Properties of photon echo on broad spectral lines

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Some features of photon echo are investigated for broad spectral lines with inhomogeneous braodening. A shift of the echo amplitude maximum and a change in the shape and duration of the echo are observed as functions of the ratio of spectral line width to the width of the exciting light pulses. The polarization properties of the photon echo depend directly on the nature of the resonant atomic transitions, and this circumstance can be used to identify the transitions experimentally.

Photon echo is produced when two short light pulses are passed in succession through the investigated medium and constitutes spontaneous coherent emission of atoms that go over, under the influence of the second exciting pulse, into the initial coherent state produced by the first pulse. Photon echo has been frequently used lately in experimental investigations of rapid relaxation processes in both solids[1,2] and gases[3-7]. In addition, the photon-echo method is used to identify atomic and molecular transitions[3,6]. This identification is based on the dependence of the polarization properties of the echo on the type of the indicated transition.

The necessary condition for the appearance of photon echo is the presence of a spread of the resonance levels about the mean value $\hbar\omega_0$. The level spread is produced in a gas because of the Doppler effect connected with the thermal motion of the molecules, and in a solid because of the thermal vibrations of the crystal-lattice atoms. An aggregate of identical atoms (molecules) with a resonant-level spread generates an inhomogeneously broadened spectral line, on which the photon echo is produced. In most experiments in gases[3,6,7] the echo was produced on a broad spectral line $1/T_0 \gg 1/T$, and 90° and 180° pulses were used for the excitation. Here T_0 and T are the reversible Doppler relaxation time and the duration of the exciting pulses, respectively.

It will be established below that the atoms inside the contour of the inhomogeneously broadened broad spectral line make unequal contributions to the photon echo. Let a first (90°) and a second (180°) exciting pulse of duration T be incident on the boundary of the medium at instants of time t = 0 and t = τ, and let their carrier frequency ω coincide with the frequency ω_0 of the atomic transition. Those atoms inside the contour of the broad spectral line which have a detuning much smaller than 1/T relative to ω should be called "strictly resonant" with the exciting light pulse. They give the maximum of the spontaneous coherent emission at the instant of time t = 2τ, with duration $2T_0$. This coincides with formation of the photon echo on a narrow spectral line $1/T_0 \ll 1/T$, when all the atoms inside the contour of the inhomogeneously broadened spectral line are "strictly resonant." Atoms with a detuning on the order of 1/T, at $1/T_0 \gg 1/T$, will be called "not strictly resonant." The latter give a maximum of the spontaneous coherent emission at later instants of time and with longer duration. The observed photon echo is a superposition of the spontaneous coherent emissions of all the atoms responsible for the inhomogeneously broadened spectral line. In the case $1/T_0 \gg 1/T$, however, the number of "strictly resonant" atoms is smaller and the main contribution to the echo is made by the "not strictly resonant" atoms. This leads to a shift of the maximum and also to a change in the waveform and duration of the light echo as a result of the broad spectral line in comparison with the narrow one.

With respect to the polarization properties of the produced echo, the atomic transitions can be broken up into two groups. For the first, the polarization of the light echo does not depend on the ratio of T and T_0, and for the second it is essentially determined by T/T_0. If the exciting light pulses have respectively circular and linear polarizations, then the first group includes atomic transitions with change of the total angular momentum $0 \rightleftarrows 1$, $1 \to 1$, and $1/2 \to 1/2$, and the second includes transitions with $J \to J$ $(J > 1)$ and $J \rightleftarrows J+1$ $(J \geq 1/2)$. In the case of exciting light pulses that are linearly polarized at an angle ψ to each other, the breakdown into two groups remains the same as before, but the atomic transitions $1/2 \rightleftarrows 3/2$ fall in the first group. The obtained features of the photon echo are useful for experimental identification of atomic and molecular transitions.

