Spectrum of polarons in a magnetic field. States in the forbidden band

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The retarded Green functions is calculated for an electron interacting with optical phonons in a magnetic field at a finite temperature and in the presence of electrons in the conduction band. New branches of the one-electron spectrum are discovered, lying below the bottom of the conduction band by an amount equal to the optical-phonon energy. The appearance of new poles in the Green function is explained by the fact that when interacting with optical phonons the electron is in an oscillating field varying with a specified frequency.

It has been shown by Levinson, Matulis, and Shcherbakov[1] that, at zero temperature, when the conduction band of a semiconductor is empty, the weak interaction of electrons with optical phonons in a magnetic field leads to the appearance of new branches of the electron spectrum, at energies close to the threshold for emission of an optical phonon. We shall show that in the case of finite temperatures and in the presence of electrons in the conduction band new branches of the electron spectrum appear near an energy lying at a distance equal to the optical-phonon energy below the bottom of the conduction band.

In this work we shall assume that the optical-phonon energy ω_0 is small compared with the width of the forbidden band, so that we can disregard two-band effects; ω_0 does not depend on the momentum; the electrons interact only with the optical phonons, and for simplicity we shall consider the case of the deformation interaction (its matrix element does not depend on the momentum); the dimensionless electron-phonon coupling constant $\alpha \ll 1$.

To calculate the spectrum at a finite temperature we shall use the diagram technique of Keldysh[2], applying it to describe the system in a magnetic field in the same manner as the zero-temperature diagram technique was applied in[1,3]. We shall find the retarded electron Green function by means of the Dyson equation

$$G^{R}(\varepsilon pl)=G_0^{R}(\varepsilon pl)+G_0^{R}(\varepsilon pl)\Sigma^{R}(\varepsilon pl)G^{R}(\varepsilon pl), \tag{1}$$

where $G_0^R(\epsilon p l)$ and $G^R(\epsilon p l)$ are the free and exact retarded Green functions corresponding to the Landau level l (it was shown in[3] that in a uniform and isotropic system $G(\epsilon p l)$ remains diagonal in l), $\Sigma^R(\epsilon p l)$ is the mass operator of the retarded Green function,

$$G_0^{R}(\varepsilon pl)=\frac{1}{\varepsilon-\varepsilon_l(p)+i\delta}, \quad \delta\to+0. \tag{2}$$

$\epsilon_l(p) = \omega_c l + p^2/2m$ is the electron spectrum in the Landau level l, $\omega_c = eH/mc$, and p is the longitudinal (with respect to the magnetic field H) momentum of the electron. The Green functions in the Keldysh technique have a matrix form[2], $G^{11}(\epsilon p l)$ being the causal electron Green function;

$$G^{12}=G^{11}-G^{R},\ G^{21}=G^{11}-G^{A},\ G^{A}=G^{R*},\ G^{22}=G^{11}-G^{R}-G^{A}.$$

The same relations are valid for the phonon Green function D. The free causal Green functions have the form

$$G_0^{11}(\varepsilon pl)=\frac{1}{\varepsilon-\varepsilon_l(p)+i\delta}+2\pi i n(\varepsilon)\delta[\varepsilon-\varepsilon_l(p)], \tag{3}$$

$$D_0^{11}(\omega)=g\left\{\frac{1}{\omega-\omega_0+i\delta}-\frac{1}{\omega+\omega_0-i\delta}-2\pi i N[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]\right\}, \tag{4}$$

where

$$n(\varepsilon)=(e^{(\varepsilon-\mu)/kT}+1)^{-1},\ N=(e^{-\omega_0/kT}-1)^{-1}$$

are the Fermi and Planck distribution functions and $g = \alpha\pi\sqrt{2\omega_0/m^3}$ is the square of the matrix element of the deformation interaction of the electrons with the phonons. The simplest vertex has the form of the matrix

$$\gamma_{ij}^{k}(l,l',\mathbf{q})=(\sigma_z)_{ij}\delta_{jk}\Lambda_{ll'}(\mathbf{q}),\ \Lambda_{ll'}(\mathbf{q})=e^{i\varphi(l-l')}Q_{ll'}(t),$$
$$t=\frac{q^2}{2m\omega_c},\quad Q_{ll'}(t)=\left(\frac{l!}{l'!}\right)^{1/2}t^{(l'-l)/2}L_l^{l'-l}(t),$$

where $\mathbf{q}$ is the phonon-momentum component perpendicular to the magnetic field, φ is its azimuthal angle, and $L_l^{l'-l}$ is a Laguerre polynomial.

