

# Investigation of the interaction between electrons and a boundary by means of transverse focusing

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A method is developed for measuring the specularity coefficient  $q$  of electrons reflected from a boundary. The method is based on focusing the electrons in a metal by a transverse homogeneous magnetic field. The coefficient  $q$  for electrons normally incident on the boundary can be determined by the method and the dependence of  $q$  on the angle of incidence  $\theta$  can be found. Results of measurement in bismuth are presented.

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Focusing of electrons in a metal by a transverse homogeneous field was observed earlier<sup>[1]</sup> with the aid of a microcontact technique.<sup>[2]</sup> The experiments were performed on bismuth samples. Besides a voltage peak on the receiving collector contact, observed in a magnetic field  $H^*$  when the diameter of the electron trajectory was comparable with the distance  $L$  between the contacts, voltage peaks were observed in<sup>[1]</sup> on the collector in the case of multiple fields  $2H^*$  and  $3H^*$ . It was established that, in multiple fields, electrons that are specularly reflected by the sample surface are focused on the collector, and it was noted that the electron focusing (EF) in multiple fields can be used to investigate the interaction of the electrons with the boundary between two media (EBI).

The use of electron focusing as a method of investigating electron-boundary interactions is of considerable interest for the following reasons. First, because of the simplicity of the experimental setup. Second, the methodological possibilities of investigating electron-boundary interactions are quite modest; among the methods that permit investigation of EBI of definite electron groups, notice should be taken of cyclotron resonance on electrons that are specularly reflected from the sample surface,<sup>[3]</sup> observation of this resonance has made it possible to establish in bismuth the specular character of reflection of electrons incident at an angle  $\sim 70^\circ$ , and the Khaikin surface-impedance oscillations,<sup>[4]</sup> so that it is possible to study the reflection of electrons traveling at small angles to the boundary (see, e.g. <sup>[5]</sup>). Third, as will be shown later on, it is possible to use the EF not only to determine the specularity coefficient  $q$  of electrons incident on a boundary at an angle  $\theta$  close to  $90^\circ$ , but also establish the dependence of  $q$  on  $\theta$ .

## PHYSICAL PRINCIPLES OF THE METHOD

Inasmuch as at present the theory of electric conductivity of a plate, with allowance for the inhomogeneity introduced by the current-conducting contacts, has been constructed only for the case when the contacts are on opposite sides of the plate,<sup>[6]</sup> we present a qualitative model analysis of EF wherein we can obtain (see below) good quantitative agreement with the measurements of EF in bismuth.

The geometry of the experiment for the observation of EF is shown in Fig. 1.<sup>[1]</sup> Two contacts B and C are placed on the samples surface M at a distance  $L$  apart. Current is passed through emitter B, and the voltage U

on the collector C is measured as a function of the magnetic field  $H$ , which is applied in the plane of the sample and is perpendicular to BC. The value of  $U$  is determined mainly by the electrons (effective) which acquire in the emitter region (in the emitter) a momentum increment  $\Delta p$  and fall in the collector region (in the collector) with this increment preserved.<sup>[2]</sup> In fields exceeding  $H^*$ , the electrons from the emitter can reach the collector only after reflection from the sample boundary, so that at  $H > H^*$  the value of  $U$  is determined to a considerable degree by the character of the EBI. The experimental conditions determine the relative contribution made to  $U$  by various groups of electrons that interact in definite fashion with the boundary. Consequently, measurements of  $U(H)$  can yield information on the EBI.

We consider for simplicity the case of cylindrical Fermi surface (FS) with  $H$  directed along its axis. The electrons reaching the collector describe an arc that bears on the surface, the chord subtending the arc having a length such that

$$s_n^{min} = n^{-1}(L-b/2) \leq s \leq n^{-1}(L+b/2) = s_n^{max}, \quad (1)$$

$n = 1, 2, 3, \dots$ ,  $b$  is the linear dimension of the collector, the emitter is assumed pointlike,  $n > 1$  is possible only in the case of specular reflection by the boundary,  $n$  determines the number of jumps, and the number of reflections is  $n - 1$ . It is assumed that the electron mean free path is  $l \gg L$ , and the limitation imposed on the number of effective electrons by their scattering in the sample volume is neglected. The number of effective electrons at a given  $H$  is given by

$$N(a) \sim \sum_{i=1}^m \int_{F(i,a)}^{M(i,a)} q^{i-1}(\theta) d\theta + \sum_{i=0}^{\infty} \int_{F(m+i,i)}^{M(m+i,i)} q^{m+i}(\theta) d\theta, \quad (2)$$