For the sake of argument, we consider a gas. However, the results are valid also for photon echo produced on a broad spectral line in a solid. To find the vector potential **A** of the photon echo, we choose as the fundamental equations the d'Alambert equation and the quantum mechanical equation for the density matrix ρ, which describes the state of an individual atom in an external field. We assume that the exciting light pulses propagate along the Z axis. We separate the rapidly-oscillating phase factors of the vector potential and of the polarization current $\mathbf{I} = \dot{\mathbf{d}}\rho$:

$$\mathbf{A}=\mathbf{a}\exp[i(kz-\omega t+\varphi)]+\text{c.c.},$$
$$\mathbf{I}=\mathbf{j}\exp[i(kz-\omega t+\varphi)]+\text{c.c.},$$

where **a** and **j** are the slowly varying amplitudes, φ is the constant phase shift, and $\dot{\mathbf{d}}$ is the operator of the derivative of the dipole moment of the atom. The equations for the slow functions take in the resonant approximation the form

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)a_\alpha=i\cdot 2\pi\lambda\!\!\!^{-}\int dv\,\mathrm{Sp}\,j_\alpha, \tag{1}$$

$$(\partial/\partial t+ikv)j_\alpha+i\cdot{}^3/_4(2J_2+1)\gamma c\lambda\!\!\!^{-}(\rho_2T_{\alpha\beta}-\rho_1^{\alpha\beta})a_\beta=0, \tag{2}$$

$$\frac{\partial}{\partial t}\rho_2+i\frac{1}{\hbar c}(a_\sigma^* j_\sigma-j_\sigma^+ a_\sigma)=0, \tag{3}$$

$$\frac{\partial}{\partial t}\rho_1^{\alpha\beta}-i\frac{1}{\hbar c}(a_\sigma^* T_{\sigma\beta}j_\alpha-j_\beta^+T_{\alpha\sigma}a_\sigma)=0, \tag{4}$$

where

$$j_\alpha=-i\omega_0R_{\mu m}\overset{\alpha}{d}_{m\mu'}\exp[i(\omega t-kz-\varphi)],$$

$$\rho_1^{\alpha\beta}=d_{\mu m}{}^\beta\rho_{mm'}d_{m'\mu'}{}^\alpha/|d_{J_1}{}^{J_2}|^2,\quad \rho_2=\rho_{\mu\mu'},$$

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$$T_{\alpha\beta}=d_{\mu m}{}^{\beta}d_{m\mu'}{}^{\alpha}/|d_{J_1}{}^{J_2}|^2,\quad \gamma=4|d_{J_1}{}^{J_2}|^2/3(2J_2+1)\hbar\lambda\!\!\!^{-3}.$$

Here $\rho_{mm'}$ and $\rho_{\mu\mu'}$ are density matrices describing the quantum-mechanical state of the atom at the lower and upper degenerate levels with total angular momentum J_1 and J_2, respectively. Further, $R_{\mu m}$ is the density matrix describing the transition between the indicated working levels, and $d^{\alpha}_{\mu m}$ and $d^{J_2}_{J_1}$ are the dipole and reduced dipole moments of this transition. Finally, γ is the probability of spontaneous emission of a quantum $\hbar\omega_0$ per unit time for an individual isolated atom. The quantities $\rho_1^{\alpha\beta}$, ρ_2, and j_α pertain to an aggregate of atoms moving in the direction of the Z axis with velocity v. The term kv takes into account the Doppler change of the frequency in this motion. The frequency ω of the electromagnetic field coincides with the frequency ω_0 of the atomic transition ($\lambda\!\!\!^{-} = c/\omega$). Summation over repeated vector and matrix indices is implied throughout, and the matrix index is taken to be the projection of the total angular momentum of the upper level. The tensor-matrix quantities $T_{\alpha\beta}$ for different transitions are given in [8].

