Nonequilibrium states in superconducting tunnel structures

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Superconducting S'-S-N and N_1-S-N_2 tunnel structures are considered. By neglecting energy relaxation processes, equations are obtained which describe the evolution of the order parameter Δ in the superconductor S on the passage of current in such systems. The rates of change of the gap ($\delta|\Delta|$) and of the phase ($\delta\arg\Delta$) of the order parameter in the presence of direct, sinusoidal (with frequency $\Omega = \Delta$) and δ-like currents are found. It is established that the gap in S decreases in the presence of a current, irrespective of its directions. The variations of the phase in S lead to the appearance of a potential difference between the superconductor S and a measuring electrode N_{meas} separated from S by a thin insulating layer.

1. INTRODUCTION

In the study of S—N or S—S′ tunnel junctions (the dash denotes an insulating layer) one usually measures the tunnel current, to calculate which, it is assumed that the metals S, N or S, S′ are in a state of thermodynamic equilibrium. Such an assumption is perfectly justified, since the deviations from equilibrium that are induced by the current passed are extremely small. However, comparatively recently direct measurements have been made of the nonequilibrium "potential difference" between pairs and quasi-particles in the superconductor S on passage of a current in the system S′—S—N [1]. The magnitude of the effect has been calculated on the basis of a semi-microscopic theory, invoking a number of assumptions [2,3]. Another equilibrium case was analyzed in the papers of Parmenter [4] and Aronov and Gurevich [5], in which the increase of the gap in a superconductor S that occurs on passage of current in the systems S′—S—S and $\mathrm{Sm_n}$—S—$\mathrm{Sm_p}$ was calculated (here $\mathrm{Sm_{n,p}}$ is a semiconductor of the n- or p-type respectively). In Parmenter's paper [4] the Josephson effect was not taken into account.

Below, on the basis of a microscopic calculation, we shall elucidate how the order parameter Δ in the superconductor S in the structures S′—S—N and N—S—N changes on passage of a current for times t that are short compared with the energy-relaxation time τ_{en}. Neglecting inelastic collisions, as we shall do, is equivalent to using the BCS Hamiltonian [6]. It is impossible to obtain stationary values of Δ by such an approach, and we can only find the rate of change of $\Delta(t)$. However, the advantage of such an approach is that the problem of the evolution of small perturbations $\delta\Delta(t)$ admits an exact solution. An expression for Im $\delta\Delta(t)$ will be found, differing from the corresponding results of Tinkham [2,3], and it will also be shown that the change in the gap ($\delta|\Delta| = \mathrm{Re}\,\delta\Delta(t)$) in the superconductor S is negative irrespective of the direction of the current. In addition, we shall study the change $\delta\Delta(t)$ when an oscillating voltage with frequency $\Omega = \Delta$ and a δ-like voltage are applied to the N—S—N structure.

2. BASIC EQUATIONS

General equations describing the kinetics of superconductors were obtained by Éliashberg [6]. Simpler equations, in which inelastic collisions were neglected, were written out in a previous paper [7]. We shall carry out an analysis of nonstationary effects in tunnel structures in accordance with the general scheme described in [7]. We shall describe the tunneling by the tunneling Hamiltonian

$$H_T=\sum_{\substack{pq\\ \alpha\beta}}(T_{pq}a_{\alpha p}^{+}\tau_{\alpha\beta}b_{\beta q}+\text{c. c.}). \tag{1}$$

Here, as in [7], we have introduced the operators (in Nambu's notation)

$$a_{1p}=a_{\uparrow p},\quad a_{2p}=\overset{+}{a}_{\downarrow,-p},\quad \tau_{\alpha\beta}=(-1)^{\alpha+1}\delta_{\alpha\beta}.$$

We shall consider the Green function in the Keldysh technique:

$$G_{\alpha\beta}^{ik}(t,t':p)=\frac{1}{i}\left\langle T_C\left[a_{\alpha p}(t_i)a_{\beta p}^{+}(t_k')\exp\left(\frac{1}{i}\int_C H_T(\theta)\,d\theta\right)\right]\right\rangle$$

(the contour C runs from $-\infty$ to $+\infty$ and back). We shall expand G in powers of H_T. Then the terms containing odd powers of H_T will give zero on averaging, and the self-energy part acquires the form

$$[\Sigma_{\alpha\beta}^{ik}(t,t';p)]_T=(-1)^{i+k+\alpha+\beta}\sum_q|T_{pq}|^2\overline{G}_{\alpha\beta}^{(0)ik}(t,t';q)$$
$$=(-1)^{i+k+\alpha+\beta}\frac{\nu}{\pi}\int d\xi\,\overline{G}_{\alpha\beta}^{(0)ik}(t,t';\xi), \tag{2}$$

where $\overline{G}$ is the Green function corresponding to the b operators (i.e., the Green function of the metals S′ and N).