$a = H/H^*$ ,  $\theta$  is the angle of incidence of the electron on the boundary,<sup>[1]</sup>  $q(\theta)$  is the specularity coefficient and is equal to the probability of specular reflection for a given  $\theta$ ;  $M(i, a)$  and  $F(i, a)$  are respectively the maximum and minimum incidence angles of the effective electrons

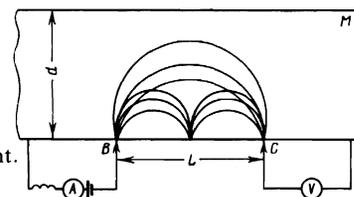


FIG. 1. Diagram of the experiment.

that execute  $i$  jumps at a given  $a$ ;  $M(1, a) = M(a)$ ,  $F(1, a) = F(a)$ ;  $m$  is determined by the condition  $s_i^{\min} \leq b$  for all  $i \geq m$ .

In the general case it is difficult to determine  $q(\theta)$  from measurements of  $U(a) \sim N(a)$  in accordance with (2), but there are a number of circumstances that facilitate this procedure. It is easy that  $M(i, a) = M(a/i)$  while  $F(i, a) = F(a/i)$ , and (2) takes the form

$$N(a) \sim \sum_{i=1}^m \int_{F(a/i)}^{M(a/i)} q^{i-1}(\theta) d\theta + \sum_{i=0}^{\infty} \int_{F(a/(m+i+1))}^{F(a/(m+i))} q^{m+i}(\theta) d\theta. \quad (3)$$

Plots of  $M(a)$  and  $F(a)$  are shown in Fig. 2a. Figure 2b shows the function  $n_1(a) = M(a) - F(a)$ . The position of the maximum  $n_1 = n_1(a_{\text{ext}}) = \pi/2 - F(a_{\text{ext}})$  is determined by the condition  $2r_H \approx L + b/2$  ( $r_H$  is the Larmor radius), and  $f(a_{\text{ext}}) \approx \pi/2 - (2b/L)^{1/2}$ ; at  $2r_H < L - b/2$  we have  $n_1 = 0$ . At fixed  $a$ , the first series in (3) begins with the  $k$ -th term,  $k$  being such that for all  $i \geq k$  we have  $2r_H > (L - b/2)i$ .

In the vicinity of  $a = k$ , only the  $k$ -th term of the first series makes a decreasing contribution to  $N(a)$  with increasing  $a$ ; the rest of the sum gives a monotonically increasing term. The decrease is due to the fact that with increasing  $a$  the electrons, whose number is proportional to

$$\int_{F(a_{\text{ext}})}^{\pi/2} q^{k-1}(\theta) d\theta, \quad (4)$$

cease to fall into the collector, in other words, the amplitude  $k$  of the peak is proportional to (4). This is a significant circumstance that makes it possible to determine directly the mean value of  $\bar{q}^{[1]}$  for the angle interval from  $F(a_{\text{ext}})$  to  $\pi/2$ , from the formula

$$\int_{F(a_{\text{ext}})}^{\pi/2} q^{k-1}(\theta) d\theta = \bar{q}^{k-1} \left[ \frac{\pi}{2} - F(a_{\text{ext}}) \right]. \quad (5)$$

We note that the difference  $\pi/2 - F(a_{\text{ext}})$  is proportional to the amplitude of the first EF line. The second series in (3) converges rapidly, and its  $i$ -th term is  $\sim 1/(m+i)^2$ .