At the initial instant of time t = 0 there is no polarization current, and the density matrices are written in the form

$$\rho_1{}^{\alpha\beta}(z,0)=\frac{n_1 f}{2J_1+1}T_{\alpha\beta},\quad \rho_2(z,0)=\frac{n_2 f}{2J_2+1}\delta_{\mu\mu'},$$

where n_1 and n_2 are the densities of the atoms on the lower and upper levels, respectively, at t = 0, while f is Maxwell's one-dimensional distribution function.

The nonlinear equations (1)–(4) were solved in the given-field approximation, i.e., neglecting the reaction of the medium on the transmitted exciting pulses. To this end, in the case of a broad spectral line in a gas, it is necessary to satisfy the condition

$$\gamma\lambda\!\!\!^{-2}L|N_0|T_0\ll 1,$$

$$N_0=\frac{n_2}{2J_2+1}-\frac{n_1}{2J_1+1},\quad T_0=\frac{c}{\omega u},$$

where L is the dimension of the gas medium, N_0 is the density of the excess population of the levels, and T_0 is the time of the reversible Doppler relaxation expressed in terms of the average thermal velocity u of the gas atoms.

Let the first exciting light pulse with amplitude $\mathbf{a}_1 = \mathbf{l}_+ a_1$ and duration T_1 have a right-circular polarization ($|\mathbf{l}_+|^2 = 1$), and let the second pulse, with amplitude $\mathbf{a}_2 = \mathbf{l}a_2$ and duration T_2, have a linear polarization ($\mathbf{l}^2 = 1$). Calculating the vector potential A, we find that the echo polarization in the atomic transitions $0 \rightleftarrows 1$ and $1 \to 1$ is directed along the polarization vector of the second exciting pulse, while in the atomic transition $1/2 \to 1/2$ it is right-circular. The values of the parameters T_1/T_0 and T_2/T_0 do not influence the polarization, and change only the amplitude of the photon echo, this being typical of this group of atomic transitions (first group). Thus, for example, the vector potential **A** of the photon echo on the atomic transition $0 \to 1$ takes the form

$$\mathbf{A}=\mathbf{l}a\exp[i(kz-\omega t-\varphi_1+2\varphi_2)]+\mathrm{c.c.},\qquad (5)$$

where

$$a=\frac{\pi}{2}\left(\frac{1}{2}\hbar\lambda\!\!\!^{-}\gamma\right)^{1/2}\lambda\!\!\!^{-}L|N_0|I,$$

$$I=\frac{1}{\sqrt{\pi}}\int_0^\infty d\eta e^{-\eta^2}\frac{\theta_1\theta_2{}^2}{\Phi_1\Phi_2{}^2}(1-\cos\Phi_2)\left[\sin\Phi_1\cos\frac{\eta t'}{T_0}+\frac{T_1\eta}{T_0\Phi_1}(1-\cos\Phi_1)\sin\frac{\eta t'}{T_0}\right],\qquad (6)$$

$$\theta_i=\left(\frac{\gamma\lambda\!\!\!^{-}a_i{}^2}{\hbar}\right)^{1/2}T_i,\quad \Phi_i{}^2=\theta_i{}^2+\frac{T_i^2}{T_0^2}\eta^2\qquad (i=1,2),$$

$$\eta=kvT_0,\quad t'=t-2\tau+T_1-T_2-z/c,\quad N_0=n_2-n_1/3.$$

Here the subscripts i = 1 and i = 2 pertain to the first and second exciting pulses, in which the constant phases are equal to φ_1 and φ_2. The term $-z/c$ takes into account the delay of the photon echo, which traverses a path z − L outside the gas medium.

