Scattering of an intense electromagnetic wave by electrons in a homogeneous magnetic field

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(Submitted January 31, 1975)
Zh. Eksp. Teor. Fiz. **69**, 94–102 (July 1975)

The process of scattering of an intense electromagnetic wave by an electron in a homogeneous magnetic field is considered. The quasi-energy spectrum of the electron is found; it depends significantly on the polarization of the electromagnetic wave. The reason for the appearance of the complex pattern of the electron band spectrum is made clear. The energy spectrum of photons emitted by the electrons in the field of a circularly polarized electromagnetic wave is investigated. It is shown that photon frequency shift $\Delta\omega$, which depends on the intensity I of the electromagnetic wave, is greatest when the electrons in the initial state possess momenta that correspond to the point of intersection of the dispersion curves in the quasi-energy spectrum. Estimates of the probabilities of the scattering process are given.

PACS numbers: 03.50.J

1. INTRODUCTION

In the present work, the emission of a photon by an electron is considered in a homogeneous magnetic field H, and in the field of a circularly polarized electromagnetic wave which is propagated along H. As in[1], the interaction of the electron with the external field is taken into account with the use of the exact solutions of the Dirac equation, and the interaction with the quantized electromagnetic field, by first-order perturbation theory.

According to the results of this research, the quasi-energy spectrum of the electron in an external field depends essentially on the polarization of the electromagnetic wave. In the case of elliptic polarization, the spectrum consists of four energy bands, and in the case of circular polarization of three bands. The difference between the spectrum of the electron for counterclockwise and clockwise circularly polarized electromagnetic waves is also significant. The features of the quasi-energy spectrum of the electron can easily be understood by starting from the classical relativistic equations of motion of a particle in an external field.[2-4] The solution of the equations of motion shows that a new mode of electron excitations is generated in the external field. This mode is characterized by the dispersion law $P_0 - P_z = \text{const}$ (P_0 and P_z are the energy and the z component of the momentum). The mixing of these excitations with the ordinary ones, which are determined by the dispersion law $P_0^2 = \mathbf{P}^2 + m^2$, also leads to a complicated picture of the band spectrum of the electrons.

The energy spectrum of the photons that are emitted by an electron in the field of a circularly polarized electromagnetic wave is determined by an equation of third order. The solution of this equation has been studied in detail for $1 - \cos\theta \ll 1$ ($\cos\theta = \mathbf{k}\cdot\mathbf{k}'/|\mathbf{k}||\mathbf{k}'|$; $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the electromagnetic wave and the wave vector of the emitted photon, respectively). It is shown that the frequency shift $\Delta\omega'$ of the photon, which depends on the intensity I of the electromagnetic wave, is most significant if the electrons in the initial state have momenta corresponding to the point of intersection of the dispersion curves in the quasi-energy spectrum, in this case $\Delta\omega' \sim \sqrt{I}$.

2. TRANSITION MATRIX ELEMENT AND THE QUASI-ENERGY SPECTRUM OF THE ELECTRON

The scattering of the electromagnetic field described by a vector potential ($A_x(\tau)$ and $A_y(\epsilon)$ are arbitrary functions of $\tau = t - z$)

$$\mathbf{A}_1 = (A_x(\tau),\ A_y(\tau),\ 0) \tag{1}$$

by an electron in the presence of a homogeneous magnetic field H, which is specified by the vector potential

$$\mathbf{A}_2 = (-yH,\ 0,\ 0), \tag{2}$$

with emission of a photon with 4-momentum $k' = (\omega'\ \mathbf{k}')$ is characterized by the matrix element

$$M_{p\to p'} = \frac{ie_0}{\sqrt{2\omega'}} \int d^4x\, \bar{\chi}_{p'}^{(+)}(x)\, \hat{e}'\, e^{ik'x}\, \Psi_p^{(+)}(x). \tag{3}$$