At sufficiently small  $b$ , in the angle interval  $M(a/i)$  to  $F(a/i)$ , we can regard  $q(\theta)$  as a constant. We then obtain from (3)

$$N(a) \sim \sum_{i=1}^m \left[ M\left(\frac{a}{i}\right) - F\left(\frac{a}{i}\right) \right] q^{i-1} \left[ F\left(\frac{a}{i}\right) \right] + \sum_{i=0}^{\infty} \left\{ F\left(\frac{a}{m+i}\right) - F\left(\frac{a}{m+i+1}\right) \right\} q^{m+i} \left[ F\left(\frac{a}{m+i+1}\right) \right]. \quad (6)$$

To determine the form of  $q(\theta)$  we can use the fact that in the vicinity  $a = k$ , at certain values of  $a$ , the quantity  $\partial N/\partial a \sim \partial U/\partial a$  takes on large absolute values and then the  $k$ -th term of the first series of (3) makes the predominant contribution to the derivative. For the derivative of the  $k$ -th term with respect to  $a$  we have

$$\frac{\partial}{\partial a} \int_{F(a/k)}^{M(a/k)} q^{k-1}(\theta) d\theta = q^{k-1} \left[ M\left(\frac{a}{k}\right) \right] \frac{\partial M(a/k)}{\partial a} - q^{k-1} \left[ F\left(\frac{a}{k}\right) \right] \frac{\partial F(a/k)}{\partial a}. \quad (7)$$

At  $b \ll L$  we have  $q[(M(a/k))] \approx q[F(a/k)]$  and

$$\frac{\partial}{\partial a} \int_{F(a/k)}^{M(a/k)} q^{k-1}(\theta) d\theta = q^{k-1} \left[ F\left(\frac{a}{k}\right) \right] \frac{\partial}{\partial a} \left[ M\left(\frac{a}{k}\right) - F\left(\frac{a}{k}\right) \right]. \quad (7')$$

## EXPERIMENT

The samples were bismuth single-crystal disks of 10 mm diameter and 2 mm thick, grown in a dismountable polished quartz mold, with  $C_3$  perpendicular to the surface and with a resistivity ratio  $\rho_{\text{room}}/\rho_{4.2^\circ\text{K}} \approx 400$  ( $\rho_{\text{room}}$  and  $\rho_{4.2^\circ\text{K}}$  are the resistivities at room temperature and  $4.2^\circ\text{K}$ ). The degree of perfection of the surface section on which the contacts were mounted was such that no surface defects could be seen in an optical microscope with magnification up to  $\sim 1000$ . The sample was placed in a magnetic field  $H$  that could be varied in magnitude and rotated in the plane of the sample. The contacts were mounted in such a way that  $BC \perp C_1$ . The collector and emitter were copper needles of 0.1-mm dia wire, sharpened by an electrochemical method. A characteristic of the contact was its resistance, which was determined in order of magnitude by the formula

$$R \approx p_F/e^2 r^2 n_0, \quad R \approx \rho l/r^2, \quad (8)$$

where  $p_F$  is the Fermi momentum,  $e$  is the electron charge,  $r$  is the radius of the contact, and  $n_0$  is the number of electrons per  $\text{cm}^3$ . The distance between the contacts was measured with a microscope. The contacts occasionally were moved during the cooling of the apparatus, the pouring of the helium, or the heating. Since contacts having lower resistance are more "stable," we used in the focusing experiments contacts of resistance  $\sim 1 \Omega$ ; usually the contact resistances ranged from 1 to 20  $\Omega$ . The contact dimension was determined in the following manner: Contacts having equal resistances were assumed to have equal dimensions. Using formula (2) at  $q \equiv 0$  and with different dimensions of the collector and emitter, we calculated the shape of the first EF line, which was determined by the ratio  $b/L$ . The calculations have shown that the shape of the first EF line is practically independent of  $q$ . Introduction of  $q \neq 0$  leads only to the appearance of an additional monotonic variation. By choosing the parameter  $b/L$  we obtained best agreement between the shape of the first EF line in the regions of the steep rise and descent of  $U$ . From the known  $b/L$  and  $L$  we determined the value of  $b$ . In order for the obtained  $b$  to coincide with those calculated from (8), we must assume for bismuth  $\rho l \approx 2 \times 10^{-7} \Omega\text{-cm}^2$ , as against  $\rho l \approx 1.5 \times 10^{-8} \Omega\text{-cm}^2$  from

