Intensity modulation of stimulated scattering under conditions of high pump intensity

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(Submitted September 26, 1974)
Zh. Eksp. Teor. Fiz. 68, 929–939 (March 1975)

The space and time evolution of various types of stimulated scattering of light is investigated as a function of the pump intensity for various values of the logarithmic decrement of the interacting waves. The analysis is carried out on the basis of generalized equations that are derived (with allowance for phase evolution) for the occupation numbers and fluxes of the interacting-wave quanta. It is shown that at pump intensities exceeding certain threshold values the scattered radiation intensity grows nonmonotonically, the stationary state being established in an oscillatory manner. The oscillation period depends on the pump intensity. The results are used to interpret the presence of a threshold for the modulation of intense laser radiation reflected by a plasma, which has been observed experimentally[3], and also the appearance of additional lines in the stimulated Raman-scattering spectrum [4] under sufficiently intense pump conditions.

1. INTRODUCTION

We investigated in this paper the evolution of the intensity of various types of stimulated scattering of light (stimulated parametric scattering—SPS, stimulated Raman scattering—SRS, and stimulated Mandel'shtam scattering—SMBS) as a function of the pump intensity. It is shown that the intensity does not increase monotonically when the pump exceeds a certain threshold value, namely, the stationary state is reached in an oscillatory fashion.

The problem of the spatial oscillations of the intensity of interacting waves is not new. Thus, for example, in[1,2] are given oscillating solutions of the problem of the interaction of three waves with damping neglected. In the present paper, this problem is solved approximately, with arbitrary wave damping. We investigate individually cases of spatial and temporal evolution. The last case can be of interest in connection with the experimental observation of the oscillations of the intensity of scattered radiation at high-power pumping.

Basov, Krokhin, et al.[3] have observed experimentally the effect of temporal modulation of the intensity of radiation reflected from a plasma exposed to a powerful laser beam. The effect has a threshold, i.e., it is observed at pump intensities exceeding a certain threshold value. In[3], the temporal modulation effect is attributed to scattering of light by nonstationary turbulence. It results from decay instability, in which each pump quantum decays in the plasma into two longitudinal plasma oscillation quanta. Thus, the modulation of the scattered radiation sets in in two stages.

In this paper we propose a different explanation of this effect. We show that the modulation of the intensity of the scattered light in time occurs in the case of sufficiently strong pumping under stimulated scattering (SPS, SRS, SMBS) by transverse or longitudinal waves in the medium. The results of the experiment[3] can be explained if the reflected radiation is regarded as stimulated scattering. The two indicated possible mechanisms of temporal modulation of the reflected radiation lead to qualitatively different dependences of the period of the modulation on the intensity.

It is also possible that the threshold splitting of the scattered-radiation line, observed in certain studies (see[4]) in liquids and in solids (the fine structure of SRS), is connected with the modulation effect.

In this paper we consider two particular problems: 1) stationary stimulated scattering, 2) the evolution of spatially homogeneous stimulated scattering in time. The first is solved on the basis of generalized (with allowance for the spatial variation of the phases) equations for the fluxes of the quanta of the interacting waves. The second problem is solved on the basis of the corresponding equations for the densities of the quanta of the interacting waves.

In a recent paper by Sparks[5] they investigated the appearance of instability (jumplike increase of the intensity in high-power pumping) of stimulated scattering on the basis of the rate equations for the densities of the quanta[6]. In these equations, no account was taken of the evolution of the phases, because they are equations of first order. The generalized equations obtained by us take into account the phase evolution and are consequently second-order equations. It follows from these equations that changes in the regime occur in the case of high-power pumping without a discontinuity.

2. EQUATIONS FOR THE DENSITIES OF THE QUANTA OF THE INTERACTING WAVES IN THE CASE OF HOMOGENEOUS DISTRIBUTION

We start here with the system of reduced equations that describe the time evolution of the complex amplitudes of the Stokes (E_S) and the laser (E_L) modes, and the amplitudes U_q of the lattice displacement in the case of SRS, SMBS, or the idling-wave amplitude in the case of SPS; it can be expressed in the form[1,2]

$$\frac{d}{dt}E_s+\gamma_s E_s=\sigma_s E_L U_q^*,\quad \frac{d}{dt}U_q+\gamma_q U_q=\sigma_q E_L E_s^*,$$
$$\frac{d}{dt}E_L+\gamma_L E_L=\sigma_L E_s U_q. \tag{2.1}$$