We shall consider the simplest term appearing in the mass operator (Fig. 1):

$$\begin{aligned}M_s^{R}&=ig\int\frac{d\varepsilon_1\,dp_1}{(2\pi)^2}[D_0^{11}(\varepsilon_1-\varepsilon)G_0^{11}(\varepsilon_1p_1s)\\&\quad-D_0^{21}(\varepsilon_1-\varepsilon)G_0^{12}(\varepsilon_1p_1s)]\\&\quad\times\int\frac{d\mathbf{q}}{(2\pi)^2}\Lambda_{ls}(\mathbf{q})\Lambda_{ls}(-\mathbf{q})\\&=\frac{\alpha\omega_c}{2\pi}\sqrt{\frac{\omega_0}{2m}}\int_{-\infty}^{+\infty}dp_1\left\{\frac{1+N-n[\varepsilon_s(p_1)]}{\varepsilon-\varepsilon_s(p_1)-\omega_0+i\delta}\right.\\&\quad\left.+\frac{N+n[\varepsilon_s(p_1)]}{\varepsilon-\varepsilon_s(p_1)+\omega_0-i\delta}\right\}.\end{aligned} \tag{5}$$

This expression diverges at $\epsilon = \epsilon_S(0) + \omega_0$ (the first term) and at $\epsilon = \epsilon_S(0) - \omega_0$ (the second term). The singularity at $\epsilon = \epsilon_S(0) + \omega_0$ was investigated by Levinson, Matulis, and Shcherbakov[1] and the second singularity was absent in the conditions of their work ($N = n = 0$). In fact, the second term of expression (5) is not small compared with ω_0 in the region of energies

$$\omega_c s-\omega_0-\frac{\alpha^2[N+n(s\omega_c)]^2\omega_c^2}{\omega_0}\leqslant\varepsilon<\omega_c s-\omega_0,$$

and this means that it is precisely in this region of energies that new branches of the spectrum can appear. In the following we shall pay attention principally to the

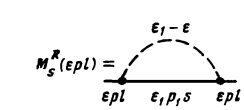

FIG. 1. Simplest diagram for the mass operator.

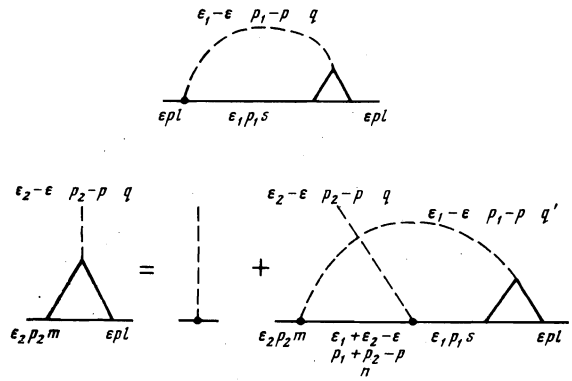

FIG. 2. Mass operator and equation for the vertex part.

study of the spectrum in the vicinity of $\epsilon = -\omega_0$. At energies close to $\pm\omega_0$, only M_0^R can be large, despite the small factor α. We shall assume that $n[\epsilon_0(p_1)]$ varies little when $p_1^2/2m \lesssim \alpha^2\omega_c^2/\omega_0$. Then the expression for $M_0^R(\epsilon pl)$ for $\epsilon \lesssim \omega_0$ has the form

$$M_0^R(\varepsilon pl) = (1+N-n_0)M^+ = -(1+N-n_0)\alpha\frac{\omega_c}{2}\sqrt{\frac{\omega_0}{\omega_0-\varepsilon}}, \tag{6}$$

and for $\epsilon \lesssim -\omega_0$ has the form

$$M_0^R(\varepsilon pl) = (N+n_0)M^- = (N+n_0)\alpha\frac{\omega_c}{2}\sqrt{\frac{\omega_0}{-\omega_0-\varepsilon}}, \tag{7}$$

where $n_0 = n(0)$. Thus, perturbation theory turns out to be inapplicable for $\epsilon \approx \pm\omega_0$ and we must take into account the higher-order diagrams that make an important contribution to the mass operator. For this we shall use a method developed for the study of the spectrum near the decay threshold[4,5,7]. The mass operator and vertex part in this approximation are determined by the following equations (Fig. 2):

