The recombination mechanism of thermal conductivity of a plasma in a strong magnetic field

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(Submitted October 7, 1974)
Zh. Eksp. Teor. Fiz. **68**, 1299–1303 (April 1975)

In the usual theory of transport processes in a high-temperature plasma the medium is regarded as consisting of electrons and ions only, that is, as completely ionized. The result is that the transverse transport coefficients of a plasma in a magnetic field decrease as the field is made stronger and can be arbitrarily small in a sufficiently strong field. Even in a high-temperature plasma, however, there is always an admixture of neutral hydrogen atoms formed through recombination. Since the magnetic field does not affect the motion of these atoms, there is a transport mechanism independent of the strength of the magnetic field. It is obvious that this mechanism imposes a lower limit on the transport coefficients, which is reached in a sufficiently strong magnetic field. In this paper these limitng values of the transport coefficients are calculated. It is shown that over a wide range of plasma parameters they are larger than the classical values, even at quite moderate field strengths ($H\sim 30$ kG).

PACS numbers: 52.25.F

In the usual theory of transport processes in a high-temperature hydrogen plasma the plasma is regarded as consisting only of electrons and ions, that is, as being completely ionized. Consequently the transverse transport coefficients of a plasma in a magnetic field decrease as the field is made stronger, and can become arbitrarily small in a sufficiently strong field.[1] Even in a high-temperature plasma, however, there are always some neutral hydrogen ions, formed through recombination. Since the magnetic field does not affect the motion of these atoms, there is a transport mechanism independent of the magnetic field strength.[1)] Obviously this mechanism gives a lower limit on the transport coefficients, which will be reached in a sufficiently strong magnetic field.[2)] In principle it is of interest to calculate the corresponding transport coefficients. The present paper is devoted to the solution of this problem.

It is obvious that in a strong magnetic field a transport coefficient (for example, the temperature-conductivity coefficient χ) can be estimated in the following way: $\chi\sim c\lambda v_{Ta}$, where c is the equilibrium fraction of neutral particles (which is assumed small), λ is their mean free path against ionization or charge transfer,[3)] and v_{Ta} is their mean velocity. Such an estimate naturally holds when the characteristic length of the problem is large compared with λ and the characteristic time is large compared with the time for establishing the equilibrium concentration of neutral atoms.[4)]

For definiteness we shall suppose the density is not too large, so that the only important process of formation of neutral atoms is radiative recombination. The size of the plasma is small enough for the radiation to escape freely and for processes of excitation of atoms by the light quanta to be negligible. Moreover we shall assume that the temperature of the plasma satisfies the inequalities

$$I_0\ll T\ll I_0M/m, \tag{1}$$

where I_0 is the ionization potential of the hydrogen atom, and M and m are the masses of an ion and an electron. The first of these inequalities allows us to use the Born approximation in calculating the cross sections for excitation and ionization; the second means that the average velocity of the ions is much smaller than the velocity of atomic electrons, and will be of use later.

The stationary value of the neutral-atom density is determined by the balance of two processes, radiative recombination and ionization of neutral particles. It turns out that in a sufficiently rarefied plasma almost all of the atoms are in the ground state, since spontaneous emission (whose probability is independent of the plasma density) leads to rapid transition of excited atoms to the ground state, which they can leave owing to processes of ionization or excitation by electron impact, which has a probability proportional to the density. To estimate the limiting density n_c at which the fraction of excited atoms is still small, we note that the probability of photorecombination into a state decreases rapidly with increase of the principal quantum number; that is, in photorecombination most of the atoms are in the ground state from the beginning. Therefore the main mechanism leading to the appearance of excited states is excitation from the ground state by electron impact, so that n_c can be estimated from the condition $n_c\sigma_{ex}v_{Te}\sim\gamma$, where γ is the probability of spontaneous emission, a characteristic value for which is of the order[5)] of $10^8 - 10^9\ \mathrm{sec}^{-1}$, and σ_{ex} is the cross section for excitation. For $T\sim 100$ eV, $\sigma_{ex}\sim 10^{-16}\ \mathrm{cm}^2$, and $n_e\sim 10^{16}\ \mathrm{cm}^{-3}$. At higher temperatures the critical density is still larger.

