Effective equation of motion of domain walls in a ferromagnet

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Methods of nonlinear mechanics are used to investigate the motion of domain walls (DW) in a uniaxial ferromagnet. In this way it is possible to reduce the description of domain wall dynamics by means of the magnetization field, on the basis of the Landau–Lifshitz equations, to a description by means of the coordinate of the DW center, and to relate the parameters that characterize the DW dynamics to concrete microscopic characteristics of the problem. On the basis of the effective equation of DW motion obtained, problems considered by way of example are the interaction of a DW with a crystallite boundary parallel to it and its interaction with substitution impurities precipitated on such a boundary. Analytical expressions are obtained for the dependence of the interaction energy on the elastic fields generated by such a boundary and on the distribution of the concentration of impurities precipitated on a dislocation boundary. The equilibrium distribution is found for a DW pinned on obstacles of this type; the effective breakaway fields from such obstacles are determined, and also the spectrum of surface waves of the Winter type.

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1. The dynamic properties of domain walls, which separate the phases of a ferromagnet with opposite orientations of the magnetic moment, determine to a significant degree the magnetization processes in ferromagnets. Domain walls (DW) are regions in which the magnetization has a spatially nonuniform distribution that corresponds to a minimum of the free energy $\mathcal{F}$ of the ferromagnet; that is, it is a solution of the equation

$$\delta\mathcal{F}=0, \tag{1}$$

where the case of a ferromagnet with anisotropy of the "easy axis" type (in the present article we shall restrict ourselves to consideration of only such ferromagnets)

$$\mathcal{F}=\int dv\left\{\frac{\alpha}{2}(\nabla\mathbf{M})^2-\frac{\beta}{2}M_z^2+\frac{\mathbf{H}_m^2}{8\pi}-\mathbf{MH}_0\right\}. \tag{2}$$

Here α is the constant of nonuniform exchange interaction, β is the constant of magnetic anisotropy energy (the Z axis is chosen along the anisotropy axis), $\mathbf{M}$ is the magnetic-moment density of the ferromagnet, $\mathbf{H}_0$ is the intensity of the external magnetic field, and $\mathbf{H}_m$ is the intensity of the static magnetic field produced by the magnetization $\mathbf{M}$ and satisfying the equations of magnetostatics

$$\operatorname{rot}\mathbf{H}_m=0,\quad \operatorname{div}(\mathbf{H}_m+4\pi\mathbf{M})=0. \tag{3}$$

In the absence of an external field, the spatially nonuniform solution of equation (1) that describes the magnetization distribution in a DW has the form (Bloch wall)

$$\cos\theta=-\operatorname{th}\frac{x-x_0}{l},\quad l=\left(\frac{\alpha}{\beta}\right)^{1/2},\quad \varphi=\frac{\pi}{2}: \tag{4}$$

θ is the angle formed by the magnetization $\mathbf{M}$ with the Z axis, φ is the azimuthal angle of the vector $\mathbf{M}$, x is the coordinate along the direction perpendicular to the plane of the DW, and x_0 is an arbitrary constant having the meaning of the center of the DW.

When an external field along the Z axis is turned on, the DW will move in such a way as to increase the volume of the domain with magnetization parallel to the external field. The motion of the DW is described by the equation of motion of the magnetization field, the Landau-Lifshitz[1] equation

$$\partial\mathbf{M}/\partial t=g[\mathbf{MH}^{eff}]+\lambda[\mathbf{M}[\mathbf{MH}^{eff}]], \tag{5*}$$

where g is the gyromagnetic ratio, λ is a relaxation constant, and $\mathbf{H}^{\rm eff}$ is the effective field acting on the magnetic moment and determined by the relation

$$\mathbf{H}^{eff}=-\delta\mathcal{F}/\delta\mathbf{M}. \tag{6}$$

During its motion in a real crystal, the DW will interact with a different sort of inhomogeneity present in the crystal. Because the characteristic relaxation times of the electronic subsystem of the crystal are significantly shorter than the relaxation times of elastic processes, in the investigation of DW motion the defects in the crystal lattice may be considered frozen. The electrons whose state determines such magnetic parameters of the crystal as the exchange-interaction constant and the magnetic-anisotropy constant adiabatically adjust themselves to the local changes of crystal-lattice parameters caused by the presence in the crystal of any defects. Thus the effect of inhomogeneities on the behavior of a DW can be described by means of a quasistatic external field, modulating the local values of the magnetic parameters of the crystal. Change of the magnetic parameters α and β of the crystal will lead, obviously, to change of the free-energy density $\mathcal{F}$ of the magnet; in other words, the dynamics of the DW with allowance for the inhomogeneities present in the crystal will be described by the equation

$$\frac{\partial\mathbf{M}}{\partial t}=g\left[\frac{\delta\tilde{\mathcal{F}}}{\delta\mathbf{M}}\mathbf{M}\right]+\lambda\left[\mathbf{M}\left[\frac{\delta\tilde{\mathcal{F}}}{\delta\mathbf{M}}\mathbf{M}\right]\right], \tag{7}$$

$$\tilde{\mathcal{F}}=\mathcal{F}+\int dv\,f(\mathbf{r},\mathbf{M}(\mathbf{r},t)), \tag{8}$$

where $f(\mathbf{r}, \mathbf{M}(\mathbf{r}, t))$ is some function that describes the change of the free-energy density of the ferromagnet that is caused by the presence of inhomogeneities in the crystal lattice.

