Lens effect due to the pressure of light on the surface of a transparent dielectric

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The dynamics and properties of a lens produced as a result of bending of a dielectric surface are discussed in connection with the experimental observation of surface self-focusing of a laser beam due to the pressure of light. The force of a light beam on a dielectric plate is considered with absorption taken into account.

1. INTRODUCTION

The use of laser sources of light makes it possible to observe directly effects due to radiation pressure. Of particular interest is the action of a laser beam, which causes the free surface of a transparent liquid, as shown by the authors earlier[1], to bend and form a nonstationary lens, which in turn changes in the reflected and transmitted light beams. The surface is bent in the direction of the medium with the lower optical density, forming generally speaking a converging lens (see Sec. 3). Thus, in the case of surface rather than volume self-focusing, a real lens is produced rather than a virtual lens, and its parameters depend on the distribution of the intensity over the cross section of the beam.

The lens effect due to light pressure was observed in a recent experiment by Ashkin and Dziedzic[2] 1). In this experiment they used the focused beam of the second harmonic of a pulsed neodumium laser ($\lambda = 0.53\ \mu$) with pulse duration 60 nsec at a power 1–4 kW at the maximum. With the beam focused on a water surface into a spot of 4.2 μ diameter, additional focusing of the transmitted light was observed (as measured in different sections of the beam). A time resolution of 10 nsec has made it possible to observe the time variation of the focal length of the lens, and a rather strong lens effect was observed, namely, the surface was lifted towards the beam to a height on the order of a micron. The focal length of the lens reached values $f \sim 10^{-2}$ cm at the instant of the maximum pulse power, and the lens developed completely within times on the order of 400 nsec, corresponding to a focal distance $f \sim 2.5 \times 10^{-3}$ cm and a 2.5 μ lift of the liquid. Thermal and volume nonlinear effects were negligibly small under the experimental conditions. Ashkin and Dziedzic observed also intense surface scattering. In all probability, this was the first observation of stimulated scattering by capillary waves[4,5], although this question was not specially discussed in [2]. The threshold of the stimulated scattering was exceeded in the experiment for both small-angle and large-angle scattering [6].

In connection with the experimental observation of surface self-focusing, it is of interest to consider in greater detail than in [1] the influence of light pressure on the motion of the surface of the liquid. We note that the direction of the bend is connected by the authors of a number of papers (see [2] for references) with the choice of the expression for the energy-momentum tensor of the field in the medium (the Abraham or Minkowski tensor [7]). As will be shown below, (see also [1]), the direction of the radiation force is determined only by the density of the momentum flux through the surface, and does not depend on the space-time components of the energy-momentum tensor.

We consider also the action of a light beam on a dielectric plate, with absorption taken into account. Allowance for the second boundary leads to the result that the total radiation force acting on the body is directed along the beam.

2. FORMATION OF A LENS ON A LIQUID SURFACE BY LIGHT PRESSURE

The surface lift $\zeta(\mathbf{r}, t)$, which determines the shape of the lens, will be obtained from the hydrodynamic equations averaged over the period of the field and linearized (for small surface slopes)

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla p' + \rho \mathbf{g}, \quad \operatorname{div} \mathbf{v} = 0, \tag{2.1}$$

with boundary conditions at $z = \zeta(\mathbf{r}, t)$

$$p'^{\mathrm{II}} - p'^{\mathrm{I}} - \alpha\Delta_\perp\zeta = p_{\mathrm{L}}, \quad v_z = \frac{\partial\zeta}{\partial t}, \quad \Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{2.2}$$

where α is the coefficient of surface tension and $\mathbf{g}$ is the acceleration due to gravity.

The light pressure[2)]

$$p_{\mathrm{L}}(\mathbf{r}, t) = (\overline{\Pi^{\mathrm{I}}_{nn}} - \overline{\Pi^{\mathrm{II}}_{nn}})_{z=\zeta} \tag{2.3}$$

is determined by the jump of the normal component of the Maxwell stress tensor, averaged over the period, the electric part of which is equal to

$$\overline{\Pi}_{ik} = -\frac{\varepsilon}{4\pi}\left(\overline{E_i E_k} - \frac{\overline{E^2}}{2}\delta_{ik}\right), \tag{2.4}$$

this expression being valid also in the presence of dispersion[8]. We see that the force exerted by the field in the case considered here, that of a transparent incompressible liquid, acts (on the average over the period) only on the surface of the medum, and the striction part simply renormalizes the pressure

$$p' = p - \rho\frac{\partial\varepsilon}{\partial\rho}\frac{\overline{E^2}}{8\pi}.$$

It is important in what follows that the light pressure is directed towards the medium with the lower optical density[1]. This circumstance, which generally speaking is not obvious, will be discussed in Sec. 5.