Allowance for the BCS interaction

$$H_{\text{BCS}}=\sum_{k,k'}Ua_{1k}^{+}a_{2k}\overset{+}{a}_{2k'}a_{k'1}$$

leads to the result that each diagram will contain a certain number of nonintersecting lines corresponding to the potential U. These lines will link points lying on a solid line (or lines). Summing the diagrams leads to Dyson's equation. Consequently, the total self-energy part consists of the sum of the tunnel self-energy part (2) and the self-energy part corresponding to the BCS interaction. For the latter we obtain in the Keldysh technique the expression

$$[\Sigma_{\alpha\beta}^{ik}(t,t')]_{\text{BCS}}=i(-1)^k\sum_p UG_{\alpha\beta}^{ii}(t,t';p)\delta_{ik}\delta(t-t')[\delta_{\alpha,\beta+1}+\delta_{\alpha+1,\beta}]$$
$$-i(-1)^k\lambda\delta_{ik}\delta(t-t')\int d\xi\,G_{\alpha\beta}^{ii}(t,t;\xi)[\delta_{\alpha,\beta+1}+\delta_{\alpha+1,\beta}], \tag{3}$$

where $\lambda = mp_0|U|/2\pi^2$. Thus, the mass operator for the BCS interaction in the self-consistent field approximation is diagonal in the time indices and off-diagonal in the spin indices.

We consider the spatially-uniform case. We perform a Fourier transformation over the space variables and write the equation for the function $G^K \equiv G_{11}^{12} + G_{11}^{21}$, which follows from Eq. (10) from [7]1):

$$\left(i\frac{\partial}{\partial t'}-\xi\right) G^K(t',t'')=\Sigma_{1\alpha}{}^R(t',\theta)\,G_{\alpha1}{}^K(\theta,t'')-\Omega_{1\alpha}(t',\theta)\,G_{\alpha1}{}^A(\theta,t''). \quad (4)$$

We shall combine Eq. (4) with the complex-conjugate equation with interchanged arguments $t' \rightleftarrows t''$, and make use of the relation $G^K(t', t'') = -[G^K(t'', t')]^*$ ([7], 6). Putting $t' = t''$ in the equation obtained, we finally find

$$i\frac{\partial}{\partial t}G^K(t,t)=2\,\mathrm{Re}\,(\Sigma_{1\alpha}{}^R G_{\alpha1}{}^K-\Omega_{1\alpha}G_{\alpha1}{}^A), \quad (5)$$

where $t = \frac{1}{2}(t' + t'')$. The equation for $F^K \equiv G_{12}^{12} + G_{12}^{21}$ has the form

$$\left(i\frac{\partial}{\partial t'}-\xi\right)F^K(t',t'')=\Sigma_{1\alpha}{}^R G_{\alpha2}{}^K-\Omega_{1\alpha}G_{\alpha2}{}^A. \quad (6)$$

We combine Eq. (6) with the equation with interchanged arguments $t' \rightleftarrows t''$. Taking into account the relation $F^K(t', t'') = F^K(t'', t')$ ([7], 7) and putting $t' = t''$, we find

$$\left(i\frac{\partial}{\partial t}-2\xi\right)F^K(t,t)=2(\Sigma_{1\alpha}{}^R G_{\alpha2}{}^K-\Omega_{1\alpha}G_{\alpha2}{}^A). \quad (6')$$

According to what has been said above, Σ^R consists of a sum of terms corresponding to the BCS interaction and the tunnel effect. Substituting the expression (3) for Σ^R_{BCS} into (5) and (6′) we obtain the required equations

$$i\frac{\partial}{\partial t}G^K=-2\,\mathrm{Re}\,(\Delta F^{K*})+J_G,\quad \left(i\frac{\partial}{\partial t}-2\xi\right)F^K=-2\Delta G^K+J_F, \quad (7)$$

$$\Delta=i\frac{\lambda}{2}\int_{-\omega_D}^{\omega_D}d\xi\,F^K(t,\xi). \quad (8)$$

Here we have made use of the relations ([7], 5–6) $[F^{K*}(t, t) = -F_{21}^K(t, t),\ G_{22}^K(t, t) = -G^K(t, t)]$, the definition of F^R and F^K, and the fact that $F^R(t, t) = F^A(t, t) = 0$. The "sources" J_G and J_F are essentially the right-hand sides of Eqs. (5) and (6), in which Σ^R and Ω must be found by means of the formulas ([7], 11) and expression (2). In the following we shall be interested in effects of first order in ν ($\nu \sim |T_{pq}|^2$), and therefore in the calculation of J_G and J_F we can substitute the equilibrium values of Σ and G into the right-hand sides of (5) and (6), without taking into account the corrections associated with the tunneling.