For SPS we have here

$$\sigma_j=2\pi i d\omega_j\ (j=s,\ q,\ L), \tag{2.2}$$

where d is the contraction of the nonlinear-susceptibility tensor. In the case of SRS and SMBS we have

$$\sigma_{s,L}=2\pi i a\omega_{s,L},\quad \sigma_q=i\frac{a}{Mn\omega_q} \tag{2.3}$$

where a is the contraction of the RS or the SMBS tensor,

n is the concentration of the scattering cells; M and ω_q are respectively the mass and frequency of the phonon oscillator.

The system (2.1) is a system of six scalar first-order equations for the amplitudes and the phases of the three waves. We shall show that it can be reduced to a system of three second-order equations for the quantum-number densities n_S, n_q, and n_L. These quantities are connected with U_S, U_q, and E_L in the following manner: in the case of SPS we have

$$n_j = E_j^* E_j / 2\pi\hbar\omega_j,\ j=s,\ q,\ L; \tag{2.4}$$

the case of SRS and SMBS we have

$$n_{s,L} = \frac{(E^*E)_{s,L}}{2\pi\hbar\omega_{s,L}},\quad n_q = \frac{Mn\omega_q}{\hbar} U_q^* U_q. \tag{2.5}$$

To obtain equations for n_S, n_q, and n_L, we multiply Eqs. (2.1) respectively by E_S^*, U_q^*, and E_L^*, and add them to the corresponding complex-conjugate equations. As a result we obtain three equations:

$$\begin{aligned}&(d/dt+2\gamma_s)E_s^*E_s = 2\operatorname{Re}\sigma_s E_s^* E_L U_q^*,\\&(d/dt+2\gamma_q)U_q^*U_q = 2\operatorname{Re}\sigma_q U_q^* E_L E_s^*,\\&(d/dt+2\gamma_L)E_L^*E_L = 2\operatorname{Re}\sigma_L E_L^* E_s U_q.\end{aligned} \tag{2.6}$$

The right-hand sides of these equations are determined by single complex function $E_S^* E_L U_q^*$. The equation for it also follows from (2.1) and takes the form

$$\begin{aligned}(d/dt+\gamma)E_s^*E_LU_q^* = {}&\sigma_s^* E_L^* E_L U_q^* U_q\\&+\sigma_q E_s^* E_s E_L^* E_L + \sigma_L E_s^* E_s U_q^* U_q.\end{aligned} \tag{2.7}$$

Here $\gamma = \gamma_S + \gamma_q + \gamma_L$.

We eliminate from (2.6) and (2.7) the function $E_S^* E_L U_q^*$ and change over to the numbers of quanta (2.4) and (2.5). As a result we obtain a closed system of three second-order equations for the three functions n_S, n_q, and n_L:

$$\begin{aligned}&(d/dt+\gamma)(d/dt+2\gamma_s)n_s = C[n_Ln_q+n_sn_L-n_sn_q],\\&(d/dt+\gamma)(d/dt+2\gamma_q)n_q = C[n_Ln_q+n_sn_L-n_sn_q],\\&(d/dt+\gamma)(d/dt+2\gamma_L)n_L = -C[n_Ln_q+n_sn_L-n_sn_q].\end{aligned} \tag{2.8}$$

The constant C is determined by the following expressions: in the case of SPS

$$C = 16\pi^3\hbar\omega_s\omega_L\omega_q d^2, \tag{2.9}$$

and in the case of SRS and SMBS

$$C = 8\pi^2 a^2 \hbar\omega_s\omega_L / Mn\omega_q. \tag{2.10}$$

The difference between Eqs. (2.8) and the first-order rate equations[5,6] is due to allowance for the explicit dependence of the triple product $E_S^* E_L U_q^*$ on the phase difference $\varphi_L - \varphi_S - \varphi_q$.

3. SYSTEM OF EQUATIONS FOR THE FLUX DENSITIES OF THE QUANTA OF THE INTERACTING WAVES IN THE STATIONARY CASE

To describe the spatial variation of the amplitudes and phases of the interacting waves in the stationary case it is necessary to use in place of (2.1) the equations

$$\begin{aligned}&dE_s/dx+\beta_sE_s = A_sE_LU_q^*,\\&dU_q/dt+\beta_qU_q = A_qE_LE_s^*,\\&dE_L/dt+\beta_LE_L = A_LE_s^*U_q;\end{aligned} \tag{3.1}$$

$$\beta_j = \gamma_j/v_j,\ A_j = \sigma_j/v_j,\ j=s,\ q,\ L. \tag{3.2}$$

Here v_j are the group velocities.