$$\Sigma^R(\varepsilon pl) = \sum_{s=0}^{\infty}\sum_{j,k=1}^{2}\int\frac{d\varepsilon_1\,dp_1\,d\mathbf{q}}{(2\pi)^4}G^{ij}(\varepsilon_1 p_1 s)D^{kl}(\varepsilon_1-\varepsilon\ p_1-p\ \mathbf{q})\Lambda_{ls}(\mathbf{q})$$
$$\times[\Gamma_{ji}^{k}(\varepsilon_1 p_1 s, \varepsilon pl, \mathbf{q}) + \Gamma_{j2}^{k}(\varepsilon_1 p_1 s, \varepsilon pl, \mathbf{q})]$$

$$\Gamma_{ij}^{k}(\varepsilon_2 p_2 m, \varepsilon pl, \mathbf{q}) = \gamma_{ij}^{k}(m, l, \mathbf{q}) + \sum_{n,s=0}^{\infty}\sum_{i',j'k'=1}^{2}\sum_{i'',j''k''=1}^{2} i\int\frac{d\varepsilon_1\,dp_1\,d\mathbf{q}'}{(2\pi)^4} \tag{8}$$

$$\times G^{j''i'}(\varepsilon_1 p_1 s)D^{k''k'}(\varepsilon_1-\varepsilon\ p_1-p\ \mathbf{q})\exp\left\{i\frac{c}{eH^2}(\mathbf{qq}'\mathbf{H})\right\} \tag{9}$$

$$\times\gamma_{ij'}^{k'}(m, n, \mathbf{q}')G^{j'i''}(\varepsilon_1+\varepsilon_2-\varepsilon, p_1+p_2-p, n)\gamma_{i''j''}^{k}(n, s, \mathbf{q})\Gamma_{i'j}^{k''}(\varepsilon_1 p_1 s, \varepsilon pl, \mathbf{q}').$$

To solve the system (8)—(9) we shall make the following approximations. First, we replace the exact phonon Green function by the free one. Since, as can be seen from (5), the form of $D(\omega, q)$ when $\omega \approx \pm\omega_0$ is important for the study of the spectrum for $\epsilon \approx \pm\omega_0$, this replacement is justified if the polarization operator of the phonon Green function has no singularities at $\omega \approx \pm\omega_0$. In the presence of poles in the electron Green function at $\epsilon \approx \pm\omega_0$, singularities in the polarization operator at $\omega \approx \pm\omega_0$ do not appear in those cases when the states at $\epsilon \approx \pm\omega_0$ and $\epsilon \approx 0$ are either completely occupied or empty, i.e., when $n(\omega_0) = n_0 = 1$ and $n(-\omega_0) = n_0 = 0$, for any N, and also when $n_0 = 1$ (or $n_0 = 0$), for $N = 0$. Secondly, we replace $G^{j''i'}(\epsilon_1 p_1 s)$ by its free value. Since the result of the integration over ϵ_1 in (8) and (9) is expressed, as in (5), in terms of the values of the electron Green function at the energies $\epsilon \pm \omega_0$, which, for the ϵ of interest to us, are far from the singularities being studied, such a replacement is permissible and is exact to within quantities of the order of α. Since the singularities at $\epsilon = \pm\omega_0$ arise only in the intergration of $G(\epsilon_1 p_1 0)$, in the sums over s we shall retain only the term with $s = 0$. After this we perform the integration over ϵ_1 and the angles in (8) and (9) and go over to the dimensionless variable $t = q^2/2m\omega_c$. The vertex part appears in the mass operator in the form of the sum $\Gamma_{j1}^{i} + \Gamma_{j2}^{i}$, and therefore we shall immediately write the equations for Γ_α:

$$\Gamma_\alpha = \{\Gamma_{11}^{1}+\Gamma_{12}^{1},\ \Gamma_{21}^{1}+\Gamma_{22}^{1},\ \Gamma_{11}^{2}+\Gamma_{12}^{2},\ \Gamma_{21}^{2}+\Gamma_{22}^{2}\},$$

$$\Sigma^{\pm}(\varepsilon pl) = \frac{\alpha\omega_c}{2\pi}\sqrt{\frac{\omega_0}{2m}}\int_{-\infty}^{+\infty}\frac{dp_1}{\varepsilon-p_1^2/2m\mp\omega_0}\sum_{\alpha=1}^{4}\int_0^\infty dt\, f_{1\alpha}^{\pm}\Gamma_\alpha^{\pm}(\varepsilon\mp\omega_0 p_1 0,\ \varepsilon pl, t), \tag{10}$$