We shall calculate the sources in the case of the S—N contact. Let the potential of the metal N with respect to S be V(t). Then

$$J_{G,S-N}=2\,\mathrm{Re}\int d\theta\exp\left(i\int_\theta V(\theta')\,d\theta'\right)[\Sigma_{11}{}^R(t-\theta)\,G^K(\theta-t)$$
$$-\Omega_{11}(t-\theta)\,G_{11}{}^A(\theta-t)]\approx 2\,\mathrm{Re}\int\frac{d\omega}{2\pi}[\Sigma_{11}{}^R(\omega+V)\,G_\omega{}^K-\Omega_{11}(V+\omega)\,G_{11}{}^A(\omega)]. \quad (9)$$

Here we have made use of the fact that $\Sigma_{\alpha\beta}$ is diagonal in the N-metal and have assumed that the frequency of variation of the potential is low compared with the characteristic frequency of the variation of the integrand in (9). The equilibrium functions in (9) have the form

$$\Sigma_\omega{}^R=-i\nu,\quad G_\omega{}^R=\frac{\omega+\xi}{(\omega+i\delta)^2-\varepsilon^2},\quad F_\omega{}^R=\frac{\Delta}{(\omega+i\delta)^2-\varepsilon^2},$$
$$G_\omega{}^K=2i\,\mathrm{th}\frac{\omega}{2T}G_\omega{}^{R''},\quad \Omega_\omega=-2i\,\mathrm{th}\frac{\omega}{2T}\Sigma_\omega{}^{R''}, \quad (10)$$
$$G_\omega{}^{R''}\equiv\mathrm{Im}\,G_\omega{}^R.$$

Calculating (9) with the aid of (10), we find

$$J_{G,S-N}=\nu\left[\mathrm{th}\frac{\varepsilon+eV}{2T}+\mathrm{th}\frac{eV-\varepsilon}{2T}+\frac{\xi}{\varepsilon}\left(\mathrm{th}\frac{\varepsilon+eV}{2T}+\mathrm{th}\frac{\varepsilon-eV}{2T}-2\mathrm{th}\frac{\varepsilon}{2T}\right)\right]$$
$$\equiv\eta(V,\varepsilon)+\frac{\xi}{\varepsilon}\rho(V,\varepsilon), \quad (11)$$

where $\epsilon^2 = \xi^2 + \Delta^2$. In an analogous way we determine J_F:

$$J_{F,S-N}=2\int\frac{d\omega}{2\pi}[\Sigma^R_{V+\omega}F_\omega{}^K-\Omega_{\omega+V}F_\omega{}^A]=\frac{\nu\Delta}{\varepsilon}\left[\mathrm{th}\frac{\varepsilon+eV}{2T}\right. \quad (12)$$
$$\left.+\,\mathrm{th}\frac{\varepsilon-eV}{2T}-2\,\mathrm{th}\frac{\varepsilon}{2T}\right]+\frac{2i\nu}{\pi}\Delta\fint\frac{d\omega}{\varepsilon^2-\omega^2}\mathrm{th}\frac{\omega+eV}{2T}\equiv\frac{\Delta}{\varepsilon}\rho(V,\varepsilon)+i\varkappa(\varepsilon,V).$$

We shall consider the S′—S contact. For simplicity we shall assume that S′ does not differ from S, and that there is no potential between the superconductors, i.e., the current across S—S is less than the Josephson critical current. However, in calculating J_{S-S} it is convenient to keep V finite and to let V tend to zero in the final expressions. We have from (5)

$$J_{G,S-S}=2\,\mathrm{Re}\left\{\int d\tau\,e^{ieV\tau}[\Sigma_{11}{}^R(\tau)\,G^K(-\tau)-\Omega_{11}(\tau)\,G^A(-\tau)]\right.$$
$$\left.-\exp\left\{2ie\int^t V\,d\theta\right\}\int d\tau\,e^{-ieV\tau}[\Sigma_{12}{}^R(\tau)\,F^{K*}(-\tau)+\Omega_{12}(\tau)\,F^{A*}(-\tau)]\right\}$$
$$=2\,\mathrm{Re}\left\{\int d\omega[\Sigma^R_{11\omega}G^K_{\omega-V}-\Omega_{11\omega}G^A_{\omega-V}]-e^{i\varphi}[\Sigma^R_{12\omega}F^{K*}_{-\omega-V}+\Omega_{12\omega}F^{A*}_{-\omega-V}]\right\}. \quad (13)$$

Next it is again necessary to express G^K, F^K and Ω in terms of the imaginary parts of G^R, F^R and Σ^R, and take into account that, in the given case, $\Sigma^{R'} \equiv \mathrm{Re}\,\Sigma^R \neq 0$, since it follows from (2) that

$$\Sigma_{\alpha\beta}{}^R=(-1)^{\alpha+\beta}\frac{\nu}{\pi}\int d\xi\,G_{\alpha\beta}{}^R\equiv\frac{\nu}{\pi}(-1)^{\alpha+\beta}\langle G_{\alpha\beta}{}^R\rangle_\xi. \quad (14)$$