Here $\Psi_p^{(+)}(x)$ ($\chi_{p'}^{(+)}(x)$) is the wave function of the electron in the external field $\mathbf{A}_1 + \mathbf{A}_2$, which describes the stationary state of the particle in the homogeneous magnetic field (2) as $t \to -\infty$ ($t \to +\infty$); $p' = p_x', p_z', n', \sigma'$; $p = p_x, p_z, n, \sigma$; $e_0 = |e|$; $e' = (e_0', \mathbf{e}')$ is the 4-vector of the polarization of the photon emitted by the system. The wave functions $\Psi_p^{(+)}(x)$ and $\chi_p^{(+)}(x)$ are expressed in the following fashion in terms of the functions $\psi_p^{(\pm)}(x)$, which are given in the Appendix:

$$\Psi_p^{(+)}(x) = \psi_p^{(+)}(x)|_{\tau_0\to-\infty}, \qquad \chi_p^{(+)}(x) = \psi_p^{(+)}(x)|_{\tau_0\to+\infty}.$$

Omitting the intermediate calculations, which are similar to those which are given in[1], we write out the formula for the matrix element (3) in the case of elliptically polarized electromagnetic wave with frequency ω (for definiteness, we shall assume that $a_1 > 0$):

$$\mathbf{A}_1 = (a_1 \cos\omega\tau,\ a_2 \sin\omega\tau,\ 0). \tag{4}$$

We have

$$M_{p\to p'} = ie_0\sqrt{2}\,\pi^3 (\omega' p_0 \bar{n} p p_0' \bar{n} p')^{-1/2} \delta(p_x' + k_x' - p_x) \sum_{s=-\infty}^{\infty} \delta(P_z' + k_z' - P_z - s\omega) \times \delta(P_0' + \omega' - P_0 - s\omega) B_{s,n'n}^{(\sigma'\sigma)}. \tag{5}$$

Here we have used the notation

$$B_{s,n'n}^{(\sigma'\sigma)} = \left(\frac{1}{\pi}\right)^{1/2} (2^n n! 2^{n'} n'!)^{-1/2} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \exp\Big\{ is\varphi + iF_{p'p} \sin 2\varphi + \frac{i}{2\beta} [-k_x'(N_{p'} + N_p) + k_{y'}(g_{p'} + g_p) - |d_{p'p}|^2 \Big\}$$

$$\times \int_{-\infty}^{+\infty} d\eta\, e^{-\eta^2} \bar{u}_{p'n'\sigma'} \hat{e}' u_{pn\sigma}|_{\zeta_{p'}\to\eta - d_{p'p};\ \zeta_p \to \eta + d^*_{p'p}},$$

$$g_p = -\frac{\beta}{\beta^2 - (kP)^2} (kPe_0a_2 + \beta e_0 a_1) \cos\varphi,$$

$$N_p = -\frac{\beta}{\beta^2 - (kP)^2} (kPe_0a_1 + \beta e_0 a_2) \sin\varphi,$$

$$d_{p'p} = \frac{1}{2\sqrt{\beta}} [g_{p'} - g_p + k_x' - i(N_{p'} - N_p + k_y')], \quad k = \omega\bar{n},$$

$$F_{p'p}=\frac{e_0^2(a_1^2-a_2^2)}{8}\frac{[\beta^2-(kP)(kP')](kP'-kP)}{[\beta^2-(kP')^2][\beta^2-(kP)^2]},$$
$$P_0-p_0=P_z-p_z=-\omega\frac{e_0^2(a_1^2+a_2^2)}{4}\frac{kP+g\beta}{\beta^2-(kP)^2}. \tag{6}$$

The parameter $g = 2a_1a_2/(a_1^2 + a_2^2)$ determines the polarization of the electromagnetic field: for $0 < g < 1$, the electromagnetic field is elliptically clockwise-polarized, and for $-1 < g < 0$, elliptically counter-clockwise polarized; the values of this parameter $g = \pm 1$ and $g = 0$ correspond to circular and linear polarizations, respectively.