$$\Gamma_\alpha^{\pm}(\varepsilon\mp\omega_0 p_2 0,\ \varepsilon pl, t) = \gamma_\alpha(0, l, t)$$
$$+\sum_{n=0}^{\infty}\sum_{\beta,\gamma=1}^{4}\frac{\alpha\omega_c}{2\pi}\sqrt{\frac{\omega_0}{2m}}\int_{-\infty}^{+\infty}\frac{dp_1}{\varepsilon-p_1^2/2m\mp\omega_0}\delta_{\alpha\beta}f_{\beta\gamma}^{\pm}G^{\alpha}(\varepsilon\mp 2\omega_0\ p_1+p_2-p\ n)$$
$$\times\int_0^\infty dt'\,Q_{in}(t)Q_{ln}(t')J_{n+l}(2\sqrt{tt'})\Gamma_\gamma^{\pm}(\varepsilon\mp\omega_0 p_1 0,\ \varepsilon pl, t'). \tag{11}$$

Here Γ^+, Σ^+ and Γ^-, Σ^- denote the vertex part and retarded mass operator near the energies $\epsilon = \omega_0$ and $\epsilon = -\omega_0$ respectively;

$$\gamma_\alpha(0, l, t) = Q_{0l}(t)\{1, 0, 0, -1\},\ G^\alpha = \{G^{11}, G^{21}, G^{12}, G^{22}\},$$

$$f_{\beta\gamma}^{+} = \begin{Vmatrix} 1+N-n_0 & -n_0 & N & 0 \\ 1+N & 0 & -(N+n_0) & n_0 \\ 1-n_0 & N+n_0 & 0 & -N \\ 0 & -(1+N) & -(1-n_0) & -(1+N-n_0) \end{Vmatrix}$$

$f_{\beta\gamma}^{-}$ is obtained from $f_{\beta\gamma}^{+}$ by replacing N by $-(1+N)$ and then changing the sign of each term of the matrix.

Since $G^\alpha(\epsilon \mp 2\omega_0)$ appears in (11), the mass operators for $\epsilon \approx \omega_0$ and $\epsilon \approx -\omega_0$ are determined in terms of each other. But under the condition $n_0 = N = 0$ the singularity at $\epsilon = -\omega_0$ disappears and the system (11) reduces to one equation. The equations that are obtained for the mass operator and vertex part coincide with those of[1], where it is shown that, in this case, new branches of the spectrum appear at $\epsilon \approx \omega_0$. For $N = 0$, $n_0 = 1$, the singularities at $\epsilon \approx -\omega_0$ are absent. Therefore, we can replace $G^\alpha(\epsilon + 2\omega_0)$ by $G_0^\alpha(\epsilon + 2\omega_0)$. But since, for $\epsilon \approx -\omega_0$,

$$G_0^{21}(\varepsilon+2\omega_0) = G_0^{12}(\varepsilon+2\omega_0) = 0,$$

provided that

$$\left|\frac{p^2}{2m}+n\omega_c-\omega_0\right| \gg \frac{\alpha^2(N+n_0)^2\omega_c^2}{\omega_0},$$

it follows from (10) and (11) that in this case, too, the vertex part is determined by one equation, which is solved in the same way as the equation for the vertex part in[1]. Taking into account that G^α and Γ^α are smooth functions of the momentum p_1 at the energies $\epsilon + 2\omega_0$ and $\epsilon + \omega_0$ respectively, we take them outside the integral at $p_1 = 0$ and put $\epsilon = -\omega_0$. We obtain

$$\Sigma^{\pm}(\varepsilon pl) = M_0^{\pm}\int_0^\infty dt\,Q_{l0}(t)\Gamma^{\pm}(\varepsilon pl, t) = M_0^{\pm}\bar{\Gamma}^{\pm}(\varepsilon pl), \tag{12}$$

$$\Gamma^{\pm}(\varepsilon pl, t) = \lambda^{\pm}(\varepsilon p)\int_0^\infty dt'\sum_{n=0}^{\infty}\frac{\sigma^{\pm}}{\sigma^{\pm}+n}Q_{0n}(t)Q_{0n}(t')$$
$$\times J_{n+l}(2\sqrt{tt'})\Gamma^{\pm}(\varepsilon pl, t)+Q_{l0}(t), \tag{13}$$

where

$$\sigma^{\pm} = \frac{p^2/2m\pm\omega_0}{\omega_c},\ \lambda^{\pm}(\varepsilon p) = \frac{M_0^{\pm}(\varepsilon)}{\mp\omega_0-p^2/2m},\ \Gamma^{\pm}(\varepsilon pl, t) = \Gamma_1^{\pm}(\varepsilon\mp\omega_0 00,\ \varepsilon pl,\ t)$$