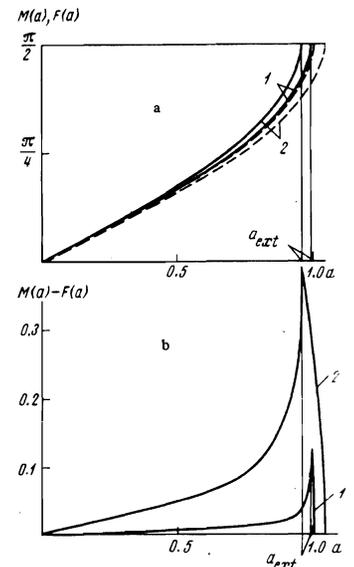


FIG. 2. a) Plots of  $M(a)$  (solid) and  $F(a)$  (dashed); b) plot of  $M(a) - F(a)$ . Curves 1 -  $b/L = 0.01$ , curves 2 -  $b/L = 0.08$ .

measurements of the resistance of thin samples (see, e.g., <sup>[7]</sup>).<sup>2)</sup> U was measured with an F118 nanovoltmeter. The measurements were performed in the temperature interval 1.3–4.2°K.

## MEASUREMENT RESULTS

The measurement results are shown in Figs. 3 and 4. Figure 3 illustrates the effect exerted on U(H) by the surface quality. Figure 3a shows a U(H) plot typical of a sample with a perfect surface. In Fig. 3, curves 2 were obtained with H directed such that the trajectories of the electrons leaving the emitter turned towards the collector, while curves 1 were recorded with the direction of H reversed. Figure 3c shows the results of measurements in the same geometry and at the same contact positions as in case 3a, except that a steel needle was used to scratch midway between the contacts a groove of width  $10^{-3}$  cm and length  $2 \times 10^{-2}$  cm. When the contacts were mounted on different sections of the sample, without changing the geometry of the experiment, we observed sometimes a "settling down" of a third, fourth, fifth, or multiple peak. A plot of such a case is shown in Fig. 3, where it is clearly seen that the amplitude of the fourth peak is smaller than that of the fifth, and the amplitude of the eighth is smaller than that of the ninth. When the contacts are mounted on different visually perfect sections of the sample, the ratio of the amplitudes of the neighboring peaks varies in the interval 0.6–0.8.

The behavior of U(H) seemed to be unaffected by the time of storage of the sample (measurement interval  $\sim 1$  year), by dust particles falling on the surface between the contacts, or by washing the sample in acetone.

When the sample surface was etched in a special solution, the form of the plot of U against H was significantly altered (see Fig. 3b). First, the amplitude of the peaks decreased abruptly with increasing number of the peak (with increasing H). Second, at the previous values of L and R the EF line broadened considerably in comparison with the case of a perfect surface. Third, a much stronger monotonic variation of U(H) appeared when H was so directed that electrons leaving the emitter could not strike the collector.

The value of U depends on the angle  $\varphi$  between H and BC (Fig. 4). It is seen from Fig. 4 that at a direction of H that prevents the electrons from the emitter to reach the collector (angle interval  $\sim 180-360^\circ$ ), the dependence of U on  $\varphi$  and H is weak. In the angle interval  $0-180^\circ$  it is much stronger. When H is inclined from the normal to BC by an angle  $\psi$ , we have  $H^*(\psi) = H^* \cos \psi = 0$  sec  $\psi$  in a wide range of angles ( $\sim 80^\circ$ ).<sup>[1]</sup> Lowering the temperature from 4.2 to 1.3°K leads to an increase of the amplitude of the peaks A by 1.3–3 times, depending on the value of L, in accordance with the formula  $\ln A = \text{const} - \beta T^2$ . The ratio of the amplitudes of different peaks remained the same in this case, and the change of the nonmonotonic part of U(H) was negligible ( $\sim 10-20\%$ ).