In analogy with the procedure used to derive (2.8) from (2.1), we obtain from (3.2) the equations for the quantum flux densities:

$$\Pi_j = v_j n_j,\ j=s,\ q,\ L. \tag{3.3}$$

These equations take the form

$$\begin{aligned}&\left(\frac{d}{dx}+\beta\right)\left(\frac{d}{dx}+2\beta_s\right)\Pi_s = D[\Pi_L\Pi_q+\Pi_L\Pi_s-\Pi_s\Pi_q],\\&\left(\frac{d}{dx}+\beta\right)\left(\frac{d}{dx}+2\beta_q\right)\Pi_q = D[\Pi_L\Pi_q+\Pi_L\Pi_s-\Pi_s\Pi_q],\\&\left(\frac{d}{dx}+\beta\right)\left(\frac{d}{dx}+2\beta_L\right)\Pi_L = -D[\Pi_L\Pi_q+\Pi_L\Pi_s-\Pi_s\Pi_q].\end{aligned} \tag{3.4}$$

Here

$$\beta = \beta_s+\beta_q+\beta_L,\quad D = C/v_sv_Lv_q,$$

where C is determined by formulas (2.9) and (2.10). These equations also differ significantly from the "rate" equations that prescribe the spatial variation of the interacting-wave quantum flux densities. Equations (3.4) are second-order equations because they take into account the spatial variation of not only the amplitudes but also of the phase difference $\varphi_L - \varphi_S - \varphi_q$.

4. SOLUTION OF THE SYSTEM (3.4) FOR THE CASE OF STIMULATED RAMAN SCATTERING

In the case SRS, the inequalities $\beta_q \gg \beta_S$ and $\beta_q \gg \beta_L$ are satisfied. The character of the solution, as we shall show, depends significantly on the pump intensity. We consider two limiting cases.

1. The functions $\Pi_{S,L}$ vary little over a distance $1/\beta_q$, i.e.,

$$\frac{1}{\beta_q}\frac{d\Pi_{s,L}}{dx} \ll \Pi_{s,L},\quad \beta_q \gg \beta_s,\ \beta_L. \tag{4.1}$$

Under these conditions Eqs. (3.4) take in the zeroth approximation in β_S/β_q and β_L/β_q the form

$$\begin{aligned}&\frac{d}{dx}\Pi_s = \frac{D}{\beta_q}[\Pi_L\Pi_q+\Pi_L\Pi_s-\Pi_s\Pi_q],\\&\frac{d}{dx}\Pi_L = -\frac{D}{\beta_q}[\Pi_L\Pi_q+\Pi_L\Pi_s-\Pi_s\Pi_q],\\&2\beta_q\Pi_q = \frac{D}{\beta_q}[\Pi_L\Pi_q+\Pi_L\Pi_s-\Pi_s\Pi_q].\end{aligned} \tag{4.2}$$

From first two formulas of (4.2) we obtain the flux conservation law:

$$\Pi_s+\Pi_L = \Pi_s^0+\Pi_L^0 \equiv \Pi, \tag{4.3}$$

where $\Pi_{S,L}^0$ represent the values of the fluxes at $x = 0$.

We note that Eqs. (4.2) for the fluxes have the same form as the corresponding equations obtained by Sparks[5] for the numbers n_S, n_q, and n_L of the quanta.

From the first equation of (4.2) we eliminate Π_q with the aid of the third equation of (4.2) and Π_L with the aid of (4.3). As a result we obtain a closed equation for Π_S:

$$\left(\frac{\beta_q^2}{D}-\frac{\Pi}{2}+\Pi_s\right)\frac{d\Pi_s}{dx} = \beta_q(\Pi-\Pi_s)\Pi_s. \tag{4.4}$$

The solution of this equation is

$$\frac{\Pi_s(x)}{\Pi_s(0)}\bigg/\left|\frac{\Pi_s(x)-\Pi}{\Pi_s(0)-\Pi}\right|^{1+\xi} = \exp(\beta_q\xi);\quad \xi = \frac{2\Pi}{2\beta_q^2/D-\Pi}. \tag{4.5}$$

In the case of weak pumping, when

$$\Pi \equiv \Pi_L^0+\Pi_s^0 \ll 2\beta_q^2/D, \tag{4.6}$$

we have from (4.5)

$$\Pi_s(x)=\frac{\Pi_s^0\Pi}{\Pi_s^0+\Pi_L^0\exp(-D\Pi x/\beta_q)}. \qquad (4.7)$$

This solution coincides with formula (4.76) of Bloembergen's book[2].