Making $V \to 0$, we convince ourselves that the first term in (13) and the coefficient of $\cos\varphi$ vanish. The remaining part gives

$$J_{G,S-S}=-\frac{2}{\pi}\sin\varphi\int d\omega\,\mathrm{th}\frac{\omega}{2T}[\Sigma_{12}^{R'}(\omega-V)F_\omega^{R''}+\Sigma_{12}^{R''}(\omega)F_{\omega+V}^{R'}]_{V\to0}$$
$$\equiv-A(\varepsilon)\sin\varphi. \quad (15)$$

In an analogous way we obtain

$$J_{F,S-S}=-i\xi A(\varepsilon)\,e^{-i\varphi}/\Delta. \quad (16)$$

In the determination of J_F it must be kept in mind that $\Sigma_{11}^{R'}(\omega) = \omega\Sigma_{12}^{R'}(\omega)/\Delta$ is an odd function of ω, and $\Sigma_{11}^{R''}(\omega) = \omega\Sigma_{12}^{R''}(\omega)/\Delta$ is an even function of ω.

Equations (7) and (8), together with the expressions (11), (12) and (15), (16) for the sources, enable us to investigate the variation of the order parameter Δ on passage of a current through the tunnel structure. We shall find the rate of change of the charge in the superconductor S in the tunnel structure S′—S—N:

$$e\dot{N}=2e\sum_p\langle\dot{\hat n}_{p\uparrow}\rangle,$$

which, because of the electrical neutrality, should be equal to zero. We integrate the first equation of (7) over ξ from $-\omega_D$ to ω_D and take into account that the first term on the right vanishes by virtue of (8) and that $iG^K(t, t) = 1 - 2\langle\hat n_{p\uparrow}\rangle$. Then

$$e\dot{N}=\frac{ep_0mL_xL_yL_z}{2\pi^2}[-\langle\eta(V,\varepsilon)\rangle_\xi+\langle A\rangle_\xi\sin\varphi], \quad (17)$$

where L_xL_y is the area of the superconductor S and L_z is its thickness, which is small compared with the energy-relaxation length. The first term in (17) is the current in the S—N system, and the second term is the Josephson current [8].

To estimate the frequency ν in (11) we shall calculate R_{NN}. Putting $\Delta = 0$ in (11) and performing the integration, we obtain, for $eV \ll T$, $(L_x L_y R_{NN})^{-1} = 2e^2 p_0 m L_z \nu/\pi^2$. With $R_{NN}L_x L_y = 10^{-5}$ ohm$\cdot$cm^2 and $L = 10^{-5}$ cm we have $\nu \approx 10^6$ sec^{-1}.

3. CHANGE OF THE ORDER PARAMETER IN THE PRESENCE OF A CURRENT

We shall consider the following problem. Suppose that for $t \le 0$ there is no current through the system S—S—N (i.e., $V = \varphi = 0$) and that the system is in a stationary equilibrium state. For $t \ge 0$ a current begins to flow in the system. We shall determine how the functions G^K, F^K and Δ will change, assuming, as in [7], that the deviations δG^K, δF^K and $\delta\Delta$ from the equilibrium values are small. The equilibrium values of G^K and F^K differ from the values of the functions for bulk superconductors:

$$G_0{}^K(t,t) = -\frac{i}{\pi}\int d\omega \, \mathrm{th}\frac{\omega}{2T} G_\omega^{R\prime} = -i\xi\chi, \quad F_0{}^K(t,t) = -i\Delta\chi, \qquad (18)$$

where $\chi = \epsilon^{-1}\tanh(\epsilon/2T)$. However, this difference is small ($\delta G_0^K \sim \nu G_0 K/T_k \ll G_0^K$) and, inasmuch as we are interested in $\delta\Delta$ in first order in ν, we can substitute the expressions (18) for the stationary values.

We linearize Eqs. (7) and (8) and, as in [7], take the Laplace transform.

Then for the Laplace transforms $f' \equiv (\mathrm{Re}\,\delta F^K)_p$ and $f'' \equiv (\mathrm{Im}\,\delta F^K)_p$ we obtain

$$(p^2+4\varepsilon^2) f' = 4\varepsilon^2\chi\Delta'' + 2\xi\chi p\Delta' + 2\Delta\eta_p + $$
$$+ p\varkappa_p - 2\varepsilon^2\Delta^{-1}A(\sin\varphi)_p + 2\xi p\Delta^{-1}(\sin^2 {}^1/_2\varphi)_p A,$$

$$(p^2+4\varepsilon^2) f'' = 2\xi\chi p\Delta'' - 4\xi^2\chi\Delta' - 2\xi\left(\frac{2\Delta}{p}\eta_p + \varkappa_p\right) - \frac{\Delta}{\varepsilon p}(p^2+4\varepsilon^2)\rho_p$$
$$-A\frac{\xi p}{\Delta}(\sin\varphi)_p - \frac{4\xi^2}{\Delta}A\left(\sin^2\frac{\varphi}{2}\right)_p. \qquad (19)$$