## ANGULAR DEPENDENCE OF q

To illustrate the influence of the form of the dependence of  $q(\theta)$  on U(a), Fig. 5 shows the values of N(a) calculated from formula (3) for several types of  $q(\theta)$ . In Fig. 5a we have  $q(\theta) \equiv \text{const} = q_0$ , and in Fig. 5b we have  $q(\theta) = 1$  at  $\theta \leq \alpha_0$  and  $q(\theta) = 0$  at  $\theta > \alpha_0$ ; the values of  $q_0$  and  $\alpha_0$  are marked next to the corresponding curves. The calculations were made for identical collector and emitter dimensions and for the ratio  $b/L = 0.08$ . The characteristic features of the curves are oscillations with a period  $\sim 1$ , superimposed on a smooth curve. At  $q \equiv 1$ , the decrease of the oscillation amplitude with increasing a is due to overlap of the EF lines with large numbers, due to the fact that the distance between the maxima is constant but their width increases in proportion to the number. We note that at large a, where  $2r_H \sim b$ , the focusing should generally vanish, since all the electrons become effective, and then the emitter resistance should acquire a dependence on H. Our results do not extend to the region of so large values of a.

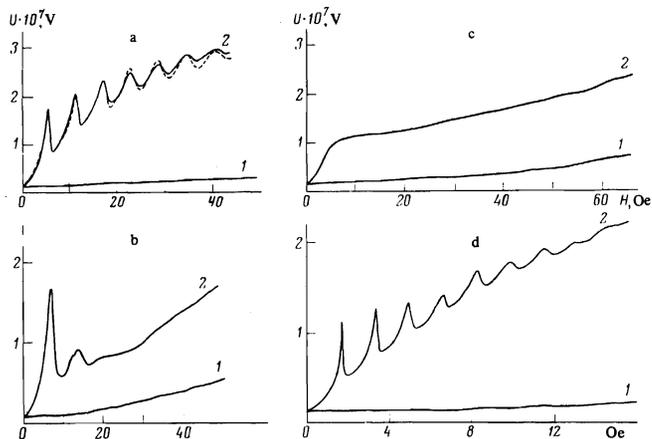


FIG. 3. Plots of U(H),  $H \perp BC$ ; a— $L \approx 0.45$  mm,  $T = 4.2$  K; b— $L \approx 0.15$  mm,  $T = 1.3$  K; c— $L \approx 0.15$  mm,  $T = 1.3$  K; d— $L \approx 0.52$  mm,  $T = 1.3$  K.

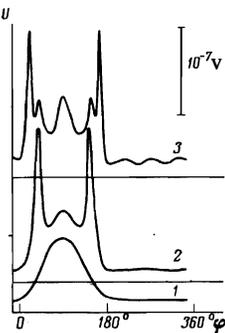


FIG. 4. Dependence of U on the angle  $\varphi$ ;  $L \approx 0.15$  mm,  $T = 1.3^\circ\text{K}$ . Curves 1–3 were plotted in fields 5.2, 12, and 24 Oe, respectively. The ordinate scale is indicated. The horizontal lines near curves 1–3 correspond to the zero values of U for the corresponding curves.

