)}\frac{\nu_e}{\omega}\frac{L_N+L_T}{k_TL_N-\gamma_1L_T}\frac{E_p^2\varphi_T}{|E_0|^2}, \tag{41}$$

we write down the solution (39) in the form

$$\delta v_s=2[\delta v_{\rm thr}\pm(\delta v_{\rm thr}^2-\delta v_{\rm thr}\,\delta v_0)^{1/2}].$$

Under the conditions of the upper ionosphere, the parameter $\gamma_1 \to 0$ and $\delta v_{thr} < 0$. The dependence of the stationary perturbation δv_s on the initial perturbation δv_0 is shown in this case in Fig. 4. We see that in the region $\delta v_0 > \delta v_{thr}$ each value of δv_0 corresponds to stationary values of δv_s, viz., $\delta v_s > 2v_{thr}$ (solid curve) and $\delta v_s < 2\delta v_{thr}$ (dashed).

If the initial perturbation is negative and exceeds in absolute magnitude the threshold $|\delta v_{thr}|$, $\delta v_0 < \delta v_{thr} < 0$, then there are no stationary solutions and the perturbation increases continuously in time. It is this phenomenon which we call resonant instability. We emphasize that the threshold value of the initial perturbation (41) decreases with increasing power of incident pump wave and with decreasing concentration gradient μ in the layer.

It is also easy to verify that only the upper stationary branch on Fig. 4 (solid curve) is stable. The lower branch, shown dashed, is unstable. Indeed, linearizing Eq. (38) near the stationary value δv_s and changing over to Laplace transforms of the small perturbations $|\delta\tilde{v}| \ll |\delta v_s|$, as defined by

$$\delta\tilde{v}(p)=\int_0^\infty e^{-pt}\delta\tilde{v}(t)dt,$$

we arrive at a dispersion equation that determines the increment p:

$$1-2KG(0,p)\delta v_s=0,$$

where G(0, p) is the Laplace transform of the Green's function G(0, t). It follows from (32) that (as $\gamma_1 \to 0$)

$$G(0,p)=-\frac{k_T}{2}\frac{L_N}{L_T(1+p\tau_N)^{1/2}+L_N(1+p\tau_T)^{1/2}}.$$

Here $G(0, p) = G_0$ from (40). From these expressions and from the definition (31) we see that in the region $\delta v_s < 2\delta v_{thr} < 0$ the increment is $p > 0$, i.e., the stationary values of δv_s shown by the dashed curve in Fig. 4 are indeed unstable. To the contrary, at $\delta v_s > 2\delta v_{thr}$ the increment $p < 0$, i.e., the upper (solid) branch in Fig. 4 is stable.

The characteristic values of the increment p at

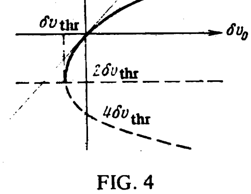

FIG. 4

$\delta v_s < 2\delta v_{thr}$ in the case of fast electronic thermal conductivity $D_T = \varkappa_e/N \gg D_a$ and $\gamma_1 \to 0$ are given by the expression

$$p=\frac{D_a N}{\varkappa_e \tau_T}\left(\frac{\delta v_s}{2\delta v_{thr\,1}}-1\right)^2-\tau_N^{-1},\ \delta v_{thr\,1}=\frac{\delta v_{thr} L_N}{L_N+L_T}. \quad (42)$$

We see that the increment increases energetically with increasing ratio $\delta v_s/\delta v_{thr}$.

Let us determine the character of the development of the resonant instability at large deviations from the initial perturbation $\delta v_0 < \delta v_{thr}$. Under the conditions $D_T \gg D_a$ and $\gamma_1 \to 0$, in the interval $t - t'' < \tau_T D_T/D_a \ll \tau_N$ the Green's function $G(0, t - t'')$ becomes much simpler[16] and Eq. (38) takes the form

$$w(\tau)=\frac{1}{4\pi^{1/2}}\int_{\tau_0}^{\tau}\frac{w^2(\tau'')}{(\tau-\tau'')^{1/2}}d\tau'' \qquad \tau_0=\max\{0,\tau-1\},$$
$$\tau=tD_a/\tau_T D_T, \quad w=\delta v/\delta v_{thr\,1}. \quad (43)$$

(The lower limit in (43) is bounded because of the condition $\tau - \tau'' < 1$.) The asymptotic solution of (43)

$$w(\tau)=4\pi^{-1/2}(b-\tau)^{-1/2}, \quad b=\text{const},$$

shows that the resonant instability has an explosive character. Therefore the characteristic time of the nonstationary processes decreases sharply if the perturbation greatly exceeds (in absolute value) the threshold perturbation.

We have considered above the conditions in the upper ionosphere, $h > 200$ km (the F-layer region) where $\gamma_1 \to 0$. In the region of the ionospheric E layer ($h \approx 100$—150 km), we have $\gamma_1 \approx 0.5$ and, in addition, in daytime $L_N \ll L_T$[50]. Here therefore, in contrast to the F layer, under daytime conditions positive concentration perturbations can become enhanced ($\delta v_{thr} > 0$, see (41)).

In ionosphere experiments[6-12], the F-layer region was perturbed at heights $h \approx 200$—300 km. Stratification of the plasma was observed, with formation of inhomogeneities that are strongly elongated along the earth's magnetic field and have a broad spectrum of transverse dimensions (from 1 km to 1 m), with a characteristic stationary perturbation $\delta N_s/N \sim 10^{-2}$. Under the conditions of these experiments we have $E_0/E_p \sim 1$, and the frequencies are $\nu_e \approx 10^2$ Hz and $\omega \approx 3 \times 10^7$ Hz (night time) or $\nu_e \approx 10^3$ Hz and $\omega \approx 6 \times 10^7$ Hz (daytime).

Consequently, the threshold value of the initial concentration perturbations δN_0 for the excitation of resonant instability amounts, according to (41), to $\delta N_{thr}/N \sim 10^{-4}$—$10^{-5}$.

These are relatively small quantities. Initial perturbations of this order frequently exist in the ionosphere under natural conditions. In addition, they can be generated by self-focusing instability in the region reflection considered by us[17] and by Valeo and Perkins[18] while small-scale perturbations can result from parametric (Perkins[19] Grach and Trakhtengerts[20]) or else drift instability.

Another characteristic feature of experimentally observed stratification of the ionosphere plasma is the effect of the prior conditioning of the ionosphere (Fialer[11]). In the case of an unperturbed ionosphere, the small-scale inhomogeneities developed over a considerable time $t \sim 2$—8 min after the field is turned on. This time is of the order of the reciprocal increment $1/p = \varkappa_e \tau_T/ND_a$ from (42). On the other hand, if the inhomogeneities have already developed, then the characteristic time of their variation, following variation of the power of the perturbing station, is much shorter, on the order of 1—5 sec. This decrease of the characteristic time agrees fully with the growth of the instability increment, $|\delta v_0| \gg |\delta v_{thr}|$, at large initial perturbations of the concentration (see (42)).

It can thus be assumed that the experimentally observed disintegration of the ionosphere layer is due to excitation of resonant instability.

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1)It is assumed that the wave frequency ω exceeds the electron cyclotron frequency ω_H. Under special conditions, particularly in the presence of an appreciable external magnetic field gradient, the extraordinary wave can reach the resonance region and be completely absorbed there. [4,5]

2)We do not consider here nonlinear absorption connected with excitation of parametric instability.

3)We note that expression (30) obtained in the case (20) remains valid, accurate to a factor on the order of unity, also if the weaker condition $v_q \gtrsim v_1 = 1 - u$ is satisfied.

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