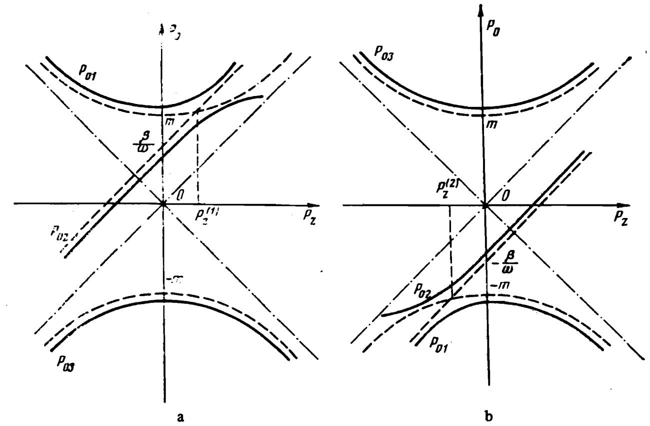
The following important point should be emphasized here. In the case of a monochromatic electromagnetic wave (4), which is switched on and off with adiabatic slowness ($a_1 \sim e^{-\epsilon|t|}$, $a_2 \sim e^{-\epsilon|t|}$ $\epsilon \to +0$), the wave function $\psi_p(x)$ behaves like $e^{-i\epsilon_p t + iP_z z}$ as $t \to -\infty$ ($\epsilon_p = [m^2 + P_z^2 + \beta(2n+1-\sigma)]^{1/2}$; We shall write out only that part of the wave function which depends on t and z; consequently, as $t \to -\infty$, the quantity P_z becomes the z component of the momentum of the electron. As is usual, we shall assume that the wave function and the vector potential $\mathbf{A}_1$ obey cyclic boundary conditions (along the z axis) and therefore the quantities P_z and $k_z = \omega$ take on the quantized values $P_z = 2\pi n_1/L$, $\omega = 2\pi n_2/L$ (L is the length of the principal region, $n_1, n_2 = 0, \pm 1, \ldots$). According to quantum mechanics (see, for example,[5]) as a result of the time evolution of the wave function under the action of the electromagnetic field, the electron energy undergoes a shift that depends on the intensity I of the electromagnetic field. It is important here that a shift in the momentum, which depends on I, cannot arise, since such a shift would mean the destruction of the boundary conditions imposed on the wave function. That no shift in momentum actually occurs can be easily established by direct calculation by computing the value of $U(t)\psi_p(x)$, where the function $\psi_p(t)$ describes the stationary state of the electron in the field (2) and U(t) the operator of the time evolution of the system under the action of the external field.

In the problem considered here, the electron energy shift that depends on I is determined from the formulas (6). Eliminating p_0 and p_z from them, and using the fact that $kp = kP$, we obtain the following dispersion equation, which connects the quasi-energy of the electron P_0 with its momentum (more accurately, the quasi-momentum) P_z:

$$P_0^2-P_z^2-m^2-\beta(2n+1-\sigma)+\frac{e_0^2(a_1^2+a_2^2)}{2}\frac{kP(kP+g\beta)}{\beta^2-(kP)^2}=0. \tag{7}$$

This equation has been studied in[1] for the case of linear polarization of the electromagnetic field ($g = 0$). According to (7), the quasi-energy spectrum of the electron in an external field depends significantly on the polarization of the electromagnetic field. The spectrum consists of four bands for $-1 < g < 1$, and of three bands for $g = \pm 1$, and is different for clockwise and counterclockwise polarizations. More significant is the difference between the spectra of the electron for counterclockwise and clockwise circularly polarized electromagnetic waves: in the case of clockwise polarization, the spectrum consists of two electron bands and one positron band, and in the case of counterclockwise polarization, of one electron band and two positron bands (see the figure).

As explained in[1], radical restructuring of the energy spectrum of the electron in an external field takes place



Quasi-energy spectrum of the electron in the field of a circularly polarized electromagnetic wave and in a homogeneous magnetic field: a—clockwise polarization; b—counterclockwise polarization.

as a result of virtual absorption processes and the simultaneous emission by the electron of several field quanta, i.e., the formation of new bands has the same nature as the Lamb shift of levels in the hydrogen atom. However, these considerations do not explain why the edges of the energy bands are not shifted under the action of the electromagnetic field, but rather new bands are formed.