Let us dwell first on the shape of the stationary bend $\zeta(\mathbf{r})$ produced by a bounded light beam. The capillary-gravitational forces balance in this case the light-pressure force, a fact expressed by the equality

$$(\rho_2 - \rho_1) g\zeta - \alpha\Delta_\perp\zeta = p_{\mathrm{L}}(\mathbf{r}). \tag{2.5}$$

This equation, which we shall find expedient to rewrite by introducing the capillary constant k_0 in the form

$$\Delta_\perp\zeta - k_0^2\zeta = -p_L/\alpha, \quad k_0^2 = (\rho_2-\rho_1)g/\alpha, \tag{2.5'}$$

has as its Green's function, which is regular at $r \to \infty$, the modified Bessel function $K_0(k_0 r)$[9], in terms of which we write down the solution

$$\zeta(\mathbf{r}) = \frac{1}{2\pi\alpha}\int d\mathbf{r}'\, K_0(k_0|\mathbf{r}-\mathbf{r}'|)\,p_L(\mathbf{r}'). \tag{2.6}$$

In the case of a small beam radius, $R \ll k_0^{-1}$, and small distances, $r \ll k_0^{-1}$, where only the capillary forces play any role, Eq. (2.5) reduces to the Poisson equation

$$\Delta_\perp\zeta = -p_L/\alpha \quad (k_0 r,\ k_0 R \ll 1), \tag{2.7}$$

and the solution is expressed in terms of the logarithmic potential

$$\zeta(\mathbf{r}) = -(2\pi\alpha)^{-1}\int d\mathbf{r}'\,\ln|\mathbf{r}-\mathbf{r}'|\,p_L(\mathbf{r}'). \tag{2.8}$$

We note that (2.8) can be obtained from (2.6) by separating the logarithmic singularity of $K_0(k_0 r)$ at zero. In the case of a broad beam $R \gg k_0^{-1}$, the capillary forces are inessential and

$$\zeta(\mathbf{r}) = p_L(\mathbf{r})/(\rho_2-\rho_1)g, \quad k_0 R \gg 1. \tag{2.9}$$

This asymptotic form also follows from (2.6) and corresponds to a slow variation of p_L over distances $\sim k_0^{-1}$.

In the one-dimensional case, when the light pressure depends only on x, we obtain for the bend

$$\zeta(x) = (2\alpha k_0)^{-1}\int_{-\infty}^{\infty} dx'\,\exp[-k_0|x-x'|]\,p_L(x'). \tag{2.10}$$

The solution of the "inverse problem," that of finding the $p_L(\mathbf{r})$ that results in a given profile $\zeta(\mathbf{r})$, is given directly by (2.5). In particular, an ideal (parabolic) lens in the central part of the illuminated region is produced under axial-symmetry conditions when

$$p_L(r) = (\rho_2-\rho_1)g\zeta(r) - 2\alpha\varepsilon^{1/2}(\varepsilon^{1/2}-1)^{-1}f^{-1}, \\ \zeta(r) = \zeta(0) + \varepsilon^{1/2}(\varepsilon^{1/2}-1)^{-1}r^2/2f, \tag{2.11}$$

f is the focal length of the lens:

$$f^{-1} = \frac{\varepsilon^{1/2}-1}{\varepsilon^{1/2}}\left.\frac{\partial^2\zeta}{\partial r^2}\right|_{r=0}. \tag{2.12}$$

For a small lens (in the capillary region), such an ideal lens is produced by a pressure that is constant over the cross section (!), and at $R \gg k_0^{-1}$ the pressure p_0 itself should vary in accordance with a quadratic law.

3. MOTION OF SURFACE UNDER THE INFLUENCE OF A LIGHT PULSE

From (2.1) and (2.2), with allowance for the small viscosity η of the medium, we obtain (see Appendix I) an equation for the bend $\zeta(\mathbf{r}, t)$ in the form

$$\int\frac{d\mathbf{r}'}{2\pi}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\left[(\rho_1+\rho_2)\frac{\partial^2\zeta}{\partial t^2} - 4(\eta_1+\eta_2)\Delta_\perp\frac{\partial\zeta}{\partial t}\right] + (\rho_2-\rho_1)g\zeta - \alpha\Delta_\perp\zeta = p_L. \tag{3.1}$$

The viscosity was introduced in accordance with the known law of the dispersion of waves on the surface of a low-viscosity liquid[3] $\Omega = \pm\Omega(q) - i\Gamma(q)$, where

$$\Omega(q) = \left[\frac{\alpha}{\rho_2+\rho_1}q^3 + \frac{\rho_2-\rho_1}{\rho_2+\rho_1}gq\right]^{1/2}, \quad \Gamma(q) = 2\frac{\eta_1+\eta_2}{\rho_1+\rho_2}q^2. \tag{3.2}$$

It is convenient to use (3.1) for large times (see Sec. 2), and for short times it is more convenient to use the equivalent equation (cf. [11])

$$(\rho_1+\rho_2)\frac{\partial^2\zeta}{\partial t^2} - 4(\eta_1+\eta_2)\Delta_\perp\frac{\partial\zeta}{\partial t} + \frac{1}{2\pi}\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \\ \times\Delta_{r'}[\alpha\Delta_{r'} - (\rho_2-\rho_1)g]\zeta(\mathbf{r}', t) = -\frac{1}{2\pi}\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\Delta_{r'}p_L(\mathbf{r}', t). \tag{3.1'}$$

Consider the motion of the surface of a liquid initially at rest, under the influence of a light pulse turned on at t = 0. Using the spatial Fourier representation, we easily obtain the solution (3.1) in the form

$$\zeta(\mathbf{r}, t) = \frac{1}{(2\pi)^2(\rho_1+\rho_2)}\int_0^t dt'\int d\mathbf{q}\, e^{i\mathbf{q}\mathbf{r}}q\frac{\sin\Omega(q)(t-t')}{\Omega(q)} \\ \times\exp[-\Gamma(q)(t-t')]\int d\mathbf{r}'\, e^{-i\mathbf{q}\mathbf{r}'}p_L(\mathbf{r}', t'), \quad \zeta(\mathbf{r}, 0) = \frac{\partial\zeta(\mathbf{r}, 0)}{\partial t} = 0. \tag{3.3}$$

This expression describes the shape of the lens and its dynamics. For the one-dimensional case $p_L = p_L(x)$ and for the axially-symmetrical case $p_L = p_L(r, t)$, the shape of the bend (3.3) becomes simpler and is given in Appendix II.