In the opposite case of strong pumping, when the parameter

$$\Pi D/2\beta_q^2\to 1, \qquad (4.8)$$

the solution (4.8) takes the form

$$\Pi_s(x)=\Pi+(\Pi_s^0-\Pi)\exp(-\beta_q x). \qquad (4.9)$$

This solution coincides in form with that obtained by Sparks[5]. It is, however, inconsistent, since the condition (4.1), which is the condition for the applicability of the initial equations (4.2), is violated in it. Consequently, in the case of strong pumping it is necessary to turn from (4.2) to the more general equations (3.4).

2. We present the solution of Eqs. (3.4) for the case of strong pumping under the boundary conditions

$$\Pi_s(0)=\Pi_s^0,\quad \Pi_q(0)=0,\quad \Pi_L(0)=\Pi_L^0,$$
$$\left.\frac{d\Pi_j}{dx}\right|_{x=0}=0,\quad j=s,q,L. \qquad (4.10)$$

The boundary conditions for the derivatives follow from equations analogous to Eqs. (2.6), with allowance for the fact that $\Pi_q(0) = 0$.

In the zeroth approximation in β_S/β_q and β_L/β_q, we obtain again from (3.4) the conservation law (4.3). Eliminating Π_L and Π_q from (3.4), we obtain one equation for Π_S:

$$\left(\frac{d}{dx}+\beta_q\right)\frac{d\Pi_s}{dx}=D\Big[(\Pi-\Pi_s)\Pi_s+(\Pi-2\Pi_s)\int_0^x\exp\{-\beta_q(x-x')\}\frac{d\Pi_s(x')}{dx'}dx'\Big]. \qquad (4.11)$$

Let us consider the solution of this equation for arbitrary pumping, but for sufficiently small x, when $\Pi_q \ll \Pi_L$ and $\Pi_S \ll \Pi_L$. In this approximation, Eq. (4.11) becomes

$$\left(\frac{d}{dx}+\beta_q\right)\frac{d\Pi_s}{dx}=D\Pi\Big(\Pi_s+\int_0^x\exp\{-\beta_q(x-x')\}\frac{d\Pi_s(x')}{dx'}dx'\Big). \qquad (4.12)$$

We write down the solution of this equation under the boundary conditions (4.10)

$$\Pi_s(x)=\frac{\Pi_s^0}{2}\Big\{\exp(x(\eta-\beta_q))\Big(1+\frac{\beta_q}{\eta}-\frac{\Pi D}{\eta^2}\Big)+\exp(-x(\eta+\beta_q))\Big(1-\frac{\beta_q}{\eta}-\frac{\Pi D}{\eta^2}\Big)+2\exp(-\beta_q x)\frac{\Pi D}{\eta^2}\Big\};$$
$$\eta=(\beta_q^2+2\Pi D)^{1/2}. \qquad (4.13)$$

In the case of weak pumping, when the condition (4.6) is satisfied, this solution takes the form

$$\Pi_s(x)=\Pi_s^0\exp(\Pi Dx/\beta_q). \qquad (4.14)$$

It coincides with (4.7) at small x, when

$$\Pi_s^0\ll\Pi_L^0\exp(-\Pi Dx/\beta_q).$$

In (4.14), the gain is proportional to the pump. In the case of strong pumping, when $2\Pi_D \gg \beta_q^2$, it follows from (4.13) that

$$\Pi_s(x)=\frac{\Pi_s^0}{2}\Big[\frac{1}{2}(e^{(2\Pi D)^{1/2}x}+e^{-(2\Pi D)^{1/2}x})+e^{-\beta x}\Big]. \qquad (4.15)$$

Thus, in the case of strong pumping, the gain is proportional to $\sqrt{\Pi_L^0}$.