The answer to this question can be obtained from a solution of the classical relativistic equations of motion. As is well known,[2-4] the quantity $P_0 - P_z$ (P_0 and P_z are the energy and the z component of the momentum of the electron is an integral of the motion of the electron in the field of the electromagnetic wave (4) and in the magnetic field (2). Upon satisfaction of the condition of cyclotron resonance ($1 - v_z = \omega_H$ ($\omega_H = e_0H/P_0$, $v_z = P_z/P_0$) or, what amounts to the same thing, the condition $kP = \beta$, the electron is accelerated in resonant fashion by the electromagnetic field. The motion of the electron, the energy and momentum of which satisfy the given condition, is infinite in the plane perpendicular to the direction of propagation of the wave,[1-6] while, in the absence of resonance the motion of the electron remains finite. What has been pointed out above means that in the external field considered by us, a new mode of electron excitations appears, characterized by the dispersion law $kP = \beta$. Under the action of the electromagnetic field, a mixing of these new excitations with the ordinary ones takes place, the dispersion law for which is given by the formula $P_0^2 = \mathbf{P}^2 + m^2$. As a result, new bands also appear in the energy spectrum.

The fact that the energy spectra of the electron are different in the case of counterclockwise and clockwise circularly polarized electromagnetic fields is also easily understood from classical considerations. Actually, in the case of clockwise polarization, the electric field of the wave, rotating in the same direction as the electron in the magnetic field, is capable of resonance acceleration of the electron, while for a positron, rotating in the opposite direction, the electric field cannot have a resonance effect. For this reason, there are two electron bands and only one positron band (see Fig. a). It is also obvious that in the case of elliptical polarization ($-1 < g < 1$), the spectrum should consist of four branches (since, of the two circularly polarized components into which the elliptically polarized wave can decompose, one acts on the electron states and the other on the positron resonantly). We note that a change in the direction of the magnetic field to the opposite direction changes the band picture: in the case of the clockwise circularly polarized wave there will be two positron bands and one electron band, and in the case of counterclockwise polarization, the opposite will hold.

In what follows, we limit ourselves to a consideration of the case of a circularly polarized electromagnetic wave ($g = \pm 1$) and assume that the condition

$$\xi \equiv \frac{e_0 a}{m^*} \ll 1 \quad (a = a_1,\ m^{*2} \equiv m^2 + \beta(2n+1-\sigma)). \tag{8}$$

is satisfied. We calculate the position of the energy bands of the electron at the momentum

$$P_z^{(1)} = \frac{\omega}{2\beta}\left[m^{*2} - \left(\frac{\beta}{\omega}\right)^2\right], \tag{9}$$

which corresponds to the intersection of the new resonance branch of electron excitations ($kP = \beta$) with the ordinary ($P_0^2 = P_Z^2 + m^{*2}$). Putting $kP_i = \omega(P_{0i} - P_Z)$ ($P_{0i} = P_{0i}(P_Z)$ is the dispersion equation for the energy bands, see the figure), we obtain the following values for this quantity in the case $P_Z - P_Z^{(1)}$ for a plane polarized wave:

$$kP_{1,2} = \begin{cases} \beta + \dfrac{e_0^2 a^2 \omega}{8m^*} \pm \dfrac{e_0 a \omega}{\sqrt{2}} & \text{for } \left|\dfrac{\omega_H - \omega}{\omega}\right| \ll 1, \\ \beta + \dfrac{e_0^2 a^2 \omega_H}{2m^*} \pm e_0 a \omega_H & \text{for } \dfrac{\omega}{\omega_H} \gg 1, \end{cases}$$

$$kP_3 = \begin{cases} -\omega m^* - \dfrac{e_0^2 a^2}{4m^*}\omega & \text{for } \left|\dfrac{\omega_H - \omega}{\omega}\right| \ll 1, \\ -\dfrac{\omega^2 m^{*2}}{\beta} - \dfrac{e_0^2 a^2}{m^*}\omega_H & \text{for } \dfrac{\omega}{\omega_H} \gg 1. \end{cases} \tag{10}$$

($\omega_H \equiv /m^*$). In the case of a counterclockwise circularly polarized wave, the branches of the spectrum intersect at $P_Z^{(2)} = -P_Z^{(1)}$ (see drawing). The location of the branches of the spectrum at this point is determined by Eqs. (10), if we replace kP_i by $-kP_i$ in them ($i = 1, 2, 3$).