For short times $t \ll \Omega(R^{-1})$, when the restoring action of the elastic forces (gravity and tension) still do not affect the motion of the surface (R is the characteristic dimension of the light beam), it is possible to omit the last term in the left-hand side of (3.1′), and also the viscous term $(\Gamma(R^{-1}) \ll \Omega(R^{-1}) \ll t^{-1})$. In this region, Eq. (3.1′) is similar in form to Newton's law, and the solution is obtained by integrating twice with respect to time:

$$\zeta(\mathbf{r}, t) = -\frac{1}{2\pi(\rho_1+\rho_2)}\int_0^t dt'(t-t')\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\Delta_{r'}p_L(\mathbf{r}', t'). \tag{3.4}$$

The same result can be obtained from the general solution (3.3) by expanding the sine function in terms of small times and returning to the spatial representation. In the important case when the intensity distribution in the beam does not change with time, i.e., $p_L(\mathbf{r}, t) = p_L(\mathbf{r})T(t)$, the shape of the bend for short times also remains constant in time, $\zeta(\mathbf{r}, t) = \zeta(\mathbf{r})\chi(t)$, and is equal to

$$\zeta(\mathbf{r}) = -\frac{1}{2\pi(\rho_1+\rho_2)}\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\Delta_{r'}p_L(\mathbf{r}'), \\ \chi(t) = \int^t dt'\,(t-t')T(t'); \quad t \ll \Omega^{-1}\left(\frac{1}{R}\right) \equiv t_R. \tag{3.5}$$

We see therefore that at large distances the bend decreases like $1/r^3$ if the light pressure decreases in power-law fashion, and decreases exponentially with exponential decrease of the pressure.

In the one-dimensional case, the bend $\zeta(x)$, as follows from (3.5) is the derivative of the Hilbert transform of the light pressure:

$$\zeta(x) = (\pi\rho)^{-1}\frac{\partial}{\partial x}\int_{-\infty}^{\infty} dx'\frac{p_L(x')}{x-x'}, \quad \rho = \rho_1+\rho_2. \tag{3.6}$$

From this we obtain the decrease of the bend over large distances $\zeta(x) \sim 1/x^2$ at a power-law asymptotic form of $p_L(x)$. If the pressure decreases exponentially, then the decrease of the bend is also exponential.

In the axially-symmetrical case $p_L(\mathbf{r}) = p_L(r)$, Eq. (3.5) reduces, after integration over the angles, to a single integral, with a complete elliptic integral as the kernal. For actual calculations, however, it is convenient to use the Fourier representation (cf. (3.3)), from which we obtain after integrating over the angles

$$\zeta(r) = \frac{1}{\rho}\int_0^\infty dq\, q^2 J_0(qr)\int_0^\infty dr'\, r' J_0(qr')\,p_L(r'), \tag{3.7}$$

where $J_0(z)$ is a Bessel function. We see from (3.5) and (3.7) that although $p_L < 0$, for short times the value of $\zeta(r)$, and all the more the curvature $\partial^2\zeta/\partial r^2$, is determined by the contribution of the regions with an appreciable light-pressure gradient. A focusing lens is produced at the center of the beam under the condition

$$\zeta''(0) = -\frac{1}{2\rho}\int_0^\infty dr\, \Delta_\perp^2 p_L(r) > 0. \tag{3.8}$$

We present examples of a focusing and defocusing lens. Thus, if

$$p_L(r) = -p_0(1+r^2/R^2)^{-3/2} \quad (p_0>0),$$

then[1]

$$\zeta(r) = -\frac{p_0}{\rho R}\frac{2-r^2/R^2}{(1+r^2/R^2)^{5/2}}, \quad \zeta''(0) = \frac{12p_0}{\rho R^3} > 0,$$

i.e., a positive lens is produced (Fig. 1a). In the other (one-dimensional) case at $p_L(x) = -p_0(1 + x^4/R^4)^{-1}$ we have

$$\zeta(x) = -\frac{p_0}{2^{1/2}\rho R}\left(1-\frac{x^2}{R^2}\right)\frac{1+4x^2/R^2+x^4/R^4}{(1+x^4/R^4)^2}, \quad \zeta''(0) = \frac{-3\sqrt{2}\,p_0}{\rho R^3} < 0,$$

i.e., the central part of the surface lens is defocusing (Fig. 1b). Further examples are given in Appendix II.