Investigation of Eq. (7) in general entails serious mathematical difficulties, since concrete expressions for the function $f(\mathbf{r}, \mathbf{M}(\mathbf{r}, t))$ in the case of a definite sort of defect in the crystal lattice have complicated form. For this reason the authors of early works[2] devoted to investigation of the DW dynamics gave up the approach based on investigation of (7). In these works the problem of the motion of a DW parallel to itself was formulated by means of a time-dependent DW coordinate χ_0. During the DW motion the magnetic moment at each point of the crystal rotates about the X axis, and in consequence there appears a component H_{mx} of the magnetic field. From the equation of motion of the free magnetic moment

$$\frac{\partial \mathbf{M}}{\partial t}=\frac{\partial \mathbf{M}}{\partial x_0}\frac{\partial x_0}{\partial t}=g[\mathbf{M}\mathbf{H}_m]$$

it follows that

$$H_{mx}=-g^{-1}\frac{\partial x_0}{\partial t}\frac{\partial \theta}{\partial x},$$

where θ is determined by the relation (4). With the field H_{mx} is associated an additional DW energy, caused by its motion,

$$\Delta\mathcal{F}=\frac{1}{8\pi}\int H_{mx}^2\,dv.$$

This kinetic energy of the DW can be easily put into the form $\frac{1}{2}\mu S(\partial x_0/\partial t)^2$, where S is the area of the DW surface, and where it is natural to identify $\mu=(2\pi l g^2)^{-1}$ with the mass of unit DW area. But in such an approach it is impossible to determine by means of the DW parameter x_0 the energy of interaction of the DW with a different kind of inhomogeneity present in the crystal, by starting from a microscopic description of the interaction of these objects with the magnetic subsystem of the medium. Therefore in[2] and in all subsequent works[3,4] devoted to the investigation of DW dynamics, the energy of interaction of a DW with obstacles was approximated by a harmonic potential well $Kx_0^2/2$, where K is a phenomenological constant describing the frequency of uniform oscillations of the DW about this defect of the crystal lattice.

For all its simplicity and lucidity, this method has an important shortcoming, namely: the problems are formulated, as a rule, without visible relation to the basic dynamic equation (7) of magnets. As a result, the statement of new problems is impeded and requires special phenomenological considerations. Therefore many interesting and important questions have remained uninvestigated—interaction with an external field, allowance for spatial dispersion, the effective field for breaking away from an obstacle, and so on.

In the present work an approach is suggested that permits a unified and physically clear formulation of a large range of problems on the dynamic properties of domain structures that occur, for example, in such media as ferromagnets, superconductors, antiferromagnets, ferroelectric materials, and so on; and a method is described for systematically obtaining the equations of motion of DW in ferromagnets, by starting from the Landau-Lifshitz equation (7)[1)]. The second section of the present work is devoted to this question. In the third section there is considered, by way of illustration, the problem of the interaction of a DW with inhomogeneities in the crystal caused by the presence of dislocations. The fourth section is devoted to discussion of the question of interaction of a DW with impurities. In the fifth section scattering of a spin wave, normally incident on a DW, is studied, and it is shown that in this situation a spin wave does not undergo reflection.

2. By transformation to dimensionless quantities, $t\to\tau/gM_0$, $r\to\tilde{r}l$, $M\to M_0\mathfrak{M}$, $H\to M_0 h$, Eq. (7) is conveniently rewritten in the form

$$\frac{\partial\mathfrak{M}}{\partial\tau}+\lambda\left[\mathfrak{M},\frac{\partial\mathfrak{M}}{\partial\tau}\right]=(1+\lambda^2)[\mathfrak{M},\mathbf{h}^{eff}]. \qquad (9)$$

(Hereafter we shall omit the tilde mark on the radius-vector $\mathbf{r}$ and its components.) Equation (9) together with the equations of magnetostatics (3) completely describes the dynamic behavior of an isolated DW in a ferromagnet, in the language of the magnetization field.