The vector potential of the photon echo in other atomic transitions of the first group is obtained from (5) by making the following substitutions: for the atomic transition $1 \to 0$

$$\gamma\to 3\gamma,\quad N_0\to n_2/3-n_1;$$

for the atomic transition $1 \to 1$

$$\gamma\to 3\gamma/2,\quad N_0\to(n_2-n_1)/3,\quad \mathbf{l}\to 2\mathbf{l};$$

for the atomic transition $1/2 \to 1/2$

$$\gamma\to 2\gamma,\quad N_0\to(n_2-n_1)/2,\quad a_2\to a_2/\sqrt{2},\quad \mathbf{l}\to\mathbf{l}_+.$$

In the case of a broad spectral line, the integrand in (6) depends essentially on the degrees θ_1 and θ_2 of the first and second exciting light pulses. We first discuss the optimal case $\theta_1 = \pi/2$ and $\theta_2 = \pi$, when the amplitude of the echo at the maximum is largest. We put also $T_1 = T_2 = T$. According to (5) and (6), "strictly resonant" atoms with detuning $\eta/T_0 \ll 1/T$ relative to the center of the spectral line produce a maximum spontaneous coherent emission at the instant of time $t \cong 2\tau + z/c$. Yet each aggregate of "not strictly resonant" atoms with detuning $\eta/T_0 \leqslant 1/T$ produces a maximum spontaneous coherent emission at a later instant of time and with a lower intensity. For example, "not strictly resonant" atoms having a detuning $\eta/T_0 = 1/T$ yield a maximum of the spontaneous coherent emission at the instant $t \cong 2\tau + z/c + 0.6T$, with an intensity smaller by a factor 0.8 than in the case when the same aggregate of atoms is strictly resonant. The contributions from different sections of the Doppler contour of the spectral line are summed by integration with respect to the variable η in formula (6). Therefore the photon echo on a broad spectral line has in the optimal case a maximum shifted relative to the instant $t = 2\tau + z/c$ towards longer times, and the duration of the duration of the echo coincides approximately with the duration of the exciting light pulses.

Figure 1 shows the results of numerical calculations of the amplitude a of photon echo for the atomic transition $0 \to 1$. The curves describe also the echo for other atomic transitions $1 \to 0$, $1 \to 1$, and $1/2 \to 1/2$, which belong to the first group. In the case of a narrow spectral line $T/T_0 \ll 1$, the echo amplitude is proportional to the factor $I = \exp[-(t'/2T_0)^2]$ and has a maximum at the instant of time $t = 2\tau + z/c$, whereas the maximum of the echo amplitude on the broad spectral line $T/T_0 = 10$ occurs at $t \cong 2\tau + z/c + 0.6T$ and is approximately smaller by a factor T_0/T than the maximum on the narrow spectral line. The decrease of the amplitude is due to the fact that only some of the atoms (which is proportional to T_0/T) inside the broad Doppler contour take direct part in the formation of the echo. A shift of the maximum of the photon echo by an amount equal to approximately half the duration of the exciting pulses was observed experimentally in ruby[9]. As indicated in [10],

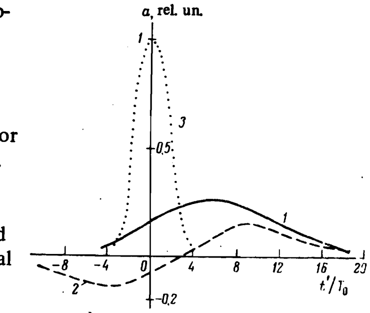

FIG. 1. Amplitude a of photon echo as a function of the time $t' = t - 2\tau - z/c$ for an atomic transition with total-angular-momentum change $0 \to 1$, connected with the vector potential A by the formula (5). Unity on the ordinate axis corresponds to the quantity $\frac{1}{2}\pi (\frac{1}{2}\gamma\lambda\hbar)^{1/2}\lambda L|N_0|$. Curves 1 and 2 correspond to a broad spectral line and to the following parameters: 1) $\theta_1 = \pi/2$, $\theta_2 = \pi$ and $T/T_0 = 10$; 2) $\theta_1 = \theta_2 = \pi$ and $T/T_0 = 10$. Curve 3 corresponds to a narrow spectral line $T/T_0 \ll 1$, $\theta_1 = \pi/2$, and $\theta_2 = \pi$.

the maximum of the optical induction produced by one exciting pulse also shifts at $T \gg T_0$.