The reconstruction of the distribution of the intensity of the field in the beam from the shape of the bend is effected with the aid of the transformation inverse to (3.5) (see (A.3)):

$$p_L(\mathbf{r}) = \frac{\rho}{2\pi}\int d\mathbf{r}'\frac{\zeta(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{3.9}$$

In axially-symmetrical or one-dimensional cases, Eq. (3.9) reduces to single integrals

$$p_L(r) = \frac{2\rho}{\pi}\left[\frac{1}{r}\int_0^r dr'\, r' K\left(\frac{r'}{r}\right)\zeta(r') + \int_r^\infty dr'\, K\left(\frac{r}{r'}\right)\zeta(r')\right], \tag{3.10}$$

where $K(k)$ is the complete elliptic integral[9], and

$$p_L(x) = -\frac{\rho}{\pi}\int_{-\infty}^\infty dx'\, \ln|x-x'|\,\zeta(x'). \tag{3.11}$$

Let us discuss now the temporal evolution of the lens (Fig. 2). For short times $\Omega(R^{-1})t \ll 1$, for a pulse duration $t \ll \tau$, the surface moves with acceleration, $\zeta(t)$ t^{n+2}, if $p_L(t) \sim t$ (3.5). At $n = 0$, the bend grows quadratically with time[1], corresponding to uniformly accelerated motion of a mass ρR^3 under the influence of a constant force $F \sim p_0 R$, i.e., $\zeta \sim p_0 t^2/\rho R$. For short pulses $\Omega(R^{-1}) \ll 1$, there exists a region $t > \tau$ in which the motion proceeds by inertia $\left(\chi(t) = t\int_0^\infty dt' T(t')\right)$ with with constant velocity, and is determined by an integral characteristic, namely the momentum of the light pressure $\int_0^\infty dt\, p_L(\mathbf{r}, t)$.

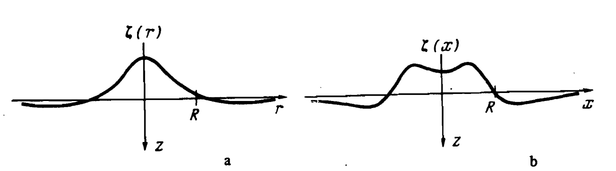

FIG. 1. Formation of a lens by light pressure over short times $t \ll t_R$: a) focusing lens (axially-symmetrical case), b) defocusing lens (one-dimensional case).

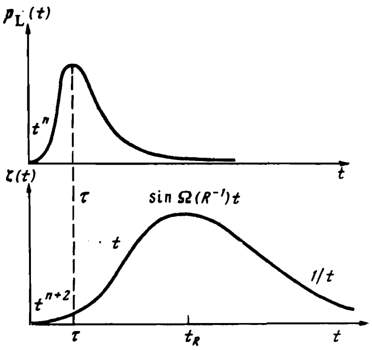

FIG. 2. Schematic dependence of the light pressure and of the bend on the time.

Over times on the order of the surface-oscillation period $\tau \ll t \lesssim t_R \equiv \Omega(R^{-1})$, both the size of the bend and the strength of the lens reach a maximum. In this region, the integrand in (3.3) is a sharp function of q near $\bar{q} \sim 1/R$, since the spatial spectrum of p_L decreases rapidly at $qR > 1$, and the phase volume is small at $qR < 1$. Taking the slowly varying time-dependent factor outside the integral sign, we obtain $\zeta \sim \sin\Omega(\bar{q})t$. The focal length of the lens f is given by

$$f^{-1} = \frac{\varepsilon^{1/2}-1}{\varepsilon^{1/2}}\frac{\partial^2\zeta}{\partial r^2}\Big|_{r=0},$$

and is minimal also at $t \sim t_R$, with the maximum strength of the lens reached before the bend reaches its maximum (owing to the additional factor q^2 in the integrand expression for $\partial^2\zeta/\partial r^2$).

We consider the free motion of the surface for large times: $\Omega(R^{-1})t \ll 1$, $t \gg \tau$. At $\Omega(k_0)t \ll 1$, where k_0 is the capillary constant, the gravitational forces still do not play any role and $\Omega(q) = (\alpha q^3/\rho)^{1/2}$. Calculating the integral (3.3) by the stationary-phase method, we obtain in the two-dimensional case

$$\zeta(r,t) = \frac{\Lambda}{3\pi\alpha t}, \quad \Lambda \equiv \int d\mathbf{r}\int_0^\infty dt\, p_L(\mathbf{r},t), \quad \frac{\rho r^3}{\alpha t^2}, \frac{\rho R^3}{\alpha t^2}, \frac{\rho^2\bar{\nu}^3}{\alpha t} \ll 1, \tag{3.12}$$

where Λ is the momentum of the light-pressure force, and $\bar{\nu} \equiv (\eta_1 + \eta_2)/(\rho_1 + \rho_2)$. Thus, at not too large distances from the center, $r \ll (\alpha t^2/\rho)^{1/3}$, the bend $\zeta < 0$ decreases like $1/t$ and is determined by the integral characteristics of the incident laser pulse. The curvature of the surface decreases like $t^{-7/3}$:

$$\frac{\partial^2\zeta}{\partial r^2} = \frac{\Lambda}{12\pi\alpha t}\Gamma\left(\frac{7}{3}\right)\left(\frac{\rho}{\alpha t^2}\right)^{2/3} < 0, \tag{3.13}$$

and the central part of the lens is defocusing.