The solution for the quantity $\bar{\Gamma}(\epsilon pl)$ appearing in the mass operator is obtained by integrating the Schmidt

formula for the solution of a Fredholm integral equation with a Hermitian kernel[6]

$$\Gamma(\varepsilon pl)=\sum_k \frac{\lambda_k}{\lambda_k-\lambda}(Q_{l0},\varphi_k)^2=\sum_k \frac{A_k}{\lambda_k-\lambda}, \qquad (14)$$

where $\varphi_k(t)$ and λ_k are the eigenfunctions and eigenvalues of Eq. (13), which depend on p and l. Since the kernel of Eq. (13) is alternating in sign, there are both positive and negative numbers amongst the eigenvalues λ_k.

The equation for the poles of the retarded Green function of the Landau level l for $\epsilon \lesssim -\omega_0$ and

$$\left|\frac{p^2}{2m}+n\omega_c-\omega_0\right| \gg \alpha^2(N+n_0)^2\frac{\omega_c^2}{\omega_0}$$

has the form

$$\Gamma(\lambda,l)=-\left(\frac{\omega_0}{\omega_0-p^2/2m}\right)^3\frac{\alpha^2\omega_c^2}{\lambda^3\omega_0^2}-\left(\frac{\omega_0+p^2/2m+l\omega_c}{\omega_0-p^2/2m}\right)\frac{1}{\lambda}, \qquad (15)$$

where $\overline{\Gamma}(\lambda, l)$ is determined by the expression (14). From Eq. (15) we find the solutions Λ_k (it can be seen from (14) that they exist), to each of which corresponds a pole of the Green function in the forbidden band, with the following momentum dependence of the energy:

$$\varepsilon=\varepsilon_{kl}^-(p)=-\omega_0\left[1+\frac{\alpha^2\omega_c^2}{4(\omega_0-p^2/2m)^2\Lambda_k^2}\right]. \qquad (16)$$

The values of λ_k and A_k were found in[1] under the conditions $\sigma \ll 1$ and $\sigma \gg 1$ (which means, in our case, $p \ll \sqrt{2m\omega_c}$ and $p \gg \sqrt{2m\omega_c}$ for $\omega_0 \ll \omega_c$). Correspondingly, we have

$$\lambda_k=(-1)^k\left(\frac{\sqrt{5}-1}{2}\right)^{-(2k+l+1)} \text{ and } \lambda_k=(-1)^k\frac{\sigma^k}{k!}.$$

In this case too, it has not been possible to find the exact value of Λ_k, but it is clear that $\lambda_{2k-1} < \Lambda_k < \lambda_{2k+1}$ and the new branches of the one-electron spectrum are a sequence of narrow sublevels that converge from below toward the energy $\epsilon = -\omega_0$ as the labels k and l increase and as $p \to \infty$. For

$$p^2/2m \approx \omega_0 - n\omega_c,$$

where n is a natural number, Eq. (13) for $\Gamma^-(\epsilon p l, t)$ is not valid; in this case, $\overline{\Gamma}^-(\epsilon p l)$ is complex, and nonattenuating states with such a momentum do not exist in the forbidden band.

We shall elucidate how the spectrum looks for $1 + N - n_0 \neq 0$ and $N + n_0 \neq 0$, when singularities are present at both $\epsilon \approx \omega_0$ and $\epsilon \approx -\omega_0$. Suppose that there is a pole of $G^1(\epsilon p l)$ at certain values of p and $\epsilon \approx -\omega_0$. In order to find Σ^- and Γ^- in the vicinity of these ϵ and p, we first find Σ^+ and Γ^+ at the energy $\epsilon + 2\omega_0 \approx \omega_0$ and for momenta close to p. Because of the integration over p_1 in Eq. (11) for Γ^+ in the vicinity of the pole of $G^1(\epsilon, p_1 + p_2 - p, n)$, the equations for Γ^+ become complex:

$$\Gamma_\alpha^+(\varepsilon pl,\ t)=\gamma_\alpha+\mu_{\alpha\beta}\hat{L}\Gamma_\beta^+,$$