Multiplying f' and f'' by $\lambda/2$ and integrating over ξ, we find the changes in the phase of Δ:

$$p^2\Phi(p)\Delta_p'' = \left\langle \frac{2\Delta\eta_p + p\varkappa_p}{p^2+4\varepsilon^2} \right\rangle_\xi - \frac{2}{\Delta}\left\langle \frac{\varepsilon^2 A(\varepsilon)}{p^2+4\varepsilon^2} \right\rangle_\xi (\sin\varphi)_p \qquad (20)$$

and in the amplitude of Δ:

$$(p^2+4\Delta^2)\Phi(p)\Delta_p' = \frac{\Delta}{p}\left\langle \frac{\rho_p}{\varepsilon} \right\rangle_\xi + \frac{4}{\Delta}\left(\sin^2\frac{\varphi}{2}\right)_p \left\langle \frac{\xi^2 A(\varepsilon)}{p^2+4\varepsilon^2} \right\rangle_\xi, \qquad (21)$$

$$\Phi(p) = \left\langle \frac{\chi}{p^2+4\varepsilon^2} \right\rangle_\xi. \qquad (22)$$

We shall determine now the change in time of $\Delta''(t) \equiv \mathrm{Re}\,\delta\Delta(t)$ and $\Delta''(t) \equiv \mathrm{Im}\,\delta\Delta(t)$ in the cases of direct and alternating currents.

4. DIRECT CURRENT

Suppose that a direct current is switched on adiabatically at $t \ge 0$, so that a constant voltage is established between the S—N contacts in the system S—S—N and a phase difference φ appears between the superconductors. The values of V and φ are related by the electrical-neutrality condition, by virtue of which the right-hand side of (17) should vanish. The characteristic switching-on time τ_{swi} satisfies the inequality $\tau_{en} \gg \tau_{swi} \gg \min(\Delta^{-1}, T^{-1})$. Then the behavior of $\delta\Delta(t)$ for $t \gg \tau_{swi}$ is determined by the poles of $\delta\Delta_p$ at $p = 0$.

The Laplace transforms of the functions appearing in the right-hand sides of the equalities (20) and (21) have, near $p = 0$, the form $\eta_p(V) = \eta(V)/p$, and so on. Taking into account the electrical neutrality condition, from (20) we find

$$\Delta_p'' = (2p^3\Phi(0)\Delta)^{-1}\left[\Delta^2\left\langle \frac{\eta(V,\varepsilon)}{\varepsilon^2} \right\rangle_\xi - \langle \eta(V,\varepsilon) \rangle_\xi\right]. \qquad (23)$$

From (21) we find the change in the gap:

$$\Delta_p' = (4p\Delta\Phi(0))^{-1}\left[\frac{1}{p}\left\langle \frac{\rho(V)}{\varepsilon} \right\rangle_\xi + \frac{\sin^2(\varphi/2)}{\Delta^2}\left\langle \frac{\xi^2 A}{\varepsilon^2} \right\rangle_\xi\right]. \qquad (24)$$

We shall calculate the change in the difference of the electrochemical potentials of the quasi-particles and pairs: $2e\,\delta V(t) = \Delta''(t)/\Delta$. It is precisely this voltage (with a certain factor—see the Appendix) that arises between the measuring electrode N_{mea} and the superconductor S in the system [1]

$$S' - S \begin{matrix} \nearrow N \\ \searrow N_{meas} \end{matrix} \qquad (24')$$

Substituting the expressions (11) and (22) into (23) and performing the inverse Laplace transformation, we obtain

$$\dot{\Delta}''(t)/\Delta = \begin{cases} 2\nu t[\Delta\arccos(\Delta/eV) - ((eV)^2-\Delta^2)^{1/2}]\,\mathrm{sign}\,V, & T \ll \Delta \\ 8\nu t(T/\Delta\pi)[\pi\Delta\,\mathrm{th}(eV/T) - 2eV], & T \gg \Delta. \end{cases} \qquad (25)$$

Thus, $\Delta''(t)$ is an odd function of V, and $\Delta'' \sim V$, if $eV \gg \Delta$ (for $T \ll \Delta$) or $eV \gg T$ (for $\Delta \ll T$). The same conclusion is also drawn in the paper by Tinkham [3]; here, however, the reason for the increase of Δ'' with increasing V is completely different. For large V the contribution to Δ'' is due to the second term in (25), which is associated with the tunneling of pairs compensating the current of quasi-particles from the N-metal. If, however, we confine ourselves to the current of quasi-particles from the N-metal, as is done in [3], the rate of growth of $\Delta''(t)$ saturates at large V. Using the value determined in [2] for the time required to establish a stationary state, and an estimate of the magnitude of ν, we find that $\delta V \approx 10^{-3}(T/\Delta)V$ for $\Delta \ll T \ll eV$.