In the one-dimensional case, the lift and the curvature decrease more slowly, like $t^{-1/3}$ and $t^{-5/3}$, respectively, and unlike the preceding case we have $\zeta(0,t) < 0$ and $\partial^2\zeta(0,t)/\partial x^2 > 0$, i.e., the lens is focusing:

$$\zeta(0,t) = \frac{\Lambda}{3\pi\alpha t}\left(\frac{\alpha t^2}{\rho}\right)^{1/3}\Gamma\left(\frac{1}{3}\right) \quad \zeta''(0,t) = -\frac{\Lambda}{3\pi\alpha t}\left(\frac{\rho}{\alpha t^2}\right)^{1/3}\Gamma\left(\frac{5}{3}\right) \tag{3.14}$$

Thus, in a low-viscosity liquid, for short pulses $\Omega(R^{-1})\tau \ll 1$, the characteristic time-dependent parameter is $t_R \equiv \Omega^{-1}(R^{-1})$—the period of the oscillations of the surface with spatial scale $\sim R$. The time of evolution of the lens is of the order of t_R ($t_R \gg \tau$), and for longer times the surface flattens out rapidly (see (3.13)) and relaxes somewhat more slowly (see (3.12)). These results conform to the experimentally observed picture of the development of the surface lens[2]. In the experiment of [2], the pulse rise time was $\tau \approx 60$ nsec, and R $= 2.1 \times 10^{-4}$ cm. For water, the lens evolution time is $t_R \approx (\rho R^3/\alpha)^{1/2} \sim 300$ nsec, which is close to the observed value 400 nsec.

4. LIGHT PRESSURE ON THE SURFACE OF A DIELECTRIC

We shall dwell first on the direction of the light-pressure force, which seems unusual at first glance[4)]. As already mentioned, (see also [1]), when a beam is incident from vacuum on the surface of a dielectric, the force is directed against the beam. Actually, as follows from the definition (2.3) of p_L and the boundary conditions for the field $[\mathbf{E}^{II} - \mathbf{E}^{I}] \times \mathbf{n} = \epsilon_2 \mathbf{E}^{II} - \epsilon_1 \mathbf{E}^{I}) \cdot \mathbf{n} = 0$ at $z = \zeta$, the light pressure can be expressed in terms of the transmitted field in the form

$$p_L = -\frac{\varepsilon_2 - \varepsilon_1}{16\pi}[|E_0^{II}|^2 + (\varepsilon - 1)|E_{0n}^{II}|^2], \quad \varepsilon \equiv \frac{\varepsilon_2}{\varepsilon_1} \tag{4.1}$$

($\mathbf{E}_0$ denotes the complex field amplitude $\mathbf{E} = \mathrm{Re}\, \mathbf{E}_0(\mathbf{r}, t) \cdot e^{i(\mathbf{kr} - \omega t)}$) and is consequently directed towards the medium with the lower optical density. Expression (4.1) coincides with the one obtained for the static field, and is responsible for the drawing of the dielectric into the region of maximum intensity.

For greater clarity, we present an independent quantum derivation of expression (4.1). The magnitude and direction of the light-pressure force can be determined by finding the change of the field momentum due to the appearance of the reflected (R) and refracted (T) waves. In terms of quantum physics, it is convenient to regard reflection and refraction as scattering: a reflected photon carrying a momentum $\hbar \mathbf{k}^R$ is produced with a probability proportional to the reflection coefficient, and a transmitted photon with momentum $\hbar \mathbf{k}^T$ is produced with the probability proportional to the transmission coefficient. At equal quantum energy $\hbar\omega$, the photon in the medium with the larger refractive index has a larger momentum $\hbar k = \hbar \epsilon^{1/2} \omega / c$ [13], which is carried away by the photon from the boundary. The light pressure is equal to the change of the normal component of the field momentum, so that[1,14]

$$p_L = \hbar k_z^i j_z^i + \hbar k_z^R j_z^R - \hbar k_z^T j_z^T, \tag{4.2}$$

where the index i pertains to the incident wave, $\mathbf{j} = \mathbf{I}/\hbar\omega$ is the photon flux, and $\mathbf{I} = c\mathbf{E} \times \mathbf{H}/4\pi$ is the Poynting vector. Introducing the energy density $W = (\epsilon E^2 + H^2)/8\pi$, we obtain

$$p_L = \cos^2\theta^i (W^i + W^R) - \cos^2\theta^T W^T, \tag{4.3}$$

where θ^i and θ^T are the incidence and refraction angles. Expressing W^R and W^T in terms of W^i with the aid of the Fresnel coefficients, we verify that (4.3) is identical with (4.1). In particular, for normal incidence ($W^R = W^i \cdot (\epsilon^{1/2} - 1)^2/(\epsilon^{1/2} + 1)^2$, $W^T = 4\epsilon W^i/(\epsilon^{1/2} + 1)^2$)

$$p_L = 2\frac{1 - \varepsilon^{1/2}}{1 + \varepsilon^{1/2}} W^i < 0 \text{ for } \varepsilon > 1. \tag{4.4}$$

We see therefore that in the case of reflection from an ideal dielectric mirror ($\epsilon \to \infty$) the force is directed, as before, towards the beam (although the calculation of the momenta in the incident and reflected beams would appear to indicate a pressure on the dielectric from the outside, just as in the case of a metal). The resolution of this paradox lies in the fact that when calculating the surface force one cannot neglect the refracted beam.

Although $E^{II} \sim \epsilon^{-1/2} \to 0$, the induction $D^{II} \sim \epsilon^{1/2} \to \infty$, so that the energy density remains finite ($W^T \to 4W^i$) and determines, in accordance with (4.3), the sign of the pressure. Of course, we are dealing with the steady state throughout, and the question of the impact forces produced when the field is turned on is not considered here.