According to these considerations, for $n\lesssim 10^{16}\ \mathrm{cm}^{-3}$ the equilibrium concentration of neutral atoms can be found by equating the rate of recombination (into all states) and the rate of ionization from the ground state. For fast electrons with energy $\mathscr{E}\gg I_0$ the cross section for photorecombination into a state with principal quantum number n is proportional to $1/n^3$, so that the total cross section for photorecombination (into all states) is given in this case by the following formula[3]

$$\sigma_r=\frac{2^7\pi\zeta(3)}{3}\alpha^3a_0^2(I_0/\mathscr{E})^{3/2},\quad \mathscr{E}\gg I_0, \tag{2}$$

where $\zeta(3)\approx 1.2$ is the Riemann zeta function, a_0 is the Bohr radius of the hydrogen atom, and α is the fine structure constant. For $\mathscr{E}\ll I_0$ the dependence of the photorecombination cross section on the number of the state is more complicated. According to data in the literature,[4] the total cross section can be written with good accuracy as

$$\sigma_r \approx 0.47 \cdot 10^2 \alpha^3 a_0^2 I_0/\mathscr{E}, \quad \mathscr{E} \ll I_0. \tag{2'}$$

It can be seen from (2) and (2′) that for $T \gg I_0$ the main contribution to the photorecombination is from electrons with energy $\mathscr{E} \sim I_0$, and since for $T \ll I_0 M/m$ the speed of the ions is much smaller than that of such electrons, the recombination probability does not depend on the speed of the ions. Therefore the velocity distribution function $g_0(v)$ of the neutral atoms will be the same as the distribution function $f_i(v)$ of the ions, i.e., a Maxwellian distribution. Averaging the quantity $\sigma_r v$ over the distribution function $f_e(v)$ of the electrons, we readily find

$$\langle \sigma_r v \rangle [\mathrm{cm}^3\,\mathrm{sec}^{-2}] \approx 2.7 \cdot 10^{-11}/T^{1/2}[\mathrm{eV}]. \tag{3}$$

The main contribution to the ionization of the atoms comes from electrons with thermal velocities. Therefore the corresponding cross section is given with good accuracy by the Born approximation[5]:

$$\sigma_i = \frac{4\pi a_0^2 I_0 \cdot 0.285}{\mathscr{E}} \ln \frac{\mathscr{E}}{0.012 I_0}$$

and after averaging:

$$\langle \sigma_i v \rangle [\mathrm{cm}^3\,\mathrm{sec}^{-1}] \approx \frac{4 \cdot 10^{-8}}{T^{1/2}[\mathrm{eV}]} \ln \{7.3 T[\mathrm{eV}]\}. \tag{4}$$

Comparing the expressions (3) and (4), we get the equilibrium concentration of neutral atoms $c(T)$:

$$c(T) = \langle \sigma_r v \rangle / \langle \sigma_i v \rangle \approx 10^{-4}/T\ [\mathrm{eV}] \tag{5}$$

(The weakly varying logarithmic factor in Eq. (4) has been replaced with a constant value corresponding to $T \sim 300$ eV).

We now proceed to find the transverse transport coefficients given by the neutral atoms. To do so we must find the correction to the already known equilibrium distribution function of the atoms which arises because of a nonuniformity of the plasma. As for the distribution functions of the electrons and ions, in a strong magnetic field their departures from the Maxwellian form can be neglected. In the temperature range (1) the main process determining the mean free path of the neutral atoms is charge transfer, which has a rather large cross section σ_* because of its resonance character.[2] Though it does not affect the concentration of neutral atoms, charge transfer in this case is the mechanism that makes their distribution function tend to become more isotropic. The following equation holds for the correction $g_1(\mathbf{v})$ to the distribution function $g_0(\mathbf{v})$ of the neutral atoms:

$$v \cos\theta \frac{\partial g_0}{\partial x} = f_i(\mathbf{v}) \int \sigma_*(u) u g_1(\mathbf{v}') d^3\mathbf{v}' - g_1(\mathbf{v}) \int \sigma_*(u) u f_i(\mathbf{v}') d^3\mathbf{v}', \quad u = |\mathbf{v} - \mathbf{v}'|,$$

$$g_0(\mathbf{v}) = c(T) f_i(\mathbf{v}), \quad f_i(\mathbf{v}) = \frac{n M^{3/2} \varepsilon(v)}{(2\pi T)^{3/2}}, \quad \varepsilon(v) \equiv e^{-Mv^2/2T}. \tag{6}$$

It is obvious that its solution can be written in the form

$$g_1(\mathbf{v}) = a(v) \varepsilon(v) \cos\theta$$

In general, $a(v)$ can be found from this only by numerical methods. However, Eq. (6) can be greatly simplified if we consider that for $\mathscr{E} \lesssim I_0 M/m$ the charge-transfer cross section $\sigma_*(u)$ depends only very weakly on the velocity, so that in the integration in Eq. (6) it can to good accuracy be replaced with an average $\sigma_*(v_{Ta})$. Then from Eq. (6) we have

$$\frac{c(T) v}{4\pi \sigma_*(v_{Ta})} \left[\left(-\frac{5}{2T} + \frac{Mv^2}{2T^2} \right) \frac{dT}{dx} + \frac{1}{n} \frac{dn}{dx} \right]$$
$$= \frac{1}{3} \left\{ \int_0^v \varepsilon(u) a(u) u^2 du \left(-u + \frac{u^3}{5v^2} \right) + \int_v^\infty \varepsilon(u) a(u) u^2 du \left(-v + \frac{v^3}{5u^2} \right) \right\}$$
$$- a(v) \left\{ \int_v^\infty \varepsilon(u) u^2 du \left(u + \frac{v^2}{3u} \right) + \int_0^v \varepsilon(u) u^2 du \left(v + \frac{u^2}{3v} \right) \right\}. \tag{7}$$

