Microwave impedance of superconductors in the mixed state

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The microwave impedance of type-II superconductors ($\kappa \gg 1$) is calculated for $T \approx T_c$. Analytic expressions are obtained for the impedance for arbitrary microwave frequencies of the external perturbation for $H \sim H_{c2}$ and throughout the complete range of variation frequencies of H in the limit of low ($\omega\tau_j \ll 1$) and high ($\omega\tau_j \gg 1$) frequencies ω of the external perturbation (τ_j is the relaxation time of the current). An expression is found for the effective mass of the flux tube.

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1. INTRODUCTION

As is well known, the finite resistance of type-II superconductors in magnetic fields exceeding the first critical field H_{c1} is due to energy dissipation associated with the motion of the vortex structure in the superconductor. This motion may take place under the influence of either a time-independent current[1] or an alternating current.[2]

The investigation of the motion of the vortex structure in an alternating (actually a microwave) field is of interest in two aspects. In the first place, for frequencies of the perturbation exceeding the so-called "depinning" frequency,[2] the various inhomogeneities of superconducting samples, which greatly impede a comparison of theoretical and experimental data on the motion of the vortices under the influence of a direct current,[3] no longer substantially influence the motion of the vortices. Secondly, the problem of the resistance of a superconductor in a microwave field has substantial applied value.

Investigation of vortex motion under the influence of microwave current has been carried out theoretically for fields $H \sim H_{c2}$,[4-6] and also in the framework of phenomenological models[2] in which, as will be shown below, several unwarranted assumptions are made. In the present work the question of the motion of the vortex structure is investigated for $T \approx T_c$, and the complex resistivity ρ of a type-II superconductor ($\kappa \gg 1$) is determined for the case when the lattice moves as a whole under the influence of microwave current. The Wigner-Seitz method, consisting in the replacement of an elementary vortex cell by a circle,[7] enabled one to determine ρ over the complete range of variation of the magnetic fields $H_{c1} \ll H \ll H_{c2}$ for small $\omega\tau_j \ll 1$ and large $\omega\tau_j \gg 1$ frequencies ω of the external perturbation (τ_j is the relaxation time of the current). In magnetic fields $H \approx H_{c2}$ the complex resistivity is calculated for arbitrary frequencies of the external perturbation, exceeding the "depinning" frequency. In conclusion the questions of the validity of the phenomenological model[2] and the effective mass of the vortex are discussed.

2. BASIC EQUATIONS

The time-dependent Ginzburg-Landau (GL) equations for the modulus $F = |\Delta|/\Delta_\infty$ of the order parameter and for the superconducting current $\mathbf{J}$ can be written in the following form:[8]

$$\nabla^2 F-(F^2-1+Q^2)F=u\dot{F}, \tag{1a}$$

$$uF^2M+\operatorname{div}(F^2\mathbf{Q})=0, \tag{1b}$$

$$\mathbf{J}=\varkappa^2 \operatorname{rot}\operatorname{rot}\mathbf{Q}=-F^2\mathbf{Q}-\dot{\mathbf{Q}}-\nabla M. \tag{1c}$$

Here $\mathbf{Q}$ and M are the gauge-invariant vector and scalar potentials, normalized to $\Phi_0/2\pi\xi$ and $2e\tau_j/\hbar$, respectively, and u is the dimensionless relaxation time of the order parameter. The distance and time in Eqs. (1) are normalized to the coherence length ξ and to the current relaxation time τ_j, and the current is normalized to the characteristic GL value: $\Phi_0/8\pi^2\delta^2\xi$.

For superconductors with a high concentration of paramagnetic impurities, the system (1) is derived from the existing microscopic theory.[9] In this case

$$u=12,\quad \tau_j=\hbar^2/2\Delta_\infty^2\tau_s,\quad \Delta_\infty^2=2\pi^2(T_c^2-T^2),$$

where τ_s is the time between collisions of an electron with impurity atoms involving spin flip.