According to [2], the direction of the light-pressure force is connected by a number of authors with the choice of the expression for the energy-momentum tensor of the field in the medium (the symmetrical Abraham tensor or the Minkowski tensor, see the reviews [7,15]). Actually, however, as follows already from [1], the direction of the light pressure does not depend on the choice of the expression for the density of the field momentum in the medium ($\mathbf{L} = \mathbf{L}^A \equiv \mathbf{E} \times \mathbf{H}/4\pi c$ according to Abraham, or $\mathbf{L} = \mathbf{L}^M = \mathbf{D} \times \mathbf{B}/4\pi c$ according to Minkowski), and is determined only by the momentum flux density, i.e., by the spatial component of the energy-momentum tensor (the components of the Maxwell stress tensor). Indeed, the equations of motion of the medium, which are not averaged over the period of the field, contain the field momentum density in the form of a derivative with respect to time[16], and the momentum density drops out after averaging. In the volume equation for a homogeneous medium, averaging over the time causes also the divergence of the Maxwellian stress tensor to drop out, for in accordance with the field equations we have

$$\frac{\partial \Pi_{ik}}{\partial x_k} = -\frac{1}{4\pi c}\frac{\partial}{\partial t}[\mathbf{D} \times \mathbf{B}]_i \quad (\varepsilon = \text{const}).$$

Thus, the influence of the field on the motion of the medium reduces to a renormalization of the striction pressure [(2.1), (2.2)] and to the radiation pressure on the interface. The surface term in the averaged equations stems from the contribution of the transition layer, where $\nabla\epsilon \to \infty$, by virtue of the equation

$$\overline{\frac{\partial \Pi_{ik}}{\partial x_k}} = \frac{\overline{E^2}}{8\pi} \nabla_i \varepsilon,$$

which leads to the expression obtained above for the surface force.

We note that the direction of the force acting on the boundary of magnetic media is determined not only by the refractive index, but also by the impedance

$$p_L = -\frac{\varepsilon_2 - \varepsilon_1}{16\pi}[|E_0^{II}|^2 + (\varepsilon - 1)|E_{0n}^{II}|^2] - \frac{\mu_2 - \mu_1}{16\pi}[|H_0^{II}|^2 + (\mu - 1)|H_{0n}^{II}|^2], \tag{4.1'}$$

where $\mu \equiv \mu_2/\mu_1$. In particular, for normal incidence we have

$$p_L = -\frac{2Z}{(Z+1)^2}[(\varepsilon - 1)Z + \mu - 1]W^i, \tag{4.5}$$

$Z \equiv (\mu/\epsilon)^{1/2}$ is the relative impedance. We note that the expression for p_L given in [15] and different from (4.5) (Appendix 2) was obtained by applying the connection between E and H in a plane wave to total fields, for which this connection does not hold because of the presence of the reflected wave.

5. PRESSURE OF LIGHT ON A PLATE

We consider first a case when there is no absorption and the media are nonmagnetic. The light pressure acting on the interface between media I and II (p_L^{12}) and II and III (p_L^{23}) is determined by the jump of the Maxwellian stress tensor (2.3) and can be reduced, by using the reflection and transmission coefficients[17] (see (5.5)) to the form (normal incidence on medium I, $\epsilon_3 = \epsilon_1$)

$$p_L^{12} = -2(\varepsilon-1)W^i(\varepsilon\cos^2\varphi+\sin^2\varphi)/[(\varepsilon-1)^2\sin^2\varphi+4\varepsilon], \quad (5.1)$$

$$p_L^{23} = 2\varepsilon(\varepsilon-1)W^i/[(\varepsilon-1)^2\sin^2\varphi+4\varepsilon], \quad (5.2)$$

where $\varphi = (\epsilon_2\mu_2)^{1/2}(\omega/c)d$ is the phase shift in the plate, d is its thickness, $\epsilon = \epsilon_2/\epsilon_1$, and $\mu = 1$. Each boundary is acted upon by a force in the direction of the medium with the smaller optical density. The resultant force f_L, according to (5.1) and (5.2), is equal to

$$f_L = p_L^{12} + p_p^{23} = 2W^i(\varepsilon-1)^2\sin^2\varphi/[(\varepsilon-1)^2\sin^2\varphi+4\varepsilon] \geqslant 0 \quad (5.3)$$

and is always directed along the light beam, this being in fact a consequence of energy conservation and valid even when absorption is taken into account (see below).

As seen from (5.1)–(5.3), the forces acting on the plate are oscillating functions of the thickness d with a period $\Delta d = \pi c/(\epsilon_2\mu_2)^{1/2}\omega = \lambda/2$, where λ is the wavelength of the light in the plate. If the thickness subtends an integer number of half waves, $d = m\lambda/2$, then $|p_L^{12}|$ and $|p_L^{23}|$ reach the maximum value $|\epsilon - 1|W^i/2$, and the total force is then minimal and equal to zero. On the other hand, if $d = (2m+1)\lambda/4$ (half-integer number of half-waves), then the resultant force is maximal:

$$f_L = 2W^i\left(\frac{\varepsilon-1}{\varepsilon+1}\right)^2.$$

We now consider a plate with absorption. Since we do not know the stress tensor in an absorbing medium (there is apparently no general expression for it, and the explicit form of the tensor depends on the employed model of the medium, see. e.g., [18] for the case of a plasma), we cannot obtain in the general case an expression for the forces acting on each surface. To find the total force, however, we can use a more general approach that is applicable to bounded bodies. The total force is equal to the change of the momentum of the field outside the body and is calculated from the stress tensor outside the body, and takes the form[5)] (4.3)