We shall consider those motions of a DW in which the deviation of the distribution of the magnetization $\mathfrak{M}$ from its equilibrium distribution in a DW is small. Then the concept of DW described by equation (4) retains its meaning, but the constant x_0, which plays the role of the coordinate of the center of the DW, becomes a function of y, z, and τ. Consequently, the solution of (9) and (3) can be represented in the form

$$\mathfrak{M}=\mathfrak{M}_0(x-x_0(\mathbf{r},\tau))+\tilde{\mathbf{m}}(\mathbf{r},\tau), \qquad (10)$$

where the vector $\mathfrak{M}_0$ has the following components:

$$\mathfrak{M}_{0x}=0,\quad \mathfrak{M}_{0y}=\sin\theta(x-x_0),\quad \mathfrak{M}_{0z}=\cos\theta(x-x_0).$$

We shall suppose that the components of the vector $\tilde{\mathbf{m}}$, the derivatives with respect to τ and to the transverse coordinates y and z, and also the amplitude h_0 of the external magnetic field and the perturbation of the free energy $f(\mathbf{r},\mathfrak{M}(\mathbf{r},\tau))$ caused by the presence of defects in the crystal, are quantities of a single order of smallness. With accuracy through quantities of the first order of smallness, the equations of magnetostatics (3) have the solutions

$$h_{mx}=-4\pi\left(\tilde{m}_x-\mathfrak{M}_{0y}\frac{\partial x_0}{\partial y}-\mathfrak{M}_{0z}\frac{\partial x_0}{\partial z}\right),$$
$$h_{mz},\ h_{my}=0. \qquad (11)$$

From the expression (11) it follows that at distances sufficiently far from the DW, there appears a finite magnetic field $\pm 4\pi\,\partial x_0/\partial z$, whose presence leads to divergent expressions in the calculation of the energy of a curved DW. This means that the DW possesses appreciably larger stiffness with respect to bending along the Z axis than with respect to bending along the Y axis, and consequently the motion of the DW will occur in such a way that $\partial x_0/\partial z$ will be appreciably smaller than $\partial x_0/\partial y$ according to the parameter l/L, where L is a characteristic radius of curvature of the wall, which is assumed in the problem to be substantially large. For this reason we shall hereafter neglect the quantity $\partial x_0/\partial z$, laying aside, for example, the problem of the interaction of a DW with a center of dilatation, where the value of $\partial x_0/\partial z$ is appreciable, because consideration of problems of this sort entails significant modification of the method being presented.

We transform to new variables by using the relations

$$m_y=\tilde{m}_y\cos\theta-\tilde{m}_z\sin\theta,\quad m_z=\tilde{m}_z\cos\theta+\tilde{m}_y\sin\theta,\quad m_x=\tilde{m}_x, \qquad (12)$$

that is, we in effect transform to a rotating system of coordinates in which the vector $\mathfrak{M}_0(x-x_0)$ at each point is directed along the Z′ axis. Then, with allowance for the fact that $\partial x_0/\partial z=0$, equation (9) can be rewritten after linearization in the following form:

$$\frac{\partial^2 m_x}{\partial x^2}-\cos 2\theta m_x-\frac{4\pi}{\beta}m_x=\frac{1}{\beta}\frac{\partial\theta}{\partial x_0}\frac{\partial x_0}{\partial\tau}-\frac{4\pi}{\beta}\frac{\partial x_0}{\partial y}\sin\theta, \qquad (13)$$

$$\frac{\partial^2 m_y}{\partial x^2}-\cos 2\theta m_y=\frac{\lambda}{\beta}\frac{\partial\theta}{\partial x_0}\frac{\partial x_0}{\partial\tau}-\frac{1}{\beta}\frac{\partial m_x}{\partial\tau}-\frac{\partial^2\theta}{\partial y^2}+[\mathfrak{M}_0,(\mathbf{h}_0+\mathbf{h}_m+\mathbf{h}^{ob})]_x, \qquad (14)$$

where the field of the obstacle is defined by the relation

$$\mathbf{h}^{ob}=-\frac{\delta}{\delta\mathfrak{M}_0}\int dv\, f\{\mathfrak{M}_0(x-x_0),\mathbf{r}\}. \qquad (15)$$

On substituting the solution of equation (13)

$$m_x=\left(\frac{\partial x_0}{\partial y}+\frac{1}{4\pi}\frac{\partial x_0}{\partial\tau}\right)\sin\theta \qquad (16)$$

into equation (14), we get

$$\beta\left(\frac{\partial^2}{\partial x^2}-\cos 2\theta\right)m_y=\lambda\frac{\partial\theta}{\partial x_0}\frac{\partial x_0}{\partial\tau}$$
$$-\sin\theta\left(\frac{\partial^2 x_0}{\partial y\,\partial\tau}-\frac{1}{4\pi}\frac{\partial^2 x_0}{\partial\tau^2}\right)+\beta\frac{\partial\theta}{\partial x_0}\frac{\partial^2 x_0}{\partial y^2}$$
$$-\beta\frac{\partial^2\theta}{\partial x_0^2}\left(\frac{\partial x_0}{\partial y}\right)^2+\sin\theta\,(h_{0z}+h_z^{ob}+h_{mz})-\cos\theta\,(h_{0y}+h_y^{ob}+h_{my}). \quad (17)$$