For the most interesting case of a superconductor without paramagnetic impurities, the system of Eqs. (1) coincides with the phenomenological relationships of Schmid,[10] which describe well the motion of the vortices under the influence of constant current, at least for not too small values of H/H_{c2}.[3] In this case

$$u=\frac{\pi^4}{14\zeta(3)}\approx 5.79,\quad \tau_j=\frac{2\hbar T_c}{\pi\Delta_\infty^2},\quad \Delta_\infty^2=\frac{8\pi^2}{7\zeta(3)}T_c(T_c-T).$$

Equations (1) together with the following relations for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$

$$\mathbf{H}=\operatorname{rot}\mathbf{Q},\quad \operatorname{rot}\mathbf{H}=\varkappa^{-2}\mathbf{J}, \tag{2a}$$

$$\mathbf{E}=-\dot{\mathbf{Q}}-\nabla M,\quad \operatorname{div}\mathbf{E}=\bar{\rho} \tag{2b}$$

form a closed system. The equation of continuity

$$\operatorname{div}\mathbf{J}=0 \tag{3}$$

combined with Eqs. (1b) and (1c) give the following equation for the determination of the scalar potential M:

$$\nabla^2 M-uF^2M=-\operatorname{div}\dot{\mathbf{Q}}, \tag{4}$$

and the smallness of κ^{-1} gives:

$$\operatorname{rot}\operatorname{rot}\mathbf{Q}=0. \tag{5}$$

In connection with the solution of the system of Eqs. (1a), (1b), (4), and (5), we shall assume that the current density does not vary along the length of the vortex filament. Such an assumption is valid, for example, for a thin superconducting film in a magnetic field perpendicular to its surface, if the thickness of the film is much less than the penetration depth of the field into the superconductor and the depth of the skin-layer of the given material. Further, we shall assume that under the influence of a microwave field the vortex lattice will be displaced as a whole, i.e., the period of the lattice is much smaller than the distance δ_e over which the magnitude of the current varies in the plane of the film

$((H/H_{c2})^{1/2} \gg \xi/\delta_e)$. Thirdly, we shall only consider the linear problem, i.e., the motion of the vortex lattice with a small velocity under the influence of small currents.

3. EXPRESSION FOR THE CONDUCTIVITY

If the conditions formulated above are satisfied, one can assume that in the zero-order approximation the vortex lattice is displaced as a whole under the influence of uhf current, i.e., F and **Q** are time-independent solutions F_0 and $\mathbf{Q}_0$, depending on the magnetic field H and on the coordinates $\mathbf{R} = \mathbf{r} - \mathbf{r}_0 e^{iat}$, where $a = \omega\tau_j$ is the dimensionless frequency of the external perturbation.

For the determination of the stationary solutions $F_0(R, H)$ and $\mathbf{Q}_0(R, H)$ it is convenient to apply the Wigner-Seitz method developed in[7], consisting in the replacement of an elementary vortex cell by a circle of radius $r_s = (2H_{c2}/H)^{1/2}$ and giving exceptionally small errors ($\sim 10^{-3}$) in comparison with allowance for the exact geometry. In such a method F_0 and $\mathbf{Q}_0$ can be regarded as functions which depend on the magnetic field and on a single coordinate r (the distance from the center of the isolated vortex).[3]

The deviations of the quantities F, **Q**, and M, associated with a deformation of the vortex lattice, give small corrections proportional to e^{iat} to the quantities F_0, $\mathbf{Q}_0$, and $M_0 \equiv 0$. The amplitudes $\tilde{f}$, $\tilde{q}$, and $\tilde{\mu}$ of these corrections are proportional to the amplitude v of the velocity of motion; for these quantities we obtain

$$\nabla^2\tilde{f} - (3F_0^2 - 1 + Q_0^2 + iau)\tilde{f} - 2F_0\mathbf{Q}_0\tilde{\mathbf{q}} = -u(v\nabla F_0),$$
$$uF_0^2\tilde{\mu} + \operatorname{div}(F_0^2\tilde{\mathbf{q}} + 2F_0\mathbf{Q}_0\tilde{f}) = 0, \qquad (6)$$
$$\nabla^2\tilde{\mu} - uF_0^2\tilde{\mu} = -ia\operatorname{div}\tilde{\mathbf{q}}, \quad \operatorname{rot}\operatorname{rot}\tilde{\mathbf{q}} = 0.$$

The correction to the current arising in this connection, which is proportional to v, is given by

$$\tilde{\mathbf{j}} = -F_0^2\tilde{\mathbf{q}} - 2F_0\tilde{f}\mathbf{Q}_0 - ia\tilde{\mathbf{q}} - \nabla\tilde{\mu} + (v\nabla)\mathbf{Q}_0. \qquad (6')$$