We consider now the character of the evolution of the solutions of (4.11) at large x. At $x = \infty$ we get from (4.11)

$$\Pi_s(\infty)=\Pi,\quad \Pi_q(\infty)=0,\quad \Pi_L(\infty)=0. \qquad (4.16)$$

To obtain the last two equations it is necessary, of course, to use the initial equations (3.4) at $\beta_S = \beta_L = 0$. Using (4.16), we represent the solution of (4.11) at large x in the form

$$\Pi_s(x)=\Pi+\delta\Pi_s(x),\quad \delta\Pi_s(x)\ll\Pi. \qquad (4.17)$$

The equation for $\delta\Pi_S$ differs in the linear approximation from Eq. (4.12) only in the sign of the right-hand side. Consequently, the solution for $\delta\Pi_S(x)$ is determined by (4.13) in which we let $D \to -D$ and $\Pi_S^0 \to \delta\Pi_S^0$, and to which we also add an analogous term proportional to the derivative $(d\delta\Pi_S/dx)^0$. The superscript zero denotes here an arbitrary value of x at which the condition (4.17) is satisfied.

From formula (4.13) as $D \to -D$ it follows that in the case of strong pumping, when

$$\Pi>\Pi_{osc}=\beta_q^2/2D, \qquad (4.18)$$

the approach to the solution (4.16) has an oscillatory character with a spatial period

$$L=2\pi/(2\Pi D-\beta_q^2)^{1/2}. \qquad (4.19)$$

Thus, in the case of strong pumping exceeding the threshold value $\Pi_{osc} = \beta_q^2/2D$, the monotonic character of the energy transfer to the scattered radiation gives way to an oscillatory character.

As a result we arrive at the following picture of the establishment of the solution at large pumps $\Pi > \Pi_{osc}$.

A monotonic approach to a level on the oder of Π takes place, as follows from (4.15), over a distance $1/\sqrt{2\Pi D} \ll 1/\beta_q$ at $\Pi \gg \Pi_{osc}$.

A nonmonotonic (oscillating, with a period) approach takes place, as follows from (4.13), over a distance $1/\beta_q$.

5. TEMPORAL EVOLUTION OF n_s, n_q, AND n_L IN THE CASE OF A SPATIALLY HOMOGENEOUS FIELD DISTRIBUTION

Two formulations of the problem of the spatial evolution of the scattered radiation are possible. These are, first, the study of the temporal evolution of the initial state in the absence of external pumping. This problem can be solved on the basis of (2.1) or equivalent equations (2.8) for the functions n_s, n_q, and n_L. A solution of this problem is analogous to that given in Secs. 3 and 4, owing to the complete analogy of Eqs. (2.8) and (3.4). More closely corresponding to the experimental conditions (particularly in[3]) is the process of the evolution of n_S, n_q, and n_L in the presence of pumping. To take the pumping into account it is necessary to add the external force $F(t)$ to the right-hand side of (2.1). This leads to corresponding changes in the system (2.8). The solution obtained for this rather complicated system of equations has shown that the main features of the process can be described in a simpler manner, by using the solution of the first two equations of (2.8) and by assuming the function n_L to be given. It should be noted that this approximation differs from the given-pump approximation when we neglect the time variation of both the amplitude and the phase of the complex function

$E_L(t)$. In the approximation considered by us, we assume only the intensity $|E_L|^2 \sim n_L$ to be constant. The change of the phase φ_L (particularly of the phase difference $\varphi_L - \varphi_q - \varphi_s$) is taken into account on going from Eqs. (2.1) to Eqs. (2.8). By virtue of this, the approximation considered by us should be called the "approximation with given pump intensity."

Thus, we consider the solution of the system (2.8) at a specified value of n_L. Had we neglected in the derivation of the equations for n_s and n_q also the change of the phase φ_L, then the first two formulas of (2.8) would not contain the terms with Cn_sn_q. We shall show that a regime of temporal modulation of the scattered radiation would be impossible without allowance for these terms.