On a broad spectral line, a change takes place also in the waveform of the echo pulse, depending on the values of θ of the exciting light pulses. In particular, at $\theta_1 = \theta_2 = \pi$ and $T/T_0 = 10$, the amplitude of the photon echo consists of two maxima, between which it passes through zero (see Fig. 1). A similar double-hump shape was observed earlier for spin echo[11], and was recently observed also in experiments on photon echo in ruby[12]. Further increase of θ of the first (and also the second) exciting pulse leads to a decrease of the maximum of the amplitude and to a more complicated waveform of the photon echo.

In contrast to the result (5), the vector potential of the photon echo on atomic transitions of the second group $J \to J$ $(J > 1)$ and $J \rightleftarrows J+1$ $(J \geq 1/2)$ does not contain a common factor of the type (6). Therefore, just like the amplitude, the polarization of the photon echo on atomic transitions of the second group also depends on the parameters T_1/T_0 and T_2/T_0. By way of example, we write down the vector potential of the photon echo on the atomic transition $1/2 \to 3/2$ for circularly and linearly polarized exciting pulses:

$$\mathbf{A}(z,t) = {}^1/_4\pi(\hbar\lambda\gamma)^{1/2}\lambda L|N_0|(\mathbf{l}_x I_1 + i\mathbf{l}_y I_2) \times \exp[i(kz - \omega t - \varphi_1 + 2\varphi_2)] + \text{c.c.}, \tag{7}$$

where

$$I_1 = \frac{1}{\sqrt{\pi}}\int_0^\infty d\eta e^{-\eta^2}\frac{\theta_2^2}{\Phi_2^2}(1-\cos\Phi_2)(U+V),$$

$$I_2 = \frac{1}{2\sqrt{\pi}}\int_0^\infty d\eta e^{-\eta^2}\frac{\theta_2^2}{\Phi_2^2}(1-\cos\Phi_2)(U-V),$$

$$U = \frac{\sqrt{3}\,\theta_1}{\Phi_1}\left[\sin\Phi_1\cos\frac{\eta t'}{T_0} + \frac{T_1\eta}{T_0\Phi_1}(1-\cos\Phi_1)\sin\frac{\eta t'}{T_0}\right],$$

$$V = \frac{\theta_1}{\sqrt{3}\,\Phi_1'}\left[\sin\Phi_1'\cos\frac{\eta t'}{T_0} + \frac{T_1\eta}{T_0\Phi_1'}(1-\cos\Phi_1')\sin\frac{\eta t'}{T_0}\right]$$

$$\theta_1 = \left(\frac{3\gamma\lambda a_1^2}{2\hbar}\right)^{1/2} T_1, \quad \theta_2 = \left(\frac{\gamma\lambda a_2^2}{\hbar}\right)^{1/2} T_2, \quad N_0 = \frac{1}{2}(n_2 - n_1/2)$$

$$\Phi_i^2 = \theta_i^2 + \frac{T_i^2}{T_0^2}\eta^2 \ (i=1,2), \quad (\Phi_1')^2 = \frac{1}{3}\theta_1^2 + \frac{T_1^2}{T_0^2}\eta^2.$$

Here l_X and l_Y are unit vectors along the Cartesian axes X and Y, with X directed along the polarization vector of the second exciting pulse.

The vector potential of the photon echo in the atomic transition $3/2 \to 1/2$ is obtained from (7) by making the substitutions $\gamma \to 2\gamma$ and $N_0 \to \frac{1}{2}(n_2/2 - n_1)$. It follows from (7) that photon echo in the atomic transitions $1/2 \to 3/2$ is elliptically polarized with a polarization ellipse axis ratio I_1/I_2, one of these axes being aligned with the polarization vector of the second exciting pulse and proportional to I_1.