As can be seen from (11) and (24), the change in the gap does not depend on the direction of the current (i.e., on the signs of V and φ) and is always negative. Indeed, from the definition of ρ it follows that

$$\left\langle \frac{\rho}{\varepsilon} \right\rangle_\xi = -\nu\,\mathrm{sh}^2\frac{eV}{2T}\left\langle \frac{1}{\varepsilon}\mathrm{th}\frac{\varepsilon}{2T}\mathrm{ch}^{-1}\frac{\varepsilon+eV}{2T}\mathrm{ch}^{-1}\frac{\varepsilon-EV}{2T} \right\rangle_\xi.$$

We shall calculate the second term in (24). We substitute the expressions (10) and (14) into (15) and carry out the integration over ξ. We find

$$\left\langle \frac{\xi^2 A}{\varepsilon^2} \right\rangle_\xi = -4\nu\Delta^3\int_\Delta^\infty d\omega \frac{\mathrm{th}(\omega/2T)}{\omega^2(\omega^2-\Delta^2)^{1/2}}.$$

Thus, both terms in (24) are negative. It can be seen also that the quasi-particle current leads to a decrease in the gap that is proportional to the time t, whereas the pair current leads to a decrease in the gap that is constant in time. The rate of decrease of the gap for $T \ll \Delta$ equals

$$\dot{\Delta}'/\Delta = -\nu\,\mathrm{arc\,ch}\,(eV/\Delta).$$

5. ALTERNATING CURRENT

In the case of an alternating current there appears at the S—S contact a voltage that we did not take into account in deriving the sources J_{S-S}. Therefore, we shall confine ourselves here to treating the system N_1—S—N_2. Then the sources in (7) consist of two terms: J_{S-N_1} and

J_{S-N_2}. We shall assume for definiteness that the temperature is close to the critical temperature. Then, as can be seen from (9) and (11), the potential can be regarded as slowly varying if the frequency of variation of the potential Ω satisfies the condition $\Omega \ll T$.

We shall analyze first the case when the voltage varies sinusoidally:

$$V=V_0 \sin \Omega t. \quad (26)$$

From (22) we find an expression for $\Phi(p)$ ($\Delta \ll T$):

$$\Phi(p)=\pi/4T(p^2+4\Delta^2)^{1/2}. \quad (27)$$

Since the Laplace transform of V(t) (26) has a pole at $p = \pm i\Omega$, a distinctive resonance[2)] arises, as can be seen from (21), if the frequency Ω is close to Δ. The difference from an ordinary resonance lies in the fact that at $p = \pm 2i\Delta$ the left-hand side of (21) has a branch point rather than a pole.

We shall find the variation of the gap in time under the action of an alternating voltage (26), with frequency $\Omega = \Delta$, on the N_1-S-N_2 contact. We shall assume that $eV_0 \ll T$. Then, for $\Delta \ll T$, we obtain

$$\left\langle\frac{\rho_2}{\varepsilon}\right\rangle_\xi=-\nu_1\left(\frac{eV_{01}}{T}\right)^2 a\frac{\Delta^2}{p(p^2+4\Delta^2)},\qquad a=\int_0^\infty\frac{dx\,\text{th}\,x}{x\,\text{ch}^2 x}.$$

We perform the inverse Laplace transformation of Δ'_p:

$$\Delta'(t)=-\frac{\nu_1 a}{\pi}\left(\frac{eV_{01}}{2\Delta}\right)^2\left(\frac{\Delta}{T}\right)\frac{1}{2\pi i}\int_{-i\infty+|\delta|}^{i\infty+|\delta|}dp\frac{e^{2\Delta tp}}{(p^2+1)^{1/2}p^2}.$$

We transform the integral I as follows:

$$I=\int\frac{e^{2\Delta tp}}{(p^2+1)^{1/2}}\left[\frac{1}{p^2}-\frac{1}{p^2+1}\right]$$
$$=\int\frac{e^{2\Delta tp}\,dp}{p^2(p^2+1)^{1/2}}-2\Delta t\int\frac{e^{2\Delta tp}\,dp}{p(p^2+1)^{1/2}}+\int\frac{e^{2\Delta tp}\,dp}{p^2(p^2+1)^{1/2}}.$$

Here we have integrated the second term in the brackets by parts. For $t \to \infty$, the principal contribution to I will be given by going round the pole $p = 0$ and round the branch points $p = \pm 2i$ (cf. [7]) in the second integral. Carrying out the corresponding calculations, we find

$$\Delta'(t)=-\frac{4a}{\pi}\nu_1 t\frac{\Delta}{T}\left(\frac{eV_{01}}{2\Delta}\right)^2\left[1+\frac{\Gamma(1/2)}{\pi(4\Delta t)^{1/2}}\cos\left(2\Delta t+\frac{\pi}{4}\right)\right]. \quad (28)$$

Thus, the amplitude of the oscillations of the gap increases with time like $t^{1/2}$, but the mean value of $\Delta'(t)$ decreases like t^{-1}.