Moreover, in determining, for example, the thermal conduction coefficient[6)] we can confine ourselves to an approximate solution of Eq. (7) in the velocity range $v \gtrsim (T/M)^{1/2}$, since in the heat flux

$$q = \frac{M}{2} 2\pi \int_0^\pi \sin\theta \cos^2\theta \, d\theta \int_0^\infty v^5 a(v) \varepsilon(v) dv$$

precisely this range of velocities makes the main contribution because of the high power of v in the last integral. At these velocities the main term in the right member of Eq. (7) is the last one, and we get

$$a(v) = -\frac{c(T) M^{3/2}}{(2\pi T)^{3/2} \sigma_*(v_{Ta})} \left[\left(-\frac{5}{2T} + \frac{Mv^2}{2T^2} \right) \frac{dT}{dx} + \frac{1}{n} \frac{dn}{dx} \right]. \tag{8}$$

Knowing the correction (8) to the distribution function of the neutral atoms, we can readily calculate the fluxes of matter and energy Q and q, associated with gradients of the density and temperature of the plasma:

$$Q = \alpha \frac{dT}{dx} + \beta \frac{dn}{dx}, \quad q = \gamma \frac{dT}{dx} + \delta \frac{dn}{dx}, \tag{9}$$

where

$$\alpha = \frac{2c(T)}{3(2\pi T)^{1/2} \sigma_*(v_{Ta}) M^{1/2}}, \quad \beta = -2\alpha \frac{T}{n},$$
$$\gamma = -2\alpha T, \quad \delta = -4\alpha T^2/n.$$

On determining the flux of heat in the absence of a flux of particles, we get the following expression for the thermal conductivity $\tilde{\varkappa}_\perp$:

$$\tilde{\varkappa}_\perp [\mathrm{cm}^{-1}\,\mathrm{sec}^{-1}] \approx \frac{1.2 \cdot 10^2}{T^{1/2}[\mathrm{eV}] \sigma_*(v_{Ta}) [\mathrm{cm}^2]}. \tag{10}$$

It is interesting to compare this thermal conduction coefficient with the classical Coulomb thermal conductivity $\kappa_\perp^{(cl)}$ of a hydrogen plasma[1]

$$\varkappa_\perp^{(cl)} [\mathrm{cm}^{-1}\,\mathrm{sec}^{-1}] \approx \frac{1.6 \cdot 10^{-2} n^2 [\mathrm{cm}^{-3}]}{H^2[\mathrm{G}] T^{1/2}[\mathrm{eV}]}.$$

Using the fact that under the condition (1) $\sigma_*(v_{Ta}) \sim 10^{-15}$ cm^2, we find

$$\tilde{\varkappa}_\perp / \varkappa_\perp^{(cl)} \approx 0.8 \cdot 10^{19} \frac{H^2[\mathrm{eV}]}{n^2[\mathrm{cm}^{-3}]} \tag{11}$$

It can be seen from this that the neutral-atom mechanism for transport processes in a plasma becomes important at very moderate magnetic fields. For example, for $n \sim 10^{14}$ cm^{-3} the field in question is $H > 3 \times 10^4$ G. The recombination diffusion of a plasma exceeds the classical value at still weaker magnetic fields, since the recombination diffusion and temperature conduction coefficients are of the same order, while the classical diffusion coefficient is smaller than the classical temperature conductivity by about a factor $(M/m)^{1/2}$.

We express our thanks to B. M. Smirnov for a discussion of the results of this work.

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1)We have learned that similar considerations were given earlier by G. I. Budker.

2)Here a "sufficiently strong" field means one in which the usual transport coefficients become negligibly small. At the same time we assume that the field is still not so large that it would appreciably affect the state of the atoms and the elementary acts of collision of atoms with electrons and ions.

3)Other processes, for example elastic scattering of the atoms, are usually unimportant.

4)This last condition clearly allows us to regard the velocity distribution functions of the electrons and ions as Maxwellian with the same temperature.

5)An exception to this is the metastable state 2s, bur numerical calculations show that the fraction of the atoms in this state is not more than a few percent. This is owing to the possibility of resonance transition of atoms from the 2s state to the 2p state in collisions with ions (cf., e.g., [2], page 95).

6)The same remark applies also to the other transport coefficients.

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Translated by W. H. Furry
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