The inhomogeneous equation (17) has a bounded solution if its right member is orthogonal to the solution of the homogeneous self-adjoint equation

$$m_y^{\rm hom}=A\sin\theta. \quad (18)$$

From the solvability condition follows also the desired effective equation of motion of the DW:

$$\frac{1}{2\pi}\frac{\partial^2 x_0}{\partial\tau^2}+4\frac{\partial^2 x_0}{\partial\tau\,\partial y}+2\lambda\frac{\partial x_0}{\partial\tau}-2\beta\frac{\partial^2 x_0}{\partial y^2}=-\frac{\partial U}{\partial x_0}, \quad (19)$$

$$U=M_0^{-2}\int_{-\infty}^{\infty}dx\,f(\mathfrak{M}(x-x_0),y,x)-2h_{0z}x_0. \quad (20)$$

Thus the problem of the motion of the magnetization, described by the Landau-Lifshitz equations (9), has been successfully reduced to the problem of the motion of curved surfaces (domain walls) which is described by equation (19). The quantity U in this equation plays the role of the energy of interaction of the DW with the external magnetic field and with inhomogeneities of a different sort in the crystal, and it therefore determines all the static and dynamic properties of the DW.

The equation of motion (19) differs from the equation obtained in[2] in that in the approach that uses the interaction function U, there is a possibility of describing more accurately the interaction of the DW with external objects in each concrete case. Furthermore, Eq. (19) takes account in an explicit manner of spatial dispersion and of the influence of an external magnetic field.

The equation of motion (19) without the dissipative term $\lambda\partial x_0/\partial\tau$ follows from a Lagrangian formalism, in which the Lagrangian is defined by the expression

$$\mathscr{L}=\int dy\left\{\frac{1}{4\pi}\left(\frac{\partial x_0}{\partial\tau}\right)^2+2\frac{\partial x_0}{\partial\tau}\frac{\partial x_0}{\partial y}-\frac{\beta}{2}\left(\frac{\partial x_0}{\partial y}\right)^2-f(x_0,y)+2x_0h_{0z}\right\}. \quad (21)$$

According to this Lagrangian function, the vector energy-momentum density has the form

$$P^0=\frac{1}{4\pi}\left(\frac{\partial x_0}{\partial\tau}\right)^2+\frac{\beta}{2}\left(\frac{\partial x_0}{\partial y}\right)^2+f(x_0,y)-2x_0h_{0z}, \quad (22)$$

$$P^y=\frac{\partial x_0}{\partial y}\left(\frac{1}{2\pi}\frac{\partial x_0}{\partial\tau}+2\frac{\partial x_0}{\partial y}\right). \quad (23)$$

It should be mentioned that the expression (22) for the Hamiltonian density P^0 of the system can be obtained directly from the expression (8) for the free energy of the ferromagnet, in which the presence of defects has been taken into account. But this is insufficient for description of the dynamics of the DW, since the Hamiltonian is not expressed in terms of canonically conjugate variables, which in the case of the Lagrangian (21) are x_0 and

$$p=\frac{1}{2\pi}\frac{\partial x_0}{\partial\tau}+2\frac{\partial x_0}{\partial y}.$$

The expression for the Hamiltonian of the system in terms of the canonically conjugate variables x_0 and p,

$$E=\int P^0\,dy=\int dy\,\{\pi p^2+2px_0\}+U(x_0,y) \quad (24)$$

or the expression (21) for the Lagrangian function can be obtained only from the dynamic equation (9). The Hamiltonian density for uniform motion of the DW, $\partial x_0/\partial y=0$, is of the form

$$P^0=\pi p^2+f(x_0,y)-2x_0h_{0z}.$$

In closing this section, we emphasize that the dynamic behavior of the DW is characterized by the following peculiarity. The dispersion law for small oscillations of an isolated (not pinned on any obstacle) DW in an ideal ferromagnet has the form

$$\omega=4\pi gM_0(k_y+(1+\beta/4\pi)^{1/2}|k_y|), \quad (25)$$

where ω and k_y are the frequency and wave vector of a surface spin wave; this coincides with the spectrum obtained in[4] 2). The spectrum is not invariant with respect to the substitution $k_y\to -k_y$; this is due to the fact that the ground state of the system is not invariant with respect to the substitution $y\to -y$ (in such a substitution the DW transforms to another DW, which adjoins the one under study).

3. We shall consider the problem of the interaction of a DW with a dislocation or a group of dislocations. The microscopic characteristics that enter into the expressions for the magnetic parameters of the crystal (the exchange constant α and the anisotropy-energy constant β) depend on the distances between the atoms in the crystal. Local stresses produced by dislocations deform the crystal lattice near the dislocations and consequently lead to local changes of the magnetic parameters that determine the energy of the DW. It is natural to suppose that these parameters depend only on the dilatational part of the stress tensor σ_{ik}. This means that a hydrostatic (for simplicity) pressure

$$p(\mathbf{r})=-{}^1/_3(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) \quad (26)$$

modulates the exchange and anisotropy constants, which now depend on the coordinates in accordance with the nonuniformity of the distribution of the hydrostatic compression near the dislocation. On the assumption that the pressure and the changes that it produces in these parameters are small and are taken into account only in a linear approximation, we get the expression for the energy density of interaction of the DW with dislocations 3)

$$f_d={}^1/_2\beta\gamma_p p(x,y)\sin^2\theta(x-x_0). \quad (27)$$

The constant γ_p describes the reaction of the magnetic parameters to hydrostatic compression:

$$\gamma_p=d\ln(\alpha\beta)/dp.$$

The nonuniform hydrostatic pressure near an isolated dislocation, so oriented that its Burgers vector b is perpendicular to the DW, while the tangent n to the dislocation is parallel to the axis of easy magnetization, is determined from (see, for example,[5])

$$p(x,y)=\frac{1+\nu}{1-\nu}\frac{\mu b}{3\pi l}\frac{y}{x^2+y^2}, \quad (28)$$

where b is the length of the Burgers vector, μ is the shear modulus, and ν is Poisson's ratio. The expression for the energy of interaction of a DW with a single dislocation is obtained after substitution of (28) and (27) into (20). An estimate of the expression thus obtained gives us

$$U_d=\beta\gamma_p\frac{\mu b}{3\pi l}\frac{1+\nu}{1-\nu}\frac{y}{x_0^2+y^2}. \quad (29)$$

Hence it follows that when an arbitrarily small external field h_{0z} is turned on, the equation of motion (19) of the DW has no stationary solution, and consequently a DW does not become pinned on an isolated edge dislocation.

It must be noted that it is an isolated DW that behaves in this manner. In the presence of domain structure the problem requires separate consideration, since here it is necessary to take account of the interaction between domain walls because of demagnetizing effects.

The expression (29) is correct at distances much smaller than the distance to the nearest dislocation. Since in a real crystal the density of dislocations is such that an appreciable number of them will occur on a single domain wall, their collective interaction with the DW will lead to pinning of it. In view of what was said above, an interesting problem is the interaction of a DW with a regular aggregate of dislocations. We shall consider, by way of example, the interaction of a DW with a dislocation boundary. The nature of the interaction of a DW with such an obstacle depends both on the type of dislocation boundary and on its orientation with respect to the DW.

Restricting ourselves to dislocation boundaries of the "pure tilt boundary" type, we shall describe its orientation by two vectors: the Burgers vector **b** and the tangent **n** to the dislocations that constitute the boundary. We shall so choose the system of coordinates that the unperturbed DW is parallel to the YZ plane (the Z axis is along the anisotropy axis).

A. $\mathbf{n} \parallel X$, **b** arbitrary. In this case $p(\mathbf{r})$ is independent of x, the potential energy U_d is independent of x_0, and consequently the DW, as was to be expected, does not interact with such a dislocation boundary.

B. $\mathbf{n} \parallel Y$, **b** arbitrary. The potential energy U_d depends on the coordinate z:

$$U_d=\frac{\beta\gamma_p}{2}\int_{-\infty}^{\infty}dx\frac{p(x,z)}{\operatorname{ch}^2(x-x_0)}=U_d(x_0,z).$$

In this problem the gradients $\partial x_0/\partial z$ become important, and it requires special consideration, which is not undertaken in the present work.

C. $\mathbf{n} \parallel Z$, $\mathbf{b} \parallel X$ (Fig. 1). By means of formulas (20) and (26) and the expression for the deformation tensor[5], we obtain the potential energy U_d of interaction with a dislocation boundary of this type:

$$U_d=-\frac{\beta\gamma_p\mu b}{6D}\frac{1+\nu}{1-\nu}\int_{-\infty}^{\infty}\frac{\sin Ny\,dx}{(\operatorname{ch}Nx-\cos Ny)\operatorname{ch}^2(x-x_0)}, \tag{30}$$

where $N=2\pi l/D$; D is the distance between dislocations.

We consider the case $N \gg 1$; here (30) is conveniently put into the form

$$U_d=\beta\Gamma\sum_{m=1}^{\infty}\sin Nmy\int_{-\infty}^{\infty}\frac{e^{-Nm|x|}\,dx}{\operatorname{ch}^2(x-x_0)}, \tag{31}$$

$$\Gamma=\frac{\mu b\gamma_p}{6l}\frac{1+\nu}{1-\nu}. \tag{32}$$

The principal contribution in this case comes from the region of small x; this enables us to convolve the series in the expression (31):

$$U_d=\beta\Gamma\Phi(y)\operatorname{ch}^{-2}x_0;\quad \Phi(y)=\begin{cases}1-\dfrac{Ny}{\pi}, & \dfrac{(2k-1)\pi}{N}<y<\dfrac{2\pi k}{N}\\ 0, & y=\dfrac{2\pi k}{N};\quad k=0,\pm1,\pm2,\ldots\end{cases} \tag{33}$$

We shall find the form of a DW pinned on the dislocation boundary under consideration. This form is described by the function $X^{st}(y)$ that is the solution of the static effective equation

$$d^2X^{st}/dy^2+h_{0z}/\beta+\Gamma\Phi(y)\varphi(X^{st})=0,\quad \varphi(X^{ст})=\operatorname{sh}X^{st}\operatorname{ch}^{-3}X^{st}. \tag{34}$$

Equation (34) has a periodic solution of the following form:

$$X^{st}=a+x(y),\quad x(y)=-\Gamma\varphi(a)[Ny^3/6\pi-y^2/2+\pi y/3N]. \tag{35}$$