The system of linearized equations (6) is satisfied by a solution of the form

$$\tilde{f} = f(a, H, r)\cos\varphi, \quad \tilde{q}_r = q_r(a, H, r)\sin\varphi,$$
$$\tilde{\mu} = \mu(a, H, r)\sin\varphi, \quad \tilde{q}_\varphi = q_\varphi(a, H, r)\cos\varphi,$$

where φ is the angle between the direction from the center of the isolated vortex (r = 0) to the point and the direction perpendicular to the field and the current, and the spatial amplitudes satisfy the following system of ordinary differential equations:

$$f'' + \frac{1}{r}f' - \left(3F_0^2 - 1 + Q_0^2 + \frac{1}{r^2} + iau\right)f - 2F_0Q_0q_\varphi = -uvF_0', \qquad (7a)$$

$$uF_0^2\mu + 2F_0F_0'q_r + F_0^2(q_r + q_\varphi)' - \frac{2F_0Q_0}{r}f = 0, \qquad (7b)$$

$$\mu'' + \frac{1}{r}\mu' - \frac{1}{r^2}\mu - uF_0^2\mu = -ia(q_r + q_\varphi)', \quad q_r = (rq_\varphi)' \qquad (7c)$$

and the boundary conditions[1)]

$$f(0) = f(r_s) = \mu(r_s) = 0, \quad (\mu/r - v/r^2)'_{r\to 0} \to 0. \qquad (8)$$

The prime denotes differentiation with respect to r.

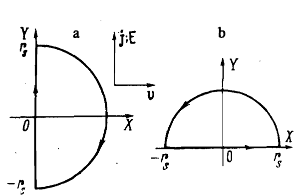

FIG. 1

The last of these conditions follows from the finiteness of the electric field at the center of the vortex.

The average electric field over the sample, arising in connection with the motion of the vortex lattice, can be found from Maxwell's equations (2b). For the integration contour shown in Fig. 1a, we have

$$\int \operatorname{div}\mathbf{E}\,ds = 0 = \oint \mathbf{E}\,d\mathbf{l} = \int_{-r_s} E_y\,dy - 2r_sE_\varphi(r_s) \qquad (9)$$

or

$$\langle E\rangle = \frac{1}{2r}\int_{-r_s}^{r_s} E_y\,dy = \left(v\frac{H}{H} - iaq_\varphi(r_s)\right)e^{iat}. \qquad (10)$$

We obtain the value of the current density, averaged over the sample, from Eq. (2a). For the integration contour shown in Fig. 1b, we obtain

$$\int [\operatorname{rot}\mathbf{j}\,ds] = 0 = \int_{-r_s}^{r_s} j_y\,dx - 2r_sj_r(r_s) \qquad (11)$$

or

$$\langle j\rangle = \frac{1}{2r_s}\int_{-r_s}^{r_s} j_y\,dx = -(F_0^2q_r + iaq_r + \mu')_{r=r_s}e^{iat}. \qquad (12)$$

From Eqs. (10) and (12) we obtain the following result for the dimensional, complex resistivity

$$\frac{\rho}{\rho_N} = \frac{\langle E\rangle}{\langle j\rangle} = -\left(\frac{vH/H_{c2} - iaq_\varphi}{F_0^2q_r + \mu' + iaq_r}\right)_{r=r_s} \qquad (13)$$

Formula (13) expresses the impedance of a superconductor, existing in the mixed state, in terms of the values of the solutions of the system (7) for $r = r_s$. The value of $j_r(r_s)$, appearing in Eq. (13), can be obtained from the first integral of the system (7):[9]

$$j_r(r_s) = -(F_0^2q_r + \mu' + iaq_r)_{r=r_s}$$
$$= uv\gamma_1 + E(0) + \frac{v}{r_s^2} - ia(u\gamma_2 + q(0)). \qquad (14)$$

Here q(0) and E(0) are the magnitudes of the vector potential and of the electric field strength at the center of the vortex

$$\gamma_1 = \frac{1}{2}\int_0^{r_s} rF_0'^2\,dr, \qquad \gamma_2 = \frac{1}{2}\int_0^{r_s} rF_0'f\,dr. \qquad (15)$$

The solution of the system simplifies considerably in the case of large magnetic fields $H \approx H_{c2}$, and also in the limit of low $a \ll 1$ and high $a \gg 1$ frequencies of the external perturbation over the complete range of variation of H.