Im $\mu_{\alpha\beta} \sim 1/\alpha$ at the energies we need:

$$\varepsilon \approx -\omega_0-\alpha^2\omega_c^2/\omega_0,$$

and $\hat{L}$ is symmetric integral operator. By means of a simple generalization of the Schmidt formula[6] it is easy to show that the solution of such a system of equations has the form

$$\Gamma_\alpha^+=\sum_k\sum_{\beta=1}^{k}\mu_k\nu_{\alpha\beta}(k)\,(\gamma_\beta,\psi_k)\psi_k,$$

where ψ_k and μ_k are the eigenfunctions and eigenvalues of the operator $\hat{L}$,

$$\nu_{\alpha\beta}(k)=[\mu_k\delta_{\alpha\beta}-\mu_{\alpha\beta}]^{-1}.$$

Since

$$|\mu_k\delta_{\alpha\beta}-\mu_{\alpha\beta}| \gtrsim 1/\alpha,$$

we have $|\nu_{\alpha\beta}| \sim \alpha$ and the vertex part $\Gamma_\alpha^+ \sim \alpha$. Therefore, at the energies ϵ and momenta p corresponding to a pole of G^1, the mass operator in $G^\alpha(\epsilon + 2\omega_0, p_1 + p_2 - p, n)$ from Eq. (11) is a quantity of order α, and we shall neglect it. Then the system (11) again reduces to one equation for Γ_1, and is solved as in the case $n_0 = 1$, $N = 0$. The only difference is that we must replace α by $\alpha(n_0 + N)$ in the expressions (12)—(16) describing Σ^-, Γ^- and the spectrum in the forbidden band. It can be shown analogously that if the Green function has a pole at $\epsilon \approx \omega_0$ and momentum p, the mass operator at energies $\epsilon - 2\omega_0$ is a complex quantity of order α, and the spectrum $\epsilon = \epsilon_{kl}^+(p)$ for $\epsilon \approx \omega_0$ is described by the equation of[1], in which it is necessary to substitute $\alpha(1 + N - n_0)$ in place of α. We note that it follows from the arguments presented that the expressions obtained for $\overline{\Gamma}^\pm(\epsilon p l)$ and $\Sigma^\pm(\epsilon p l)$ are not solutions of Eqs. (10)—(11) at momentum values satisfying the condition

$$\varepsilon_{kl}^+(p)=\varepsilon_{kl}^-(p)+2\omega_0 \qquad (17)$$

or in a region of order

$$\Delta p \sim \alpha(\omega_c/\omega_0)\sqrt{2m\omega_0}.$$

surrounding these values. There are no solutions of the system (10)—(11) that would correspond to the presence of non-attenuating (to within quantities of order α) states satisfying the condition (17).

The concentration of electrons in the new states in the forbidden band is determined by the expression

$$\frac{m\omega_c}{i(2\pi)^3}\int_{-\infty}^{+\infty}dp\int_{-\infty}^{-\infty}d\varepsilon\, G^{12}(\varepsilon p)$$

and, for $\omega_c \gg \omega_0$, if $n(\epsilon)$ varies little in the range of energies in which these levels are located, amounts in order of magnitude to

$$\alpha^2(n_0+N)^2n(-\omega_0)\,(m\omega_c^2/\omega_0)^{3/2}. \qquad (18)$$

The appearance of new states in the forbidden band can have an effect, e.g., in the measurement of the optical absorption. Additional absorption bands appear, associated with transitions between these states and the Landau levels of the conduction and valence bands. The absorption coefficient here turns out to be proportional to the square of the small coupling constant. In particular, for transitions from the valence Landau level $l = 0$ to the states $\epsilon_{k0}^-(p)$,

$$K(\nu)=\left(\frac{ep_{cv}}{m_0}\right)^2\frac{m\omega_c}{4\pi^2cn\nu}\int dp\,d\varepsilon\, G^{21}(\varepsilon p0)\,G_h^{12}(\varepsilon-\nu p0), \qquad (19)$$