The value of a, the mean distance of the DW from the dislocation boundary, is determined from the condition for a maximum of U_{eff}:

$$U_{eff}=-\beta\frac{\pi^2\Gamma^2}{90N^2}\varphi^2(a)-h_{0z}a. \tag{36}$$

As is evident from Fig. 2, on which is shown a graph of the function $U_{eff}(a)$ for various fields, $U_{eff}(a)$ at small fields has two minima, corresponding to two stable states, a_1 and a_2. With increase of field, one minimum first disappears at field $h_{0z}=h_1=0.024\beta M_0\Gamma^2/N^2$, which is the field for breaking away of the DW from the dislocation boundary. With further increase of the field, at a certain value $h_{0z}=h_2=0.042\beta M_0\Gamma^2/N^2$ (the field for surmounting of the dislocation boundary by the domain wall), the second minimum disappears. On substituting the value of the parameter $\gamma_p \approx 6\times10^{-6}\,\mathrm{bar}^{-1}$, we get, for $D/b \sim 10$ to 100, the following values of the critical breakaway fields $h_{cr} \sim 10^{-6}$ to 10^{-8} (Ni), 10^{-5} to 10^{-7} (Fe).

The start fields h_1 and h_2 are in essence effective quantities that characterize an individual isolated DW in an infinite crystal. In a comparison of the theory with experimental data, obtained for example in the investigation of Barkhausen jumps, it is necessary to bear in mind that during the motion of a DW in the case of a periodic domain structure, the magnetic field inside the specimen changes in consequence of change of the boundary conditions on the specimen surface. Consequently the effective field for breakaway of an isolated DW from an obstacle will differ from the effective breakaway field of an individual DW in a many-domain structure.

We give the following rough estimate from above for the breakaway field of a DW in a many-domain structure. We assume for simplicity that at $h_{0z}=0$ all the DW, distributed uniformly with density η per unit length,

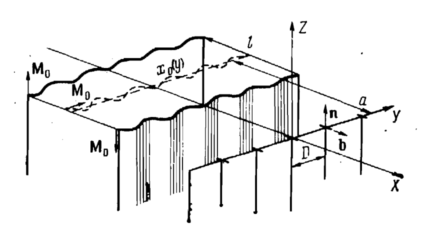

FIG. 1. Structure of a domain wall near a dislocation boundary.

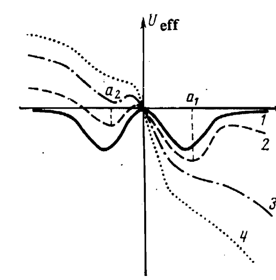

FIG. 2. Effective energy of interaction of a domain wall with a dislocation boundary. Curve 1, $h_{0z}=0$; 2, $h_{0z}<h_1$; 3, $h_{0z}=h_1$; 4, $h_{0z}=h_2$.

are located exactly at the centers of identical potential wells near dislocation boundaries. We shall suppose that after turning on of an external field $h_{0z} \neq 0$, all the DW in the ferromagnet move simultaneously through such a distance Δx as to compensate the magnetic flux through the specimen. This determines the displacement of each DW in field h_{0z}.

$$\Delta x = h_{0z}/8\pi\eta. \quad (37)$$

On substituting for Δx the value of the critical distance, we obtain an estimate of the breakaway field in a many-domain structure (the thickness of the crystal is of the order of 1 mm):

$$h_{cr} = 8\pi\eta[a(0) - a(h_z)]. \quad (38)$$

Thus, for example, $h_{cr} = 8.0$ Oe for Ni and $h_{cr} = 31.3$ Oe for Fe.

In closing this section, we shall consider small oscillations of a DW pinned on an obstacle of the "pure tilt dislocation boundary" type, oriented as shown in Fig. 1. We express the solution of the equation of motion of the DW, $x_0(y, t)$, in the form

$$x_0 = X^{st}(y) + \tilde{x}(y, \tau), \quad (39)$$

where $X^{st}(y)$ is the solution of the static effective equation (35), and where $\tilde{x}(y, \tau)$ is a small addition that describes the oscillations of the DW. We shall seek a solution of the linearized effective equation

$$\frac{1}{2\pi}\frac{\partial^2\tilde{x}}{\partial\tau^2} + 4\frac{\partial^2\tilde{x}}{\partial y\partial\tau} - 2\beta\frac{\partial^2\tilde{x}}{\partial y^2} - 2\beta\Gamma\frac{\partial\varphi}{\partial x_0}(X^{st})\tilde{x} = 0 \quad (40)$$

in the form

$$\tilde{x}(\tau, y) = [\Xi(y) + \xi(y)]e^{i\Omega\tau}, \quad (41)$$

where the functions $\Xi(y)$ and $\xi(y)$ are, respectively, slowly and rapidly (in comparison with the function $\Phi(y)$) oscillating functions of the coordinate y.