The shift of the energy bands of the electron, which depends on the intensity I of the electromagnetic field, is more significant for those bands which arise upon mixing of the resonant and ordinary branches of excitations. According to (10), the shift of the bands mentioned is proportional to $\sqrt{I}$ while the shift of the remaining positron band is proportional to I. It can be shown that, far away from the point of intersection of the branches ($P_Z = P_Z^{(1)}$), the shift of all three bands is proportional to I.

3. ENERGY SPECTRUM OF THE EMITTED PHOTONS AND THE SCATTERING PROBABILITY

From the conservation laws

$$P' + k' = P + sk,$$

where $P = (P_0, P_Z)$, $P' = (P_0', P_Z')$, $k' = (\omega', k_Z')$, $s = 0, \pm 1, \ldots$, corresponding to the process considered, it is easy to obtain the following equation, which determines the energy spectrum of the photons irradiated by the electron in the external field:

$$k'^2 - 2k'(P+sk) + \beta(2n - \sigma - 2n' + \sigma') + 2skP \mp e_0^2 a^2 \frac{\beta k k'}{(\beta \mp kP)(\beta \mp kP \pm kk')} = 0. \tag{11}$$

Here the upper sign corresponds to clockwise and the lower to counterclockwise circular polarization of the electromagnetic wave.

Using the relations (5) and (6), we obtain the following formula for the probability (per unit time) of emission of a photon, summed over the momenta of the final states:

$$W_{n'\sigma',n\sigma}(P_z) = \frac{\alpha}{16\pi}\sum_i \sum_{s=-\infty}^{+\infty} \int d\Omega \left|B_{s,n'n}^{(\sigma'\sigma)}\right|^2 \frac{\omega_i \omega}{g(\omega_i')} \frac{\Theta(kP - kk_i')}{p_0 kP(kP - kk_i')},$$

$$g(\omega') = \left|2\frac{\omega'}{\omega}(1 - \cos^2\theta) - 2\frac{P_0 - P_z\cos\theta}{\omega} - (1 - \cos\theta)\left[2s \pm \frac{e_0^2 a^2 \beta}{(\beta \mp kP \pm kk')^2}\right]\right|, \quad \cos\theta = \frac{k_z'}{\omega'}, \tag{12}$$

where $d\Omega$ is the element of solid angle in the direction of the emitted photons, Σ_i denotes the sum over all roots of the dispersion equation (11).

In the general case, Eq. (11) has three roots for fixed values of the number s and of the quantum numbers of the electron, while at $\cos\theta = 1$, this equation has only a single root and at $\cos\theta = -1$, two roots. At the same time, two of the three roots of the dispersion equation (as $\cos\theta \to 1$) become infinite or, as $\cos\theta \to -1$, one does so. The fact that the roots of the dispersion equation become infinite is connected with the fact that the order of this equation decreases for $\cos\theta = \pm 1$: for $\cos\theta = \pm 1$, the superfluous roots take on infinitely large values. It is easy to show, using Eq. (12), that the probability of photon emission with frequency $\omega' \to \infty$ is infinitely small; therefore, the infinitely large (in magnitude) roots of (11) cannot be considered. It is easy to obtain solutions of Eq. (11) for $1 - \cos\theta \ll 1$ (in accord with what was said above, one should take into account only a single root (ω_1') of Eq. (11) in this case. Making use of the fact that in the case considered $kk' \ll |\beta - kP|$, we have (for clockwise circular polarization of the electromagnetic wave)

$$\omega_1' = \frac{1}{F}\left[\beta\left(n - n' - \frac{\sigma - \sigma'}{2}\right) + skP\right],$$

$$F = \frac{kP}{\omega} + \left[P_z + s\omega + \frac{e_0^2 a^2 \beta \omega}{2(\beta - kP)^2}\right](1 - \cos\theta). \tag{13}$$