where ν is the light frequency, n is the refractive index, p_{cv} is the matrix element of the momentum for a transition between the valence band and the conduction band, m_0 is the free-electron mass, and G_h^{12} is the Green function of the holes. For a parabolic valence band in the case when the states $\epsilon_{kl}^-(p)$ are not occupied ($n_0 = n(-\omega_0) = 0$), and under the same conditions for which the expression (18) was obtained, for frequencies

$$\left|\omega_0-\frac{m_h}{m}(\nu-\varepsilon_g+\omega_0)\right| \gg \frac{\alpha^2N^2\omega_c^2}{\omega_0}$$

we obtain from (19):

$$K(\nu)\sim\frac{\alpha^2N^2e^2p_{cv}^2m\sqrt{m_h}\,\omega_c^3\omega_0}{cn\nu[\omega_0-m^{-1}m_h(\nu-\varepsilon_g+\omega_0)]^2[\omega_0+m^{-1}m_h(\nu-\varepsilon_g+\omega_0)]}$$
$$\cdot\sum_{k=0}^{\infty}\frac{\theta[\nu-\varepsilon_g-\varepsilon_{k0}^-(p_1)]}{\Lambda_{k0}^2(p_1)[\nu-\varepsilon_g-\varepsilon_{k0}^-(p_1)]^{1/2}}$$

where m_h is the effective mass of the holes, ϵ_g is the width of the forbidden band in the magnetic field, and

$$p_1=[2m_h(\nu-\varepsilon_g+\omega_0)]^{1/2}.$$

In conclusion, we note that, of course, it does not follow from the appearance of the new poles in the Green function that the ground-state energy of the system is decreased by ω_0 when the weak interaction of the electrons with the phonons is taken into account. For weak coupling, the ground-state energy of the whole system is decreased by a small amount. But, because an electron interacting with phonons is in a field that is periodic in time and, consequently, should possess a quasi-energy determined to within $\mp k\omega_0$ (k is a natural number), the electron density matrix contains terms oscillating with frequencies $\epsilon_0 \pm k\omega_0$ and proportional to powers of the coupling constant. This is conveniently illustrated by means of the exactly soluble problem of the interaction of an electron that can be in a state with energy ϵ, with an assembly of linear oscillators of frequency ω_0. This problem is described by the Hamiltonian

$$\hat{H}=\sum_i\left(\frac{\hat{P}_i^2}{2M}+\frac{M\omega_0^2}{2}\hat{x}_i^2\right)+\varepsilon a^+a+\sum_i\gamma_i\sqrt{M\omega_0^3}\,a^+a\hat{x}_i,$$

in which the last term describes the interaction. This Hamiltonian can be brought to diagonal form by the transformation

$$S=\exp\sum_i\left(-\frac{i\gamma_i}{\sqrt{M\omega_0}}\hat{P}_ia^+a\right)$$

The ground-state energy of the system in this problem is

$$\tilde{\varepsilon}=\varepsilon-\sum_i\gamma_i^2\omega_0.$$

But when we separate out the purely electronic energies, we find a set of energies $\tilde{\epsilon} \pm k\omega_0$. In fact, the Heisenberg electron annihilation operator can be represented in the form

$$a(t)=S\exp\left[\sum_i\frac{\gamma_i}{\sqrt{2}}(b_i^+e^{i\omega_0t}-b_ie^{-i\omega_0t})\right]aS^+e^{-i\tilde{\varepsilon}t}$$

(b_i^+ and b_i are phonon creation and annihilation operators). Its time dependence contains the frequencies $\tilde{\epsilon} \pm k\omega_0$ and it is clear from the form of a(t) that the electron Green function should have poles corresponding to these frequencies. In particular, for weak coupling ($\gamma_i \ll 1$) the greatest amplitude corresponds to the frequencies $\tilde{\epsilon}$, $\tilde{\epsilon} \pm \omega_0$:

$$a(t)=\left[a+\sum_i\frac{\gamma_i}{\sqrt{2}}(b_i-b_i^+)\right]e^{-i\tilde{\varepsilon}t}-\sum_i\frac{\gamma_i}{\sqrt{2}}b_iae^{-i(\tilde{\varepsilon}+\omega_0)t}$$
$$+\sum_i\frac{\gamma_i}{\sqrt{2}}b_i^+ae^{-i(\tilde{\varepsilon}-\omega_0)t}.$$

The example considered shows that additional poles of the electron Green function at energies $\tilde{\epsilon} - k\omega_0$ should also appear in the problem, investigated in the papers[8,9], of the interaction of an impurity-center electron with optical phonons in semiconductors.

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