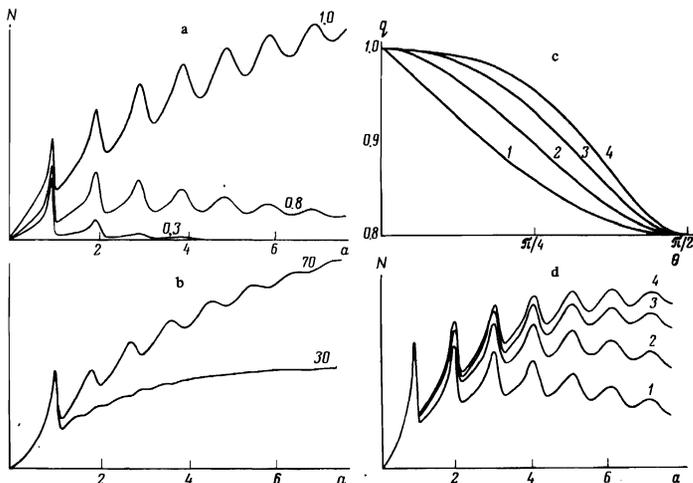


FIG. 5

As seen from Fig. 5a, the ratio of the amplitudes of neighboring EF lines is equal to  $q_0$  with good accuracy, in accord with (4). The steplike plot of  $q(\theta)$  (Fig. 5b) leads to a change of the period of oscillations at a  $> 1$ , which in this case is not equal to  $a_{\text{ext}}$ . The reason is that the first line is due to focusing of electrons traveling towards the surface at an angle close to  $90^\circ$ , while the rest are due to weak focusing of the electrons specularly reflected from the sample surface and traveling at an angle close to  $\alpha_0$ . This is the cause of the abrupt decrease of the amplitude of the second peak in comparison with the first, whereas the amplitudes of the succeeding peaks decrease smoothly with increasing  $a$ . In the region  $a \lesssim 1$ , the function  $N(a)$  is influenced by the form of  $q(\theta)$  only in the angle interval  $\sim 0-30^\circ$  (see Fig. 5b), and from among the electrons leaving the emitter and colliding with the surface, only those traveling at an angle from the indicated interval can reach the collector. The exact value of the maximum angle is determined by the dimensions of the collector and emitter and by  $L$ .

One of the methods of solving Eq. (3) is to substitute  $q$  in the form of a series

$$q(\theta) = \sum_{i=0}^l a_i \theta^i \quad (9)$$

and to choose the coefficients  $a_i$  to obtain the necessary agreement between  $U(a)$  and the form calculated from (3). At the present time, however, the author knows of no convincing physical considerations concerning the form of the series (9) for arbitrary  $\theta$ . At the same time, theoretical investigations of the scattering of glancing electrons have shown that the series (9) begins with the linear term.<sup>[5,8,9]</sup> When determining  $q(\theta)$  it is natural to stipulate, besides the usual equality  $q(0) = 1$ , also  $q'(\pi/2) = 0$ . The ratio of the amplitudes of neighboring peaks (formula (5)) yields the value of  $q$  at  $\theta = \pi/2$ . Finally, one more condition that can be used is the ratio of the amplitude of the first EF line to the value of the monotonic component in the vicinity of the first line.

The foregoing conditions enable us to leave in (9) four terms and to represent  $q$  in the form

$$q(\theta) = 1 + a_1 \theta + a_m \theta^m + a_n \theta^n$$

Figure 5a shows plots of  $q(\theta)$  satisfying the foregoing relations ( $q(\pi/2) = 0.8$ ) for  $(m, n) = (2, 3) (3, 4) (4, 5)$ . Curve 1 is for  $q(\theta) = 1 - 0.2 \sin \theta$ , curve 2 is for  $q(\eta) = 1 - 0.025\eta - 0.55\eta^2 + 0.375\eta^3$ , curve 3 for  $q(\eta) = 1 - 0.025\eta - 0.725\eta^3 + 0.55\eta^4$ , and curve 4 for  $q(\eta) = 1 - 0.025\eta - 0.9\eta^4 + 0.725\eta^5$ . The coefficient of  $\eta = \theta(\pi/2)$  was determined, in order of magnitude, from the ratio of the amplitude of the first EF line to the monotonic component, the coefficients of  $\eta^m$  and  $\eta^n$  were determined from the conditions  $q(\pi/2) = 0.8$  and  $q'(\pi/2) = 0$ . Figure 5d shows the results of calculations by formula (3) for identical collector and emitter direction, and for the ratio  $b/L = 0.06$ . Curves 1-4 of Fig. 5c correspond to the curves having the same numbers as in Fig. 5d. The dashed curve of Fig. 3a is curve 4 of Fig. 5d, which agrees well with the experimental data.