As was shown in the previous section, the amount of the shift of the energy bands of the electron is greater close to the point of intersection of the dispersion curves. Therefore, the greatest interest attaches to the study of the scattering of an electromagnetic wave by electrons with momenta P_Z near $P_Z^{(1)}$ (for clockwise polarization of the wave). With the help of Eq. (13), we obtain, for $P_Z = P_Z^{(1)}$

$$\frac{\omega_1'}{\omega} \approx s + \left(n - n' - \frac{\sigma - \sigma'}{2}\right)\left(1 \mp \frac{1}{\sqrt{2}}\xi\right) \text{ for } \omega \approx \omega_H,$$

$$\frac{\omega_1'}{\omega} = \left[1 + (1 - \cos\theta)\frac{\omega^2}{\omega_H^2}\right]^{-1}\left\{\left(n - n' - \frac{\sigma - \sigma'}{2} + s\right)\left(1 \mp \xi\frac{1 - {}^1/_2(1 - \cos\theta)\omega^2/\omega_H^2}{1 + (1 - \cos\theta)\omega^2/\omega_H^2}\right) \pm s\xi\right\} \text{ for } \omega \gg \omega_H. \tag{14}$$

According to (14), the frequency shift of the photon (which depends on the electromagnetic field) is proportional to $\sqrt{I}$, while, in the absence of a homogeneous magnetic field ($H = 0$) the frequency shift of the photon in the Compton effect is proportional to I.[7] We also note the very strong dependence of the photon frequency ω_1' on the value of the angle θ.

We give the formula for the coefficients $B_{s,n'n}^{(\sigma'\sigma)}$ for the case in which we can neglect the dependence on φ of the function $\rho \equiv 2|d_{p'p}|^2$. The latter can be done for $\xi^2 \ll 1 - \cos\theta \ll \xi$ if $\omega = \omega_H$ or for $\xi^2 \leq 1 - \cos\theta \leq \xi$, if $\omega \gg \omega_H$. We get (taking into account only the polarizations x and y of the emitted photon, $a_1 = a_2 = a$)

$$B_{s,\bar{n}'n}^{(\sigma,-\sigma)} = 2m\bar{n}k' D_{s-n'+n} I_{n'n} \cdot \begin{cases} e_+ & \text{for } \sigma = 1, \\ e_- & \text{for } \sigma = -1, \end{cases}$$

$$B_{s,n'n}^{(11)} = 2\bar{n}Pe_- D_{s-n'+n+1}[\mp\alpha_0 m^* I_{n'n} + \sqrt{2n'\beta}\, I_{n'-1,n}] + 2\bar{n}Pe_+ D_{s-n'+n-1} \cdot$$

$$[\mp\alpha_0 m^* I_{n'n}+\sqrt{2n\beta}\, I_{n',n-1}],$$

$$B^{(-1,-1)}_{s,n'n}=2\bar{n}Pe_-D_{s-n'+n+1}[\mp\alpha_0 m^* I_{n'n}+\sqrt{2(n+1)\beta}\, I_{n',n+1}] \qquad (15)$$

$$+2\bar{n}Pe_+D_{s-n'+n+1}[\mp\alpha_0 m^* I_{n'n}+\sqrt{2(n'+1)\beta}\, I_{n'+1,n}].$$

Here $e_\pm = e_x \pm ie_y$;

$$D_s\equiv D_s(\varkappa_p)=(2\pi)^{-1}\int_0^{2\pi} d\varphi \exp\left\{ is\varphi+i\frac{\varkappa_p}{\omega'}(k_x'\sin\varphi-k_y'\cos\varphi)\right\},$$

$$\varkappa_p=e_0 a\omega'\frac{\beta-kP+{}^1/_2 kk'}{(\beta-kP)(\beta-kP+kk')},\quad \alpha_0=\begin{cases} 2^{-1/2} & \text{for } \omega\approx\omega_H,\\ 1 & \text{for } \omega\gg\omega_H,\end{cases}$$

$$I_{n'n}\equiv I_{n'n}(\rho)=\left(\frac{n!}{n'!}\right)^{1/2}\rho^{(n'-n)/2}e^{-\rho/2}L_n^{n'-n}(\rho),$$

$$\rho=\frac{1}{2}(e_0 a)^2\frac{\beta(kk')^2}{(\beta-kP)^2(\beta-kP+kk')^2}.$$

In Eqs. (15), the upper and lower signs refer to the upper and lower electron bands on figure a, respectively.

