# Experimental possibility of determining the strain potential

V. I. Makarov and G. V. Sukhina

*Physico-technical Institute, Ukrainian Academy of Sciences*
(Submitted April 8, 1974; resubmitted December 9, 1974)
Zh. Eksp. Teor. Fiz. **68**, 1311–1314 (April 1975)

It is shown how the strain potential $\lambda_{ik}$ can be determined experimentally on the basis of its averaged values, which can be found by studying the amplitude of the ultrasound-absorption-coefficient oscillations that are produced in a magnetic field by quantization of the conduction-electron energy spectrum.

PACS numbers: 72.55., 62.80.

If a sound wave propagates in a crystal, the dispersion of the conduction electrons $\epsilon(\mathbf{p})$ can be written in the form[1]

$$\varepsilon(\mathbf{p})=\varepsilon_0(\mathbf{p})+\lambda_{ik}(\mathbf{p})u_{ik}, \tag{1}$$

where $u_{ik}$ is the strain tensor due to the propagation of the sound wave in the crystal and $\lambda_{ik}(\mathbf{p})$ is the strain potential. At the present time, the dispersion $\epsilon_0(\mathbf{p})$ has been experimentally determined for most metals. Yet the strain potential $\lambda_{ik}(\mathbf{p})$ has not been determined experimentally[1)].

This paper is devoted to one of the experimental possibilities of determining $\lambda_{ik}(\mathbf{p})$ from the average values of $\lambda^*_{ik}(\mathbf{p})$, which can be determined by studying the oscillations of the ultrasound absorption coefficient in a magnetic field $\Gamma_{osc}(H)$ due to quantization of the energy spectrum of the conduction electrons.

In the case of an arbitrary dispersion law, the quantity $\Gamma_{osc}(H)$ is given by[2)]

$$\Gamma_{osc}=\text{const}\frac{\omega}{q_z}\sum_m\frac{[m^*(\varepsilon_F,p_z^m)]^2}{\partial^2S_m/\partial p_z^2}|\lambda_{ik}^*(p_z^m)u_{ik}|^2\cdot$$
$$\cdot\sum_{k=1}^{\infty}\psi(\lambda k)\cos\left(k\frac{cS_m}{e\hbar H}\right)\cos\left(\pi k\frac{m^*(\varepsilon_F,p_z^m)}{m_0}\right), \tag{2}$$

where

$$\psi(z)=\frac{z}{\operatorname{sh} z},\qquad \lambda=2\pi^2\frac{Tcm^*(\varepsilon_F,p_z^m)}{e\hbar H},$$
$$m^*(\varepsilon_F,p_z^m)=\frac{1}{2\pi}\frac{\partial S_m(\varepsilon,p_z^m)}{\partial\varepsilon}\Big|_{\varepsilon=\varepsilon_F}$$

and c, e, $\hbar$, and $m_0$ are respectively the speed of light, the electron charge, Planck's constant, and the mass of the free electron, T is the temperature, $\omega$ is the frequency of the ultrasound, $q_z$ is the projection of the wave vector on the axis $z \parallel H$, $S_m(\epsilon_F, p_z^m)$ is the extremal cross of the Fermi surface, and $\epsilon_F$ is the Fermi energy. The quantity $p_z^m$ is determined from the equation

$$\hbar\omega+\varepsilon_n(p_z)-\varepsilon_n(p_z+q_z)=0,$$

where $p_z$ is the projection of the electron momentum on the axis $z \parallel H$.

The function $\lambda^*_{ik}(p_z^m)$ is defined by the expression

$$\lambda_{ik}^*(p_z^m)=\int_0^{2\pi}d\varphi\,\lambda_{ik}(p_z^m,\varphi), \tag{3}$$

where $\varphi$ is the dimensionless time of revolution of the quasiparticle on its orbit in the magnetic field.

Formula (2) describes the oscillations of Gurevich, Skobov, and Firsov in the limit when the broadening of the levels can be neglected, i.e., the parameter

$$\min\{\omega\tau,\ ql\sqrt{\hbar\omega_c/\varepsilon_F}\}\gg1,$$

where $\omega_c = eH/m^*c$, $l$ is the electron mean free path, and $\tau$ is the electron free path time.