$$f_L = \cos^2\theta^i(W^i+W^R) - \cos^2\theta^T W^T,$$

where, however, the superscript T pertains now to the beam that has passed through the plate. We present here the expression for f_L in the case of normal incidence

$$f_L = 2W^i[(|Z|^2-1)^2\sin^2\varphi' + (|Z|^2+1)^2\operatorname{sh}^2\varphi'' + Z'(|Z|^2+1)\operatorname{sh}2\varphi'' + Z''(|Z|^2-1)\sin 2\varphi']/|D|^2, \quad (5.4)$$

which is obtained by using the explicit forms of the reflection coefficient $R = E^R/E^i$ and the transmission coefficient $T = E^T/E^i$ of the plate[17]:

$$R = (Z^2-1)\sin\varphi/D, \quad T = 2iZ/D, \quad D = (Z^2+1)\sin\varphi + 2iZ\cos\varphi, \quad (5.5)$$

where $\varphi = \varphi' + i\varphi'' = nkd$, $n = n' + in''$ is the relative refractive index, and $k = (\epsilon_1\mu_1)^{1/2}\omega/c$ is the wave number of the light outside the plate. In the absence of absorption, expression (5.4) with $\mu = 1$ reduces to (5.3). Absorption causes the amplitude of the oscillations of the total force to decrease with increasing plate thickness.

Let us consider the limiting case of strong absorption over the thickness of the plate, $\varphi'' \gg 1$. In this case $f_L = 2W^i(|Z|^2 + 1)/|Z + 1|^2$ and coincides with the expression obtained when account is taken of only the incident and reflected waves. We note that taking the limit as $\varphi'' \to \infty$ in the case of finite absorption corresponds to a transition from a plate to a half-space ($d \to \infty$). Then f_L, of course, does not coincide with the light pressure, inasmuch as f_L takes into account not only the surface force but also the volume force. It is curious that a dielectric plate with $\epsilon' \to \infty$ and a metallic plate with $\epsilon'' \to \infty$ are acted upon by equal total forces $f_L = 2W^i$. Exactly the same force acts also on a thick plate of low optical density $|\epsilon_2/\epsilon_1| \ll 1$, $\varphi'' \gg 1$.

APPENDIX I

We present a derivation of an equation for the bend $\zeta(\mathbf{r}, t)$ (2.1). Introducing the potential $v = \nabla\varphi$ and eliminating with the aid of (1.3) the pressure

$$p' = -\rho\frac{\partial\varphi}{\partial t} + \rho g z,$$

we obtain in the approximation linear in ζ

$$\Delta\varphi = 0, \quad \left.\frac{\partial\varphi}{\partial z}\right|_{z=0} = \frac{\partial\zeta}{\partial t},$$
$$\left(-\rho_2\frac{\partial\varphi_2}{\partial t} + \rho_1\frac{\partial\varphi_1}{\partial t}\right)_{z=0} + (\rho_2-\rho_1)g\zeta - \alpha\Delta_\perp\zeta = p_L(\mathbf{r}, t). \quad (A.1)$$

From the first two equations we obtain the potential $\varphi_{1,2}$ with the aid of Green's function of the Laplace equation for the Neumann boundary-value problem in a half-space $z \leqslant 0$:

$$\varphi_{1,2}(\mathbf{r}) = \pm\frac{1}{2\pi}\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial\zeta(\mathbf{r}', t)}{\partial t}. \quad (A.2)$$

Eliminating with the aid of (A.2) the potential from the last equation of (A.1), we get

$$\frac{\rho_1+\rho_2}{2\pi}\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial^2\zeta(\mathbf{r}', t)}{\partial t^2} - [\alpha\Delta_\perp - (\rho_2-\rho_1)g]\zeta(\mathbf{r}, t) = p_L(\mathbf{r}, t). \quad (A.3)$$

Applying to (A.3) the operator

$$\int\frac{d\mathbf{r}''}{|\mathbf{r}-\mathbf{r}''|}\Delta_{r''}$$

and bearing in mind that

$$\int\frac{d\mathbf{r}''}{|\mathbf{r}-\mathbf{r}''|}\Delta_{r''}\frac{1}{|\mathbf{r}'-\mathbf{r}''|} = -(2\pi)^2\delta(\mathbf{r}-\mathbf{r}'), \quad (A.4)$$

we obtain (with allowance for the small damping of the surface oscillations) an equation of motion of the surface in a form similar to "Newton's law" (3.1′) (cf. also [19]).

APPENDIX II

We present here formulas for the bend of the surface $\zeta(\mathbf{r})$ in an axially-symmetrical ($p_L(\mathbf{r}, t) = p_L(r)T(t)$) and the one-dimensional ($p_L(\mathbf{r}, t) = p_L(x)T(t)$) cases for short times (3.5) for model distributions of the pressure. We introduce for convenience the dimensionless functions $p_L(r) = -p_0 f(r/R)$, $f(0) = 1$, $\zeta(r) = -(p_0/\rho R)\Phi(r/R)$, and analogous functions for the one-dimensional case.