4. THE APPROXIMATION OF LARGE FIELDS $H \approx H_{c2}$

In the limit of large magnetic fields $H \approx H_{c2}$, one can seek the solution of the system (7) in the form of a series in powers of the small parameter $c \sim (1 - H/H_{c2})^{1/2}$.

In the model of a circular lattice, the solution of the time-independent problem was found in[7]:

$$F_0 = \frac{cr}{r_s}\exp\left\{-\frac{r^2}{2r_s^2}\right\}, \quad c^2 = \frac{1 - H/H_{c2}}{\beta_A(1 - 2e^{-1})}, \quad \beta_A = 1.1576. \qquad (16)$$

For convenience of the solution of the system (6) in the case under consideration, we introduce the scalar potential $\tilde{z}$, which is related to $\tilde{\mu}$ by the equation

$$\tilde{z} = \tilde{\mu} - v\mathbf{Q}_0. \qquad (17)$$

Then the correction to the current is represented in the form

$$\tilde{j} = F_0^2\tilde{\mathbf{q}} - 2F_0\tilde{f}\mathbf{Q}_0 - \nabla\tilde{z} - ia\tilde{\mathbf{q}} + [\mathbf{v}\times\mathbf{H}]/H_{c2}. \qquad (18)$$

Since in the zero-order approximation in c, the quantity $\tilde{j}$ is equal to the normal current in the film

$$\tilde{j} \approx -ia\tilde{q} + [v \times H]/H_{c2}, \qquad (19)$$

it follows at once from Eq. (3) that $\tilde{q}$ is a constant vector.

Substituting the constant values of the amplitudes q_r and q_φ into the system (7), we have

$$q_r = q_\varphi = -uv/2. \qquad (20)$$

We obtain the combination of the amplitude of the potential z and the corrections of order c^2 to q_r and q_φ, necessary for the determination of ρ, from Eq. (7c):

$$(z' + ia(q_r - q_\varphi))_{r=r_s} = \tfrac{1}{2}uv[F_0^2(r_s) - \langle F_0^2\rangle]. \qquad (21)$$

Substituting the found quantities (20) and (21) into expression (13) for the complex resistivity $\rho = \rho_1 + i\rho_2$, we have:

$$\frac{\rho}{\rho_N} = 1 - \frac{u}{2}\frac{\langle F_0^2\rangle}{1 + iau/2}, \quad \langle F_0^2\rangle = \frac{1}{\beta_A}\left(1 - \frac{H}{H_{c2}}\right). \qquad (22)$$

In the limits of low and high frequencies of the external perturbation, expression (22) agrees with the result obtained by Maki and Thompson[4-6].

5. THE APPROXIMATION OF HIGH FREQUENCIES $a \gg 1$

In the approximation under consideration, one can seek the unknown quantities entering into the system (7) in the form of a series in powers of the small parameter a^{-1}. In fact, it is seen from Eq. (6′) that for $a \gg 1$ the correction to the superconducting current $\tilde{j}$ in the zero-order approximation is given by

$$\tilde{j} \approx -ia\tilde{q}. \qquad (23)$$

Therefore condition (3), just as in the preceding case, gives

$$q_r = q_\varphi = q_\infty = \text{const}. \qquad (24)$$

Substituting the constant values q_r and q_φ into the system (7), we find the solution of this system to the first approximation in a^{-1}:

$$j = -iaq_\infty, \quad \mu = 2F_0' q_\infty / uF_0. \qquad (25)$$

We obtain the relation between the constant q_∞ and the amplitude of the vortex velocity from the boundary condition for the scalar potential μ at the center of the vortex:

$$q_\infty = -uv/2. \qquad (26)$$

We obtain the combination of the corrections to q_r and q_φ of order a^{-1}, which is needed for the determination of ρ, from Eq. (7c):

$$ia(q_{r=r_s} = -\tfrac{1}{2}uv(F_0^2(r_s) - \langle F_0^2\rangle) - \mu'(r_s). \qquad (27)$$

Substituting the found quantities into the expressions for ρ and j, we have:

$$\rho/\rho_N = 1 + i\langle F_0^2\rangle / a,$$
$$j = \tfrac{1}{4}uv\langle F_0^2\rangle - \tfrac{2}{3}F_0'''(0)/F_0'(0) + \tfrac{1}{2}iauv. \qquad (28)$$

The dependences of $\langle F_0^2\rangle$ and $2F_0'''(0)/3F_0'(0)$ on the applied magnetic field are shown in Fig. 2. It is seen that in the limit of large fields $H \approx H_{c2}$, the results obtained in this section coincide with the results of the preceding section in the limit of high frequencies.