Thus, the amplitude $\Gamma_{osc}(H)$ of the ultrasound absorption coefficient contains a new quantity $\lambda^*_{ik}(\mathbf{p})$ in addition to the known quantities $S_m$, $m^*$, and $\partial^2S_m/\partial p_z^2$, which enter, for example, in the amplitude of the de Haas-van Alphen effect[3,4]. For an experimental determination of $\lambda^*_{ik}(\mathbf{p})$ it is therefore necessary to carry out an amplitude analysis of the experimental $\Gamma_{osc}(H($ dependence; it is then possible to determine the values of $\lambda_{ik}(\mathbf{p})$ of the corresponding extremal sections.

This raises the problem of how to determine $\lambda_{ik}(\mathbf{p})$ knowing the value of $\lambda^*_{ik}(p_z^m)$ for different directions of the magnetic field.

Let the magnetic field **H** be directed along the vector $\boldsymbol{\xi}$. From among all the values $\lambda^*_{ik}(p_z^m)$ pertaining to different extremal sections of the Fermi surface, we choose the value pertaining to the central section (i.e., $p_z^m = 0$). In this case expression (3) can be written in the form

$$\lambda_{ik}^*(\boldsymbol{\xi})=\int d\Omega_e\delta(\mathbf{e}\boldsymbol{\xi})\frac{\rho(\mathbf{e})}{|\mathbf{v}(\mathbf{e})|(\mathbf{en})}\lambda_{ik}(\mathbf{e}), \tag{4}$$

where $|\mathbf{v}(\mathbf{e})|$ is the modulus of the velocity on the Fermi surface, **n** is the normal to the Fermi surface, $\rho(\mathbf{e})$ is the radius vector drawn from the center of the Fermi surface in the **e** direction, $\delta(\mathbf{e}\cdot\boldsymbol{\xi})$ is the Dirac function, and $d\Omega_\mathbf{e}$ is the unit-sphere element.

We shall henceforth assume that the function $\lambda_{ik}(\mathbf{p})$ pertains to a Fermi-surface cavity having a symmetry center and possessing the property that any ray drawn from the center encounters the surface only at one point. We emphasize that this situation is encountered frequently. To determine the function $\lambda_{ik}(\mathbf{e})$ from (4) in this case we can employ the relations obtained by I. Lifshitz and Pogorelov[5].

In our case, the sought function $\lambda_{ik}(\mathbf{e})$ is of the form

$$\lambda_{ik}(\mathbf{e})=\left\{\int d\Omega_\xi\lambda^*(\boldsymbol{\xi})-\int_{z^2>\mu^2}d\Omega_\xi\frac{\lambda_{ik}(\boldsymbol{\xi})z}{\sqrt{z^2-\mu^2}}\right\}$$
$$\times\left\{\int d\Omega_\xi m^*(\boldsymbol{\xi})-\int_{z^2>\mu^2}d\Omega_\xi\frac{m^*(\boldsymbol{\xi})z}{\sqrt{z^2-\mu^2}}\right\}^{-1}, \tag{5}$$

where $z = (\mathbf{e}\cdot\boldsymbol{\xi})$; $\mu$ is an arbitrary number satisfying the condition $\mu \ll 1$; $m^*(\boldsymbol{\xi})$ is the effective mass of the quasiparticle and corresponds to the central section of the surface $\epsilon_0(\mathbf{p})$. It is thus seen from (5) that to determine experimentally the strain potential $\lambda_{ik}(\mathbf{e})$ it is necessary to know the quantities $\lambda^*_{ik}(\boldsymbol{\xi})$ and $m^*(\boldsymbol{\xi})$ at different directions of the magnetic field.

The values of $m^*(\boldsymbol{\xi})$ can be obtained by investigating the temperature dependence of the amplitude of the quantum oscillations of the absorption coefficient of ultrasound or the amplitude of the de Haas-van Alphen effect. In the indicated phenomena one determines experimentally the quantity $|\lambda^*_{ik}(\boldsymbol{\xi})|$, while formula (5) contains the quantity $\lambda^*_{ik}(\boldsymbol{\xi})$. This raises the question of whether formula (5) is suitable for an experimental determination of $\lambda_{ik}(e)$.