## DISCUSSION OF RESULTS

We note a number of factors that offer evidence in favor of the proposed EF model. The necessary condition for the observation of sharp focusing is satisfaction of the inequality  $b \ll L$ , which limits the number

of effective electrons to a strip of the central section of the Fermi surface, which is almost cylindrical in shape. A favorable fact in this connection is that the electron "ellipsoid" of bismuth has in its central part deviations from ellipsoidal shape, which bring the shape of this part closer to cylindrical,<sup>[10]</sup> and this contributes to the enhancement of the EF in the bismuth. The assumption that the dominant contribution to  $U$  is made by the effective electrons is experimentally confirmed by the weak dependence of  $U$  on  $H$  at  $H$  directions that prevent the effective electrons from reaching the collector (Fig. 4 and curves 1 of Fig. 3), and this takes place practically in an entire angle interval equal to  $\pi$  (Fig. 4). The main evidence in favor of the model is that the numerical values (dashed in Fig. 3a) agree with the experimental ones (solid curve 2 of Fig. 3a) for a perfect sample surface. A reconciliation between experimental data obtained from surfaces with defects (Figs. 3b and c) with those calculated from the model is apparently inadvisable for a number of reasons. For example, it must be borne in mind that the surface of an etched surface can constitute a mosaic system of disoriented small "mirrors," that admit of the existence of effective electrons not accounted for by formula (2). The presence of a mosaic system can determine the broadening of the EF line and the increasing monotonic variation of  $U$  in strong fields in the cases represented in Figs. 3b and 3c, while the rapid fall-off of the EF line amplitude with increasing number may possibly be due to disorientation of the "mirrors" (Fig. 3b). The absence of EF (Fig. 3c) is due to volume irregularities of the metal lattice. The small amplitude of the fourth peak (Fig. 3d) seems to be caused by a local surface defect as confirmed by the small amplitude of the eighth (multiple) peak. The presence of the local defect possibly causes a certain deviation of the shape of  $U(H)$  of Fig. 3d from the case shown in Fig. 3a.

An experimental study of the law of reflection of conducting electrons makes it possible, besides determining the causes of the diffuseness of the scattering (atomic roughnesses,<sup>[5,8]</sup> microscopic unevennesses,<sup>[5]</sup> Umklapp processes<sup>[9]</sup>), also to establish the structure of a perfect, i.e., thermodynamic-equilibrium, surface.<sup>[5]</sup> Unfortunately, the results of theoretical studies<sup>[5,8,9]</sup> of the angular dependence of  $q$  are valid only for glancing electrons, when their velocity is almost parallel to the surface, so that it is impossible to carry out a sufficiently detailed comparison of the experimental data with the theory. The fact of practically specular reflection of electrons normally incident on the surface (for the most perfect sections of the sample surface, the mean value of  $\bar{q}$  in the angle interval  $85-90^\circ$  was 0.8), offers evidence that the dimensions of the surface roughnesses are smaller than the de Broglie wavelength of the electrons in bismuth,  $\sim 10^{-5}$  cm. Using the formula for the specularity coefficient of<sup>[5]</sup>, which is valid for small  $\theta$ ,

$$q(\theta) = 1 - 2\alpha \hbar^{-1} p_F \sin \theta, \quad (10)$$

and approximating the obtained dependence

$$q(\theta) = 1 - 1.6 \cdot 10^{-2} \theta - 1.5 \cdot 10^{-4} \theta^3 + 7.6 \cdot 10^{-2} \theta^5$$

by a linear one in the region of small  $\theta$ , we obtain the order-of-magnitude value  $\alpha \sim 10^{-8}$  cm, i.e.,  $\alpha$  is of the order of the interatomic distances, as is typical of surfaces with atomic roughnesses.<sup>[5]</sup>

Thus, the dimensions of the roughnesses of the in-

investigated surfaces are apparently of the order of the interatomic distances. We note that the representation of  $q$  in the form (10), which is valid also for the non-glancing electrons, assuming a quadratic spectrum for bismuth,<sup>[5]</sup> describes the experimental data much worse. For this case the deviation of the model-calculated values from the experimental ones is represented in practice by the difference between curves 1 and 4 of Fig. 5d.

The observed  $U(H)$  dependence (curves 1 of Fig. 3) confirms qualitatively the conclusions of the theory of the static skin effect.<sup>[11]</sup> With increasing  $H$ , the collector voltage increases negligibly in comparison with the value of  $U$  at  $H = 0$ , whereas the bulk conductivity of bismuth decreases sharply with increasing  $H$  (in a 50-Oe field, the transverse magnetoresistance increases 30 times<sup>[12]</sup>). The weak dependence on  $H$  is due to the shunting action of the near-surface layer of the sample. In accordance with the conclusions of the theory,<sup>[11]</sup> after the sample is etched the dependence of  $U$  on  $H$  becomes stronger (Fig. 3b, curve 1).