The first two equations of (2.8) have a stationary solution[1)]:

$$\bar{n}_s = n_L\frac{\gamma_s+\gamma_q}{\gamma_s} - \frac{2\gamma\gamma_q}{C}, \quad \bar{n}_q = n_L\frac{\gamma_s+\gamma_q}{\gamma_q} - \frac{2\gamma\gamma_s}{C}. \tag{5.1}$$

From this we get the threshold value of n_L, at which the existence of a stationary regime is possible:

$$n_{\text{thr}} = 2\gamma\gamma_s\gamma_q/(\gamma_s+\gamma_q)C. \tag{5.2}$$

At short times, when $n_s \ll n_L$ and $n_q \ll n_L$, the system of the first two equations of (2.8) becomes linear. The form of the solution is determined by the characteristic equation

$$\begin{gathered}[p^2+(\gamma+2\gamma_s)p+2\gamma\gamma_s][p^2+(\gamma+2\gamma_q)p+2\gamma\gamma_q]\\ -2Cn_L[p^2+(\gamma+\gamma_s+\gamma_q)p+\gamma(\gamma_s+\gamma_q)]=0.\end{gathered} \tag{5.3}$$

Let us investigate this equation for two cases.

1. $\gamma_s = \gamma_q = \gamma_L \equiv \gamma_0$ (SPS, SMBS). The solution that determines the growth is of the form

$$p = -\frac{5}{2}\gamma_0 + \left(\frac{\gamma_0^2}{4} + 2Cn_L\right)^{1/2}. \tag{5.4}$$

Growth takes place at $n_L > 3\gamma_0^2/C$. Thus, the growth threshold coincides with the threshold of the stationary state (5.2).

2. $\gamma_q \gg \gamma_s$, γ_L (SPS, SMBS). If $p < \gamma_q$, then we get from (5.3)

$$p = Cn_L/\gamma_q - 2\gamma_s. \tag{5.5}$$

We see therefore that the buildup threshold again coincides with the threshold for the existence of the stationary state (5.2).

It follows from (5.5) that at $n_L \gg 2\gamma_s\gamma_q/C$ (but $n_L \ll \gamma_q^2/C$) we have

$$p = Cn_L/\gamma_q, \tag{5.6}$$

and consequently the growth increment is proportional to n_L. At $p \gg \gamma_q$ we obtain from (5.3)

$$p = \sqrt{2Cn_L}, \quad n_L \gg \gamma_q^2/C. \tag{5.7}$$

Thus, during the initial section of the time evolution the linear increase of the growth rate with increasing n_L gives way to an increase like $n_L^{1/2}$. This result is analogous to that obtained in Sec. 4 in the study of the spatial growth of the function $\Pi_s(x)$ with small x (see (4.13) and (4.15)).

We now consider the time evolution of $n_s(t)$ near the stationary state $\bar{n}_s$, $\bar{n}_q$. To this end we represent the functions n_s and n_q in the form

$$n_s(t) = \bar{n}_s + \delta n_s(t); \quad n_q(t) = \bar{n}_q + \delta n_q(t). \tag{5.8}$$

The time evolution of the functions δn_s and δn_q is determined in the linear approximation by the characteristic equation that follows from the first two equations of (2.8) and (5.8):

$$\begin{gathered}[p^2+(\gamma+2\gamma_s)p+2\gamma\gamma_s][p^2+(\gamma+2\gamma_q)p+2\gamma\gamma_q]\\ +C(\bar{n}_q-n_L)[p^2+(\gamma+2\gamma_q)p+2\gamma\gamma_q]\\ +C(\bar{n}_s-n_L)[p^2+(\gamma+2\gamma_s)p+2\gamma\gamma_s]=0.\end{gathered} \tag{5.9}$$

This equation goes over respectively into (5.3) at $n_q = 0$ and $n_s = 0$.

We consider again two cases.

1. $\gamma_q = \gamma_s = \gamma_L \equiv \gamma_0$ (SPS, SMBS). From (5.9) we have

$$\begin{aligned}p &= -{}^5/_2\gamma_0 \pm [{}^1/_4\gamma_0^2 + C(2n_L - \bar{n}_q - \bar{n}_s)]^{1/2}\\ &= -{}^5/_2\gamma_0 \pm [{}^{49}/_4\gamma_0^2 - 2Cn_L]^{1/2}.\end{aligned} \tag{5.10}$$