After substitution of (41) in (40) and separation of the slowly and rapidly oscillating terms, we obtain the following system of equations:

$$\frac{d^2\Xi}{dy^2} - 2\frac{i\Omega}{\beta}\frac{d\Xi}{dy} + \frac{\Omega^2}{4\pi\beta}\Xi + \Gamma\frac{d^2\varphi(a)}{da^2}\langle x\Phi\rangle\Xi - \Gamma\frac{d^2\varphi(a)}{da^2}\langle x\Phi\xi\rangle + \Gamma\frac{d\varphi(a)}{da}\langle\Phi\xi\rangle = 0, \quad (42a)$$

$$\frac{d^2\xi}{dy^2} - 2\frac{i\Omega}{\beta}\frac{d\xi}{dy} + \frac{\Omega^2}{4\pi\beta}\xi + \Gamma\left\{\frac{d\varphi(a)}{da}\Phi(y) + \frac{d^2\varphi(a)}{da^2}[x\Phi - \langle x\Phi\rangle]\right\}\Xi = 0, \quad (42b)$$

where $\langle\ldots\rangle$ means an average over the period of the function $\Phi(y)$.

On substituting into equation (42a) the solution of (42b)

$$\xi(y) = -\Gamma\Xi\sum_{m\neq 0} B_m e^{imNy},$$

$$B_m = \frac{d\varphi/da}{\pi i N^2 g(m,\Omega)} - \frac{d^2\varphi(a)}{da^2}\frac{\Gamma\varphi(a)}{N^4 g(m,\Omega)}\left[\frac{1}{3m^2} - \frac{4}{\pi^2 m^4}\right], \quad (43)$$

where[4)]

$$g(m,\Omega) = \frac{\Omega^2}{4\pi\beta N^2} + 2\frac{\Omega m}{\beta N} - m^2 \neq 0,$$

we get the equation for $\Xi(y)$:

$$\frac{d^2\Xi}{dy^2} + 2\frac{i\Omega}{\beta}\frac{d\Xi}{dy} + \left\{\frac{\Omega^2}{4\pi\beta} + \frac{d^2U_{\text{eff}}(a)}{da^2}\right\}\Xi = 0. \quad (44)$$

(Equation (44) was obtained on the assumption that $\Omega \ll \beta^{1/2}N$, which, as will be seen below, is well fulfilled for small k_y.)

Equation (44) has a solution of the type $\Xi \sim e^{iky}$ with dispersion law

$$\omega = gM_0\Omega = gM_0[-4\pi k_y + (d^2U_{\text{eff}}(a)/da^2 + 4\pi(4\pi+\beta)k^2)^{1/2}]. \quad (45)$$

As estimate tells us that for $N \sim 10$ to 100, the gap in the spectrum has the order $\omega(0) \sim 1$ to 10 MHz.

4. In this section we shall consider the problem of the interaction of a DW with nonmagnetic impurities. The presence of impurities leads to a change of the magnetic parameters of the crystal. Since the DW width $l \sim (10^5$ to $10^3)$ a_0 (a_0 is the atomic distance), we can describe the modulation of the local values of the constants α and β by means of a continuous function of the coordinates $c(\mathbf{r})$, the impurity concentration. In other words, in the expression (20) for the energy of interaction U_s of a DW with impurities, the function f_s has the form

$$f_s = \gamma_s\beta c(\mathbf{r})\sin^2\theta(x - x_0), \quad (46)$$

where $\gamma_s = d\ln(\alpha\beta)/dc$ describes the reaction of the magnetic parameters of the system in the presence of impurities. It is obvious that a DW will interact only with inhomogeneities in the distribution of impurities; these may be produced by various physical causes: the presence of dislocations, of dislocation centers, of block boundaries, etc. In the present work we shall restrict ourselves to consideration of the problems of the interaction of a DW with substitution impurities, precipitated on an isolated dislocation and on a group of dislocations of the "pure tilt boundary" type, oriented as is shown in Fig. 1.

The distribution of impurities in a crystal at not too low temperatures, in an inhomogeneous stress field, far from the dislocation cores, is described by the expression[5]

$$c(\mathbf{r}) = c_0\exp\left\{-\frac{p(\mathbf{r})(v_s - v_a)}{k_B T}\right\}, \quad (47)$$

where c_0 is the relative concentration of impurities in the crystal in an unperturbed region, v_s and v_a are the atomic volumes of the atoms of the substitution impurity and of the matrix, T is the temperature, and k_B is Boltzmann's constant.

A. In the case of impurities precipitated on an edge dislocation, so oriented that the hydrostatic pressure p is described by equation (28), the expression for the interaction energy U_s^d is obtained after substitution of (46), (47), and (28) in (20). Asymptotic estimates of this expression lead us to

$$U_s^d = \begin{cases} \gamma_s\beta c_0\left[\exp\left(-\frac{T_s^d}{T}\frac{y}{x_0^2+y^2}\right) - 1\right], & |y| \gg 1 \\ -\gamma_s\beta c_0\,\mathrm{ch}^{-2}x_0\frac{|T_s^d|}{T}, & |y| \ll 1,\quad T_s^d y > 0 \\ \gamma_s\beta c_0\,\mathrm{ch}^{-2}x_0\left[\left(\pi T y^2\left|\frac{y}{T_s}\right|\right)^{1/2}\exp\left(-\frac{T_s^d}{Ty}\right) - 1\right], & |y| \ll 1,\quad T_s^d y < 0 \end{cases} \quad (48)$$

where

$$T_s^d = \frac{\mu b(v_s - v_a)}{3\pi l k_B}\frac{1+\nu}{1-\nu};$$

or a wide class of magnets, $T_s^d \sim 0.1$ to 1 K.