If the photon echo in the atomic transitions $1/2 \rightleftarrows 3/2$ is formed on a narrow spectral line, then we obtained from (7), in accordance with [13],

$$I_1 = \frac{1}{2}\left(\sqrt{3}\sin\theta_1 + \sin\frac{\theta_1}{\sqrt{3}}\right)(1-\cos\theta_2)\exp\left[-\left(\frac{t'}{2T_0}\right)^2\right],$$

$$I_2 = \frac{1}{4}\left(\sqrt{3}\sin\theta_1 - \sin\frac{\theta_1}{\sqrt{3}}\right)(1-\cos\theta_2)\exp\left[-\left(\frac{t'}{2T_0}\right)^2\right].$$

Consequently, on a narrow spectral line, the amplitude of the echo is maximal for $\theta_1 = 100°$ and $\theta_2 = 180°$. Here

$$I_1 \cong 2.55\exp[-(t'/2T_0)^2], \quad I_2 \cong 0.43\exp[-(t'/2T_0)^2],$$
$$I_1/I_2 \cong 5.93,$$

and the direction of rotation of the echo polarization vector coincides with the direction of rotation of the polarization vector of the first exciting pulse.

In the case of a broad spectral line, individual atoms inside the Doppler contour, depending on the detuning η/T_0, give a maximum of the spontaneous coherent emission at different instants of time and with different polarizations. For example, atoms with detuning $\eta/T_0 = 1/T$ at $T_1 = T_2 = T$, $T = 10T_0$, $\theta_1 = 5\pi/9$, and $\theta_2 = \pi$ emit a photon echo at the instant of time $t \cong 2\tau + z/c + 0.6T$, with a polarization-ellipse axis ratio $I_1/I_2 \cong 5.52$. The total contribution to the echo can be determined only by numerically integrating (7) over all the detunings η/T_0.

Figure 2 shows the results of a numerical calculation of the quantities I_1, I_2, and I_1/I_2 as functions of the time for fixed parameters $\theta_1 = 5\pi/9$, $\theta_2 = \pi$, $T_1 = T_2 = T$, and $T/T_0 = 10$. For the indicated values of the parameters, the echo has a maximum at the instant $t \cong 2\tau + z/c + 0.6T$, and its approximate duration is T. Consequently, the already-noted shift of the maximum of the echo is observed in this case, too. However, at the same time, the polarization also changes. The axis ratio I_1/I_2 of the polarization ellipse changes during the time of the photon echo, and differs in magnitude from

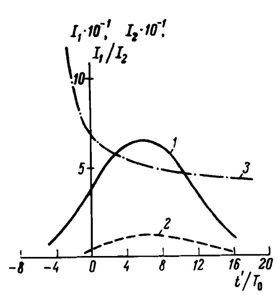

FIG. 2

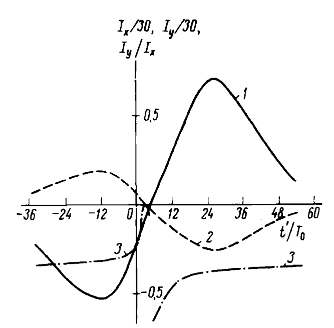

FIG. 3

FIG. 2. Dependence of the quantities I_1, I_2, and I_1/I_2 on the time t' (curves 1, 2, and 3, respectively) for atomic transition with a total-angular-momentum change $1/2 \rightleftarrows 3/2$. The first exciting pulse is circularly polarized, and the second linearly.

FIG. 3. Dependence of the quantities I_X, I_Y, and I_Y/I_X on the time t' (curves 1, 2, and 3, respectively), for atomic transitions with total-angular-momentum change $1 \rightleftarrows 2$. The exciting light pulses are linearly polarized at an angle $\psi = \pi/4$ to each other. The quantity I_Y/I_X is equal to the tangent of the angle between the polarization vectors of the photon echo and the second exciting pulse.

the value obtained for a narrow spectral line at the same values of θ_1 and θ_2.