We see therefore that at pumps

$$n_L > \frac{49}{8}\frac{\gamma_0^2}{C} \equiv n_{\text{osc}} \approx 2n_{\text{thr}} \tag{5.11}$$

oscillations appear. In (5.11) n_{osc} is the threshold value at which oscillations appear, and n_{thr} is the threshold of the stationary state (5.2). Thus, the period of the oscillations is determined by the expression

$$T = 2\pi/\omega = 2\pi/[2C(n_L - n_{\text{osc}})]^{1/2}. \tag{5.12}$$

This expression can be represented in the form

$$T = \frac{4\pi}{7\gamma_0}\left(\frac{I_{\text{osc}}}{I - I_{\text{osc}}}\right)^{1/2}, \tag{5.13}$$

where $I = v_L\hbar\omega_L n_L$ is the flux of the pump quanta and $I_{osc} = {}^{49}/_8 v_L\hbar\omega_L\gamma_0^2/C$ is the threshold value of the flux at which the oscillations take place.

2. $\gamma_q \gg \gamma_s, \gamma_L$ (SMBS, SRS). In analogy with (5.10), we assume that $|p| \sim \gamma_q$ at the threshold of the appearance of oscillations. At $\gamma_q \gg \gamma_s$ and $\gamma_q \gg \gamma_L$ we get from (5.1)

$$\bar{n}_q - n_L = -2\gamma_q\gamma_s/C, \quad \bar{n}_s - n_L \approx n_L\gamma_q/\gamma_s - 2\gamma_q^2/C.$$

Taking this into account, we get from (5.9) the equation

$$p^2 + 3\gamma_q p + 2\gamma_q^2 + C(\bar{n}_s - \bar{n}_L) = p^2 + 3\gamma_q p + Cn_L\gamma_q/\gamma_s = 0. \tag{5.14}$$

Therefore

$$\begin{aligned}p_{1,2} &= -\frac{3}{2}\gamma_q \pm \left[\frac{\gamma_q^2}{4} + C(n_L - \bar{n}_s)\right]^{1/2}\\ &= -\frac{3}{2}\gamma_q \pm \left(\frac{9}{4}\gamma_q^2 - Cn_L\frac{\gamma_q}{\gamma_s}\right)^{1/2}.\end{aligned} \tag{5.15}$$

The oscillation threshold is

$$n_{\text{osc}} = \frac{9}{4}\frac{\gamma_q\gamma_s}{C} = \frac{9}{8}n_{\text{thr}}.$$

The period of the oscillations is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(Cn_L\gamma_q/\gamma_s - {}^9/_4\gamma_q^2)^{1/2}} = \frac{4\pi}{3\gamma_q}\left(\frac{I_{\text{osc}}}{I - I_{\text{osc}}}\right)^{1/2}. \tag{5.16}$$

Here $I_{osc} = {}^9/_4 v_L\hbar\omega_L\gamma_q\gamma_s/C$ is the threshold value of the flux at which the oscillations take place.

6. DISCUSSION OF RESULTS

From the foregoing analysis of the temporal evolution of the intensity of the scattered radiation we obtain the following picture.

At pumps $n_{thr} < n_L < n_{osc}$ there is observed a monotonic increase of the intensity of the scattered radiation with time. The growth rate first increases in proportion to n_L and then in proportion to $n_L^{1/2}$. The stationary state sets in with monotonic increase of $n_s(t)$.

At pumps $n_L > n_{osc}$ there is, during the initial stage, a monotonic increase with growth rates determined by formulas (5.4) and (5.7). Near the stationary state, the evolution is nonmonotonic (an oscillatory regime sets in). The period of the oscillations is determined by formulas (5.12) and (5.16). At large excesses $n_L \gg n_{osc}$, the modulation period $T \approx 2\pi/\sqrt{2Cn_L}$ is of the same order as the buildup time during the initial section.

The period of the oscillations decreases monotonically with increasing n_L. The theory proposed in[3] leads to a nonmonotonic dependence. It is important that in the considered theory the oscillation threshold n_{osc} is always higher than the threshold n_{thr} for the appearance of stimulated scattering. Thus, $n_{osc} = 2n_{thr}$ at $\gamma_S = \gamma_q = \gamma_L$ and $n_{osc} = (9/8)n_{thr}$ at $\gamma_q \gg \gamma_L$ and $\gamma_q \gg \gamma_S$.