## CONCLUSION

In conclusion, notice should be taken of certain methodological possibilities afforded by the EF. It is of interest to investigate the dynamics of strongly-nonequilibrium electrons generated in the emitter at large accelerating voltage. With the aid of EF it is relatively simple to study the electron-boundary interaction by sputtering or precipitating on the region between the contacts various substances, to investigate the Andreev reflection<sup>[13]</sup> from the boundary between a normal metal and a superconductor, and investigate therein the dependence on the energy of the incident electrons. Bismuth is particularly convenient in this connection, for the large resistance of the contact (1–20  $\Omega$ ) makes it possible to vary in a wide range the energy acquired by the electron in the emitter.

It is possible in principle to observe a quantum size effect in EF. The motion of the hopping electrons is quantized,<sup>[14]</sup> and at fixed  $H$  and in the case of specular reflection by the boundary, only trajectories with definite  $s_m$  are admissible. The condition (1) for the electrons that reach the collector takes at  $n > 1$  the form

$$n^{-1}(L-b/2) \leq s_m \leq n^{-1}(L+b/2). \quad (11)$$

The  $U(H)$  plot should reveal voltage spikes if (11) is satisfied.

I am grateful to Yu. V. Sharvin for a number of critical remarks and to N. P. Tsoi for numerous calculations and for a mathematical reduction of the measurement results.

<sup>1</sup>For simplicity, we consider only half the effective electrons with  $0 \leq \theta \leq \pi/2$ .

<sup>2</sup>It must be borne in mind that we are measuring a quantity larger than  $R$  in (8), since the experimental value includes also the resistance due to the contaminants in the layer between the Cu and the Bi.

<sup>1</sup>V. S. Tsoi, ZhETF Pis. Red. **19**, 114 (1974) [JETP Lett. **19**, 70 (1974)].

<sup>2</sup>Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. **48**, 984 (1965) [Sov. Phys.-JETP **21**, 655 (1965)].

<sup>3</sup>M. S. Khaikin and V. S. Edel'man, Zh. Eksp. Teor. Fiz. **47**, 878 (1964) [Sov. Phys.-JETP **20**, 587 (1965)].

<sup>4</sup>M. S. Khaikin, Zh. Eksp. Teor. Fiz. **39**, 212 (1960) [Sov. Phys.-JETP **12**, 152 (1961)].

<sup>5</sup>A. F. Andreev, Usp. Fiz. Nauk **105**, 113 (1971) [Sov. Phys.-Uspekhi **14**, 609 (1972)].

<sup>6</sup>S. A. Korzh, Fiz. Tverd. Tela **16**, 29 (1974) [Sov. Phys.-Solid State **16**, 17 (1974)].

<sup>7</sup>I. N. Zhilyaev and L. P. Mezhev-Deglin, ZhETF Pis. Red. **19**, 461 (1974) [JETP Lett. **19**, 248].

<sup>8</sup>R. M. More and D. Lessie, Phys. Rev. **B8**, 2527 (1973).

<sup>9</sup>R. M. More, Phys. Rev. **B9**, 392 (1974).

<sup>10</sup>V. S. Edel'man and M. S. Khaikin, Zh. Eksp. Teor. Fiz. **49**, 107 (1965) [Sov. Phys.-JETP **22**, 77 (1966)].

<sup>11</sup>V. G. Peschanskiĭ and M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **55**, 1980 (1968) [Sov. Phys.-JETP **28**, 1045 (1969)].

<sup>12</sup>T. Hattori, J. Phys. Soc., Japan **23**, 19 (1967).

<sup>13</sup>A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964); **51**, 1510 (1966) [Sov. Phys.-JETP **19**, 1023 (1967)].

<sup>14</sup>T. W. Nee and R. E. Prange, Phys. Lett. **25A**, 582 (1967).

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