Let us investigate photon echo from two exciting light pulses that are linearly polarized at an angle ψ to each other. By determining the vector potential of the photon echo in the atomic transitons $0 \rightleftarrows 1$, $1 \to 1$, $1/2 \to 1/2$, and $1/2 \rightleftarrows 3/2$, we verify that the polarization properties of the echo on these transitions do not depend on the values of the parameters T_1/T_0 and T_2/T_0. Yet the intensity of the echo is proportional to the factor of the type (6) and is determined essentially by the quantities T_1/T_0 and T_2/T_0. Atomic transitions with these properties constitute the first group. We consider further atomic transitions $1 \rightleftarrows 2$, which pertain to the second group $J \to J$ $(J > 1)$ and $J \rightleftarrows J+1$ $(J \geq 1)$. Solving Eqs. (1)–(4), we obtain the vector potential of the photon echo on the atomic transition $1 \to 2$ in the form

$$A(z,t)=\frac{3\pi}{4\sqrt{10}}(\hbar\lambda\gamma)^{1/2}\lambda L|N_0|\mathbf{I}\exp[i(kz-\omega t-\varphi_1+2\varphi_2)]+\text{c.c.}, \tag{8}$$

where the components of the vector $\mathbf{I}$ are given by the formulas

$$I_x=\cos\psi\int_0^\infty d\eta e^{-\eta^2}\Big\{(2U\cos^2\psi+3V\sin^2\psi)\frac{3\theta_2^2}{\Phi_2^2}\times\sin^2\left(\frac{1}{2}\Phi_2\right)+[2U\sin^2\psi+V(2-3\sin^2\psi)]\frac{4\theta_2^2}{(\Phi_2')^2}\sin^2\left(\frac{1}{2}\Phi_2'\right)\Big\},$$

$$I_y=-\sin\psi\int_0^\infty d\eta e^{-\eta^2}[2U(2-\sin^2\psi)+3V\sin^2\psi]\times\frac{\theta_2^2}{\Phi_2\Phi_2'}\sin\left(\frac{1}{2}\Phi_2\right)\sin\left(\frac{1}{2}\Phi_2'\right),$$

$$U=\frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2}\frac{\theta_1}{\Phi_1}\left[\sin\Phi_1\cos\frac{\eta t'}{T_0}+\frac{T_1\eta}{T_0\Phi_1}(1-\cos\Phi_1)\sin\frac{\eta t'}{T_0}\right],$$

$$V=\frac{1}{\sqrt{3\pi}}\frac{\theta_1}{\Phi_1'}\left[\sin(\Phi_1')\cos\frac{\eta t'}{T_0}+\frac{T_1\eta}{T_0\Phi_1'}(1-\cos\Phi_1')\sin\frac{\eta t'}{T_0}\right],$$

$$\theta_i=\left(\frac{6\gamma\lambda a_i^2}{5\hbar}\right)^{1/2}T_i,\quad \Phi_i^2=\frac{3}{4}\theta_i^2+\frac{T_i^2}{T_0^2}\eta^2,$$

$$(\Phi_i')^2=\theta_i^2+\frac{T_i^2}{T_0^2}\eta^2\quad (i=1,2),\quad N_0=\frac{1}{3}n_2-\frac{1}{5}n_1.$$

The X axis coincides here with the polarization vector of the second pulse.

The vector potential of the photon echo on the atomic transition $2 \to 1$ is obtained from the formula (8) by making the substitutions $\gamma \to 5\gamma/3$ and $N_0 \to n_2/5 - n_1/3$. As follows from (8), the photon echo on the atomic transitions $1 \rightleftarrows 2$ at $\psi = 0$ is polarized along the polarization vectors of the exciting light pulses, and at $\psi = \pi/2$ it is polarized along the polarization vector of the first pulse. In the intermediate case, the photon-echo polarization depends in a very complicated manner on ψ, θ_1, θ_2, T_1/T_0, and T_2/T_0, and varied from point to point over the extent of the entire echo. For the concrete numerical calculations we have assumed $T_1 = T_2 = T$, $\psi = \pi/4$, and $T/T_0 = 30$. The optimal exciting pulses are then those with $\theta_1 = 11\pi/18$ and $\theta_2 = 19\pi/18$. The parameter $T/T_0 = 30$ agrees approximately with the experimental value (see [6]). In this case an echo of duration T produces with a maximum amplitude at the instant of time $t \cong 2\tau + z/c + 0.7T$. The echo polarization vector at the maximum point makes an angle of 19.4° with the polarization of the second exciting pulse and lies outside the angle between the polarization vectors of the exciting pulses. The direction of the polarization of the echo changes somewhat from point to point over the entire echo, making angles 19.7° at $t = 2\tau + z/c + 0.1T$ and 19.3° at $t = 2\tau + z/c + 1.1T$ with the polarization of the second exciting pulse.