6. THE CASE OF LOW FREQUENCIES $a \ll 1$

From expressions (13) and (14) it is seen that in the limit of low frequencies $a \ll 1$ of the external perturbation, the quantity ρ will be determined by the functions

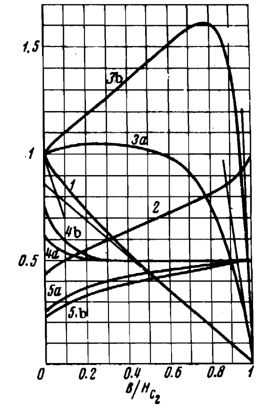

FIG. 2. The dependence of various quantities describing the structure of a moving vortex on the magnetic field. The quantities $\langle F_0^2\rangle$ (curve 1) and $-2F_0'''(0)/3F'(0)$ (curve 2) enter into formula (28) for ρ and j with $\omega\tau_j \gg 1$. The remaining quantities: $-(q(0) + u\gamma_2)/u$, $-q_\varphi(r_s)/u$ (curves 4 and 5, respectively) are found for $\omega = 0$ and enter into formula (30) for ρ with $\omega\tau_j \ll 1$ (curves 3 correspond to ρ_2/a). Furthermore, the quantity $q(0) + u\gamma_2$ determines the effective mass of the vortex. The index a corresponds to $u \approx 5.79$, and b corresponds to $u = 12$.

$\tilde{f}(0, H, r)$, $\tilde{q}(\sigma, H, r)$, and $\tilde{\mu}(0, H, r)$, which describe the motion of the vortex lattice under the influence of a current which is constant in time:

$$\frac{\rho_1}{\rho_N} = \frac{vH/H_{c2}}{vH/2H_{c2} + E(0) + uv\gamma_1}, \qquad (29)$$

$$\frac{\rho_2}{\rho_N} = a\frac{\rho_1}{\rho_N}\frac{H_{c2}}{H}\left\{\frac{q(0) + u\gamma_2}{v}\frac{\rho_1}{\rho_N} - q_\varphi(r_s)\right\}. \qquad (30)$$

The expression for the real part of the resistivity agrees with our previously obtained result ([3], Fig. 1b). In the neighborhood of H_{c2} expressions (29) and (30) agree with relationship (22) for ρ in the region of low frequencies.

In order to determine the imaginary part of the resistivity, it is necessary to solve the system (7) with $a = 0$ and with the boundary conditions (8). The functions $\beta(4)$ and $\alpha(r)$ were introduced by us for convenience in the numerical integration of the system (7):

$$f(r) = uvr\beta(r), \quad F_0(r)q_\varphi(r) = uv\alpha(r), \qquad (31)$$

satisfying the system of equations

$$\beta'' + \frac{3}{r}\beta' - (3F_0^2 - 1 + Q_0^2)\beta - \frac{2Q_0}{r}\alpha = -\frac{F_0'}{r},$$
$$\alpha'' + \frac{3}{r}\alpha' - (F_0^2 - 1 + Q_0^2)\alpha - \frac{2Q_0}{r}\beta = -\frac{F_0}{r}\mu, \qquad (32)$$
$$\mu'' + \frac{1}{r}\mu' - \frac{1}{r^2}\mu - uF_0^2\mu = 0$$

with the boundary conditions

$$\alpha(0) = \beta(0) = \beta(r_s) = 0, \quad (\mu/r - 1/r^2)' \underset{r\to 0}{\to} 0, \quad \alpha'(r_s) = 0. \qquad (33)$$

The system of Eqs. (32) and (33) was solved by the method of matrix trial runs.[11] The value of θ was automatically determined in the computation process by the condition

$$\lim_{r\to 0}(\alpha(r) + \beta(r) + \tfrac{1}{2}F_0(r)) \to 0. \qquad (34)$$

Plots of $q_\varphi(r_s)$, $q(0) + u\gamma_2$, and $\rho_2(H)$ are shown in Fig. 2 for different values of the magnetic field H. In the limit of small magnetic fields the functions f(r), $j_r(r)$, and $j_\varphi(r)$ agree with the calculations by Cohen and Rickaysen,[12] and in the limit of large fields—they agree with the results calculated in Sec. 4 of the present article.