As in the case of direct interaction with an isolated dislocation, the effective equation (19) has no static solution if U_s^d is described by the expressions (48).

B. The energy U_s^b of interaction of a DW with impurities precipitated on the dislocation boundary depicted in Fig. 1 has the form

$$U_s^b = \frac{\gamma_s\beta c_0}{2}\int_{-\infty}^{\infty}\mathrm{ch}^{-2}(x-x_0)\left\{\exp\left[-\frac{T_s^b}{T}\frac{\sin Ny}{\mathrm{ch}\,Nx-\cos Ny}\right]-1\right\}dx, \quad (49)$$

where $T_s^b = \pi l T_s^d/D$; in most crystals, for $D/b \sim 10$ to 100, $T_s^b \sim 10$ to 100 K.

If we consider only $T \gg T_s^b$, we get

$$U_s^b = -\frac{c_0\gamma_s}{\gamma_p}\frac{v_s - v_a}{k_B T}U_d. \quad (50)$$

It is clear that the expression for U_s^b will lead to a static solution of the type (35) and to effective breakaway fields that differ from the field for surmounting the dislocation boundary by a factor A, where

$$A \approx 10c_0^2(D/b)^2(T_s/T)^2, \quad (51)$$

and also to a dispersion law, analogous to (45), for small oscillations; the gap changes by a factor $\sqrt{A}$. For the parameter values $c_0 = 0.04$, $D/b = 80$, and $T = 500$ K, the constant A takes the value 0.2. It must be noted that in the calculation of the obstacle parameters, both effects (impurity and dilatational) are combined.

5. We shall show that the transmission coefficient of a spin wave normally incident on a DW is unity. For this purpose we shall seek a solution of equation (5) in the form

$$\mathbf{M} = \mathbf{M}_0(x-x_0) + \boldsymbol{\mu}(x, t), \quad (52)$$

where μ far from the DW ($x \to \pm\infty$) describes a spin wave with the dispersion law (see, for example,[6])

$$\omega = gM_0[(\beta+\alpha k^2)(\beta+4\pi+\alpha k^2)]^{1/2}.$$

On transforming to the variables (12), we obtain from equation (5) the following system of linearized equations:

$$\partial m_y/\partial\tau = \alpha\Delta m_x - 4\pi m_x - \beta m_x\cos 2\theta(x-x_0),$$
$$\partial m_x/\partial\tau = -\alpha\Delta m_y + \beta m_y\cos 2\theta(x-x_0). \quad (53)$$

The exact solution of this system having the asymptotic form $e^{i(\omega t+kx)}$ for $x \to +\infty$ (transmitted wave) has the form

$$m_{x,y} = Be^{i(\omega t+kx)}(\mathrm{th}\,(x-x_0)/l + ikl). \quad (54)$$

Hence it is evident that the solution (54) has no terms $\sim e^{i(\omega t-kx)}$ (reflected wave); that is, the coefficient of transmission of a spin wave through a domain wall is unity.

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1) This method is essentially a generalization of the method proposed by Landau and Lifshitz [1].

2) Winter [3] studied the spectrum of small oscillations of a DW by direct investigation of the Landau-Lifshitz equation, and he took account of the interaction with obstacles by rewriting the interaction energy $Kx_0^2/2$ in the language of the magnetization field. Allowance for the energy of interaction with obstacles leads to the appearance in the spectrum of a gap $\Delta = 4\pi gM_0K$.

3) The arguments presented above are of course correct only in the case of those dislocations that produce at each point of the crystal a change of the specific volume, for example edge dislocations. As regards the class of dislocations (including, for example, screw dislocations) that generate a stress tensor with zero trace, they require special consideration. We note only that apparently, by virtue of the considerations enumerated, they influence the behavior of DW to a lesser degree than do edge dislocations.

4) It is obvious that values of Ω that are solutions of the equation $g(m, \Omega) = 0$ are forbidden, and close to these energy values the method of solution used is not suitable. But we are interested only in $k_y \ll 1$ (the function $\Xi(y)$ must be smooth in comparison with $\Phi(y)$, and consequently the frequency is close to $(d^2U_{eff}/da^2)^{1/2} \ll \Omega^*$ where $\Omega^* = 4\pi N[(1 + \beta/4\pi)^{1/2} - 1]$ is the lowest forbidden frequency value.

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*$[\mathbf{MH}^{eff}] \equiv \mathbf{M} \times \mathbf{H}^{eff}$.

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238