The influence of the parameters T/T_0, θ_1, and θ_2 on the polarization and waveform of the echo is reflected in Fig. 3, which shows the results of a numerical calculation of the echo amplitude at fixed parameters $T_1 = T_2 = T$, $\psi = \pi/4$, $T/T_0 = 30$, and $\theta_1 = \theta_2 = 19\pi/18$. The relation $\theta_1 = \theta_2 = 19\pi/18$ leads to a double-humped profile of the echo. The first and second maxima are produced at the instants of time $t \cong 2\tau + z/c - 0.3T$ and $t \cong 2\tau + z/c + 0.9T$, while the echo polarization vectors at these instants of time make angles 18° and 20.1° with the polarization vector of the second exciting pulse, respectively, and lie outside the angle between the polarization vectors of the exciting pulses.

It follows from our investigation that for an experimental identification of the atomic transitions one must bear in mind the dependence of the echo polarization on the values of the parameters T_1/T_0 and T_2/T_0. This dependence must be taken into account also in theoretical and experimental investigations of the influence of the mixing atomic collisions on the polarization of the photon echo formed on broad spectral lines. The latter remark is particularly important in connection with the influence, noted in [14], of the mixing atomic collisions on the polarization of the photon echo produced on a narrow spectral line. On broad spectral lines, the polarization of the photon echo depends not only on the mixing atomic collisions, but also on the parameters T_1/T_0 and T_2/T_0. However, a quantitative comparison of these effects calls for a separate investigation.

[1] A. Compaan, Phys. Rev. 5B, 4450 (1972).
[2] L. Q. Lambert, Phys. Rev. 7B, 1834 (1973).
[3] C. K. N. Patel and R. E. Slusher, Phys. Rev. Lett. **20**, 1087 (1968).
[4] B. Bölger and J. C. Diels, Phys. Lett. **28A**, 401 (1968).
[5] R. G. Brewer and R. L. Shoemaker, Phys. Rev. Lett. **27**, 631 (1971).
[6] S. S. Alimpiev and N. V. Karlov, Zh. Eksp. Teor. Fiz. **63**, 482 (1972) [Sov. Phys.-JETP **36**, 255 (1973)].
[7] J. R. Meckley and C. V. Heer, Phys. Lett. **46A**, 41 (1973).
[8] A. I. Alekseev and I. V. Evseev, Zh. Eksp. Teor. Fiz. **56**, 2118 (1968); **57**, 1735 (1969) [Sov. Phys.-JETP **29**, 1139 (1968); **30**, 938 (1969)].
[9] I. D. Abella, N. A. Kurnit, and S. R. Hartmann, Phys. Rev. **141**, 391 (1966).
[10] S. M. Zakharov and E. A. Manykin, in: Kvantovaya élektronika (Quantum Electronics), Vol. 2, Nauka 1973.
[11] W. B. Mims, Phys. Rev. **141**, 499 (1969).
[12] P. F. Liao and S. R. Hartmann, Phys. Lett. **44A**, 361 (1973).
[13] C. V. Heer and R. H. Kohl, Phys. Rev. **1A**, 693 (1970); **2A**, 549 (1970).
[14] C. H. Wang, Phys. Rev. **1B**, 156 (1970).

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