The formula (28) determines the evolution of the gap when tunneling of quasi-particles to only one electrode N_1 is taken into account. In the system N_1-S-N_2 the variation of $\Delta'(t)$ will be determined by a sum of two terms, differing in the parameters $\nu_{1,2}$ and $V_{01,2}$. These parameters are related to each other by the electrical-neutrality condition. For $\Delta \ll T$ this relationship has the simple form $\nu_1 V_{01} = \nu_2 V_{02}$.

As follows from (20), the phase of the order parameter displays no such resonance behavior. If $\Omega = 2\Delta$, the amplitude of the oscillations of $\Delta''(t)$ will damp with time like $t^{-1/2}$.

Finally, we shall investigate the evolution of $\delta\Delta(t)$ in the case when a current pulse is passed through the system N_1-S-N_2. If the duration τ_{pul} of the pulse satisfies the condition $1/T \ll \tau_{pul} \ll 1/\Delta$, it can be assumed that the potential applied has a δ-like form. Such a problem was solved in general form in [7]. It is not difficult to see that the only singular points of the function $\Delta''(p)$ are first- and second-order poles at the point $p = 0$, and therefore $\Delta''(t)$ has a constant part and a part increasing linearly with time [7]. The function $\Delta'(p)$ (21) has, beside the first-order pole at $p = 0$, a branch point of the type $(p^2 + 4\Delta^2)^{-1/2}$, and therefore the gap $\Delta'(t)$ approaches a constant value, while experiencing oscillations that are damped in a power-law fashion. It is interesting to note that the function $f(\epsilon, t)$ is not damped, but oscillates with time with frequency 2ϵ. Thus, the situation here is analogous to that which is realized in a plasma, in which the perturbation of the distribution function oscillates in time without damping while the electric field, determined by an integral of the distribution function over the velocities, oscillates and is damped [10].

6. CONCLUSION

Thus, when quasi-particles are injected into or extracted from a superconductor a nonequilibrium phase of the order parameter appears and the gap changes. The magnitude of the change in the gap is fairly small. It is determined by the relationship between the "frequency" ν of the tunneling and the inverse energy relaxation time τ_{en}^{-1} and, for $\nu \approx 10^6$ sec^{-1} and $\tau_{en}^{-1} \approx 10^9$ sec^{-1}, is equal to $\delta\Delta'/\Delta = 10^{-3}\ln(eV/\Delta)$ ($eV \gg \Delta$). The gap will also be changed by virtue of the directed motion of the Cooper pairs, which we have not taken into account. However, the gap change $\delta\Delta_{GL}$ induced by the current passing through the film will be less than the gap change $\delta\Delta_T$ induced by direct tunneling. Indeed, $\delta\Delta_{GL}/\Delta = j^2/j_{GL}^2$ (j_{GL} is the critical current for the film,

$$j=e(p_0 m)\nu(eV/T)\Delta L_z f(\Delta, T),$$

L_z is the film thickness, and f is a dimensionless function of order unity). Taking $\delta\Delta_T/\Delta \approx \nu\tau_{en}$ and assuming that $\Delta \approx T$ and $L_z \approx p_0/m\Delta$, we obtain $\delta\Delta_{GL}/\delta\Delta_T \approx (\nu/\tau_{en}\Delta^2) \ll 1$.

APPENDIX

We shall consider the system (24′). A current flows across the contacts $S'-S-N$, so that the order parameter Δ changes in the superconductor S, and there is no current across the contacts in $S-N_{meas}$. We shall determine what voltage V_{meas} is established between the contacts in $S-N_{meas}$. We average the first equation (7) over ξ. Then, substituting the equilibrium values (10) in place of Σ^R and Ω, we obtain

$$\langle\eta(V_{meas})\rangle_\xi=2\nu\,\text{Re}\,i\int\frac{d\omega}{2\pi}\left[\langle\delta G_\omega{}^K\rangle_\xi+2\,\text{th}\frac{\omega}{2T}\langle\delta G_\omega{}^A\rangle_\xi\right],$$
$$\delta G_\omega{}^K=\int d(t'-t'')e^{i\omega(t'-t'')}\delta G^K(t',t''). \quad (A.1)$$

Here $\delta G^K(t', t'')$ is the nonequilibrium correction to the Green function, induced by the current passed. We differentiate the equality (A.1) with respect to the time and take into account that the first term in the right-hand side vanishes by virtue of the electrical-neutrality condition already used. We obtain

$$\dot V_{meas}\frac{d\langle\eta(V_{meas})\rangle_\xi}{dV_{meas}}=-4\nu\int\frac{d\omega}{2\pi}\,\text{th}\frac{\omega}{2T}\,\text{Im}\langle\delta\dot G_\omega{}^A\rangle_\xi. \quad (A.2)$$