A. One-Dimensional Case

1. $f(u) = (1 + u^2)^{-n-1}$, $n \geq 0$ is an integer,

$$\Phi(u) = 2\frac{(-1)^{n+1}}{n!}\left[\frac{d^n}{dp^n}\frac{1}{(1+p)^{n+1}}\frac{u^2-p^2}{(u^2+p^2)^{n+1}}\right]_{p=1}. \quad (A.5)$$

At any value of n, a focusing lens $\Phi''(0) < 0$ is produced. At [1] $n = 0$ we have $\Phi(u) = (1-u^2)/(1+u^2)^2$, $\Phi''(0) = -6$, and at $n = 1$ we have $\Phi(u) = (3 - 6u^2 - u^4)/2(1+u^2)^3$, $\Phi''(0) = -15$.

2. $f(u) = e^{-u^2}$ [1] (Gaussian beam),

$$\Phi(u) = \frac{2}{\pi}\Phi\left(1, \frac{1}{2}, -u^2\right), \quad \Phi(0) = \frac{2}{\pi}, \quad \Phi''(0) = -\frac{8}{\pi} < 0, \quad (A.6)$$

where

$$\Phi(\gamma, \alpha; z) = 1 + \frac{\gamma}{\alpha}\frac{z}{1!} + \frac{\gamma(\gamma+1)}{\alpha(\alpha+1)}\frac{z^2}{2!} + \ldots$$

is a confluent hypergeometric function[9]. A focusing lens is produced (in the central part), and as $r \to \infty$ the bend decreases exponentially.

3. $f(u) = 1/\cosh^2 u$. Focusing lens

$$\Phi(u) = (1 - \mathrm{sh}^2 u)/\mathrm{ch}^3 u, \quad \Phi''(0) = -5 < 0. \qquad \text{(A.7)}$$

B. Two-Dimensional Case

1. $f(u) = (1 + u^2)^{-\mu-1}$, $\mu > 0$,

$$\Phi(u) = \sqrt{\pi}\frac{\Gamma(\mu+3/2)}{\Gamma(\mu+1)} F\left(\frac{3}{2}, \mu + \frac{3}{2}; 1; -u^2\right),$$

$$\Phi(0) = \sqrt{\pi}\frac{\Gamma(\mu+3/2)}{\Gamma(\mu+1)}, \quad \Phi''(0) = -3\sqrt{\pi}\frac{\Gamma(\mu+5/2)}{\Gamma(\mu+1)} < 0, \qquad \text{(A.8)}$$

where

$$F(\alpha, \beta; \gamma; z) = 1 + \frac{\alpha\beta}{\gamma}\frac{z}{1!} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)}\frac{z^2}{2!} + \ldots$$

is the Gauss hypergeometric function[9]. At arbitrary μ, a focusing lens is produced. At [1] $\mu = 1/2$ we get

$$\Phi(u) = \frac{2 - u^2}{(1+u^2)^{5/2}}.$$

In general, for half-integer $\mu = 1/2 + k$, the function $\Phi(u)$ is elementary, ($F(3/2, k+2; 1; z)$, and can be expressed with the aid of recurrence relations in terms of $F(3/2, 1, 1; z) = (1 - z)^{-3/2}$).

2. $f(u) = (1 + u^4)^{-1}$,

$$\Phi(u) = \frac{\pi\sqrt{2}}{4}(\mathrm{Re} + \mathrm{Im}) F\left(\frac{3}{2}, \frac{3}{2}; 1; iu^2\right),$$

$$\Phi(0) = \frac{\pi\sqrt{2}}{4}, \quad \Phi''(0) = \frac{9\pi\sqrt{2}}{8} > 0. \qquad \text{(A.9)}$$

In this case the central part of the bend is a defocusing lens (cf. Fig. 1b in the one-dimensional case).

3. $f(u) = e^{-u^2}$ (Gaussian beam) [1]

$$\Phi(u) = \pi^{1/2}\Phi(3/2, 1; -u^2), \quad \Phi(0) = \pi^{1/2}, \quad \Phi''(0) = -3\pi^{1/2} < 0. \qquad \text{(A.10)}$$

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1) Observation of the lens effect under conditions of total internal reflection was reported also by Ivanova, Chernov, and Shepelev [3].

2) It follows from (2.3) that the light pressure depends on the shape of the bend $\zeta(\mathbf{r}, t)$, but the principal term of the expansion in the small surface slopes $|\nabla_\perp\zeta| \ll 1$, namely the quantity $p_L = (\Pi^{I}_{zz} - \Pi^{II}_{zz})_{z=0}$, does not depend on ζ and is determined by the intensity of the field in the beam. The next term of the expansion (linear in $\nabla_\perp\zeta$), which is responsible for the stimulated scattering by the surface waves [4-6] and also for the renormalization of the surface tension, will not be taken into account here.

3) The case of a liquid with large viscosity, which is of interest in connection with observation of the lens effect on glasses [10], calls for a special analysis.

4) We note that the direction of the light-pressure force on a transparent dielectric was known already to Poynting [11], who performed the first measurement of this force after Lebedev measured the light pressure on ideally absorbing and reflecting surfaces (the authors learned of Poynting's work from Ashkin's article [2], who apparently likewise did not know of this work earlier [12]). Poynting's reasoning, based on consideration of wave trains, are close to the quantum (!) derivation.

5) If media I and III are identical, it follows immediately that the force is directed along the beam. Indeed, at $\theta^i = \theta^T$, using the energy conservation law $W^i \geqslant W^R + W^T$, we obtain $f^L \geqslant 2W^R \cos^2\theta^i \geqslant 0$.

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