7. PHENOMENOLOGICAL MODEL AND THE VORTEX MASS

In the articles by Gittleman and Rosenblum,[2] and also in a number of subsequent articles, a phenomenological model was proposed, describing the motion of the vortex lattice as a whole under the influence of a microwave current. The problem of the lattice's motion was reduced to the problem of the motion of an isolated vortex, whose dynamical properties were described with the aid of two phenomenological coefficients–the effective mass m of the vortex and the coefficient of viscosity η. It was assumed that, not the superconducting density but the total current J(t) flowing in the superconductor enters into the Lorentz force acting on the isolated vortex, as a consequence of which it was related to the vortex velocity by the equation:

$$\Phi_0 J(t)/c=\eta v+m\dot{v}. \quad (35)$$

Here Φ_0 is the quantum of magnetic flux. The relation between the average electric field E and the vortex-lattice velocity was given in the form

$$E(H)-E(0)=vH/c. \quad (36)$$

The coefficients η and m in Eq. (35) were determined from a comparison of the resistivity

$$\rho(H)-\rho(0)=\Phi_0 H/c^2(\eta+i\omega m) \quad (37)$$

following from this model with the experimentally determined quantity ρ(H).[2,13,14]

The calculations carried out above show that the coefficients m and η entering into the expression for ρ do not have, generally speaking, anything in common with the corresponding coefficients in expression (35) for the current. In fact, from Eqs. (10) and (36) it is seen that the relationship (36) between E and v, which is utilized in the phenomenological model, is valid only in the case of motion under the influence of a current which is constant in time (a = 0), since the quantity $q_\varphi(r_s)$ appearing in (10) depends on the applied magnetic field. As a consequence of this, it is found that in the framework of the phenomenological model under consideration, the concept of effective mass per unit length of the vortex can be introduced by two methods, by comparing the expressions obtained by us for ρ and j, respectively, with formulas (37) or (35).

It is clear from a comparison of the expressions for ρ(H) that formula (37), giving a directly proportional dependence of ρ on the magnetic field, is valid only in the region of small magnetic fields $H \lesssim 0.1\, H_{c2}$. In the limit of high frequencies ($a \gg 1$) of the external perturbation, we obtain

$$m_\rho=\tau_j^2 H_{cm}^2 H/H_{c2}(1-\langle F_0^2\rangle). \quad (38)$$

For low frequencies of the external perturbation ($a \ll 1$), the value of m_ρ turns out to be negative, although also small. This already indicates that, at arbitrary frequencies the dispersion law for ρ(H) does not coincide with that predicted in the phenomenological model.

Now proceeding to the "current mass," from a comparison of expressions (19), (14), and (25) with (35) we find that the phenomenological model predicts the correct dispersion dependence for the current in the limit of large magnetic fields for arbitrary frequencies of the external perturbation, with the mass m_J given by

$$m_J={}^1/_2 u\tau_j^2 H_{cm}^2 \quad (39)$$

In the high-frequency limit the value of the mass does not depend on the applied magnetic field and coincides with m_J. Finally, for low frequencies of the external perturbation, from Eq. (14) we obtain

$$m_{J_1}=-\frac{q(0)+u\gamma_2}{v}\tau_j^2 H_{cm}^2, \quad (40)$$

which, as Fig. 2 indicates, is also close to (39).

Thus, for arbitrary fields and frequencies the effective current mass per unit length of the vortex is essentially given by expression (39). Therefore, for the solution of problems related to the motion of flux tubes in the presence of an uhf field in type-II superconductors, it is quite feasible to use the phenomenological relation (35), in which the coefficient m does not depend on temperature and is given by

$$m={}^1/_2\pi\hbar^2 N(0)$$

for both paramagnetic and normal impurities. In order of magnitude, this mass is equal to $m_e n^{1/3} \sim 10^{-20}$ g/cm.

[1)] The error which we made earlier in writing the boundary conditions for the system (6) led to an erroneous result for the complex conductivity σ in the limits of high ($a \gg 1$) frequencies of the external perturbation in theses NT-18.

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