We shall find $\langle\delta\dot G_\omega^A\rangle_\xi$. The equations from $G^A(t', t'')$ and $F^A(t', t'')$ have the form

$$\left(i\frac{\partial}{\partial t'}-\xi\right)G^A(t',t'')=\delta(t'-t'')+\Delta(t')F^{A+}(t',t''),$$
$$\left(i\frac{\partial}{\partial t'}-\xi\right)F^A(t',t'')=-\Delta(t')G^{A+}(t',t''). \quad (A.3)$$

Writing the analogous equations for $G^R(t', t'')$ and $F^R(t', t'')$ and using the relations $G^{A*}(t'', t')$ and $G^R(t', t'')$, $F^A(t', t'') = F^R(t'', t')$, we obtain the equation

$$-\left(i\frac{\partial}{\partial t''}+\xi\right)G^A(t',t'')=\delta(t'-t'')+\Delta^*(t'')F^A(t',t''),$$
$$\left(i\frac{\partial}{\partial t''}-\xi\right)F^A(t',t'')=-\Delta(t'')G^A(t',t''). \quad \text{(A.4)}$$

We subtract Eq. (A.4) from (A.3) and take the Fourier transform over the difference-variable $\tau = t' - t''$. Then

$$i\dot{G}_\omega^A=\left(\Delta+\frac{\dot\Delta}{2i}\frac{\partial}{\partial\omega}-\frac{\ddot\Delta}{8}\frac{\partial^2}{\partial\omega^2}\right)F_{-\omega}^{A*}-\left(\Delta^*-\frac{\dot\Delta^*}{2i}\frac{\partial}{\partial\omega}-\frac{\ddot\Delta^*}{8}\frac{\partial^2}{\partial\omega^2}\right)F_\omega^A,$$
$$2\omega F_\omega^A=\left(\Delta-\frac{\dot\Delta}{2i}\frac{\partial}{\partial\omega}-\frac{\ddot\Delta}{8}\frac{\partial^2}{\partial\omega^2}\right)G_\omega^A-\left(\Delta+\frac{\dot\Delta}{2i}\frac{\partial}{\partial\omega}-\frac{\ddot\Delta}{8}\frac{\partial^2}{\partial\omega^2}\right)G_{-\omega}^{A*}. \quad \text{(A.5)}$$

Here we have expanded $\Delta(t)' = \Delta(t + \frac{1}{2}\tau) = \Delta(t) + \frac{1}{2}\tau\dot\Delta + \tau^2\ddot\Delta/8$. We then linearize Eqs. (A.5) and determine the connection between δG_ω^A, δF_ω^A and the deviations of the order parameter ($\Delta'(t)$ and $\Delta''(t)$) from its equilibrium value. We obtain

$$\langle\delta\dot{G}_\omega^A\rangle_\xi=\frac{\ddot\Delta''(t)}{2\omega}\frac{\partial}{\partial\omega}\langle F_\omega^{A_0}\rangle_\xi-\dot\Delta'(t)\frac{\partial}{\partial\omega}\langle F_\omega^{A_0}\rangle_\xi. \quad \text{(A.6)}$$

We substitute (A.6) into (A.2) and take into account that, according to (10),

$$\operatorname{Im}\langle F_\omega^{A_0}\rangle_\xi=\frac{\pi\Delta\operatorname{sign}\omega}{2(\omega^2-\Delta^2)^{1/2}}\theta(|\omega|-\Delta)$$

is an odd function of ω. Then

$$\dot V_{\text{meas}}=\frac{d}{dV_{\text{meas}}}\langle\eta(V_{\text{meas}})\rangle=-\nu\ddot\Delta''(t)\Delta\int_\Delta^\infty d\omega\frac{\operatorname{th}(\omega/2T)}{\omega}\frac{\partial}{\partial\omega}\frac{1}{(\omega^2-\Delta^2)^{1/2}} \quad \text{(A.7)}$$

By expressing the left-hand side of (A.7) in terms of the differential conductivity $\sigma'_{NS} = dI/dV$ and the conductivity σ_{NN} and integrating by parts in the right-hand side in (A.7), we find, finally,

$$\dot V_{\text{meas}}=\frac{\sigma_{NN}}{4\sigma'_{NS}}\Delta\left(\int_\Delta^\infty\frac{d\omega}{(\omega^2-\Delta^2)^{1/2}}\frac{\partial}{\partial\omega}\frac{\operatorname{th}(\omega/2T)}{\omega}\right)\ddot\Delta''(t).$$

1)We denote this equation by the symbol ([7], 10). The function G^K was introduced by L. V. Keldysh.

2)This kind of resonance behavior of the gap under the action of radiation on the superconductor was considered by Ivlev [9].

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