Results were presented above for two regions: the initial section and the approach to the steady state. A complete description of the evolution calls for a solution of a system of nonlinear equations for the functions $n_S(t)$ and $n_q(t)$. In the particular case when $\gamma_S = \gamma_L = \gamma_q \equiv \gamma_0$ and $n_S(t=0) = n_q(t=0)$, we have at any instant of time $n_q(t) = n_S(t)$. Consequently, the complete description of the time evolution is given by a single equation

$$\frac{d^2n_s}{dt^2}+5\gamma_0\frac{dn_s}{dt}+(6\gamma_0^2-2Cn_L)n_s+Cn_s^2=0.$$

We see that the evolution is described by the nonlinear-oscillator equation, in which the sign of the "elasticity coefficient" depends on n_L. The nonlinear term in this equation is due to allowance for the change of the phase φ_L, and describes the departure of the quanta n_S to the pump. It determines the feasibility of the stationary state at a given pump intensity, and also the feasibility of the appearance of the modulation effect.

It was already noted above that in[3] there was observed the appearance of oscillations of the intensity of the radiation reflected from the plasma in the case of high-power pumping. For a complete quantitative description of this phenomenon it is necessary to use a system of initial equations more complicated than in the present work. It is of interest, however, to make a numerical estimate of the modulation threshold by means of formula (5.11) and carry out a comparison with the experimental data of[3]. Assume that stimulated Raman scattering by plasmons takes place. The damping of all three waves is determined by the frequency ν_e of the electron-electron collisions, i.e., $\gamma_S = \gamma_q = \gamma_L \equiv \gamma_0 = \nu_e$. The collision frequency is $\nu_e \sim \omega_e\mu \sim \omega_e/r_e^3 n_e$. Here ω_e is the electron plasma frequency, μ is the plasma parameter, r_e is the Debye radius for the electrons, and n_e is the electron concentration. At $n_e = 10^{21}$ cm^{-3} and $T_e = 10^3$ eV $= 10^7$ K we obtain $\omega_e \sim 2\times10^{15}$ sec^{-1}, $r_e \sim 6\times10^{-7}$ cm, and $\mu \sim 4\times10^{-3}$. It follows therefore that $\nu_e \sim 6\times10^{12}$ sec^{-1}. Thus, $\gamma_0 \sim 6\times10^{12}$ sec^{-1}.

The quantity C for a plasma can be determined from formula (2.9) in which $\omega_L \sim \omega_S \sim \omega_q = 2\times10^{15}$ sec^{-1}, $d = \epsilon^{nl}/4\pi$, and $\epsilon^{nl} \lesssim e/m\omega_e^2 r_e \sim 5\times10^{-7}$. At these values we obtain $C \sim 5\times10^6$ cm^3/sec^2.

According to formula (5.11), $n_{osc} \approx 6\gamma_0^2/C \sim 4\cdot10^{19}$ cm^{-3}. From this we obtain for the threshold pump flux at which the oscillations set in

$$I_{osc}=\hbar\omega_L v_L n_{osc}\sim2.5\cdot10^{11}\ \mathrm{W/cm^2}$$

We have obtained the lower bound, since the wave number k was replaced in the formula for ϵ^{nl} by $1/r_e$. If $k \sim r_e/3$, then $I_{osc} \sim 2\times10^4$ W/cm^2.

Our numerical results agree with the experimental data of[3]. We note that if the pump pulse duration exceeds the time of flight through the scattering region l/c and the duration of the monotonic section of the establishment of the stationary state $1/\sqrt{2Cn_L}$, then the spectrum of the scattered radiation at $I > I_{osc}$ will contain (in the given-pump-intensity approximation employed in the present paper) three components ω_S and $\omega_S \pm \omega$, where ω is determined by formulas (5.10) and (5.12) or (5.15) and (5.16). This seems to explain the threshold appearance of the three lines in the fine structure of the SRS line in the case of high-power pumping.

Taking into account the modulation of the additional mode n_q, the number of lines in the spectrum can increase to five.

A more general analysis, with allowance for the temporal evolution of the pump mode intensity (see the start of Sec. 5) shows that the pump mode is also subject to a time modulation with a threshold. Therefore, for a sufficiently strong pump, the number of lines of the spectrum of the scattered radiation can increase to seven. An increase in the number of lines of the emission spectrum with increasing pump is observed experimentally.

1)In a more accurate analysis (see the start of Sec. 5), the quantity n_L in (5.1) is replaced by $(\gamma_L^2/\gamma^2)n_L$, and therefore $n_S < n_L$ and $n_q < n_L$.

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Translated by J. G. Adashko
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