We consider the case when $\lambda^*_{ik}(\xi)$ as a function of $\xi$ reverses sign. We mark on the unit sphere the points where $\lambda^*_{ik}(\xi)$ vanishes. Then the surface of the unit sphere breaks up into closed regions in which the function $\lambda^*_{ik}(\xi)$ has opposite signs. Choosing the sign of $\lambda^*_{ik}(\xi)$ in one of the regions, we can uniquely determine the function $\lambda_{ik}(e)$ itself from the experimental values of $\lambda^*_{ik}(\xi)$. By the same token, one can use formula (5) to determine $\lambda_{ik}(e)$, including also the distribution of the sign of this function. Only the general sign of $\lambda_{ik}(e)$ remains unknown.

Thus, by investigating the oscillations, due to quantization of the energy spectrum of the conduction electrons, of the ultrasound absorption in a magnetic field we can experimentally reconstruct the $\lambda_{ik}(\mathbf{e})$ dependence.

In conclusion, let us assess the information that can be obtained from a study of the influence of small impurity concentrations on the amplitude of the de Haas-van Alphen effect. It is known[6,7] that the presence of impurities in a metal causes the amplitude of the de Haas-van Alphen effect to decrease. This decrease is described by the Dingle factor. It can be shown (see, e.g.,[8]) that in the case when the Larmor radius exceeds the effective radius of the impurities, the Dingle factor is equal to

$$\frac{1}{\tau_D}=\Gamma^*(p_z^m)=\int_0^{2\pi}d\varphi\,\Gamma(p_z^m,\varphi), \tag{6}$$

where $\Gamma(\mathbf{p})$ is a quantity characterizing the damping of the quasiparticles in the absence of external fields. We see that expression (6) is analogous to expression (3). Therefore $\Gamma(\mathbf{e})$ is determined by the following expression:

$$\Gamma(\mathrm{e})=\left\{\int d\Omega_\xi\Gamma^*(\xi)-\int_{z^2>\mu^2}d\Omega_\xi\Gamma^*(\xi)\frac{z}{\sqrt{z^2-\mu^2}}\right\}\times\left\{\int d\Omega_\xi m^*(\xi)-\int_{z^2>\mu^2}d\Omega_\xi m^*(\xi)\frac{z}{\sqrt{z^2-\mu^2}}\right\}^{-1}. \tag{7}$$

Thus, study of the amplitudes of the quantum oscillations of the ultrasound absorption and of the magnetic susceptibility makes it possible to determine two new functions $\lambda_{ik}(\mathbf{p})$ and $\Gamma(\mathbf{p})$ characterizing the interaction of electrons with phonons and the interaction of electrons with impurities.

The authors are grateful to I. M. Lifshitz, A. A. Slutskin, and A. M. Kadigrobov for a discussion of the results.

1)Walther [2] first proposed the form of the $\lambda_{ik}(p)$ dependence, and then determined numerical values for it.

2)Gurevich, Skobov, and Firsov [3] derived an expression for $\Gamma_{osc}(H)$ for the case of a quadratic dispersion law $\epsilon_0(p)$ that is easily generalized when it is recognized that

$$\left.\frac{\partial^2\varepsilon_n(p)}{\partial p_z^2}\right|_{p_z=p_z^m}\cong-\frac{1}{m^*}\left.\frac{\partial^2 S_m}{\partial p_z^2}\right|_{p_z=p_z^m}.$$

1A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskiĭ, Zh. Eksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys.-JETP 5, 685 (1957)].
2K. Walther, Phys. Rev., 174, 782 (1968).
3V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. 40, 786 (1961) [Sov. Phys.-JETP 13, 552 (1961).
4V. G. Skobov and E. A. Kaner, Zh. Eksp. Teor. Fiz. 46, 1809 (1964) [Sov. Phys.-JETP 19, 1219 (1964)].
5I. M. Lifshitz and A. V. Pogorelov, Dokl. Akad. Nauk SSSR 96, 1143 (1954).
6Yu. A. Bychkov, Zh. Eksp. Teor. Fiz. 39, 1401 (1960) [Sov. Phys.-JETP 12, 971 (1961)].
7R. B. Dingle, Proc. Roy. Soc., A211, 517 (1952).
8A. A. Slutskin, Fiz. Tverd Tela [Sov. Phys.-Solid State] (in press).

Translated by J. G. Adashko
140