In accord with Eqs. (15), the role of the s-quantum processes of emission and absorption increases with increase in the value of the angle θ. It can also be shown that the probability of emission of a photon decreases with increase in the angle θ and at $\theta = \pi$ becomes exponentially small.

In conclusion, we write down the numerical estimate of the probability of emission of the photon within the solid angle defined by the inequality $\xi^2 < \cos\theta < \xi$, for $H = 10^5$ Oe, $\omega = 1.9 \times 10^{14}\ \mathrm{sec}^{-1}$ and $E = 3 \times 10^6$ V/cm (E is the intensity of the electric component of the electromagnetic field). Taking it into account that in the approximation considered here,

$$|B^{(1,1)}_{s,n'n}|\approx|B^{(-1,-1)}_{s,n'n}|,\quad |B^{(\sigma,-\sigma)}_{s,n'n}|\ll|B^{(1,1)}_{s,n'n}|,$$

we get (we set $n = n' = 0$)

$$\int d\Omega W_{01,01}\sim\frac{\alpha\omega_H^2}{\omega}\approx 10^8\ \mathrm{sec}^{-1}\ (\xi\approx 10^{-3}).$$

The authors express their gratitude to Ya. B. Zel'dovich for interest in the research.

APPENDIX

WAVE FUNCTIONS OF THE ELECTRON

The exact solutions of the Dirac equations in the external field $A_1 + A_2$ can be represented in the form (see[1])

$$\psi_p^{(\pm)}(x)=C_{pn\sigma}\exp\left\{-ip_0 t+ip_x x+ip_z z+iyN_p-\frac{i}{2\bar{n}p}\int_{\tau_0}^{\tau} d\tau' R_p(\tau')\right\}$$

$$\times\exp\left\{\frac{-\zeta_p^2}{2}\right\}u_{pn\sigma}|_{p_0=\pm p_{0\sigma}};$$

$$u_{pn1}=\begin{pmatrix}[g_p+e_0A_x-i(N_p+e_0A_y)]H_n+\sqrt{\beta 2n}H_{n-1}\\ (\bar{n}p+m)H_n\\ [g_p+e_0A_x-i(N_p+e_0A_y)]H_n+\sqrt{\beta 2n}H_{n-1}\\ (\bar{n}p-m)H_n\end{pmatrix}$$

$$u_{pn-1}=\begin{pmatrix}(\bar{n}p+m)H_n\\ -[g_p+e_0A_x+i(N_p+e_0A_y)]H_n-\sqrt{\beta}H_{n+1}\\ (-\bar{n}p+m)H_n\\ [g_p+e_0A_x+i(N_p+e_0A_y)]H_n+\sqrt{\beta}H_{n+1}\end{pmatrix} \qquad (A.1)$$

Here

$$\beta=e_0H;\quad H_n\equiv H_n(\zeta_p);\quad \zeta_p=(-\beta y+p_x-g_p)/\sqrt{\beta},$$

$$g_p\equiv g_p(\tau)=-\frac{\beta}{\bar{n}p}\int_{\tau_0}^{\tau} d\tau'\left[e_0A_y(\tau')\cos\frac{\beta}{\bar{n}p}(\tau-\tau')+e_0A_x(\tau')\sin\frac{\beta}{\bar{n}p}(\tau-\tau')\right],$$

$$N_p\equiv N_p(\tau)=-\frac{\beta}{\bar{n}p}\int_{\tau_0}^{\tau} d\tau'\left[e_0A_y(\tau')\sin\frac{\beta}{\bar{n}p}(\tau-\tau')\right.$$

$$\left.-e_0A_x(\tau')\cos\frac{\beta}{\bar{n}p}(\tau-\tau')\right],$$

$$\bar{n}=(1,0,0,1),$$

$$R_p(\tau)=(N_p+e_0Ay)^2-g_p^2+(e_0A_x)^2+2p_x(g_p+e_0A_x).$$

The rest of the notation has the same meaning as in[1] (see Eq. (3)).[1)]

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1) We note that the wave functions used in [1] describe the behavior of the electrons in a uniform magnetic field directed along the negative axis and in the field of a linearly polarized electromagnetic wave propagating in the z direction.

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Translated by R. T. Beyer
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